String Core Effect on the Axion Dark Matter Abundance

Ken'ichi Saikawa

Deutsches Elektronen-Synchrotron (DESY), Hamburg, Germany*

DOI: http://dx.doi.org/10.3204/DESY-PROC-2017-02/saikawa_kenichi

In this contribution, we highlight a problem in estimation of the mass of axion dark matter in the scenario where the Peccei-Quinn symmetry is broken after inflation. In order to obtain a definite prediction for its mass, it is necessary to evaluate its relic abundance precisely by taking account of the production from cosmic strings and domain walls. After reviewing a long-standing controversy about the axion production efficiency from decaying strings, several recent approaches aiming at resolving the uncertainty are discussed.

The axion [1, 2] is a pseudo Nambu-Goldstone boson associated with spontaneously broken global Peccei-Quinn (PQ) symmetry [3]. There is a strong motivation to search for the axion, as it provides the elegant solution to the strong CP problem in quantum chromodyamics (QCD) and can be a good candidate of dark matter in the universe [4, 5, 6]. However, there are some complications when we consider the evolution of the axion dark matter in the early universe. In particular, it is known that topological defects such as strings and domain walls are formed in the early universe if the PQ symmetry is broken after inflation, and that axions produced from these defects may have an impact on the relic dark matter abundance [7]. Therefore, it is important to know about the efficiency of axion production quantitatively in order to give an accurate estimate of its relic abundance and obtain a definite prediction for its mass m_a.

The formation and evolution of topological defects are described by a complex scalar field Φ called the PQ field. The global $U(1)$ PQ symmetry is spontaneously broken when the PQ field acquires a vacuum expectation value $|\langle\Phi\rangle|^2 = v_{\rm PQ}^2/2$ due to the potential $V(\Phi) = \lambda(|\Phi|^2 - v_{\rm PQ}^2/2)^2$, where λ is the self-coupling, and $v_{\rm PQ}$ represents the energy scale of PQ symmetry breaking, which is related to the axion decay constant, $f_a \propto v_{\rm PQ}$. At that time, vortex-like objects called strings are formed. Furthermore, these strings are attached by sheet-like objects called domain walls around the epoch of the QCD phase transition, because of the existence of the effective potential for the axion field induced by the non-perturbative effects in QCD. If the effective potential has only one non-degenerate minimum, these string-wall systems eventually collapse due to the tension of domain walls [8].

Axions are copiously produced from the collapse of such string-wall systems around the epoch of the QCD phase transition, and the axion number is expected to be frozen after they decay away. Therefore, the axion density at the present time $t_{\rm today}$ can be written as

$$\rho_a(t_{\rm today}) = m_a n_a(t_{\rm decay}) \left(\frac{R(t_{\rm decay})}{R(t_{\rm today})}\right)^3, \tag{1}$$

*Current affiliation: Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), München, Germany where $R(t)$ represents the scale factor of the universe, and $t_{\rm decay}$ represents the time at which the string-wall systems decay away. Assuming that the decay proceeds sufficiently faster than the cosmological timescale, we can roughly estimate the number density of axions at $t_{\rm decay}$ as

$$n_a(t_{\rm decay}) \sim \frac{\rho_{\rm defects}(t_{\rm decay})}{\langle E_a(t_{\rm decay})\rangle}, \tag{2}$$

where $\rho_{\rm defects}$ represents the energy density of the defects and $\langle E_a\rangle$ is the mean energy of axions radiated from them. Therefore, it is helpful to know about two factors in order to understand the axion production efficiency: One is the energy density of defects, which is believed to follow the scaling solution, $\rho_{\rm defects}(t) \approx \rho_{\rm strings}(t) = \xi\mu/t^2$, where μ is the energy of strings per unit length and ξ is a dimensionless factor. The other is the mean energy $\langle E_a(t_{\rm decay})\rangle$, which depends on the energy spectrum of radiated axions.

In the literature, there has been a long-standing controversy about the estimation of axion production efficiency from topological defects [9, 10, 11, 12, 13, 14, 15]. The authors of Refs. [9, 11, 15] claimed that the spectrum of radiated axions is hard (*i.e.* the mean energy $\langle E_a(t_{\rm decay})\rangle$ is enhanced), and that it suppresses the relic axion abundance according to Eq. (2). However, other authors [10, 12, 13, 14] did not confirm this feature, arguing that most of radiated axions have a lower frequency corresponding to the horizon size, $\langle E_a\rangle \sim 2\pi/t$. If this is the case, the axion abundance becomes larger than that estimated based on the vacuum re-alignment mechanism [4, 5, 6].

The most straightforward way to address this controversy is to perform large scale field theoretic lattice simulations [14, 16, 17, 18]. In this approach, the classical equation of motion for the complex scalar field Φ in the expanding universe is solved numerically on discretized coordinates. Based on this approach, the whole process of the evolution of defects, including the formation of strings, that of domain walls, and the collapse of the string-wall systems, was investigated in Ref. [17], and the spectrum of axions radiated from them was estimated by applying the masking analysis method introduced in Ref. [16]. The results indicated that there are $\mathcal{O}(1)$ strings per horizon volume [*i.e.* $\xi \sim \mathcal{O}(1)$], and that radiated axions are mildly relativistic. In particular, the spectrum of radiated axions peaks at lower frequencies, which supports the conclusion of Refs. [10, 12, 13, 14]. These results were refined in Ref. [18], and the prediction for the axion mass $m_a = (0.8\text{–}1.3)\times 10^{-4}\,\mathrm{eV}$[1] was obtained by imposing the assumption that the axion becomes the main constituent of dark matter.

In contrast to the preceding argument, there still remains an unresolved issue associated with the technical limitations of lattice simulations. In the field theoretic lattice simulations, we must consider two extremely different length scales: One is the width of the string core δ_s, which is inversely proportional to the PQ scale, $\delta_s \sim (\sqrt{\lambda}v_{\rm PQ})^{-1}$, and the other is the Hubble radius $H^{-1} \sim t$, which corresponds to the typical distance between two neighboring strings. In reality, there is a huge hierarchy between these two scales: $H^{-1}/\delta_s \sim 10^{30}$ at $t \sim t_{\rm decay}$. However, it is impossible to realize such a huge hierarchy in the lattice simulations, due to the limitation of dynamical ranges. For simulations with 512^3 grid points, we only realize $H^{-1}/\delta_s \lesssim 300$, which is far smaller than realistic values. There is a possibility that this difference gives rise to a nontrivial consequence [19], which originates from the fact that the energy of string cores acquires a large logarithmic correction due to the gradient energy of surrounding axion fields, $\mu \approx \pi v_{\rm PQ}^2 \ln(H^{-1}/\delta_s)$. When $H^{-1} \gg \delta_s$, This core energy becomes

[1] If we just use maximum and minimum values for numerical factors such as ξ and $\langle E_a(t_{\rm decay})\rangle/(2\pi/t_{\rm decay})$ without using the propagation of uncertainty law, we obtain $m_a = (0.6\text{–}1.5)\times 10^{-4}\,\mathrm{eV}$.

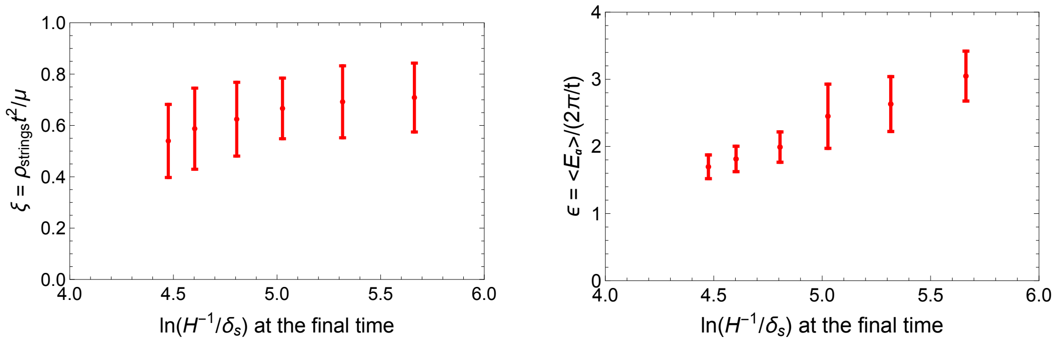

Figure 1: The renormalized string density (left) and mean energy of radiated axions (right) for various values of $\ln(H^{-1}/\delta_s)$ at the final time of the simulations.

larger than the string radiation power $P \sim v_{\rm PQ}^2$ [20], and we expect that the radiation damping becomes less efficient [21]. Therefore, it is still unclear whether strings evolve similarly to what we observe in the conventional field theoretic simulations if we take $H^{-1} \gg \delta_s$.

In order to check the dependence on the ratio H^{-1}/δ_s, we performed followup studies based on the field theoretic simulations. The setup of the simulations was the same with Ref. [17] except for that we set $m_a = 0$.[2] We varied the ratio between the string core width $\delta_s \sim (\sqrt{\lambda} v_{\rm PQ})^{-1}$ and the Hubble radius H^{-1} at the final time of the simulations by varying the self coupling of the PQ field within the range $0.093 \leq \lambda \leq 1$. From these simulations, we evaluated the density parameter of strings $\xi = \rho_{\rm strings} t^2/\mu$ and the ratio between the mean energy of radiated axions and the horizon scale $\epsilon \equiv \langle E_a \rangle/(2\pi/t)$ at the final time of the simulations. The results are shown in Fig. 1. We observed that both the string density and mean energy slightly increase with $\ln(H^{-1}/\delta_s)$, which agrees with the features pointed out in Ref. [19]. However, it is still dangerous to extrapolate these results to the realistic value, $H^{-1}/\delta_s \sim 10^{30}$. In order to investigate what happens in such cases, it is necessary to develop some alternative methods.

Recently, a couple of new approaches are proposed to investigate the dynamics of axionic strings with a large core energy. One possible way is to implement string cores as *smeared* external objects instead of directly solving the equation of motion for the complex scalar field Φ. This was performed in 2+1 D in Ref. [22], whose results showed that strings exhibit significantly different behavior at large values of $\ln(H^{-1}/\delta_s)$. However, there remains the question whether such results are peculiar to the simulations in 2+1 D spacetime. The other method was proposed in Ref. [23], which still relies on the field theoretic simulations but realizes a large string core energy by introducing additional field degrees of freedom. The axion production was investigated in Ref. [24] based on this simulation method, and the prediction for the axion mass $m_a = (2.62 \pm 0.34) \times 10^{-5}\,$eV was obtained based on the assumption that the axion becomes the main constituent of dark matter. It should be noted that both of these modified simulation methods are designed such that they realize the large-scale behavior of axionic strings without evaluating the dynamics of the PQ field Φ around the string core via first principles. Therefore, they are regarded as effective theories in the sense that they can correctly describe the dynamics

[2] If we take $m_a \neq 0$, strings start to collapse due to the tension of domain walls. For simplicity, here we do not consider this collapsing process, and focus only on the evolution of strings in the scaling regime.

of strings on large scales but break down at some smaller distance scales.

Let us try to figure out the implications of these new simulation results by going back to the initial argument based on energy conservation [Eqs. (1) and (2)]. In both types of modified simulation methods [22, 24], the results showed that, while strings become denser, the axion production becomes less efficient for realistic values of the string core energy, leading to a lower value of the axion mass. These facts imply that a large factor of ρ_{defects} in Eq. (2) should be compensated by producing more energetic axions. Therefore, it is possible that the results are sensitive to physics at smaller scales. Given the fact that these modified simulation methods can describe the infrared behavior of axionic strings but fail to describe ultraviolet modes, it is important to investigate the axion production from the decay of small scale strings in more detail. Understanding such dynamics of axionic strings is crucial to obtain a robust prediction for the mass of axion dark matter and interpret the results of forthcoming experiments.

Acknowledgments

The author would like to thank the organizers of *the 13th Patras Workshop on Axions, WIMPs and WISPs (Axion-Wimp 2017)* for a very motivating conference.

References

[1] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978).

[2] F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).

[3] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).

[4] J. Preskill, M. B. Wise and F. Wilczek, Phys. Lett. B **120**, 127 (1983).

[5] L. F. Abbott and P. Sikivie, Phys. Lett. B **120**, 133 (1983).

[6] M. Dine and W. Fischler, Phys. Lett. B **120**, 137 (1983).

[7] R. L. Davis, Phys. Lett. B **180**, 225 (1986).

[8] A. Vilenkin and A. E. Everett, Phys. Rev. Lett. **48**, 1867 (1982).

[9] D. Harari and P. Sikivie, Phys. Lett. B **195**, 361 (1987).

[10] R. L. Davis and E. P. S. Shellard, Nucl. Phys. B **324**, 167 (1989).

[11] C. Hagmann and P. Sikivie, Nucl. Phys. B **363**, 247 (1991).

[12] R. A. Battye and E. P. S. Shellard, Nucl. Phys. B **423**, 260 (1994) [astro-ph/9311017].

[13] R. A. Battye and E. P. S. Shellard, Phys. Rev. Lett. **73**, 2954 (1994) Erratum: [Phys. Rev. Lett. **76**, 2203 (1996)] [astro-ph/9403018].

[14] M. Yamaguchi, M. Kawasaki and J. Yokoyama, Phys. Rev. Lett. **82**, 4578 (1999) [hep-ph/9811311].

[15] C. Hagmann, S. Chang and P. Sikivie, Phys. Rev. D **63**, 125018 (2001) [hep-ph/0012361].

[16] T. Hiramatsu, M. Kawasaki, T. Sekiguchi, M. Yamaguchi and J. Yokoyama, Phys. Rev. D **83**, 123531 (2011) [arXiv:1012.5502 [hep-ph]].

[17] T. Hiramatsu, M. Kawasaki, K. Saikawa and T. Sekiguchi, Phys. Rev. D **85**, 105020 (2012) Erratum: [Phys. Rev. D **86**, 089902 (2012)] [arXiv:1202.5851 [hep-ph]].

[18] M. Kawasaki, K. Saikawa and T. Sekiguchi, Phys. Rev. D **91**, no. 6, 065014 (2015) [arXiv:1412.0789 [hep-ph]].

[19] L. Fleury and G. D. Moore, JCAP **1601**, 004 (2016) [arXiv:1509.00026 [hep-ph]].

[20] A. Vilenkin and T. Vachaspati, Phys. Rev. D **35**, 1138 (1987).

[21] A. Dabholkar and J. M. Quashnock, Nucl. Phys. B **333**, 815 (1990).

[22] L. M. Fleury and G. D. Moore, JCAP **1605**, no. 05, 005 (2016) [arXiv:1602.04818 [hep-ph]].

[23] V. B. Klaer and G. D. Moore, arXiv:1707.05566 [hep-ph].

[24] V. B. Klaer and G. D. Moore, arXiv:1708.07521 [hep-ph].