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Evidence of different fractal behavior in EPOS3 simulated p-p events at $\sqrt{s}$ = 0.9 TeV and 7 TeV: a study using visibility graph technique

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We have performed a study to search for the evidence of fractality in multiparticle production process at LHC energy regime using a non conventional tool visibility graph technique. P-p events at centre of mass energy 0.9 TeV and 7 TeV are generated using EPOS3 MC without hydro dynamical evolution option. Fractal properties have been studied by mapping the event-by-event pseudo rapidity distributions of produced charged particles into scale-free network in detail. The analysis reveals the presence of fractal nature in fluctuation pattern of charged particles produced in p-p events at two different energies and also manifests that the fractal behavior depends on centre of mass energy.

Keyword: Fractal, Visibility graph, Power of scale-freeness, EPOS3 monte carlo

1 Introduction

B B Mandelbrot[1] first used the term "fractal" from the word fractus whose meaning in Latin is broken or fragmented. Fractal patterns are infinitely complex patterns and they are self-similar across different scales. Studies on self-similarity in multiparticle production process have a long history. Using techniques based on the fractal theory several procedures have been developed to analyze the fractal structure in multiparticle production process. The most popular of them have been developed by Hwa (Gq moment)[2] & Takagi (Tq moment)[3]. After this many rigorous techniques like detrended fluctuation analysis (DFA) method[4] and multifractal-DFA (MF-DFA) method[5] have been proposed. The latest addition in this kind of analysis is power of scale-freeness of visibility graph – PSVG. The special feature of this method is that it can be applied to finite time series.

Visibility graph converts a time series into an equivalent network. Several properties of the time series are carried by this network and the study of the network reflects nontrivial information about the series. It has been reported that the periodic time series can be transformed into regular graphs and random series corresponding to random graphs[6] and fractal series into scale-free graphs[7,8].

The method of PSVG has already been adopted for fractal analysis in multiparticle production processes[9-12]. All the analyses use event averaged relevant distribution as input to visibility graph algorithm. However, it is well known that two types of fluctuation exist in multiparticle production, one is spatial fluctuation for each interaction (event) and this fluctuation pattern also fluctuates from event-to-event. In our earlier communication we have reported a study[13] on PSVG by capturing event-by-event fluctuation of spatial fluctuations in case of ^{16}O-AgBr interactions data at 60 A GeV from CERN SPS.

Motivated by our earlier work[13] we have applied the method of PSVG on EPOS3 simulated p-p events at two different centre of mass energies 0.9 TeV and 7 TeV to search for fractal behavior in multiparticle production process at LHC energy regime.

2 Visibility Graph

The visibility graph algorithm maps time series X to its visibility graph[14]. Let i^{th} point of the time series is X_i. Each data value is considered to be a node. Two nodes are connected if they can see each other and a straight visibility line exists between them. Figure 1 represents schematically a visibility graph for time series X.

Formally, two arbitrary data values (t_a, X_a) and (t_b, X_b) are visible to each other if any other point (t_c, X_c) between them satisfies the criterion

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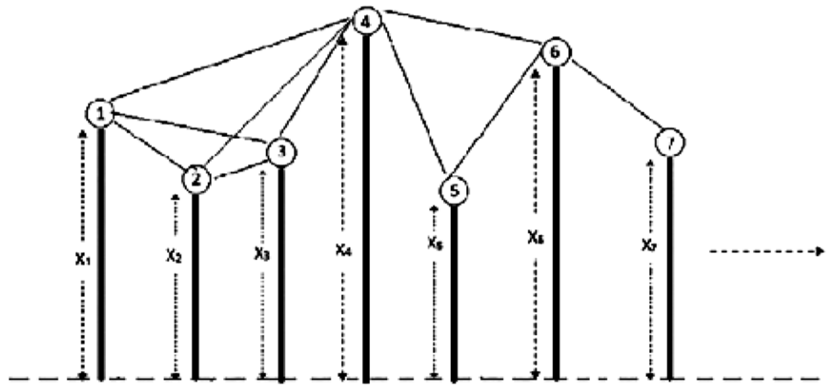

Fig. 1 – Schematic representation[13] of visibility graph for time series *X*.

$$X_c \leq X_b + (X_a - X_b)\frac{t_b - t_c}{t_b - t_a} \quad \ldots (1)$$

From the graph theory, we know that the degree of a node is the number of connections or edges that the node has with other nodes. The degree distribution, say $P(k)$, of a network formed from the time series, is defined as the fraction of nodes with degree k in the network. Thus, if there are n nodes in total in a network and n_k of them have degree k, then $P(k) = \frac{n_k}{n}$.

The scale-freeness property of visibility graph states that the degree distribution of its nodes satisfies a power law behavior as under:

$$P(k) \sim k^{-\lambda_p} \quad \ldots (2)$$

where, λ_p is a constant and it is known as power of the scale-freeness in visibility graph PSVG. It corresponds to the amount of complexity and fractal nature of the time series[14-16].

3 EPOS3 Monte Carlo Simulations

EPOS3[17] is a hybrid monte-carlo event generator used for high energy interactions. It has three main features: a flux-tube initial condition, *3+1D* viscous hydrodynamics and a hadronic afterburner modeled via UrQMD. It utilizes similar technique for particle production at the LHC energy in proton − proton, proton − nucleus and nucleus − nucleus collisions.

EPOS3 is a real event generator based on "Gribov-Regge theory"[18]. In EPOS3 each binary interaction is represented by a parton ladder. There are two types of parton ladders: open (for inelastic collisions) and closed (for elastic collisions). In this model beam remnants can possibly take part in the collision process enhancing the particle yield.

In this paper we have simulated 30 Lakh p-p events (minimum bias) at √s = 0.9 TeV and 7 TeV without hydro option. Events are chosen with a minimum of two charged particles (proton, pion & kaon) in the kinematic interval pT > 0.1 (GeV/c) and |η|<2.5 in order to include hard as well as soft particles produced in wide range of pseudo rapidity space.

4 Results and Discussion

To capture the event by event fluctuations we have mapped the single event η distribution into the corresponding visibility graph. We use the method of successive difference for the bin counts: $y(\eta_i) = dn(\eta_{i+1}) - dn(\eta_i)$, and as required by the VG technique, project the modified distribution onto a positive (y, η) plane (Fig. 2).

On an event-by-event basis, modified η-distributions are converted into the corresponding VGs. The quantity degree denoted by k represents the number of connections that one particular node possesses with the other nodes present in the VG of a single event. An overall degree distribution $P(k)$ for the entire event sample is obtained thereby after combining the single event distributions together. $P(k)$ Vs k graph for p-p interactions at 0.9 TeV & 7 TeV are shown, respectively in Fig. 3(a and b). It is evident from Fig. 3(a and b) that the power law relationship of Eq. (2) is satisfied for both the cases. This result provides evidence in favour of the fractal nature of fluctuation patterns in mutiparticle production in p-p interactions at 0.9 TeV & 7 TeV.

To extract the value of PSVG, λ_p we have performed non linear curve fit for both the plots of Fig. 3. The fit parameter λ_p and the goodness of the

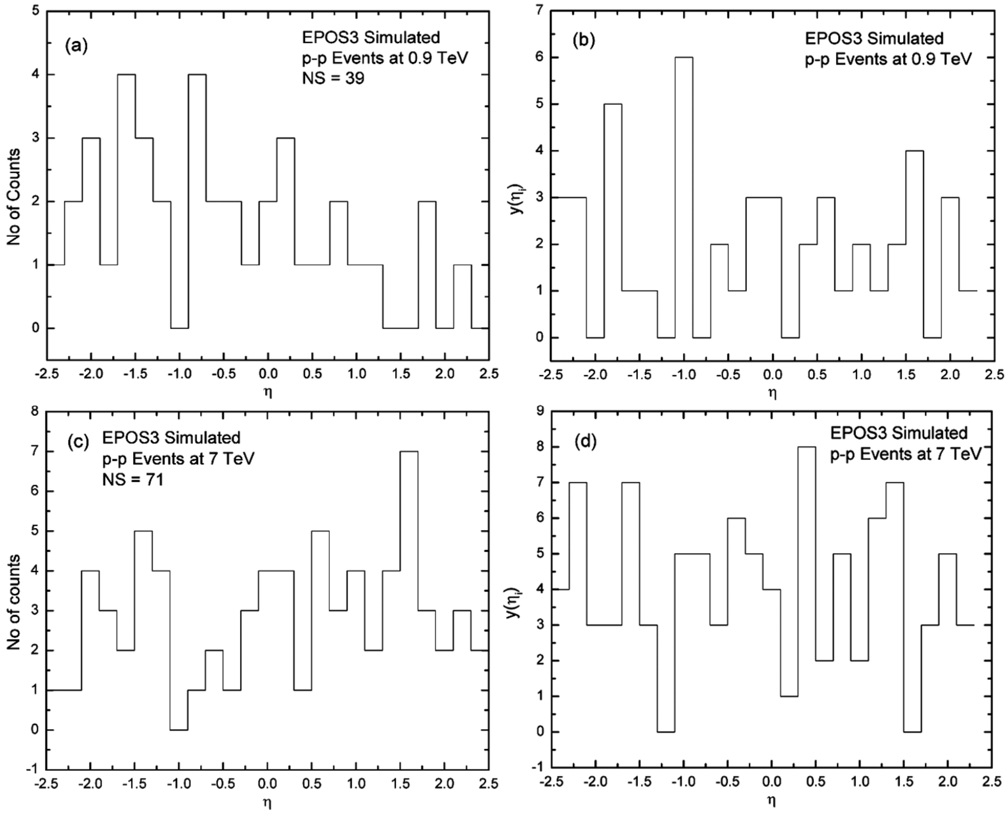

Fig. 2 – The single event (a) pseudo rapidity distribution, (b) modified pseudo rapidity distribution for p-p events at √s = 0.9 TeV, (c) pseudo rapidity distribution and (d) modified pseudo rapidity distribution for p-p events at √s = 7 TeV.

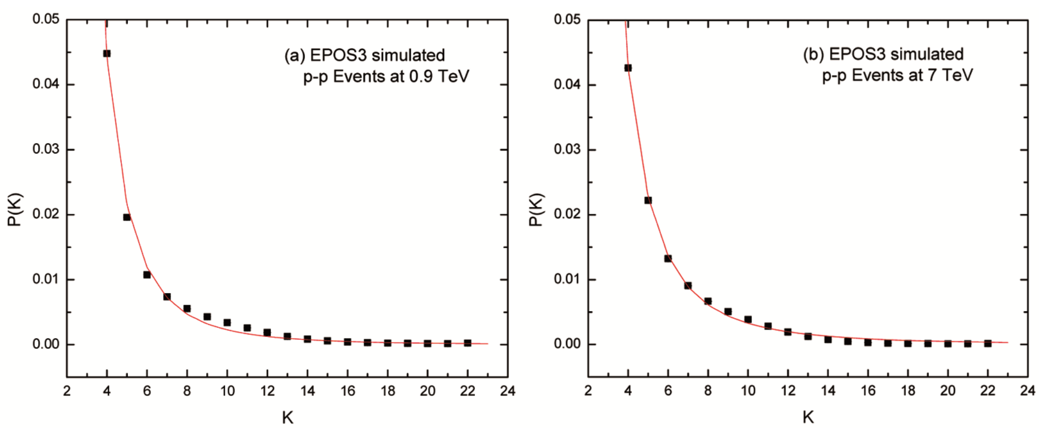

Fig. 3 – Degree distribution of all the VGs generated from the p-p events of (a) 0.9 TeV and (b) 7 TeV.

fit R^2 are tabulated in Table 1. As the power of scale-freeness of visibility graph (PSVG) is a measure of fractality, it can be seen from Table 1 that fractal behaviour decreases with the increase of interactions energy. Similar, studies need to be performed with available LHC data, so that a comparative scenario may be revealed.

Table 1 – Details of separation of events and linear fit parameters.

Events	√s	λ_p	R^2
p-p	0.9 TeV	2.79 ± 0.06	0.997
	7 TeV	3.16 ± 0.09	0.995

5 Conclusions

This study reveals that the event-by-event fluctuation of charged particles is fractal in nature and the power of scale-freeness of visibility graph (PSVG) increases with the increase of interactions energy. It shows the evidence of different fractal behaviour in multiparticle production process of p-p interactions at 0.9 TeV & 7 TeV.

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