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Special Issue
Selected Papers from Shapes and Symmetries in Nuclei: From Experiment to Theory (SSNET'21 Conference)

Edited by
Prof. Dr. Costel Petrache and Prof. Dr. Jerzy Dudek

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https://doi.org/10.3390/sym15051075

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Wobbling Motion in Nuclei

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Abstract: Wobbling motion as an exotic collective mode in nuclei without axial symmetry, was intensively discussed during the last few years. The observation of the newly proposed transverse wobbling, first reported in ^{135}Pr and soon after in nuclei from other mass regions, was considered as a significant discovery in low-spin nuclear structure. However, both the reported experimental results and the proposed theoretical models were actively questioned in work devoted to the study of the low-spin wobbling mode in the same nuclei. We recently re-measured the electromagnetic character of the $\Delta I = 1$ transitions connecting the one- to zero-phonon and the two- to one-phonon wobbling bands in ^{135}Pr, showing their predominant $M1$ magnetic character, which is in contradiction with the wobbling interpretation. These new experimental results, which were reproduced by either the quasiparticle-plus-triaxial-rotor model and interacting boson-fermion model calculations, are against the previously proposed wobbling nature of the low-spin bands in ^{135}Pr. On the other hand, we obtained conclusive experimental evidence for the theoretically proposed transverse wobbling bands at medium spin in ^{136}Nd. The comparison of the experimental data with calculations using the triaxial projected shell model as well as a new particle-rotor model with frozen orthogonal geometry of the active nucleons, supports the description in terms of transverse wobbling of medium-spin bands in triaxial even-even nuclei.

Keywords: γ-ray spectroscopy; wobbling; collective models

Citation: Lv, B.; Petrache, C.M. Wobbling Motion in Nuclei. *Symmetry* **2023**, *15*, 1075. https://doi.org/10.3390/sym15051075

Academic Editor: Anna Cimmino

Received: 15 April 2023
Revised: 26 April 2023
Accepted: 9 May 2023
Published: 12 May 2023

1. Introduction

Nuclear wobbling motion is one of the exotic rotational motions in which the axis of rotation does not coincide with any of the inertia axes of a deformed nuclear body. It was first discussed by Bohr and Mottelson for even-even nuclei at high spin in nuclei without axial symmetry, and without active particles resulting from broken pairs [1]. This type of collective motion is the quantum mechanics analog to the precession of a free asymmetric top in classical mechanics. The triaxially deformed nucleus always rotates around the axis having the largest moment of inertia (MoI) and in the limit of very high angular momentum the rotation axis executes harmonic oscillations about the space-fixed angular momentum vector. Wobbling motion is considered as an unambiguous fingerprint of stable triaxiality, not necessarily rigid (see ref. [2]), of a deformed rotating nucleus which rotates simultaneously around all of its three axes with associated unequal moments of inertia.

Wobbling motion was identified via the observation of a series of rotational bands composed of $E2$ transitions with the excitation energies increasing with the number of wobbling oscillation quanta. The signature quantum number ($\alpha = 0$ or 1 for even-even nuclei and $\alpha = \pm 1/2$ for odd-even nuclei) of two consecutive bands is different, lowest and first excited bands corresponding to zero- and one-phonon wobbling bands, respectively. The connection between the first excited and lowest bands is realized by $\Delta I = 1$ dipole transitions; unlike the signature partners of a given configuration corresponding to spin up and down $s = \pm 1/2$ of the particle, the multipole $E2/M1$ mixing ratios δ are very large because the wobbling motion involves the entire nuclear charge, resulting in a collectively enhanced $E2$ component. The nuclear wobbling motion was first reported in the odd-even nuclei $^{161-167}$Lu and ^{167}Ta two decades ago [3–8]. In all these nuclei the wobbling mode is recognized in triaxial strongly deformed bands and assigned to the $\pi i_{13/2}$ intruder configuration with a large deformation of $\varepsilon_2 \approx 0.4$. It should be noted that the spin of the odd nucleon is assumed parallel to that of the core in all these nuclei. Frauendorf and Dönau have recently refined the study of the wobbling motion in odd-mass nuclei [9]. Two types of wobbling modes, transverse wobbling (TW) and longitudinal wobbling (LW), were suggested based on whether the odd nucleon aligns its angular momentum perpendicular or along the axis of collective rotation.

The wobbling mode gives rise to a series of rotational bands built on the same configuration, each one having different tilt angles of the core angular momentum vector with respect to the principal axis of the intrinsic system. The band with lowest energy corresponds to the rotation about the axis of the largest moment of inertia, and has the wobbling-phonon quantum number $n = 0$. The bands corresponding to increasing tilt angles with respect to that axis have $n = 1, 2, \ldots$. In transverse wobbling the odd nucleon has the angular momentum orthogonal to the core angular momentum, and the wobbling energy E_{wob} is defined as

$$E_{\text{wob}}(I) = E(n=1, I) - \frac{E(n=0, I+1) + E(n=0, I-1)}{2}, \tag{1}$$

The transverse wobbling mode was firstly investigated using the triaxial particle-rotor model under the assumption of frozen orthogonal geometry for two low-spin bands in ^{135}Pr, and the high-spin bands of the odd-mass Lu isotopes [9]. For longitudinal wobbling the angular momentum of the odd particle is parallel to the axis with the largest moment of inertia and the wobbling energy E_{wob} increases with increasing angular momentum. Recently, the first identification of transverse wobbling bands at low spin was reported in ^{135}Pr [10,11]. Soon after these works, transverse wobbling bands were reported in other odd-A nuclei, e.g., ^{105}Pd [12], ^{133}Ba [13], ^{183}Au [14]. Very recently, the longitudinal wobbling, for which the wobbling energy increases with increasing spin, has been reported in ^{133}La, ^{187}Au, and ^{127}Xe [15–17]. All these nuclei have an odd nucleon occupying a high-j orbital (neutron for the ^{105}Pd, ^{127}Xe, ^{133}Ba, and proton for all the other nuclei). However, the transverse wobbling description of the low-spin bands in all these nuclei is seriously questioned based on new experimental data and new calculations with geometric and algebraic models, see Section 2.

Wobbling motion has been widely observed in odd-A nuclei of various mass regions, while there are only two even-even nuclei over the nuclear chart in which wobbling bands were suggested: ^{112}Ru and ^{104}Pd [18,19], in which the "γ-bands" were interpreted as one- and two-phonon wobbling bands based on the odd-even staggering pattern of the level energies which is characteristic of rigid triaxiality. In fact, these nuclei are γ-soft, and the experimental evidence for wobbling interpretation is not solid enough. At medium spin the triaxial shape can be more stable due to the polarization induced by the particles resulting from the breaking of a nucleon pair. Possible one- and two-phonon transverse wobbling bands above $I = 10$ were initially suggested for the first time in the two even-even nuclei ^{134}Ce and ^{136}Nd [20]. However, that conjecture was not supported experimentally because the mixing ratios and transition probabilities of the linking $\Delta I = 1$ transitions between the hypothesized wobbling bands were lacking. Until very recently, the only observation of wobbling bands above spin $I = 10$ built on the $\pi h^2_{11/2}$ configuration was reported in the even-even nucleus ^{130}Ba [21]. The experimental findings supported the transverse wobbling interpretation of two-quasiparticle bands in ^{130}Ba, and were in good agreement with calculations employing the constrained triaxial covariant density-functional theory and quantum particle rotor model [21]. The possible interpretation as transverse wobbling of two bands based on the $\pi h^2_{11/2}$ configuration in the even-even nucleus ^{136}Nd were recently investigated theoretically using the triaxial projected shell model [22]. Very recently, we investigated experimentally the two-quasiparticle bands based on the $\pi h_{11/2}^2$ configuration in ^{136}Nd by measuring the mixing ratios of the $\Delta I = 1$ transitions between the wobbling bands [23], see Section 3.

In this article, the discussion of the present experimental results and their theoretical interpretation using quasiparticle triaxial rotor model (QTR) and interacting boson-fermion model (IBFM) of the low-spin wobbling motion in odd-A nuclei, in particular in ^{135}Pr, is presented in Section 2. The present experimental results and the theoretical interpretation using the particle rotor model (PRM) and projected shell model (PSM) of the medium spin two-quasiparticle wobbling bands of even-even nuclei, in particular in ^{136}Nd, is presented in Section 3, and a summary is given in Section 4.

2. Wobbling Motion in Odd-A Triaxial Nuclei

The negative-parity states of ^{135}Pr were firstly reported in ref. [24]. New states were identified in refs. [10,11], which were combined with previously known negative-parity states forming new bands. To investigate the nature of the bands, the $\Delta I = 1$ character of the interband transitions were extracted via angular distribution and polarization measurements. Large mixing ratios corresponding to high E2 admixtures for the connecting transitions were obtained, which corroborated with a decrease of the wobbling energy supports to the interpretation as one- and two-phonon transverse wobbling bands. This marks the first observation of wobbling in a mass region other than A $\approx$ 160. New results on negative-parity states in ^{135}Pr were also recently reported in ref. [25], in which the electromagnetic characters of some key transitions for the wobbling interpretation of the low-spin non-yrast bands were in contradiction with those reported in ref. [10], being predominantly $M1$ instead of $E2$ as expected for wobbling bands. These results raised numerous questions regarding the proposed transverse wobbling interpretation of the low-spin bands in ^{135}Pr [26–29].

Experimentally, the main argument is focused on the precision measurement of the $E2/M1$ mixing ratios δ of the ΔI = 1 connecting transitions between the excited and lowest wobbling bands. It is generally known that the mixing ratios of ΔI = 1 transitions obtained from the χ^2 fit to the experimental angular distributions always yields two solutions, $|\delta_1| > 1$ and $|\delta_2| < 1$, and therefore, excepting measurements with very high statistics, is not conclusive. In fact, ΔI = 1 connecting transitions between the excited and lowest wobbling bands are weak and the statistics not sufficient to definitely conclude on their electromagnetic character based on only angular distributions; but this approach was exactly that used in refs. [10,11]. The ambiguity on the electromagnetic character of ΔI = 1 transitions can be solved by measuring additional observables, like the linear polarization. The linear polarization has been measured for only two out of the six linking transitions in ^{135}Pr in ref. [10]. However, only the sign of the detected linear polarization values were employed to draw conclusions about their predominant $E2$ character, not the magnitude of the measured linear polarization. For the two-phonon wobbling band in ^{135}Pr reported in ref. [11], the wobbling interpretation was based on only angular distributions, without linear polarization measurements. Furthermore, the polarization data for the two linking transitions presented in ref. [10] are inconsistent with the results reported in ref. [25] which was published shortly after ref. [10] and was performed using the same reaction and experimental setup. Following these works, an erratum paper was published [25] in which similar polarization results as in ref. [10] were reported. However, the questions about the analysis remained unanswered. Theoretically, the validity of transverse wobbling motion in odd-mass nuclei has been debated since the time when the mode was proposed. In particular, the frozen approximation used in ref. [9] seems to not take into account properly the effect of the Coriolis force on the odd nucleon which has the angular momentum coupled orthogonally to that of the core. Very recently, the interpretation in terms of wobbling motion of the low-lying bands in odd-mass nuclei was studied in detail in refs. [29,30], and a new collective mode called tilted precession (TiP) was proposed in ref. [29]. It was demonstrated that the three-dimensional rotation of a triaxial odd-mass nucleus represents in fact a precession of the total angular momentum around a certain tilted axis rather than wobbling motion. The TiP collective mode was first applied to ^{135}Nd [30]. The level energies and the electromagnetic transition probabilities of the excited bands built on the $\pi h_{11/2}$ configuration were well described using quasiparticle-plus-triaxial-rotor model calculations, supporting the interpretation as tilted precession bands rather than wobbling bands.

In this context, a new experiment was performed at the University of Jyväskylä, Finland, using ^{100}Mo(^{40}Ar, 1p4n)^{135}Pr reaction at a beam energy of 152 MeV. The low-lying negative-parity bands in ^{135}Pr have been investigated in detail. Prompt γ-rays were detected by the JUROGAM II spectrometer comprising Compton-suppressed Germanium detectors: 24 clovers [31] and 15 phase-one detectors [32]. To overcome the aforementioned drawbacks in refs. [10,11], a new method combining both linear polarization and angular correlation analyses was used to accurately extract the mixing ratios of the transitions connecting low-lying bands in ^{135}Pr. Mixing ratio values $|\delta| < 1$ have been obtained for the 747-, 813- and 450-keV transitions in ref. [33], see Figure 1. These results are in contradiction with the $|\delta| > 1$ values reported in refs. [10,11], and claims in favor of a non-wobbling nature of the low-spin bands in ^{135}Pr. In addition, the assignment of the cascade of 827-, 764- and 1009-keV transitions as two-phonon wobbling band in ref. [11] is not easy to accept because the energies of the transitions in the band do not follow the $I(I+1)$ dependence of a rotational band. The second excited $19/2^-$ state in ^{135}Pr has to be assigned as the bandhead of the new band, consisting of the strong 688- and 871-keV transitions which follow the $I(I+1)$ dependence (see ref. [33]).

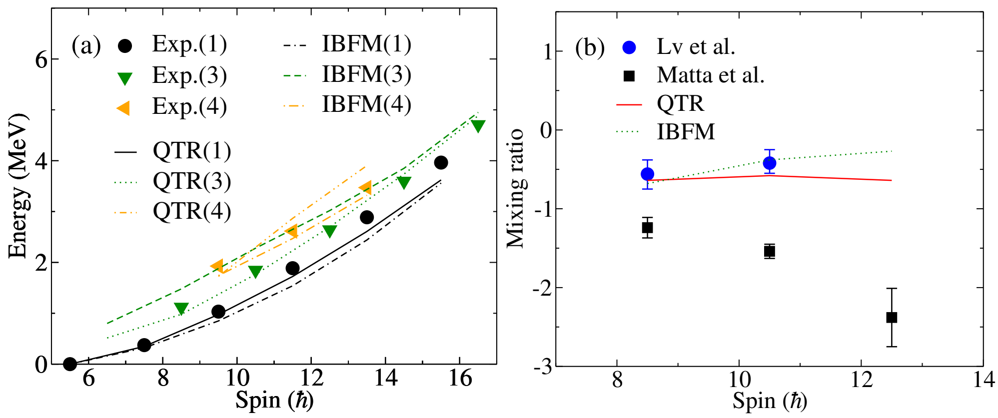

Figure 1. (**a**) Comparisons of theoretical QTR and IBFM model calculations with the experimental excitation energy spectra of bands 1, 3, 4 of ^{135}Pr. (**b**) Experimental mixing ratios compared with QTR and IBFM calculated values for the 747- and 813-keV transitions between bands 3 and 1 from Lv et al. [33]. Also the results from Matta et al. [10] for $|\delta| > 1$ are shown for comparison. The notations "1", "3", and "4" for the experimental bands corresponds those used in Ref. [33].

The nature of the low-lying bands in ^{135}Pr has been investigated using the quasiparticle-plus-triaxial-rotor (QTR) and the interacting boson-fermion (IBFM) models [33,34]. In contrast with the QTR calculations of refs. [10,11], the frozen approximation of the particle angular momentum have not been adopted in the present work, and the relative magnitude of the irrotational-flow moments of inertia have not been modified. The present QTR model predicts rotational bands that result from a mixture of collective and single-particle excitations. It resulted that in all bands the single-particle angular momentum rapidly realigns from the short to the intermediate axis, while the total angular momentum changes orientation toward the intermediate axis. The calculated bands have different relative components of the single-particle and collective excitations. This is in contrast with the previous QTR calculations [10,11,33], in which the bands were described as a precession of the total angular momentum around the short axis. In ref. [34], an alternative interpretation of the reported low-lying excited bands in ^{135}Pr was also discussed. The predicted mixing ratios of the ΔI = 1 transitions between those excited bands and lowest band previously interpreted as wobbling bands are significantly smaller than the experimental values, indicating a predominant magnetic character. The earlier wobbling interpretation is thus severely questioned.

Figure 1 provides the experimental excitation energy [$E(I)$] versus spin [I] of bands 1, 3, and 4, as well as the mixing ratios for the ΔI = 1 transitions linking the claimed wobbling bands 3 to 1 of ^{135}Pr in ref. [33]; the data are in excellent agreement with these two models without invoking the wobbling mode. Figure 2 gives the comparisons of the calculated $B(E2; I \rightarrow I-1)_{out}/B(E2; I \rightarrow I-2)_{in}$ and $B(M1; I \rightarrow I-1)_{out}/B(E2; I \rightarrow I-2)_{in}$ ratios with the new experimental results reported in ref. [33], showing a good agreement, but being significantly different from the results of ref. [10]. In the present QTR calculations, it is found that the total angular momentum of the bands rapidly realign from the short to the intermediate axis instead of maintaining a fixed orthogonal geometry of the particle and core angular momenta as described in previous work [9,10]. Based on these new experimental results and theoretical analysis, we conclude that the nucleus ^{135}Pr is not a low-spin wobbler. It is also worth mentioning that the results of IBFM calculations also strongly challenged the previous interpretation of low-lying bands in ^{133}La, ^{127}Xe, and ^{105}Pd as wobbling bands.

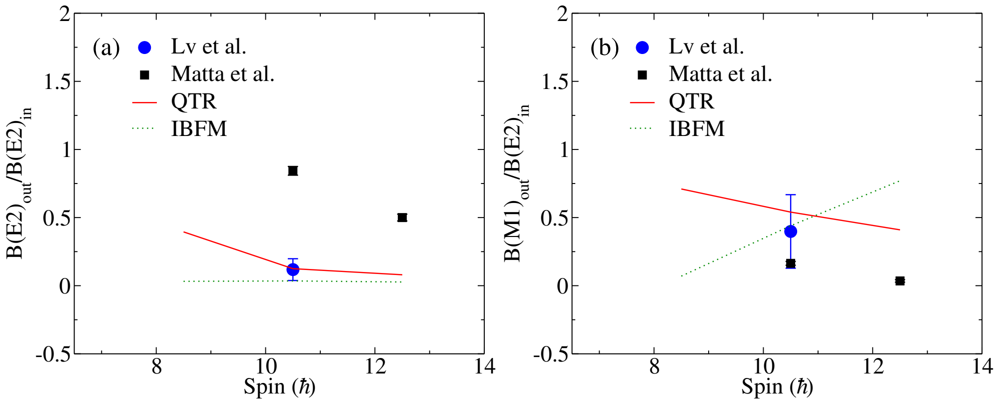

Figure 2. Ratios of transition probabilities (filled-blue circle) for (**a**) the 813-keV ($21/2^- \rightarrow 19/2^-$) and (**b**) 726-keV ($21/2^- \rightarrow 17/2^-$) transitions in comparison with QTR and IBFM calculations (solid and dashed lines) from Lv et al. [33], and with the experimental results (black squares) from Matta et al. [10].

In ref. [35], the triaxial particle rotor model has been used to investigate the transverse wobbling bands in odd-A nuclei. In particular, various parameter sets for the moments of inertia were employed for ^{105}Pd, all leading to a good agreement with the experimental data. Analyzing the azimuthal geometry of the angular momenta of the core and valence nucleon, distinct modes of rotational excitation were revealed. These modes are confirmed to be sensitive to the ratio J_m/J_s between the moment of inertia around the m and s axes. The TW occurs when the ratio J_m/J_s corresponds to the rigid-body MoIs, i.e., when the wobbling of the total angular momentum is around the s axis. The wobbling around the m axis occurs for both of the lowest and excited bands when the ratio is between the aforementioned two. The conclusion was drawn that tunneling and precession are two components of the quantum wobbling motion. In the excited bands of ^{105}Pd, the tunneling feature predominates and the transverse wobbling description is more complex.

Out of the mass regions of A $\approx$ 110, 130, and 160, two wobbling bands in ^{183}Au were identified in the A $\approx$ 190 mass region [14]. Both bands were interpreted as the TW bands with wobbling energies behaving differently as a function of spin, that of the positive parity band is increases with spin, whereas that of the negative parity band decreases with spin. Calculations using a particle rotor model with triaxial deformation successfully reproduce the experimental results. For the measured TW bands, a critical value of spin, I_c, was found: The wobbling energy decreases with spin above I_c and increases with spin below I_c. Thus, it is declared that ^{183}Au is the only nucleus with wobbling bands having both the increasing and the decreasing components of the wobbling energy, providing the experimental evidence of the complete transverse wobbling phenomenon. A pair of longitudinal wobbling-bands have been reported in ^{187}Au [15]. The longitudinal wobbling nature of the bands is supported by the $\Delta I = 1$ transitions between these bands with mainly $E2$ character and an increasing trend of the wobbling energy. It is found that the particle rotor model describes the experimental observations well. It proves that this exotic collective mode is a widespread phenomenon over the nuclear chart [15]. However, the extraction of mixing ratio values of the transitions between these bands only used the angular distribution method, which is known to give rise to more than one solution from the fit of experimental data with low statistics. In order to check if the low-spin bands in ^{187}Au have a wobbling nature, an experiment was performed at the Heavy Ion Research Facility in Lanzhou (HIRFL), China. Using combined measurements of linear polarization and of angular correlation ratio, the mixing ratios of the transitions between the claimed longitudinal wobbling bands were determined in the work [36]. It was found that the new measured mixing ratio values were significantly different from the values reported in ref. [15], and were in good agreement with the values previously obtained from internal conversion coefficient measurements. This shed serious doubts on the longitudinal wobbling interpretation of the low-lying bands in ^{187}Au. Moreover, after analyzing the experimental proofs reported in all low-spin wobbling bands of odd-A nuclei, it was pointed out that the previous, flawed study paradigm has lead to the erroneous identification of low-spin bands as wobbling bands.

3. Wobbling Motion in Even-Even Triaxial Nuclei

The interpretation in terms of wobbling motion of the bands built on the $\pi h^2_{11/2}$ configuration in ^{130}Ba were first investigated in ref. [21] using the constrained triaxial covariant density functional theory combined with the quantum particle rotor model calculations. The experimental level energies and the electromagnetic transition probabilities were well reproduced. It was shown that the band properties satisfy the character of transverse wobbling motion, demonstrating that two-quasiparticle wobbling bands can exist in even-even nuclei. The detailed analysis of the probability density distributions for the orientation of the angular momenta with respect to the body-fixed frame and of the total angular momentum geometry, further supports the wobbling nature of the bands. In addition, the analysis of the data shows that the transverse wobbling regime is much more stable than for the known cases with one odd quasiparticle [21].

Very recently, two medium-spin bands based on the $\pi h^2_{11/2}$ configuration (L1 and L3) in ^{136}Nd [23] were investigated using the triaxial projected shell model (PSM) in ref. [22], investigating their interpretation in terms of wobbling bands. By analyzing the angular momentum geometry, the components of the wave functions were extracted, the probability density distribution profiles for the tilted angles of the angular momentum vector (θ, ϕ) with respect to the principal axes were calculated, and the distribution of the angular momentum component on the short axis was obtained [22]. That detailed analysis revealed that the transverse wobbling mode is valid at the bandhead of the bands L1 and L3 in ^{136}Nd, but is eroded with increasing spin. Two possible reasons for the erosion of the TW mode were suggested. First, due to the large moment of.inertia along the i axis, the orientation of the rotational angular momentum changes from the s axis to the i axis. This leads to the decrease of the wobbling energy with increasing spin, which is a fingerprint of the transverse wobbling. The second reason is the alignment of the quasiparticle angular momentum which is along the i axis, instead of the s axis. This effect is present in the PSM results, and is induced by configuration mixing. In addition, the effect of the quasiparticles alignment on the transverse wobbling geometry was also explored, showing that the zero-phonon band is more affected than the one-phonon band, which works against the decreasing trend of the wobbling energy expected in the TW mode. The effect of the rotational alignment of the quasiparticles on the transverse wobbling scenario suggested the collapse of transverse wobbling with increasing spin, which is more complex than the frozen alignment approximation adopted in ref. [9].

We recently studied the nature of bands L1 and L3 in ^{136}Nd experimentally. High spin states in ^{136}Nd were populated using the ^{100}Mo (^{40}Ar, 4n) reaction. We deduced the mixing ratio of the $\Delta I = 1$, 751-keV transition connecting the wobbling bands by combining the linear polarization and angular correlation analyzes. The obtained mixing ratio was $|\delta| < 1$, corresponding to a 19% $E2$ component, similar to the (≈25%) $E2$ component of the transitions between the wobbling bands in ^{130}Ba. The value was then used to experimentally extract the transition probabilities ratios $B(E2)_{out}/B(E2)_{in}$. It is crucial to note that for two-quasiparticle wobbling bands the connecting $\Delta I = 1$ transitions can be predominantly magnetic because the gyromagnetic factors of two nucleons contributes to $B(M1)$, whereas in the one-quasiparticle wobbling bands the gyromagnetic factor of only one nucleon contributes.

In order to further investigate the nature of the two-quasiparticle bands L1 and L3 in ^{136}Nd, Budaca et al. [37] performed calculations with a new developed particle-rotor model which rigidly couples in an orthogonal geometry the total angular momentum of two quasiparticles to that of a triaxial core. This new model was also successfully applied to the reported two-quasiproton wobbling bands of ^{130}Ba, ^{134}Ce, and ^{138}Nd, and to the two-quasineutron hole band of ^{138}Nd,.

Figure 3 shows the comparison between the calculated energy spectra and wobbling energy of the proposed wobbling bands in ^{136}Nd using the PRM [37] and PSM [22] with the experimental data. Both models reproduce the experimental energy spectra well. For the wobbling energy, the PRM exhibits a global good agreement with all experimental data points, while the PSM only reproduces the low spin region, overestimating the high-spin part band. More important is that the $B(E2)_{out}/B(E2)_{in}$ and $B(M1)_{out}/B(E2)_{in}$ ratios of the $\Delta I = 1$ transitions between bands L3 and L1 of ^{136}Nd calculated using both the PRM and PSM shown in Figure 4, are in good agreement with the experimental data. It is also worthwhile to mention that the possible wobbling motion based on two-quasiparticle configurations in ^{130}Ba, ^{134}Ce, and ^{138}Nd have also been investigated using a triaxial rotor with a rigidly aligned pair of quasiparticles in ref. [37]. A good agreement between the calculated wobbling bands and the experimental data has been obtained for all studied nuclei. However, the confirmation of these predictions for ^{134}Ce and ^{138}Nd are yet to be provided by crucial measurements of the mixing ratios of the connecting transitions between the claimed wobbling bands.

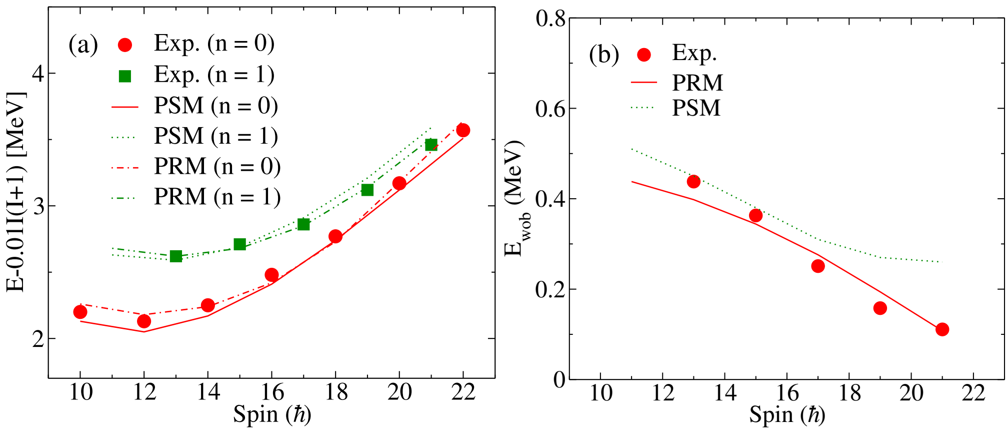

Figure 3. (**a**) Energies minus a rigid-rotor reference of bands L1 and L3 in ^{136}Nd obtained from PSM and PRM calculations compared with the experimental data [23]. (**b**) Wobbling energies of bands L1 (n = 0) and L3 (n = 1) obtained from PSM and PRM calculations compared with the experimental results.

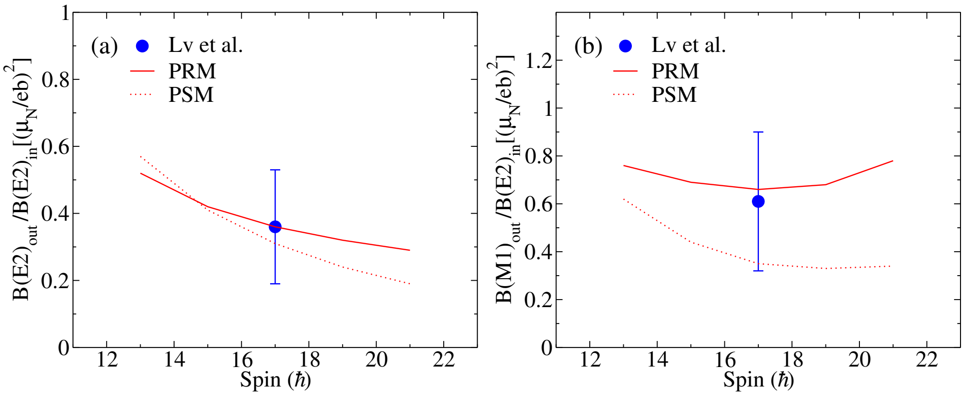

Figure 4. (**a**) $B(E2)_{out}/B(E2)_{in}$ and (**b**) $B(M1)_{out}/B(E2)_{in}$ ratios for the ΔI = 1 transitions between bands L3 and L1 of ^{136}Nd from PSM and PRM calculations (lines) compared with the experimental data (symbols) from Lv et al. [23]

4. Conclusions

This article presents the status of the experimental results and their theoretical interpretation on low-spin wobbling bands in odd-even nuclei, in particular in ^{135}Pr, and on medium-spin two-quasiparticle wobbling bands in even-even nuclei, in particular in ^{136}Nd. In summary, transverse wobbling and longitudinal wobbling modes have been theoretically suggested for low-spin bands in a series of odd-A nuclei. However, the validity of the reported experimental results supporting these predictions has been seriously questioned by recent results. New experiments were performed to re-examine the low-spin bands in ^{135}Pr and ^{187}Au, and results in contradiction with the wobbling interpretation were obtained. These new experimental results were in excellent agreement with several theoretical models which do not involve the wobbling motion, giving a strong support against the wobbling interpretation of the bands in ^{135}Pr and ^{187}Au, and shedding doubts on the existence of the low-spin wobbling motion. On the other hand, the mixing ratios of the ΔI = 1 transition connecting the proposed wobbling bands L1 and L3 in ^{136}Nd have been measured, and the ratios of reduced transition probabilities have been deduced. The results of the PSM and a newly developed PRM are in good agreement with experimental data, supporting the interpretation as transverse wobbling bands in even-even nuclei. This work sheds new light on the nature of low-lying bands in odd-A nuclei and puts the phenomenon of wobbling on a solid experimental basis, providing reliable new experimental data to test different theoretical interpretations.

Author Contributions: C.M.P. and B.L. contributed to this article. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Special Research Assistant Project of the Chinese Academy of Sciences, and by the French Ministry of Foreign Affairs and the Ministry of Higher Education and Research, France (PHC PROTEA Grant No. 42417SE).

Data Availability Statement: All data related to this work are available from the corresponding authors upon reasonable request.

Acknowledgments: We thank the JYFL Accelerator laboratory staff for excellent operation of the K130 cyclotron.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

TW	Transverse wobbling
LW	Longitudinal wobbling
E_{wob}	Wobbling energy

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