DISCUSSION

NOYES: I wonder if there is an experimentalist here who is familiar with the details of the K_{e3} spectrum? As I recall, most methods have a very strong experimental bias against measurements of high energy electrons in this decay. I wonder if someone can make an experimental comment on that point, as the evidence given for a 350 MeV $\pi^{0\prime}$ depends entirely on whether or not such bias is present.

HEISENBERG: There seems to be no answer.

FEINBERG: I would like to make a comment on another point. The $\pi^{0\prime}$ could be looked for by the same type of experiment that was described the other day, to look for violations of isotopic spin conservation, in particular by the reaction $d+d \to \alpha+\pi^{0\prime}$. If the $\pi^{0\prime}$ interacts strongly, then one would expect that the cross section for this, well above threshold, would be, maybe, 10^{-29} cm^2, which is much larger than the cross section for production of α+photon, so I think finding this particle, if it exists, might be possible by such an experiment.

(The following remark by Dr. Chamberlain was added after the discussion.)

CHAMBERLAIN: If the $\pi^{0\prime}$ mass is greater than two pion masses, it will be difficult to distinguish its production from double π^0 production in the $d+d$ reaction.

RECENT RESEARCH ON THE NONLINEAR SPINOR THEORY OF ELEMENTARY PARTICLES

W. Heisenberg

Max-Planck-Institut für Physik und Astrophysik, München, Germany

During the last year research on the nonlinear spinor theory has been carried out in Munich mainly along three lines which may be characterized by the three topics: indefinite metric in Hilbert space, group theory, and approximation methods for the calculation of eigenvalues; some work has also been done on the analytical behavior of matrix-elements.

1. INDEFINITE METRIC IN HILBERT SPACE

(a) The question of whether the probability interpretation of quantum theory is compatible with the use of an indefinite metric in Hilbert space has been taken up from a general mathematical viewpoint in three papers by Schlieder. Starting from given symmetry groups Schlieder studies bilinear forms which are invariant under the given transformations, and constructs the corresponding fundamental metric tensor. He then states supplementary conditions which are sufficient for the probability interpretation of the asymptotic waves. While for finite and compact groups one comes back in this way essentially to the conventional theory, infinite and non-compact groups may lead to more general representations in a space with indefinite metric which still are compatible with the probability interpretation of the asymptotic waves. Besides the special cases that have been studied in connection with Quantum electrodynamics by Bleuler and Gupta, and in connection with the Lee model by the author of the present report, Schlieder mentions the case of systems in which superselection rules exist. These rules divide the space of states into different incoherent sectors. It is

easily seen that one would not run into any difficulties with the probability interpretation if e.g., the metric in one sector is positive definite, while it is negative definite in another sector.

Schlieder discusses finally the representation of the scale transformation in a Hilbert space with indefinite metric. This transformation may actually be used here for defining a quantum number, while in a definite metric it would only lead to trivial results. The reason is seen from the following argument: if one wants to introduce a quantum number for the scale transformation, one has to say that the Hilbert vector of a state with quantum number λ is multiplied by η^{λ} in a scale transformation (η being the scale factor). In a Hilbert space with definite metric the relation

$$\langle\phi_i|AB|\phi_f\rangle = \sum_n \langle\phi_i|A|\chi_n\rangle\langle\chi_n|B|\phi_f\rangle \tag{1}$$

shows that $|\chi_n\rangle\langle\chi_n|$ must be invariant under the scale transformation. This would only be possible for $\lambda = 0$, since $|\chi_n\rangle$ may be considered as the complex conjugate of $\langle\chi_n|$; the scale transformation would therefore become trivial. In an indefinite metric however, relation (1) is to be replaced by

$$\langle\phi_i|AB|\phi_f\rangle = \sum_n \langle\phi_i|A|\chi^n\rangle\langle\chi_n|B|\phi_f\rangle \tag{2}$$

This equation is invariant under the scale transformation, if only $|\chi^n\rangle$ transforms like $\eta^{-\lambda}$ when $\langle\chi_n|$ transforms like η^{λ}. This is a possible, and in fact necessary, assumption, since $|\chi^n\rangle$ is the inverse vector to $|\chi_n\rangle$ defined by

$$\langle\chi^m|\chi_n\rangle = \delta_{nm}\,. \tag{3}$$

Therefore λ can be used as a quantum number.

(b) The possible presence of a dipole ghost of mass zero in the mass-spectrum of the vacuum-expectation values has been the object of an investigation by Mitter. By means of the Tamm-Dancoff method one can derive for the S_F function a nonlinear integro-differential equation, which in the lowest approximations reduces to a nonlinear differential equation. From a discussion of the solutions of this nonlinear equation Mitter tries to check whether the conditions

$$\int_0^\infty \xi|\rho|d\rho = 0 \text{ and } \int_0^\infty \xi|\rho|\rho d\rho = 0 \tag{4}$$

are actually enforced by the nonlinear character of the equations. The calculations have so far been carried out only for a somewhat simplified version of the theory, in which for symmetry reasons one could not expect the occurrence of mass eigenvalues different from zero. In this case Mitter gets as the only solution obeying the boundary conditions:

$$S_F = \int d^4p e^{ip(x-x')} \frac{p_\nu \gamma_\nu}{(p^2)^2} \tag{5}$$

which corresponds to a pure dipole ghost. In this solution the condition $\int_0 \xi|\rho|d\rho = 0$ is satisfied; the other condition $\int_0 \xi|\rho|\rho d\rho = 0$ however is not satisfied. The extension of the calculations to the realistic case where mass eigenvalues different from zero are possible has not yet been carried out.

2. GROUP THEORY

(a) Conservation of parity.

The fundamental equation

$$\gamma_\nu \frac{\partial\psi}{\partial x_\nu} \pm l^2 \gamma_\mu\gamma_5\psi(\overline{\psi}\gamma_\mu\gamma_5\psi) = 0 \tag{6}$$

is invariant under the transformation

$$\psi(\mathbf{r}, t, l) = \gamma_4\psi(-\mathbf{r}, t, l)\,.$$

But this transformation cannot be used as space reflection parity, since it does not commute with the Touschek transformation applied here for defining the baryonic number. Therefore a "parity of the second kind" had been introduced:

$$\psi(\mathbf{r}, t, l) = -i\frac{p_\nu\gamma_\nu}{|p|}\gamma_4\psi(-\mathbf{r}, t, l) \tag{7}$$

which came out as a natural consequence of the fact that the fermion wave functions obey a Klein-Gordon equation instead of a Dirac equation. This transformation is primarily defined on the energy shell of the nucleon. If one generalizes it as in Eq. (7) then Eq. (6) seems however not to be invariant under the non-local operation (Eq. (7)), therefore the parity of the second kind defined in this way can at most be approximately conserved. A similar situation had been reported by Thirring for his theory based upon the use of two Weyl spinors for the nucleons.

Duerr has studied the question whether one cannot for a theory starting from Eq. (6) introduce space reflection by an operation different from Eq. (7) and coinciding with it on the nucleon energy shell so that the equation should be strictly invariant under this operation. If this is possible parity should be strictly conserved for strong and electromagnetic interactions. The non-conservation of parity for the weak interactions should then be interpreted as a lack of symmetry in the ground state " world ", from which the particles are produced. It seemed convenient for this investigation to rewrite Eq. (6) in a different form closely related to a form suggested by Schremp and Gürsey. Instead of the spinor ψ_α one introduces a new spinor $\chi_{\alpha\tau}$ by the relation

$$\chi_{11} = \psi_1; \quad \chi_{21} = \psi_2; \quad \chi_{12} = -\psi_4^*; \quad \chi_{22} = \psi_3^* \tag{8}$$

The first index of χ refers to the Dirac spin, the second to the isospin. In this notation Eq. (6) takes the form

$$\sigma_\nu \frac{\partial}{\partial x_\nu}\chi \pm l^2 \sigma^\mu \chi(\chi^* \sigma_\mu \chi) = 0 \tag{9}$$

where $\sigma_\nu = (\sigma_k, 1)$ and $\sigma^\nu = (\sigma_k, -1)$. The bracket is to be interpreted as $\chi^*_{\alpha\tau}\sigma_{\mu,\alpha\beta}\chi_{\beta\tau}$. The advantage of the form (9) as against Eq. (6) lies in the fact that the isospin-group is seen more clearly in Eq. (9) than in Eq. (6); the σ_μ-matrices in Eq. (9) indicate that we have to do with a two-dimensional representation of the proper Lorentz-group. If one introduces the transposition matrices C_σ and C_τ in the conventional way by

$$\sigma_k^T = -C_\sigma^{-1}\sigma_k C_\sigma; \qquad \tau_k^T = -C_\tau^{-1}\tau_k C_\tau \tag{10}$$

the operation $\psi(\mathbf{r}, t, l) \rightarrow \gamma_4 \psi(-\mathbf{r}, t, l)$ can be expressed for the χ spinor as

$$\chi(\mathbf{r}, t, l) = C_\sigma^{-1} C_\tau^{-1} \chi(\mathbf{r}-, t, l) \tag{11}$$

Eq. (9) is invariant under this operation.

This is essentially a CP transformation.

It had been suggested in an earlier paper that the field operators should be considered as functions not only of x_μ but also of l and that the operators at different values of l may be connected by the commutator $\{\chi(x, l), \chi^*(x', l')\}$. If this is done, one realizes that Eqs. (6) and (9) are invariant under the transformation

$$\chi(\mathbf{r}, t, l) \rightarrow \chi(\mathbf{r}, t, -l) \tag{11'}$$

This transformation can be connected with space reflection in the following manner: the states of the system are usually characterized by the momenta $p_\nu = (p_k, p_0)$ where by definition the energy $p_0 > 0$. What one measures in an experiment is, however, not the values x_ν or the momenta p_ν but the ratios x_ν/l and the momenta $\pi_k = p_k l$. Since it is necessary to keep the energy positive one may define:

$$\rho_k = \frac{x_k}{l}, \qquad \rho_0 = \frac{x_0}{|l|}; \qquad \pi_k = p_k l, \ \pi_0 = p_0 |l|.$$

A change of sign in l reserves the sign of the space coordinates ρ_k but not of the time ρ_0. Therefore the operation (11′) represents reflection in space, connected with a reflection of l:

$$\chi(\rho_k, \rho_0, l) \rightarrow \chi(-\rho_k, \rho_0, -l) \tag{12}$$

To bring out this symmetry more clearly, one may define a new 4×2 component spinor by

$$\mathbf{X}(\boldsymbol{\rho}_k, \rho_0, l) = \begin{pmatrix} \chi(\boldsymbol{\rho}_k, \rho_0, l) \\ \chi(\boldsymbol{\rho}_k, \rho_0, -l) \end{pmatrix} \tag{13}$$

and at the same time go over from the Pauli-matrices σ_k to Dirac-matrices Γ_ν by which one represents the complete Lorentz-group in the conventional manner. The Γ_ν therefore have a different physical meaning from the γ_ν in Eq. (6):

$$\Gamma_4 \mathbf{X}(\boldsymbol{\rho}_k, \rho_0, l) = \mathbf{X}(\boldsymbol{\rho}_k, \rho_0, -l) \tag{14}$$

For this operator $\mathbf{X}$ which now can be restricted to positive values of l, Duerr obtains the differential equation:

$$\Gamma_\nu \frac{\partial}{\partial x_\nu}\mathbf{X} \pm \frac{l^2}{2}\{\Gamma_\mu \mathbf{X}(\overline{\mathbf{X}}\Gamma_\mu \mathbf{X}) + \Gamma_\mu \Gamma_5 \mathbf{X}(\overline{\mathbf{X}}\Gamma_\mu \Gamma_5 \mathbf{X})\} \tag{15}$$

The operation P of space reflection (11′) or (12) may now be written in the conventional form

$$\chi(\boldsymbol{\rho}_k, \rho_0) \rightarrow \Gamma_4 \mathbf{X}(-\boldsymbol{\rho}_k, \rho_0) \tag{16}$$

Eq. (15) is identical with Eq. (9), written once for $\chi(\rho_k, \rho_0, |l|)$, and once in the corresponding form for $\chi(\rho_k, \rho_0, -|l|)$. The brackets in Eq. (15) contain, as in Eq. (9), the unit-matrix in isospin space. The isospin group takes the conventional form

$$\mathbf{X} \rightarrow e^{i\alpha_e \tau_e} \mathbf{X} \tag{17}$$

The introduction of the operator $\chi(\rho_\nu, -l)$ besides $\chi(\rho_\nu, l)$ would in itself lead to a doubling of the number of states, if the states produced by $\chi(\rho_\nu, -l)$ were not by definition identified with the states produced by $\chi(\rho_\nu, l)$. This identification is formally carried out by introducing the commutator between $\chi(\rho_\nu, l)$ and $\chi(\rho_\nu, -l)$, which appears as the mass term in the commutator of $\chi(\rho_\nu, l)$. The sign of the mass term is arbitrary in the beginning: it decides the parity of the nucleon, which can be chosen arbitrarily. But when the choice has been made, the number of states is the same as before the introduction of $\chi(\rho_\nu, -l)$.

Finally one gets for the vacuum expectation value of $\mathbf{X}\overline{\mathbf{X}}$:

$$\langle \Omega | \mathbf{X}_{\alpha\sigma}(x) \overline{\mathbf{X}}_{\beta\rho}(x') | \Omega \rangle = \int d^4 p e^{ip(x-x')} \times$$

$$\times \left[\delta_{\sigma\rho} \left(\frac{\Gamma^\nu_{\alpha\beta} p_\nu + i\kappa\delta_{\alpha\beta}}{p^2 + \kappa^2} \right) + \begin{matrix}\text{regularizing}\\ \text{terms}\end{matrix} + \begin{matrix}\text{contributions from}\\ \text{continuous spectrum}\end{matrix} \right] \tag{18}$$

In this way one sees that Eq. (16) defines a space reflection parity which is strictly conserved and coincides on the energy shell of the nucleons with the parity of the second kind defined by Eq. (7). The non-conservation of parity for the weak interactions must be due to a lack of symmetry in the ground-state " world ", which would show up in the higher approximations of some vacuum-(or rather " world ") expectation values.

(b) Representation of the strange particles.

It has been emphasized in an earlier paper that the non-conservation of isospin in the electromagnetic forces must be due to an asymmetry of the ground state " world ". The " world " possesses a very big total isospin, and this ground state is, therefore, highly degenerate. Nambu has in a recent paper pointed to the analogy of this assumption with the situation in superconductivity. The analogy is actually quite close, since the lowest state of a superconductor has lost the invariance under the gauge-transformation; the gauge-transformation, however, belongs to the isospin group in the representation of Eq. (6). In this connection I might mention a paper of Yamazaki in which he tries to apply the Tamm-Dancoff method to the problem of superconductivity in the form given to it by Bogolubov. Yamazaki intends to check in this way the validity of the Tamm-Dancoff method. He has been able to show that in this case the Tamm-Dancoff method leads exactly to the results of Bogolubov.

If the ground state of the world has a big isospin, the strange particles may be produced by attaching some part of this isospin to a nucleon or a π-meson. In a similar manner the outer valency electron of the calcium atom produces a triplet spectrum instead of a doublet spectrum, since a spin $\frac{1}{2}$ of the inner shells has been attached to the valency electron.

For a mathematical representation of this possibility one has to express the degeneracy of the ground state. If particles of strangeness 1 are considered, it is sufficient to label only the last isospin $\frac{1}{2}$ of the ground state by an index having the values ± 1. This isospin can be attached to the particles if an isospin interaction exists of the same general type as the spin-spin interaction in the outer atomic shells. From Eq. (9) one would not immediately expect such an interaction. But the large mass difference between the triplet and the singlet π meson derived from this equation shows that such an interaction is indirectly produced by exchange terms in a manner similar to the spin-spin interaction in the outer atomic shells; e.g. the energy difference between the orthohelium and the parahelium terms. Therefore, the isospin interaction may also show up in the vacuum expectation values.

If one introduces the notation $|\Omega_\alpha\rangle$ for the two different ground states ($\alpha = \pm 1$), one expects for the vacuum expectation value of the product of two field operators the general form:

$$\langle \Omega_\alpha | \chi_\mu(x, l) \; \chi_\nu^*(x', l) | \Omega_\beta \rangle = a\delta_{\alpha\beta}\delta_{\mu\nu} + b\tau^i_{\mu\nu}\tau^i_{\beta\alpha} \tag{19}$$

where the indices refer to the isospin and a, b are functions of the space-time coordinates and the Dirac spin. Eq. (19) is invariant under rotations in isospin-space if the rotation is applied on the operator χ_μ and the ground state simultaneously. The second term on the right hand side is due to the isospin interaction; its size should be determined later on by a condition of consistency.

Since Eq. (9) is invariant under the CP transformation (11) and since there is no experimental evidence against this symmetry in the ground state, Eq. (19) should also be invariant under the operation (11) without any corresponding transformation in the ground state. If χ and χ^* in Eq. (19) refer to the same value of l this condition however can only be fulfilled by $b=0$, since the term $\tau^i_{\mu\nu}\tau^i_{\alpha\beta}$ would change sign under the CP transformation. On the other hand if χ and χ^* refer to opposite values of l, the second term in (19) can be made invariant under the CP transformation. Therefore, using again the spinor $\mathbf{X}$ instead of χ, the general form of the vacuum expectation value could, in analogy to (18), be expressed by

$$\langle\Omega_\lambda|\mathbf{X}_{\alpha\sigma}(x,l)\bar{\mathbf{X}}_{\beta\rho}(x',l)|\Omega_\mu\rangle = \int\rho(\kappa^2)d\kappa^2\int d^4p e^{ip(x-x')}\times$$
$$\times\left[\frac{\delta_{\lambda\mu}\delta_{\sigma\rho}\Gamma^\nu_{\alpha\beta}p_\nu+i\kappa(a\delta_{\sigma\rho}\delta_{\lambda\mu}\delta_{\alpha\beta}+b\Gamma^5_{\alpha\beta}\tau^i_{\rho\sigma}\tau^i_{\mu\lambda})}{p^2+\kappa^2}+\right.$$
$$\left.+\text{regularizing terms}\right] \tag{20}$$

The change of sign in the expression $\tau^i_{\sigma\rho}\tau^i_{\mu\lambda}$ under the operation C is compensated for by the change of Γ^5 under P in the complete transformation CP.

Eq. (20) contains two important items of information:

i) If one carries out calculations in an approximation in which one uses only operators for one value of l and as contraction functions the vacuum expectation values of products of only two field operators with the same l, (13), then an interaction between the isospin of the particle and of the ground state cannot be expressed, and the strange particles cannot appear as eigenvalues. In order to get the strange particles one must either use the operators $\chi(\rho_\nu, l)$ and $\chi(\rho_\nu, -l)$ simultaneously or one must include vacuum expectation values of products of at least four field operators. The vacuum expectation values of products of four χ-operators of the same l may in fact express an interaction in a CP invariant manner:

$$\langle\Omega_\alpha|\chi_\lambda(x,l)\chi_\mu{}^*(x',l)\chi_\rho(x'',l)\chi_\sigma{}^*(x''',l)|\Omega_\beta\rangle =$$
$$= c\delta_{\alpha\beta}\delta_{\lambda\mu}\delta_{\rho\sigma}+id\boldsymbol{\tau}_{\beta\alpha}\cdot[\boldsymbol{\tau}_{\lambda\mu}\times\boldsymbol{\tau}_{\rho\sigma}] \tag{21}$$

where c and d are functions of x, x', x'', x'''

ii) While Eq. (20) is invariant under the CP transformation, applied only on the field operators, it is not invariant under P or C separately. Therefore, a single strange particle cannot have well defined space reflection parity. Only a pair of such particles of opposite strangeness may have a parity. This seems to be the reason for the well-known fact that a K-meson can disintegrate both into two or three π-mesons.

A calculation of the mass eigenvalues for the strange particles has so far not been carried out, since such a calculation would, according to i), either require a very substantial extension of the Tamm-Dancoff method or an entirely different, new approach.

3. APPROXIMATION METHODS FOR THE CALCULATION OF EIGENVALUES

In a theory with indefinite metric in Hilbert space, where the field operators obey a nonlinear differential equation, the anticommutator, say, of $\chi(x)$ and $\chi^*(x')$ will generally not behave like a δ-function at the point $x=x'$. Therefore, the operator representing translation in time (the Hamiltonian) cannot generally be expressed by the field variables at a given instant of time. Consequently, the conventional methods of calculating eigenvalues, as the Ritz method, fail. This situation occurs already in the Lee model after renormalization; therefore, Sekine has studied approximation methods in the Lee model.

If one introduces a finite interval of time, Δt, the equations of motion in the Lee model can be expressed as integro-differential equations for the renormalized operators without any infinite constants. The anticommutator between $\psi^*(\mathbf{r}, t)$ and, say, $\int_{t-\Delta t}^{t} dt'\psi(\mathbf{r}', t')$ behaves approximately like $\delta(\mathbf{r},\mathbf{r}')\dfrac{\text{const}}{\log|\Delta t|}$ and vanishes for $\Delta t\to 0$.

Sekine introduces an approximate Hamiltonian H' with the following properties: the matrix-elements of H' between any two states of energy, E_i and E_f, are equal to those of the exact Hamiltonian H, if only

$$E_i \ll \frac{1}{\Delta t} \text{ and } E_f \ll \frac{1}{\Delta t}.$$

H' depends only on the operators at a given instant of time $t = t_0$. The commutation of H' with $\int_{t-\Delta t}^{t} \psi(\mathbf{r}', t')dt'$ produces the equations of motion in a sufficient approximation for all matrix elements in the region of small energies $(E \ll 1/\Delta t)$.

When these conditions can be fulfilled, the conventional methods of variation may be applied on H' and lead to eigenvalues which for $E \ll 1/\Delta t$ are identical with the corrrect eigenvalues of the system. Formally, the calculations of Sekine are closely related to a conventional cut-off procedure, but this procedure is used only as a mathematical tool, while the eigenvalues and matrix elements are independent of it.

The apparent success of this method in the Lee model suggests the following procedure in the nonlinear spinor theory: the wave equation and the corresponding local Hamiltonian may be modified by some cut-off process which can be non-relativistic (e.g. limitation of integrals in momentum space by a maximum value K) and which changes the theory into a non-local one. For the non-local theory the anticommutator will not vanish everywhere for a given time $t = t_0$; it will still contain δ-like terms which, however, go to zero in the limit $K \rightarrow \infty$. In this modified theory the conventional methods of variation may be applied; they will — so one may hope — lead to eigenvalues which converge towards the correct eigenvalues in the limit $K \rightarrow \infty$. The theory would be relativistically invariant only in this limit.

This extension of the results from the Lee model to the nonlinear spinor theory has, however, not yet been investigated in detail; therefore it is still unknown whether a useful method of approximation can be constructed in this way.

4. VACUUM EXPECTATION VALUES OF THE PRODUCT OF FOUR FIELD OPERATORS

In analogy to the investigations by Lehmann of the vacuum expectation value of the product of two field operators, Montaldi has studied the vacuum expectation value of the product of four field operators of the type:

$$\langle \Omega | \psi_\alpha(x) \bar{\psi}_\beta(x') \psi_\gamma(x'') \bar{\psi}_\delta(x''') | \Omega \rangle \tag{22}$$

Generally the analytic form of such expressions may be very complicated. But in the special case of Eq. (6), where the invariance under the Pauli-Gürsey group and the Touschek group are required, as well as the Lorentz invariance, then on account of the postulate of microcausality, Eq. (22) can be reduced to only two functions of 6 mass-variables. These two functions must fulfill a number of symmetry conditions resulting from the underlying groups. Besides that, through Eq. (6), they can be connected with the mass-spectrum $\xi(\kappa^2)$ in the vacuum expectation value of two field operators. Finally these functions must obey restrictive conditions of the type of Eq. (4), in order to avoid divergencies due to the nonlinear character of the fundamental field equation.

The results of Montaldi can be stated in the following formula:

$$\begin{aligned}
&\langle \Omega | T\psi_\alpha(x_1)\psi_\beta(x_2)\bar{\psi}_\lambda(x_3)\bar{\psi}_\mu(x_4)|\Omega\rangle = \\
&= \int_0^\infty d\kappa_1^2 \ldots \int_0^\infty d\kappa_6^2 K^F_{-1,\alpha\beta}(x_1-x_2,\kappa_1) K^F_{1,\lambda\mu}(x_3-x_4,\kappa_6) \Delta_F(x_1-x_3,\kappa_2)\Delta_F(x_1-x_4,\kappa_3)\Delta_F(x_2-x_3,\kappa_4) \times \\
&\qquad \times \Delta_F(x_2-x_4,\kappa_5) M_1(\kappa_1^2,\ldots\kappa_6^2) + \\
&+ \int_0^\infty d\kappa_1^2 \ldots \int_0^\infty d\kappa_6^2 K^F_{0,\alpha\lambda}(x_1-x_3,\kappa_2) K^F_{0,\beta\mu}(x_2-x_4,\kappa_5) \Delta_F(x_1-x_2,\kappa_1)\Delta_F(x_1-x_4,\kappa_3)\Delta_F(x_2-x_3,\kappa_4) \times \\
&\qquad \times \Delta_F(x_3-x_4,\kappa_6) M_2(\kappa_1^2\ldots\kappa_6^2) + \\
&+ \int_0^\infty d\kappa_1^2 \ldots \int_0^\infty d\kappa_6^2 K^F_{0,\alpha\mu}(x_1-x_4,\kappa_3) K^F_{0,\beta\lambda}(x_2-x_3,\kappa_4) \Delta_F(x_1-x_2,\kappa_1)\Delta_F(x_1-x_3,\kappa_2)\Delta_F(x_2-x_4,\kappa_5) \times \\
&\qquad \times \Delta_F(x_3-x_4,\kappa_6) M_3(\kappa_1^2\ldots\kappa_6^2) +
\end{aligned}$$

+ three similar terms, with the K functions replaced by H functions and M_1, M_2, M_3 replaced by M_4, M_5, M_6 respectively. (23)

The K and H functions are defined as follows: (C = charge conjugation matrix)

$$K_0^{\pm}(x) = \gamma_\nu \frac{\partial}{\partial x_\nu} \Delta^{\pm}(x) \qquad \mathrm{H}_0^{\pm}(x) = \gamma_5 K_0^{\pm}(x)$$

$$K_{-1}^{\pm}(x) = \gamma_\nu C^{-1} \frac{\partial}{\partial x_\nu} \Delta^{\pm}(x) \qquad \mathrm{H}_{-1}^{\pm}(x) = \gamma_5 K_{-1}^{\pm}(x)$$

$$K_1^{\pm}(x) = C\gamma_\nu \frac{\partial}{\partial x_\nu} \Delta^{\pm}(x) \qquad \mathrm{H}_1^{\pm}(x) = K_1^{\pm}(x)\gamma_5 \tag{24}$$

furthermore,

$$Z_F(x) = \begin{cases} Z^+(x) & \text{for } x_0 > 0 \\ -Z^-(x) & \text{for } x_0 < 0 \end{cases}$$

where $Z_F = K^F$, H^F or Δ_F.

For simplicity, we do not quote here the symmetry conditions that must be satisfied by the spectral functions $M_i(\kappa_1^2 \ldots \kappa_6^2)$ due to the above mentioned requirements. We shall limit ourselves to giving the conditions analogous to Eq. (4) for the two independent mass spectra (which we shall denote by N and Q), in terms of which the functions M_i can be explicitly constructed: ($A = N$ or Q):

$$\int_0^\infty d\kappa_1^2 \ldots \int_0^\infty d\kappa_6^2 A(\kappa_1^2 \ldots \kappa_6^2) = 0$$

$$\int_0^\infty d\kappa_1^2 \ldots \int_0^\infty d\kappa_6^2\, \kappa_j^2\, A(\kappa_1^2 \ldots \kappa_6^2) = 0$$

$$\int_0^\infty d\kappa_1^2 \ldots \int_0^\infty d\kappa_6^2\, \kappa_j^2\, \kappa_l^2 A(\kappa_1^2 \ldots \kappa_6^2) = 0$$

$$\vdots$$

$$\int_0^\infty d\kappa_1^2 \ldots \int_0^\infty d\kappa_6^2\, \kappa_1^2 \ldots \kappa_6^2\, A(\kappa_1^2 \ldots \kappa_6^2) = 0$$

The vacuum expectation value of products of four field operators should contain terms that can be interpreted as due to the creation and annihilation of π-mesons; therefore, the spectra N and Q should in some way contain not only the mass of the nucleon but also the mass of the π-meson.

DISCUSSION

BLOKHINTSEV: Concerning the work of Mitter on these commutation rules, I should like to know what kind of initial conditions are taken for the commutation rules?

HEISENBERG: As far as I know, in ordinary theory, the initial condition never has any effect on the commutation rules.

DUERR: May I comment on this point? What Mitter actually did was to derive, in an approximate fashion, not really the anticommutator but rather the vacuum expectation value of the T-product, that is the propagation function, under the following conditions: first, that the anticommutator has the symmetry properties connected with the theory; second, that the anticommutator has the property of being zero for space-like distances; third, that the energies are positive; forth, that you have invariance with respect to time reversal. Under these conditions, he then derived the stated form of the propagation function.

BREIT: I want to ask whether any of these theories have consequences for quantum electrodynamics? Presumably, if there are consequences they would be in the nature of some interaction of another field with the electron-positron and electromagnetic fields. If there were consequences, perhaps they could be tested, because some quantum electrodynamics tests are carried out very precisely. Of course, in the same connection, there is the question of the muon. I wonder if any of these theories would have bearing on the behavior of the muon?

HEISENBERG: May I answer very briefly: I am convinced there are consequences but they have not been worked out. Actually, one can probably only approach this problem in the following order: the most trivial thing one can do is to treat the nucleons and pions and the strong interactions. The next step must be the strange particles. After that, one may come to quantum electrodynamics, namely, to the existence of the photon and only at this stage one may discuss these consequences.

DUERR : I want to make a short comment on this l-parity, i.e., the parity which is connected with the reversal of l. I want to give a " nonmystical " interpretation of this. In order to incorporate parity in a strict fashion, you are forced, in a way, to double the number of components of the field operator. If you double the number of components of the field operator, however, you immediately run into the difficulty that you can write down five different invariant fourth order expressions, i.e., you have five Fermi interaction terms which will involve five different coupling constants. So you look for a method of doubling the components which does not increase the number of coupling constants. Now, in this special case which we have investigated, we introduce the doubling of the components in a very special fashion in the following sense : in the original equation l enters only quadratically. We now could slightly modify this theory in stating that the theory may also depend on the absolute value of l, i.e., it may depend on the sign of the square root of l^2. If you use this degree of freedom in that way, then of course, this is equivalent to doubling the number of components, and parity can be introduced, but you do not introduce new kinds of interactions.

DYNAMICAL THEORY OF ELEMENTARY PARTICLES SUGGESTED BY SUPERCONDUCTIVITY

Y. Nambu

The Enrico Fermi Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

My talk is based on some work done in collaboration with Dr. Jona-Lasinio.

We would like to propose here a theory of elementary particles which is based on a mathematical analogy between the dynamics of relativistic particles and that of superconductors in the theory of Bardeen, Cooper and Schrieffer [1]. That there can exist such an analogy is not surprising. Both relativistic quantum field theory and solid state physics deal with many body problems of large media, and in fact we already know many instances where field theoretical techniques have been successfully applied to problems of solid state physics. We shall see presently that this interaction of the two branches of physics can be reciprocal, and that solid state physics can provide us with useful models which help us understand the dynamics of elementary particles.

I. We start with the comparison of the Dirac equation for a nucleon, say, and the Bogolubov-Valatin relation [2, 3] for an elementary excitation (quasi-particle) in a superconducting medium. They are given respectively by

$$\begin{aligned} E\psi_1 &= \boldsymbol{\sigma}\cdot\mathbf{p}\psi_1 + m\psi_2 \\ E\psi_2 &= -\boldsymbol{\sigma}\cdot\mathbf{p}\psi_2 + m\psi_1 \\ E &= \pm\sqrt{p^2+m^2} \end{aligned} \tag{1}$$

and

$$\begin{aligned} E\psi_{p+} &= \varepsilon_p\psi_{p+} + \phi\psi^\dagger_{-p-} \\ E\psi^\dagger_{-p-} &= -\varepsilon_p\psi^\dagger_{-p-} + \phi\psi_{p+} \\ E &= \sqrt{\varepsilon_p{}^2+\phi^2} \end{aligned} \tag{2}$$

Here the Weyl representation is used for the Dirac equation : ψ_1, ψ_2 correspond to the eigenstates of chirality $\gamma_5 = \mp 1$. In Eq. (2), $\psi_{p\pm}$ is the wave function of an electron with momentum p and spin $\pm$ (up or down), so that $\psi^\dagger_{-p-}$ effectively represents