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J. E. A. Lys, C. T. Murphy, and M. Binkley
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

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S. Dado, A. Engler, G. Keyes, R. W. Kraemer, and G. Yekutieli
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

and

A. Brody and J. Hanlon
State University of New York at Stony Brook, Stony Brook, New York 11794

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A. A. Seidl, W. S. Toothacker, and J. C. Vander Velde
University of Michigan, Ann Arbor, Michigan 48104

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A. A. Seidl, W. S. Toothacker, and J. C. Vander Velde
University of Michigan, Ann Arbor, Michigan 48104[†]

ABSTRACT

We have studied rescattering in 100 GeV/c proton and positive pion interactions with deuterons in the Fermilab 30-in. bubble chamber. We study specifically events in which the deuteron breaks up and meson production occurs. After attempting to take account of elastic rescattering, we determine the fraction of events in which rescattering occurs, and the multiplicity distribution of the rescatter events. The rescatter fraction found for 100 GeV/c π^+d events is consistent with the values found in 21 GeV/c and 205 GeV/c π^-d experiments while the fraction found for 100 GeV/c pd events is slightly larger. The rescatter events' multiplicity distributions are in reasonable agreement with 205 GeV/c pp or π^-p multiplicity distributions. We compare our results to predictions of energy flux cascade and coherent tube models.

I. INTRODUCTION

We present results on a study of rescattering in proton-deuteron and positive pion-deuteron interactions at 100 GeV/c incident hadron momentum in the Fermilab 30-in. bubble chamber.

We use the word rescattering to describe interactions in which both nucleons in the deuteron take active part, as opposed to interactions in which one nucleon--the spectator nucleon--emerges with a momentum distribution essentially determined by the deuteron wave function. Rescattering may be thought of in terms of a model in which some product of a beam particle-nucleon interaction subsequently interacts with the second nucleon in the deuteron. In such a model there is explicit rescattering. Alternatively, a rescatter event may be thought of as an interaction of a beam particle with a single target that has a baryon number of two. Evidence for rescattering is provided by a deficiency in the observed fraction of events with spectator-like protons.

From our data we calculate the fraction of rescatter events, after correcting for deuteron final state events and for elastic rescattering, so that our results refer to inelastic rescattering in deuteron break up events. The calculation involves reasonable assumptions about the properties of spectator nucleons. We then calculate the multiplicity distribution for rescatter events.

Interactions with nuclear targets have received considerable attention recently because they present the possibility of studying the space-time evolution of multiparticle production. Experimental results (reviewed in

Refs. 1 and 2) clearly rule out naive cascade models; it is concluded that in hadron-hadron interactions the individual particles found in the asymptotic final state are not active instantaneously. Various models[3-7] have been suggested to explain the available data.

Because of the simplicity of the deuteron, hadron-deuteron interactions can contribute uniquely to the understanding of hadron-nucleus interactions. For example, in heavier nuclei it is necessary to make some sort of average over the number of nucleons that interact, while with the deuteron we can hope to study interactions in which a specific number of nucleons, namely two, take part in the interaction.

The existence of rescattering, its approximate magnitude, and some of its manifestations in high energy ($\gtrsim$ 20 GeV/c) experiments, have been pointed out recently by a number of authors.[8-13] The present paper gives one of the first detailed studies of rescattering in the deuteron at high energies, with an emphasis on multiplicity. For a review of rescattering in the deuteron at lower energies, see Ref. 14.

Notation

In this paper we consistently use the symbol M to denote number of events, and N to denote charged particle multiplicity. We use P_N to denote the probability that an inelastic interaction (of a specified type) will have charge multiplicity N, with normalization such that the sum over all possible N values is unity. We take P_N (hd) to refer to hadron-deuteron events in which the deuteron breaks up and meson production occurs, thus excluding deuteron final state and quasi-elastic events, and always add one to the prong count if odd to take account of the presumed invisible proton. For rescatter events, N is the multiplicity of the whole event, not the presumed multiplicity of the interaction with the "first" nucleon in the deuteron; we will use N_1 at one point to denote the latter multiplicity.

II. EXPERIMENTAL DETAILS

The data were obtained from an exposure of the Fermilab 30-in. deuterium bubble chamber to a beam of 100 GeV/c positive particles. The beam composition was approximately 57% proton, 39% π^+, 2% μ^+, and 2% K^+, and a tagging system[15] allowed a determination of the mass and location of each beam track in the bubble chamber. Results on multiplicity distributions have been published.[10]

The present paper uses a 23,000-picture sample of the film in which all tracks with momentum $\lesssim$ 1.5 GeV/c were measured in events with $N \geq 3$. Positive tracks with momentum below 1.4 GeV/c (as determined by the geometry reconstruction program) were assigned a mass, assumed either proton or pion, in a special ionization scan.

Because of data processing losses, a multiplicity-dependent and institution-dependent weight was assigned to each event. At least two independent prong counts were made on each event and discrepancies were edited by an experienced scanner or physicist. About 2.5% of the events found could not be assigned a unique prong count; such events are excluded from the data sample and a multiplicity-dependent weight corrects for this loss.

Short proton (or deuteron) tracks with lengths as small as 1-2 mm in the bubble chamber were measured. However, for this paper all tracks of length less than 5 mm have been classified as invisible and removed from the data, and prong counts amended accordingly (for a proton, a range < 5 mm corresponds to a momentum < 120 MeV/c). This was done to ensure that our results are not affected by any short track scanning bias.

The final numbers of weighted odd-and even-prong events are given in Table I for proton and pion beam particles. The average weight per event was 1.20. Also given in Table I are the numbers of even-prong events with a backward proton (here and elsewhere in this paper, a backward proton means a backward hemisphere proton and is defined as an outgoing proton with $\cos\theta < 0.0$, where θ is the laboratory angle between the incident beam direction and the outgoing proton).

The multiplicity distributions of the weighted events in Table I are presented in Table II for pd and Table III for π^+d events. The distributions in Tables II and III are close to, but not identical with, our best estimates of the odd-and even-prong multiplicity distributions as determined in this experiment, since the event samples used are slightly different.

The mean multiplicity of the backward proton events from pd (π^+d) interactions is 7.15±0.20 (7.45±0.29), implying a mean multiplicity for the presumed pn (π^+n) interactions of 6.15±0.20 (6.45±0.29) for $N \geq 3$. In comparison, the mean multiplicity of the odd-prong events with $N \geq 3$, after removal of the deuteron final-state contribution, is 6.70±0.10 (6.97±0.15) for pn (π^+n) interactions. This apparent discrepancy reduces to less than one standard deviation when the following two factors, both resulting from the fermi motion of the target nucleons, are taken into account. First, a struck proton can recoil backwards in the laboratory. Second, the average total center-of-mass energies of the hadron-neutron collisions that contribute to the odd-prong events and to the backward proton events are slightly different.

Estimates of One- and Two-Prong Multiplicities and Deuteron Final State Cross Sections

In what follows we need values for the quantities $P_1(pn)$, $P_1(\pi^+n)$, $P_2(pd)$, and $P_2(\pi^+d)$, and the cross sections at each multiplicity N for deuteron final-state events, i.e., for the reactions $hd \rightarrow dX$. It is not possible to measure these quantities reliably in this experiment.

As before,[10] we use π^-p data[16] and pp data[17-20] to estimate $P_1(hn)$. We take $P_1(pn) = (0.6\pm0.1)\ P_2(pp)$ and $P_1(\pi^+n) = (0.6\pm0.1)\ P_2(\pi^-p)$, resulting in $P_1(pn) = 0.088\pm0.015$ and $P_1(\pi^+n) = 0.056\pm0.012$.

We estimate $P_2(hd)$ assuming the relation:

$$P_2(hd) = 0.5\,[P_1(hn) + P_2(hp)]\,F_2, \tag{1}$$

where F_2 is a deuteron final-state correction which we take to be 0.90 ± 0.05. Such a relation is suggested by simple impulse approximation considerations, and similar relations for higher multiplicities, with $F = 1$, were found to be in reasonable agreement with the data.[10] The resulting values are $P_2(pd) = 0.105\pm0.020$ and $P_2(\pi^+d) = 0.081\pm0.020$ (we have arbitrarily doubled the errors to take account of uncertainties in the assumed relation).

To obtain deuteron final-state cross sections, we first extrapolate published $pd \rightarrow dX$ data[21, 22] to include the whole kinematic region, and assume a smooth variation of the cross section with beam momentum. The result is $\sigma(pd \rightarrow dX) = 2.0 \pm 0.3$ mb at 100 GeV/c. Then, since approximately 90% of the cross section occurs with recoil mass squared $M_x^2 < 20\ \mathrm{GeV}^2$, we assume the same multiplicity distribution as[23] for $pp \rightarrow pX$ events with $M_x^2 < 20\ \mathrm{GeV}^2$, and arrive at partial cross sections of 0.9 mb ($N = 2$), 0.9 mb ($N = 4$), 0.2 mb ($N = 6$), and < 0.05 mb ($N \geq 8$). The t-distribution of these events[21, 22] is such that 76% will have an invisible (range < 5 mm) deuteron and hence be classed as odd prongs.

No measurements of $\pi^+ d \rightarrow dX$ are available near our energy, so we make use of $\pi^- p$ diffraction dissociation data,[24] plus charge symmetry. We assume:

$$\sigma(\pi^- d \rightarrow d\pi^{-*})/\sigma(pd \rightarrow dp^*) = \sigma(\pi^- p \rightarrow p\pi^{-*})/\sigma(pp \rightarrow pp^*), \qquad (2)$$

where a star superscript indicates a diffractively produced excited state, and we assume that all deuteron final-state events are diffractively produced. The result is $\sigma(\pi^+ d \rightarrow dX) = 1.3 \pm 0.3$ mb, with individual multiplicity contributions, from π^{-*} multiplicity distributions, of 0.40 mb ($N = 2$), 0.63 mb ($N = 4$), 0.22 mb ($N = 6$), 0.05 mb ($N = 8$), and < 0.05 mb ($N \geq 10$).

The final numbers of odd- and even-prong deuteron breakup events, obtained after subtracting the deuteron final-state contributions, are given in Table I. In the remainder of this paper we use only deuteron breakup events. The largest of the deuteron final-state partial cross sections, for $N = 4$, is only 8% of the deuteron breakup cross section for $N = 4$, so the corrections are small.

III. RESCATTER FRACTIONS

Rescatter Fraction, pd

We make the simplifying assumption that deuteron breakup events can be divided into three classes: proton spectator events, neutron spectator events, and rescatter events. It is assumed that the ratio of event numbers in the first two classes equals the ratio of the inelastic free neutron and free proton cross sections, which is approximately unity. The assumed properties of proton spectators allow us to determine the number of proton spectator events, and hence the number of neutron spectator events. The remaining events are assigned to the rescatter class.

To determine the number of spectator proton events in our sample, we assume that all odd-prong and backward proton events, apart from a small fraction which is calculated below, are spectator proton events. We can then use either the number of odd-prong events or the number of backward proton events to estimate the number of forward proton spectators. Use of backward proton event numbers may be preferable, since the result is less dependent on knowledge of the deuteron wave function.

Before making a determination of the rescatter fraction, two effects must be considered. These are spectator neutron events with an invisible or backward proton, and elastic rescattering.

The contributions of spectator neutron events to odd prong and backward proton events are caused mainly or wholly by the fermi motion of the struck proton. We have used 100 GeV/c pp data,[17] and smeared the target proton momentum in accordance with the Hulthen wave-function prediction (see e.g., Fridman[25]; we use the same parameter values as Ref. 26) to determine these contributions. The deuteron form factor was included appropriately to take account of those events where the deuteron does not break up. The resulting fractions of neutron spectator events with $N \geq 4$ that are expected to give odd prong and backward proton events are each 0.002 ± 0.001, in both cases 90% with $N = 4$, 10% with $N = 6$. Appropriate corrections are made below in determining the rescatter fraction.

We now consider elastic rescattering in detail. In a model with explicit rescattering, one special type of rescatter is an elastic scatter on what we will call the pseudo-spectator nucleon. The scattered particle may be a beam particle (pre-scattering) or a product of an inelastic interaction on the other nucleon; we refer to both as elastic rescattering. The forward peaking in elastic scattering experiments means that the resulting pseudo-spectator momentum distribution will peak at a low (~300 MeV/c) value, and since pseudo-spectator and true spectator nucleons are indistinguishable there can be interference. Dean[27] has suggested how to take account of these elastic rescatters within Glauber theory, with neglect of any spin or isospin effects. Dean assumes that when the scattering follows an inelastic interaction the scattered particle has beam-like properties, that is, has the same elastic scattering amplitude as the beam particle. We have evaluated Dean's formula for the vector momentum distribution of the spectator and pseudo-spectator nucleon [Eq. (9) of Dean's paper]. In our evaluation we used an imaginary elastic scattering amplitude, exponential in momentum transfer, and we included the longitudinal momentum transfer in elastic scattering. In the elastic rescattering term we used the Hülthen deuteron wave function, while in the no-rescatter term we added a 6.5% D-wave component (explicitly, a multi-gaussian fit to the Reid soft-core wave function[28]). Appropriate Møller flux factor[29] terms were also included.

Predictions from the evaluation of Dean's formula are compared to our data in Fig. 1. Also shown are the predictions of a pure spectator model, that is, with no pseudo-spectator term but still including D-wave and flux factor terms. In Fig. 1(a), which shows the momentum distribution of backward protons from even-prong events, the predicted values are normalized to the expected number of odd-prong events that have an invisible backward proton. Figure 1(b) shows the angular distribution of seen spectator protons that have a momentum in the range 120-300 MeV/c. Here a seen spectator is defined to be a proton from an even-prong event, and if an event has two protons we select the backward proton if there is one, or the slower proton if neither is backwards. The predicted curves in Fig. 1(b) are normalized to the cos $\theta < 0.0$ region; in the cos $\theta > 0.0$ region we expect a considerable contribution from proton recoils from neutron spectator events. The reasonable agreement between the data and the predictions from Dean's formula suggests that the latter does not give a gross misestimate of the elastic rescattering effect.

In order to use the backward proton events to estimate the number of forward proton spectator events, we need a theoretical value for the ratio r of forward to backward visible spectator (including pseudo-spectator) protons. Our evaluation of Dean's formula gives a value of 1.45, to be compared with values of 1.29 if elastic rescattering is omitted, and 1.23 if both the D-wave component and elastic rescattering are omitted. The latter two values reflect simply the Møller flux factor (all values refer to spectator momenta above 120 MeV/c). Much of the contribution of the larger r value when elastic rescattering is included comes from proton momenta above 300 GeV/c, and is not apparent in Fig. 1(b). Simply weighting our backward proton events with appropriate Møller flux factors leads to r = 1.27. We then take r = 1.45±0.16. The error is such that the no-elastic rescatter value is only one standard deviation away.

We expect that the rescatter fraction calculated with this value of r will refer only to inelastic rescatters, that is, events in which new particle production occurs on both nucleons. We note that we may not have taken into account elastic rescatters by low energy particles; however, the small change in the value for r above (1.29 to 1.45) suggests that our result will not be substantially altered by low energy elastic rescatters.

We now proceed to calculate F_{rs}, the fraction of rescatter events, where we now explicitly refer to inelastic rescatters. Denoting the numbers of proton spectator, neutron spectator, and rescatter events with $N \geq 3$ by M(psp), M(nsp), M(rs) respectively, we write:

$$M(psp) = M(odd) + (1 + r)M(back\ pr) \qquad (3)$$

$$M(nsp) = M(psp) \times \sigma(pp,\ N \geq 4)/\sigma(pn,\ N \geq 3) \qquad (4)$$

$$M(rs) = M(total) - M(psp) - M(nsp), \qquad (5)$$

where M(odd), M(back pr) and M(total) are respectively the number of odd-prong events, backward proton events, and total events in our data sample.

The rescatter fraction, for $N \geq 3$, is defined by:

$$F_{rs}(\geq 3) = M(rs)/M(total) \tag{6}$$

or using Eqs. (4) and (5):

$$F_{rs}(\geq 3) = 1 - \frac{M(psp)}{M(total)}\left[1 + \frac{\sigma(pp,\ N \geq 4)}{\sigma(pn,\ N \geq 3)}\right]. \tag{7}$$

The rescatter fraction for all events (we mean all deuteron breakup events in which meson production occurs) is given by:

$$F_{rs} = (1 - P_2)F_{rs}(\geq 3) + P_2 F_{rs}(2), \tag{8}$$

where $F_{rs}(2)$ is the rescatter fraction for $N = 2$ events. We assume $F_{rs}(2) = 0.10 \pm 0.05$, a value half way between zero and the value of $F_{rs}(\geq 3)$, and consistent with any reasonable extrapolation of the number of rescatter events at each N determined below. A non-zero value for $F_{rs}(2)$ is indicated by the small value deduced later on for $\langle N_2 \rangle$, the mean multiplicity in the presumed incoherent interaction on the second nucleon in the deuteron.

Finally, the results from our data, utilizing Eqs. (3), (7), and (8), and assuming that the ratio of pn to pp inelastic cross sections is the same as for total cross sections,[30] are $F_{rs}(\geq 3) = 0.20 \pm 0.02$ and $F_{rs} = 0.19 \pm 0.02$. The errors include contributions from errors in $P_2(pd)$, $P_1(pn)$, $P_2(pp)$, r, $F_{rs}(2)$, and the pn and pp total cross sections, as well as the statistical errors in the event numbers. The last-named is the largest contributor. The error in r of ± 0.16 by itself gives an error in F_{rs} of ± 0.010, so that if $r = 1.29$ then $F_{rs} = 0.20 \pm 0.02$.

Rescatter Fraction, π^+d

We have determined the π^+d rescatter fraction in the same way as for pd. Evaluation of Dean's formula gives a value for the forward-backward visible spectator ratio r of 1.34, smaller than in the pd case because of the smaller pion-nucleon elastic cross section. Hence we take $r = 1.34 \pm 0.05$. The cross sections in the analogies of Eqs. (4) and (7) are now π^+p and π^+n cross sections, and from charge symmetry we take the latter as equal to the π^-p cross section. For $F_{rs}(2)$ in Eq. (8) we take now a value of 0.08 ± 0.04. Finally we obtain $F_{rs}(\geq 3) = 0.15 \pm 0.03$ and $F_{rs} = 0.14 \pm 0.03$. Again the statistical error is the largest contributor to the errors in F_{rs}. The error in r of ± 0.05 by itself gives an error in F_{rs} of ± 0.004.

The pd and π^+d rescatter fractions are summarized in Table IV.

Comparison with Rescatter Fractions from Other Experiments

Two π^-d experiments[12,13] have reported rescatter fractions for $N \geq 3$, calculated with the assumption that $\sigma(\pi^-p, N \geq 4) = \sigma(\pi^-n, N \geq 3)$, and making use of odd prong plus backward proton events. We have recalculated the rescatter fractions from the published data without making the aforementioned assumption. The results are given in Table V. Our calculations used published cross sections[30-33] and used the same equations as for our data, with the same assumptions for $P_1(\pi n)$ and $P_2(\pi d)$, plus $F_{rs}(2) = 0.08 \pm 0.04$. Deuteron final-state events were assumed to contribute equally to odd- and even-prong events. Elastic rescattering corrections were not included. For comparison, Table V also gives rescatter fraction values from the present experiment when elastic rescattering corrections are omitted (we expect that the elastic rescattering correction will be approximately independent of beam momentum above ~20 GeV/c).

The values in Table V suggest that the rescatter fraction for πd interactions is constant over the incident beam momentum interval 21-205 GeV/c. In contrast, the πp mean charged particle multiplicity changes from 4.6 to 8.0 over this interval. These points argue against a simple cascade model in which the rescatter probability is proportional to the hadron-nucleon mean multiplicity, but cannot rule out some multiplicity dependence.

The results of the present experiment suggest that the rescatter fraction is larger for pd than for πd interactions. This suggestion is reinforced if we take the average πd value in Table V. Thus it appears that the rescatter fraction, while independent of beam momentum above ~20 GeV/c, does depend on the nature of the beam particle. Clearly it is important for other deuterium experiments to confirm these interesting observations. The connection with models of hadron-nuclei interactions is examined in a later section of this paper.

IV. MULTIPLICITY DISTRIBUTIONS OF RESCATTER EVENTS

In this section we determine the numbers of rescatter events at each charge multiplicity N, after making a few reasonable assumptions.

At each value of N (N even) we consider those even-prong events that do not have a visible backward proton, of number $M_N(a)$. We assume that all rescatter events, $M_N(rs)$, and almost all neutron spectator events, $M_N(nsp)$, (the exceptions were discussed earlier) are included in $M_N(a)$. Some proton spectator events are also included in $M_N(a)$, and we assume that their number may be obtained from the N-1 prong events by the equation:

$$M_N^a(psp) = Hf_N M_{N-1}. \tag{9}$$

Here H is the ratio of all visible forward proton spectator events to all odd-prong events, and our evaluation of Dean's formula leads to H = 0.161 for pd events and H = 0.164 for $\pi^+ d$ events. Thus H results from the deuteron wave function, the Møller flux factor, and the elastic rescattering effect. The term f_N in Eq. (9) corrects for the fact that the average beam-neutron total center-of-mass energy for the slow neutrons that give odd-prong events is different from that for the faster, backward-moving neutrons that give visible forward proton spectators. We evaluated f_N assuming that the average multiplicity of hn events has the same s-dependence as found for pp events,[34] and assuming that hn multiplicity distributions follow the same "universal curve" that Slattery[35] fit to pp distributions. Values for f_N varied from $f_4 = 0.96$ to $f_{20} = 1.37$. Odd prong rather than backward spectator events were used to determine forward spectator numbers because the f_N depart less from unity for the former.

We assume that the numbers of neutron spectator events are given by:

$$M_N(nsp) = \frac{F_p M(\text{total}) P_N(hp)(1 - F_{rs})}{1 - P_2(hd)} \tag{10}$$

$$F_p = \frac{\sigma(hp,\ \text{inel.})}{\sigma(hn,\ \text{inel.}) + \sigma(hp,\ \text{inel.})}. \tag{11}$$

Again M(total) is the total number of events in our sample. The hp inelastic multiplicity probabilities $P_N(hp)$ in Eq. (10) are taken from pp and $\pi^+ p$ experiments.[17-20]

Finally, we have:

$$M_N(rs) = M_N(a) - M_N^a(psp) - M_N(nsp). \qquad (12)$$

Equations (9) - (12) were used to determine $M_N(rs)$ for $N \geq 4$. Values for $M_2(rs)$ follow from the values for $F_{rs}(2)$ assumed earlier. The results, together with the means for N and N^2 and the dispersion D, are given in Table VI. Errors include statistical errors, errors in F_p and $P_N(hp)$, and an assumed error of ±0.03 in H; the statistical errors dominate. The rescatter events' multiplicity distributions (normalized to sum to unity) are plotted in Fig. 2, along with 205 GeV/c pp and $\pi^- p$ distributions.[32,36]

Discussion of Rescatter Event Multiplicities

In Table VII some properties of the rescatter events' multiplicity distributions are compared to hd and hp distributions[10, 17-20, 32, 36] (the comparison is made for $N \geq 4$, since we do not experimentally measure $N = 2$). We see reasonable agreement between the rescatter events' moments at 100 GeV/c and the respective 205 GeV/c moments. Such agreement is interesting in that the center-of-mass energies of 100 GeV/c hd collisions and 205 GeV/c hp collisions are approximately equal. The comparison can also be seen in Fig. 2 (where $N = 2$ is now included).

It is of interest to determine a quantity $\langle \Delta N \rangle$, defined as the difference between the mean multiplicity of hadron-deuteron rescatter events and that of hadron-nucleon interactions at the same incident beam energy. Our results give $\langle \Delta N \rangle = 1.95 \pm 0.30$ (pd) and 1.69 ± 0.45 ($\pi^+ d$). If, in a model with explicit rescattering, the rescatter is an interaction between a materialized hadron from a beam-nucleon collision and the second nucleon in the deuteron then the mean multiplicity of the materialized hadron-second nucleon interaction is given by $<N_2> \approx <\Delta N> + 0.67$. The term 0.67 comes from assuming that in 67% of the rescstters the aforementioned materialized hadron is charged. We find $<N_2> = 2.62 \pm 0.30$ (pd) and 2.36 ± 0.45 ($\pi^+ d$). These values are comparable with the mean multiplicity of ~2 GeV/c πp interactions.

In Eq. (10) above, it was implicitly assumed that the multiplicity distribution of neutron spectator events was the same as that of free hadron-proton interactions. This assumption might be called a "no-cascade" assumption. In a model with explicit rescattering, an equivalent assumption is that the rescatter probability is independent of the multiplicity of the first interaction. We could alternatively take a cascade-type model, in which the probability of a rescatter following a beam nucleon interaction of charged plus neutral multiplicity N_1' increases with N_1', although evidence from experiments with heavier nuclei tends to argue against such models.[1,2,37] To indicate the effect that such a cascade-type model would have on our results, we have recalculated the rescatter event multiplicities under the rather extreme assumption that the rescatter probability is equal to $\beta N_1'$, with $\beta = F_{rs}/<N_1'> = 0.020$ (pd) and 0.014 ($\pi^+ d$). The resulting rescatter events mean multiplicities would be 8.60 ± 0.30 (pd) and 8.70 ± 0.45 ($\pi^+ d$). We see that cascade-type models can increase the calculated mean multiplicities by up to ~0.3 units. However, after recalculating appropriately the mean multiplicities of the beam-first nucleon interactions in rescatter events, we find that cascade-type models will reduce the calculated values of $<N_2>$ by ~0.8 units.

V. COMPARISON WITH THEORETICAL MODELS

In this section we compare our results with predictions of Energy Flux Cascade (EFC) models and with Coherent Tube models.

In the EFC model,[1-3] the result of a hadron-nucleon interaction is an energy flux which, after travelling one mean free path in a nucleus, corresponds on the average to two components, a "head" and a "tail." The head has energy $\sim E_b$ (the beam energy), and undergoes $(\bar{\nu} - 1)$ further collisions, while the tail has energy $\sim E_b^{1/3}$ and is assumed not to undergo any multiplicity increasing collisions. The parameter $\bar{\nu}$ is defined, and estimated to be 1.096 and 1.069 respectively for pd and πd interactions, with some theoretical uncertainty, in the Appendix. Thus in deuterium the EFC model predicts a fraction $(\bar{\nu} - 1)$ of rescatters by the head component. Our data, however, yield a fraction which is twice as large. Explicitly, $F_{rs}/(\bar{\nu} - 1)$ is 2.0±0.3 (pd) and 2.1±0.4 (π^+d). There is no inconsistency here if the tail component rescatters independently of and approximately as often as the head. In this picture then, since $\bar{\nu}$ is almost independent of beam momentum above ~20 GeV/c, we expect F_{rs} to be similarly independent; since $\bar{\nu}$ (pd) > $\bar{\nu}$(πd) we expect F_{rs}(pd) > F_{rs}(πd). The data agree with both these expectations.

Each rescatter by the head component, in the EFC model, produces a mean multiplicity increase of η times the hadron-nucleon mean multiplicity, with the parameter η variously estimated[2-6,38] at ~0.25 to 0.50. Heavy nuclei experiments[2,39] suggest $\eta \approx 0.5$. If tail-nucleon rescatters produce negligible multiplicity increase, then the assumed fraction $(\bar{\nu} - 1)$ of head rescatters produces a mean multiplicity increase given by $\langle \Delta N \rangle F_{rs}/(\bar{\nu} - 1)$. Dividing by the hadron-nucleon mean multiplicity, our experiment[40] then yields values for η of 0.60±0.12(pd) and 0.52±0.17(π^+d). Again, if our picture of tail rescatters is correct there is agreement with the EFC model and with heavy nuclei experiments.

In the coherent tube model[7] the incident hadron is assumed to see a target composed of i nucleons, where i can be 1 to A (A = atomic number). The multiplicity distribution, for a given i value, is a function of the total energy, and averaging over i gives the overall multiplicity distribution. Hence our rescatter events, taken as i = 2 events, should have the same multiplicity distribution as 200 GeV/c hadron-nucleon interactions, in reasonable agreement with the data. The rescatter fraction in deuterium in this model is just ($\langle i \rangle - 1$), but it is not clear[41] what the prediction is for $\langle i \rangle$.

We conclude that our results are in agreement with simple predictions of both the EFC and the coherent tube models. Stronger conclusions must await either more experimental data on rescatter events, for example, rapidity distributions of outgoing particles, or more specific predictions from the models.

VI. CONCLUSIONS

Our main conclusions are as follows:

We have determined rescatter fractions for events in which the deuteron breaks up and meson production occurs. We have attempted to take account of elastic rescatters, so that these fractions refer to inelastic rescatters. We find F_{rs} = 0.19±0.02(pd) and 0.14±0.03 (π^+d).

We have determined the multiplicity distributions of inelastic rescatter events. The mean multiplicities are 8.3±0.3 (pd) and 8.4±0.4 (π^+d).

Our 100 GeV/c π^+d rescatter fraction is in agreement with recalculated 21 GeV/c and 205 GeV/c π^-d rescatter fractions.

The pd rescatter fraction is consistent with being larger than the πd rescatter fraction.

The data are consistent with the relation $F_{rs} = 2(\overline{\nu} - 1)$.

The rescatter events' multiplicity distributions appear to agree with predictions of energy flux cascade and coherent tube models.

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APPENDIX

The parameter $\overline{\nu}$, for a hadron-nucleus collision, is the average number of inelastic collisions the hadron would make with nucleons in the nucleus if, following each collision, the hadron remained as a single hadron.[1,2] The value of $\overline{\nu}$ can be obtained from the formula $\overline{\nu} = A\sigma(\text{hN, inel.})/\sigma(\text{hA, inel.})$, where $\sigma(\text{hN})$ and $\sigma(\text{hA})$ are respectively the average hadron-nucleon cross section and the hadron-nucleus cross section. For deuterium we take $\sigma(\text{hd, inel.})$ as the cross section for production of new particles, and since no measurements of this quantity are available we estimate it using the Glauber formula:[42]

$$\sigma(\text{hd,inel.}) = \sigma(\text{hp,inel.}) + \sigma(\text{hn,inel.}) - 2\sigma(\text{hp,inel.})\sigma(\text{hn, inel.})\langle r^{-2}\rangle/4\pi. \quad \text{(A1)}$$

We then have:

$$\frac{1}{\overline{\nu}} = 1 - \frac{2\sigma(\text{hp, inel.})\sigma(\text{hn, inel.})\langle r^{-2}\rangle}{4\pi[\sigma(\text{hp, inel.}) + \sigma(\text{hn, inel.})]} \quad \text{(A2)}$$

or approximately:

$$\overline{\nu} \approx 1 + \frac{\sigma(\text{hN,inel.})\langle r^{-2}\rangle}{4\pi}. \quad \text{(A3)}$$

Inserting appropriate inelastic cross sections[30,33] into Eq. (A2) we obtain, at 100 GeV/c, $\overline{\nu}(\text{pd}) = 1.096$ and $\overline{\nu}(\pi\text{d}) = 1.069$, where we have taken values[30] for $\langle r^{-2}\rangle$ of 0.035 mb^{-1} (pd) and 0.040 mb^{-1} (πd). Equation (A3) shows that $\overline{\nu}$ is relatively independent of beam momentum in the range ~20 to 200 GeV/c, where cross sections are relatively constant. Uncertainties in our $\overline{\nu}$ values arise from uncertainties in the values of $\langle r^{-2}\rangle$ and in the validity of Eq. (A1).

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*Permanent address: Weizmann Institute of Science, Rehovoth, Israel.

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TABLE I. Weighted events ($N \geq 3$).

	pd	π^+d
Odd prongs	2263	1103
Even prongs	4394	1913
Even prongs with a backward proton	237	122
Odd prongs, deuteron final-state events removed	2160	1041
Even prongs, deuteron final-state events removed	4361	1893

TABLE II. Multiplicity distributions of weighted pd events; deuteron final-state events not removed.

N	Odd prongs (N - 1)	Even prongs	Even prongs with a backward proton
4	529	1030	59
6	584	1143	79
8	504	941	43
10	337	636	33
12	180	341	12
14	77	175	9
16	37	96	3
18	10	25	0
20	4	6	0
22	2	0	0

TABLE III. Multiplicity distributions of weighted π^+d events; deuteron final-state events not removed.

N	Odd prongs (N - 1)	Even prongs	Even prongs with a backward proton
4	245	419	29
6	264	480	32
8	242	439	26
10	158	288	18
12	107	165	11
14	60	76	4
16	23	29	1
18	2	11	0
20	0	4	0
22	0	1	0
24	0	1	0

TABLE IV. Rescatter fractions, F_{rs}, from this experiment. The quantity r is the value assumed for the ratio of forward to backward visible spectators; the smaller r values neglect elastic rescattering corrections (see text).

N range	r (pd)	F_{rs}(pd)	r (π^+d)	$F_{rs}(\pi^+d)$
All N	1.45±0.19	0.187±0.022	1.34±0.05	0.142±0.026
	1.29	0.197±0.019	1.29	0.145±0.026
N ≥ 3	1.45±0.19	0.197±0.022	1.34±0.05	0.148±0.026
	1.29	0.208±0.019	1.29	0.152±0.026

TABLE V. Rescatter fractions, F_{rs}. No elastic rescattering corrections are included. The values in parentheses are those given in Refs. 13 and 12 respectively.

N range	21 GeV/c π^-d [a]	100 GeV/c π^+d	205 GeV/c π^-d [b]	100 GeV/c pd
All N	0.139±0.019	0.145±0.026	0.146±0.018	0.197±0.019
N ≥ 3	0.148±0.019	0.152±0.026	0.150±0.019	0.208±0.019
	(0.11±0.01)		(0.14±0.01)	

[a] Data from Ref. 13
[b] Data from Ref. 12

TABLE VI. Multiplicity distributions of inelastic rescatter events, with means $\langle N \rangle$, mean squares $\langle N^2 \rangle$, and dispersions D.

N	M_N(rs), pd	M_N(rs), π^+d
2	77±42	21±12
4	189±41	68±24
6	287±44	70±27
8	295±39	115±25
10	199±32	70±21
12	136±23	47±16
14	84±16	28±11
16	63±13	12±7
18	17±7	7±5
20	3±6	3±3
22	0	1±1
$\langle N \rangle$	8.27±0.29	8.36±0.44
$\langle N^2 \rangle$	82.6±4.9	83.7±8.0
D	3.76±0.19	3.71±0.28

TABLE VII. Comparison of multiplicity moments for $N \geq 4$.

	$\langle N \rangle$	$\langle N^2 \rangle$	D
100 GeV/c pd rescatters	8.65±0.23	87.3±4.5	3.53±0.14
100 GeV/c pd[a]	7.65±0.04	68.3±0.8	3.13±0.03
100 GeV/c pp[b]	7.14±0.03	59.3±0.6	2.87±0.03
205 GeV/c pp[c]	8.23±0.06	80.2±1.2	3.54±0.04
100 GeV/c π^+d rescatters	8.68±0.42	87.7±8.0	3.51±0.25
100 GeV/c π^+d[a]	7.75±0.07	70.2±1.3	3.17±0.06
100 GeV/c π^+p[b]	7.37±0.05	62.8±0.9	2.91±0.04
205 GeV/c π^-p[d]	8.52±0.06	85.4±1.2	3.58±0.05

[a]Ref. 10
[b]Refs. 17-20
[c]Ref. 36
[d]Ref. 32

FIGURE CAPTIONS

Fig. 1. Proton distributions from (weighted) pd events. (a) The momentum distribution of backward protons in even-prong events. The solid circles and crosses are respectively the predictions with and without elastic rescattering (see text), normalized using the number of odd-prong events. (b) The angular distribution of seen spectator protons (see text) with momentum between 120 and 300 MeV/c. The curves are the predictions with (solid curve) and without (broken curve) elastic rescattering, both normalized to the cos $\theta < 0.0$ region.

Fig. 2. The normalized charged particle multiplicity distributions in 100 GeV/c pd and πd rescatter events. Also shown are 205 GeV/c pp and π^-p distributions from Refs. 36 and 32 respectively. The errors on the pp and π^-p points are less than 0.01.

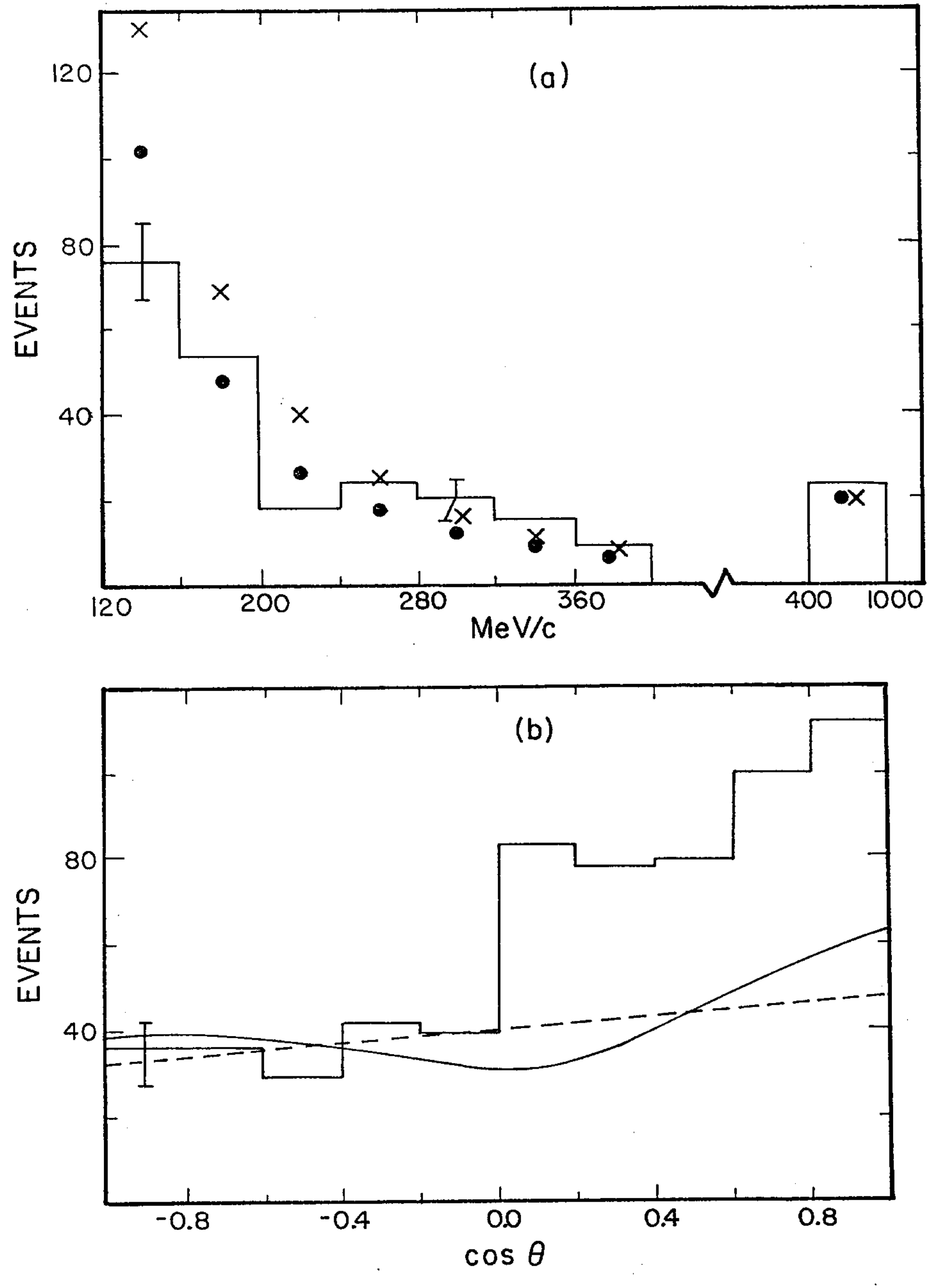
(a)
EVENTS
120
80
40
120
200
280
360
400
1000
MeV/c
(b)
EVENTS
80
40
-0.8
-0.4
0.0
0.4
0.8
cos θ

Fig. 1

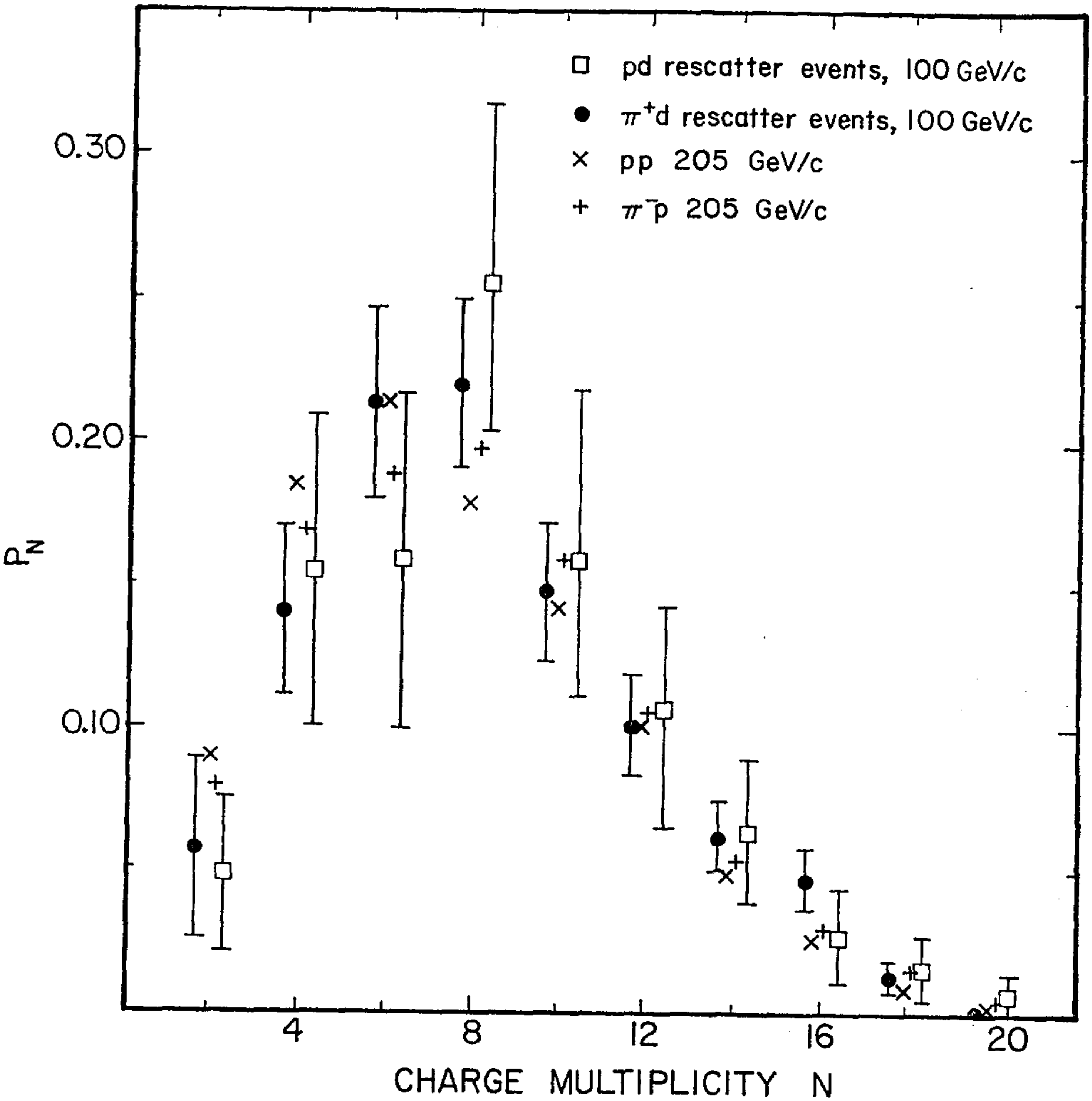
pd rescatter events, 100 GeV/c
π⁺d rescatter events, 100 GeV/c
pp 205 GeV/c
π⁻p 205 GeV/c
0.30
0.20
0.10
P_N
4
8
12
16
20
CHARGE MULTIPLICITY N

Fig. 2