ON THE REGULARIZATION IN THE CALLAN-SYMANZIK EQUATION

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There is a number of different approaches to the Callan-Symanzik (CS) equation, as roughly classified to the following three categories:

(I) Method of response equation[1], renormalization group approach.
(II) Dispersion theory[2].
(III) Canonical theory of broken scale invariance (CTBSI)[3].

In the approaches of the third category, regularization procedure is indispensable, either by Pauli-Villars method or by the dimensional regularization technique. What we want to do here is to push further the most conservative approach of CTBSI with the Pauli-Villars regulators. The reason is that this old-fashioned but well-defined method still seems to provide us with the solid basis of the much intuitive understanding of the various anomalies, yet has never been fully examined in the literatures. We confirm that the CS equation is derived in this approach in a completely general manner.

From CTBSI follows a "dimensional equation" for the unrenormalized Green's function Γ; the right-hand side is another Green's function with an insertion of the trace of the energy-momentum tensor. If the naive dimensional analysis (NDA) was justified, this equation would be put into the form

$$m_0 \frac{\partial}{\partial m_0} \Gamma = -\Delta\Gamma \ , \qquad (1)$$

from which the CS equation follows immediately. Without having regulator fields, however, NDA obviously fails due to the presence of divergent integrals. As a consequence a close tie between the CS equation and CTBSI is lost. One may start with (1) which can be obtained as a formal relation among the divergent Feynman amplitudes without any reference to CTBSI. We want, however, to restore the above mentioned tie by including the regulators.

Incidentally we can use the CS equation to derive the Pais-Epstein formula[4] which expresses the unwanted "self-stress" S by the amount of failure of NDA for the self-mass δm;

$$S = \frac{1}{3}(1 - m_0 \frac{\partial}{\partial m_0})\, \delta m \,.$$

The "unregularized" δm results in a nonvanishing S; an immediate contradiction with the Lorentz covariance of a one-particle state. This is perhaps the most unambiguous argument to show that the regularization is necessary. The same argument shows also that the insertion in the right-hand side of the CS equation is not the trace of the true energy-momentum tensor.

If we introduce the regulators in the Lagrangian, NDA holds ture. Consequently we have (m; the renormalized mass)

$$m\frac{\partial}{\partial m}\,\Gamma_r = -\Delta\Gamma_r \;, \qquad (2)$$

for the renormalized Green's functions. This equation looks similar to the one which was dismissed by Coleman[5] as being false. In the right-hand side of (2), however, we have an insertion of the trace of the true energy-momentum tensor that includes the contribution from the regulators as well as the one from the physical fields. To the former part one cannot apply Weinberg's theorem on the asymptotic behavior. To separate the part to which one can apply Weinberg's theorem, we pick up the term of the physical fields multiplied by a divergent coefficient so that the result is finite. We subtract this term from the right-hand side of (2). The result is then transferred to the left-hand side. This yields exactly the anomalous terms given by the CS equation. Every step of the calculation is well-defined.

We can extend the method easily to the more complicated cases where there are two (or more) massive fields. In separating the finite part expressed only in terms of the physical fields, we make full use of the BPH theorem. The procedure is essentially the generalization of the previous lowest-order calculation[6]. Our derivation is completely general and is independent of the detailed properties of the regulator fields.

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DISCUSSION

Schroer: Do you have an "intuitive picture" for the method of dimensional regularization?

Fujii: Yes, there are almost one-to-one correspondences between the calculations with the two regularization methods. For example, to our regulator contribution in $\Delta\Gamma_r$ corresponds a term proportional to n-4, which is cancelled by a denominator 1/(n-4) to result in the anomalies.

Symanzik: I should like to give some arguments in favor of the response equation interpretation of the PDE discussed by Fujii. If one renormalizes a family of Lagrange theories such that different members of the family correspond to different values of the parameters (masses, renormalized coupling constants, etc.) then adding to the Lagrangian a finite local scalar operator such that hereby only a change of the Lagrangian within the considered family of Lagrangians is effected, then the infinitesimal operation to this insertion leads to a response equation, and the finiteness of the parametric function occuring therein is a direct consequence of the finiteness of the insertion. One is indeed led to the whole family of such insertions. They have been given for ϕ^4 theory by Lowenstein, who calls them generalized vertex operations, and are equivalently obtained by Nishijima's use of homogeneous generalized unitarity relations reported here. That some insertions may have a (treacherous!) similarity to the trace of the energy-momentum tensor at zero momentum transfer is accidental and plays no role whatsoever for the derivation of the response equation, nor does Poincaré invariance.

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