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# The $\cos 2\phi$ azimuthal asymmetry in $e^+e^- \to \pi^+\pi^- X$ process

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ABSTRACT

We study the transverse momentum dependent $\cos 2\phi$ azimuthal asymmetry in $e^+e^- \to \pi^+\pi^- X$ process. Particularly, we take into account the contribution of the acoplanarity to the azimuthal asymmetry due to the digluon radiation correction effect. As a comparison, we also consider the contribution from Collins effect within the transverse momentum dependent factorization. We perform the calculation at $\sqrt{s} = 10.58$, accessible at Belle and BaBar Collaborations; and we choose in the Gottfried-Jackson frame, which is more convenient in comparing theoretical calculations with current experimental measurement. In the calculation we apply a recent parametrization on the Collins function. We adopt the Drell-Yan parametrization for the mean value of $\cos 2\phi_1$. We find that in the region $q_T \ll Q$ region, the asymmetry is dominated by the Collins effect, while the acoplanarity effect dominates in the large $q_T$ region ($q_T/Q > 0.5$) and is negligible in the small $q_T$ region. In the intermediate region the two contributions are comparable.



## 1. Introduction

Understanding the origin of azimuthal asymmetries in (semi-)inclusive high energy process involving hadrons has become one of the main goals in QCD and hadronic physics. A notable example is the $\cos 2\phi$ asymmetry of the final-state dilepton in the Drell-Yan process, which has the general angular dependence:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\Big(1+\lambda\cos^2\theta+\mu\sin^2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\Big), \quad (1)$$

with $\lambda$, $\mu$, and $\nu$ are the coefficients describing the sizes of different angular dependence. Of particular interest, $\nu$ denotes the asymmetry of the $\cos 2\phi$ azimuthal angular distribution of the dilepton, with $\phi$ the angle between the lepton plane and the hadron plane. Early theoretical study on the angular distribution of the Drell-Yan process was carried out by Lam and Tung [1]. They proved a relation for $\lambda$ and $\nu$ up to the leading order of QCD: $1-\lambda-2\nu = 0$, the so-called Lam-Tung relation. The measurements on the pion-induced fixed target Drell-Yan by the NA10 [2] and the E165 [3] Collaborations decades ago showed a large value of $\nu$, near 30% in the region $Q_T \sim 3$ GeV, demonstrating a clear violation of the Lam-Tung relation. The recent collider mode measurement performed by the CMS collaboration in the large $q_T$ region also shows that the Lam-Tung relation is violated in the *pp* Drell-Yan process. It is known that gluon radiation processes may give rise to a non-zero $\cos 2\phi$ asymmetry, which in the case of $q\bar{q}$ annihilation dominance is given by $\nu = Q_T^2/(Q^2+3Q_T^2/2)$ [4]. However, the analysis in Refs. [2,5] showed that the magnitude and the $Q_T$ dependence of the asymmetry cannot be explained up to next-to-leading order perturbative QCD correction from gluon radiations. Boer [6] proposed that one special transverse momentum dependent (TMD) distribution, the Boer-Mulders function, can account for the $\cos 2\phi$ asymmetry in the low $q_T$ region, in the framework of TMD factorization. Recently, the violation of the Lam-Tung relation was interpreted in Refs. [7,8] as a consequence of the acoplanarity of the partonic subprocess. The acoplanarity mainly arises from the perturbative gluon radiation beyond $\mathcal{O}(\alpha_s)$ such that the axis of the annihilating quark-antiquark pair (natural axis) no longer necessarily resides on the colliding hadron plane. The acoplanarity are adopted to explore the angular dependence of the dilepton and the degree of violation of the Lam-Tung relation at both colliders and fix-target experiments [9]. Indeed, in Ref. [10] detailed perturbative-QCD calculations up to next-to-leading order (NLO) were performed, showing that QCD radiative effects at NLO (or beyond) can provide physical explanation on data from collider as well as fixed target experiments.

In this letter, inspired by the method in Refs. [7–9], we will study the effect of acoplanarity to the $\cos 2\phi$ azimuthal asymmetry

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in the process of unpolarized hadron pair production in $e^+e^-$ annihilation: $e^+e^- \to h_1 h_2 X$. Similar to the case of Drell-Yan process, the perturbative di-gluon radiation can also generate acoplanarity for this process, that is, the produced quark pairs do not reside on the plane of the final-state hadron pair. The perturbative contribution to the angular dependence of hadron pair in electron-positron annihilation has been studied in details in Ref. [11]. Up to LO QCD, a relation similar to the Lam-Tung holds: $1-\lambda-2\nu=0$, with $\lambda$ and $\nu$ the angular coefficients following Eq. (1).

Apart from the perturbative contribution, the Collins function [12] can also account for the $\cos 2\phi$ azimuthal asymmetry in $e^+e^- \to h_1 h_2 X$. It is a spin-dependent TMD fragmentation function which describes the number density of the unpolarized hadron in a transversely polarized quark. Experimentally, the first nonzero asymmetry contributed by the Collins function was measured by HERMES [13], and COMPASS [14] in single transversely polarized semi-inclusive deep inelastic scattering (For a review, see Ref. [15]). As the distribution of the produced hadron is asymmetric in the transverse plane with respect to the quark momentum, the convolution of two Collins functions for each hadron gives rise to the $\cos 2\phi$ asymmetry. In recent years the BELLE [16,17] and BABAR [18] Collaborations have measured the $\cos 2\phi$ asymmetry in $e^+e^- \to \pi\pi X$ process. In order to extract the Collins function from experimental data, the observable is defined [17,19,20] by applying the ratio of the unlike-sign pair (the two hadrons in the final state have the opposite electric charge) production over the like-sign pair (the two hadrons have the same electric charge) production. In this way the asymmetry from the radiation effect is suppressed in the ratio since it is supposed to be flavor-blind.

In this letter, we will consider both the acoplanarity caused gluon radiation effect and the Collins effect to study the $\cos 2\phi$ asymmetry in $e^+e^- \to \pi^+\pi^- X$ process. Although the radiation effect is usually neglected in the phenomenological analysis to ensure more clean extraction of the Collins function, the effect still deserves further study as it contains very useful information on QCD dynamics. Furthermore, it is also interested to compare the contributions from the gluon radiation effect and the Collins effect to reveal the kinematical region sensitive to each effect.

The remained contents of the paper are as follows. In Section 2, we derive the formula of the non-coplanarity effect caused by digluon radiation. In Section 3, we provide the formalism of the Collins effect which can also account for the asymmetry in $e^+e^- \to \pi^+\pi^- X$ process. In Section 4, according to the expressions obtained in the section 2 and section 3, we calculate the numerical result of the $\cos 2\phi$ azimuthal asymmetry in $e^+e^- \to \pi^+\pi^- X$ process. In the last Section, we summarized the work.

## 2. Gluons radiation correction in $e^+e^- \to \pi^+\pi^- X$ process

As we all know, the gluon bremsstrahlung will lead to the azimuthal asymmetry in $e^+e^- \to \pi^+\pi^- X$ process [11]. In this section, we first discuss the radiation correction contribution of a single gluon, that is, an extra gluon radiation at the final state of $e^+e^- \to q\bar{q}g$, and then consider the non-coplanarity between the quarks and the hadron ($\pi$ mesons) planes caused by digluon radiation, so as to modify the scattering cross-section formula. The radiation of a single gluon from a parton is depicted in the two Feynman diagrams in Fig. 1. Note that the discussion here is limited to two-jet events ($M^2 \ll q_T^2 \ll Q^2$, with $M$ the mass of the final state hadron, which is a typical hadronic scale), i.e., we will not consider hadrons from gluon fragmentation, which must be considered when $Q_T \sim Q$, as well as in three jet events [11].

The angle distribution of the lepton pair production Drell-Yan process in the Collins-Soper frame was first investigated in [4]. As shown in left panel of Fig. 2, the $z$ axis bisects the angle between $\boldsymbol{P}_2$ and -$\boldsymbol{P}_1$ in the Collins-Soper frame [21]. The asymmetry in

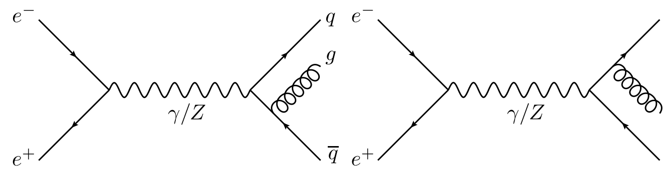


**Fig. 1.** Feynman diagram for correction of single gluon radiation.

$e^+e^- \to q\bar{q}g \to \pi^+\pi^- X$ process is similar to the case in Drell-Yan [4,7,8,11]:

$$\frac{dN}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}[1+\lambda\cos^2\theta + \mu K(\zeta_1,\zeta_2,q_T^2/\tilde{s})\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi], \tag{2}$$

where the function $K(\zeta_1,\zeta_2,Q_T^2/\tilde{s})$ is given by Eq. (168) in Ref. [11]

The coefficients $\lambda$, $\mu$, $\nu$ in formula are the three structure constants in the Collins-Soper frame, and their specific forms are as follows:

$$\lambda_{CS} = \frac{Q^2 - \frac{1}{2}q_T^2}{Q^2 + \frac{3}{2}q_T^2}, \tag{3}$$

$$\mu_{CS} = \frac{q_T Q}{Q^2 + \frac{3}{2}q_T^2}, \tag{4}$$

$$\nu_{CS} = \frac{q_T^2}{Q^2 + \frac{3}{2}q_T^2}. \tag{5}$$

Thus, it is easy to verify that the Lam-Tung relation [1,22] will also be satisfied in $e^+e^- \to q\bar{q}g \to \pi^+\pi^- X$:

$$1-\lambda-2\nu=0, \tag{6}$$

When this relation is satisfied, accompanied by the radiation of a single gluon, i.e., in the $\alpha_s$ order, $\pi$ meson plane is coplanar with quark plane. This is the case of azimuthal asymmetry of single gluon in $e^+e^- \to q\bar{q}g \to \pi^+\pi^- X$.

In the following, we will calculate the double-gluon radiation correction, which will lead to acoplanarity between the quarks and the $\pi$ mesons. The acoplanarity effect in the Drell-Yan process has been studied in Refs. [7,8]. As shown in the right panel of Fig. 2, there is an angle $\phi_1$ between quark plane and hadron plane. This conclusion will be used to calculate the azimuthal asymmetry in $e^+e^- \to q\bar{q}gg \to \pi^+\pi^- X$.

Since quarks are non-coplanar to $\pi$ mesons, structure constants will have some differences with Eq. (2). They are connected by the following formulas, and are denoted by $\mu'$ and $\nu'$

$$\mu' = \mu\cos\phi_1, \tag{7}$$

$$\nu' = \nu\cos 2\phi_1, \tag{8}$$

and $\lambda$ remains unchanged.

It is clear that the acoplanarity between quarks and $\pi$ mesons will modify the asymmetry, for which we would like to also consider in the process $e^+e^- \to \pi^+\pi^- X$. According to Ref. [11], we can find the asymmetry of this process in the Collins-Soper frame:

$$\frac{dN}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}[1+\lambda\cos^2\theta + \mu' K(\zeta_1,\zeta_2,q_T^2/\tilde{s})\sin 2\theta\cos\phi + \frac{\nu'}{2}\sin 2\theta\cos 2\phi]. \tag{9}$$

As the phenomenological analysis of the experimental measurement of $e^+e^- \to \pi^+\pi^- X$ process is usually performed in the

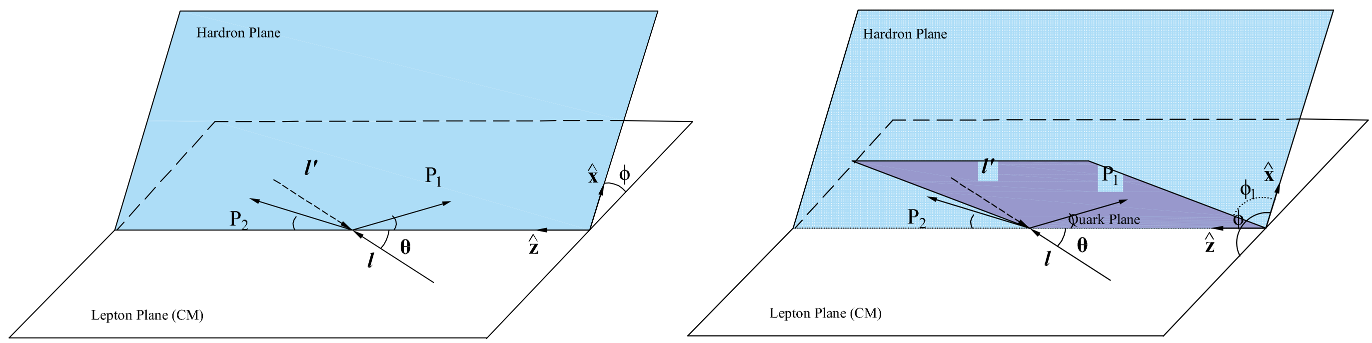


**Fig. 2.** Kinematics of the annihilation process in the lepton center of mass frame, the analogue of the Collins-Soper frame. Here $\phi$ is the angle between the hadron plane and the lepton plane, and $\phi_1$ denotes the angle between the hadron plane and the quark plane.

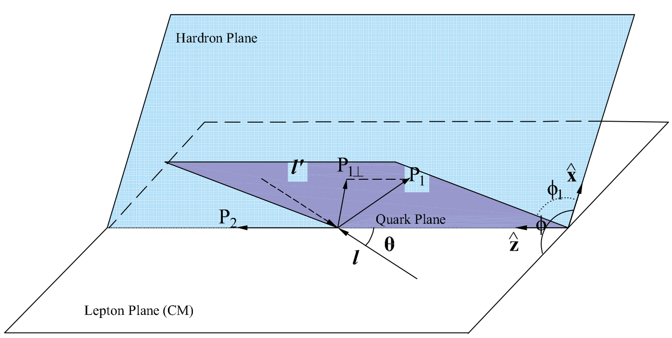


**Fig. 3.** Kinematics of the annihilation process in the lepton center of mass frame, the analogue of the Gottfried-Jackson frame.

frame equivalent to the Gottfried-Jackson (GJ) frame [23], we need to transform the CS frame result Eq. (9) to that in the GJ frame. The GJ frame is shown in Fig. 3, in which $\boldsymbol{P}_2$ is along the $z$ axis.

Fortunately, the structure constants of the two frames can be connected by a matrix transformation [11]:

$$\begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix}_{GJ} = \frac{1}{\Delta_{CS}} \begin{pmatrix} 1-\frac{1}{2}\rho^2 & -3\rho & \frac{3}{4}\rho^2 \\ \rho & 1-\rho^2 & -\frac{1}{2}\rho \\ \rho^2 & 2\rho & 1+\frac{1}{2}\rho^2 \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix}_{CS}, \tag{10}$$

where $\rho = q_T/Q$ and

$$\Delta = 1+\rho^2+\frac{1}{2}\rho^2\lambda+\rho\mu-\frac{1}{4}\rho^2\nu. \tag{11}$$

After some algebra, we find:

$$\begin{aligned} \frac{dN}{d\Omega} &= \frac{3}{4\pi}\frac{1}{\lambda_{GJ}+3}[1+\lambda_{GJ}\cos^2\theta_{GJ} \\ &+ \mu'_{GJ} K(\zeta_1,\zeta_2,q_T^2/\tilde{s})\sin 2\theta_{GJ}\cos\phi_{GJ} \\ &+ \frac{\nu'_{GJ}}{2}\sin^2\theta_{GJ}\cos 2\phi_{GJ}], \end{aligned} \tag{12}$$

where

$$\lambda_{GJ} = \frac{Q^4-4Q^2q_T^2+q_T^4}{Q^4+4Q^2q_T^2+q_T^4}, \tag{13}$$

$$\mu'_{GJ} = \frac{2Qq_T\left(Q^2-q_T^2\right)}{Q^4+4Q^2q_T^2+q_T^4}\cos\phi_1, \tag{14}$$

$$\nu'_{GJ} = \frac{4Q^2q_T^2}{Q^4+4Q^2q_T^2+q_T^4}\cos 2\phi_1. \tag{15}$$

The coefficients $\lambda_{GJ}$, $\mu'_{GJ}$, $\nu'_{GJ}$ represent the structure constants corresponding to the double gluon radiation in GJ frame, respectively. Obviously, because of the non-coplanarity between quark plane and the $\pi$ pair plane, $\mu'_{GJ}$ and $\nu'_{GJ}$ will violate the Lam-Tung relation.

## 3. Collins effect in $e^+e^- \to \pi^+\pi^- X$ process

The transverse polarization effect in fragmentation was first discussed by Collins [12], who proposed the chiral-odd polarization fragmentation function $H_1^\perp$, the Collins function. It describes the number density of an unpolarized hadron fragmented from a transversely polarized quark [24]:

$$\begin{aligned} &D_{\pi/q^\uparrow}(z,\boldsymbol{P}_\perp) - D_{\pi/q^\uparrow}(z,-\boldsymbol{P}_\perp) \\ &= \Delta^N D_{\pi/q^\uparrow}(z,-\boldsymbol{P}_\perp^2)\frac{(\hat{\boldsymbol{k}}\times\boldsymbol{P}_\perp)\cdot\boldsymbol{S}_q}{zM_\pi}, \end{aligned} \tag{16}$$

where $\boldsymbol{P}_\perp$ is the transverse momentum of the hadron, $\hat{\boldsymbol{k}}$ and $\boldsymbol{S}_q$ are the quark transverse momentum and transverse spin vector respectively, and $z, M_h$ represent the light cone momentum fraction and the mass of the hadron respectively. According to the Trento convention [25], $H_1^{\perp q}(z,p_\perp)$ and $\Delta D_{\pi/q^\uparrow}(z,p_\perp)$ can be connected by

$$\Delta D_{\pi/q^\uparrow}(z,p_\perp) = (2p_\perp/zM_h)H_1^{\perp q}(z,p_\perp) \tag{17}$$

$p_\perp = |\boldsymbol{P}_\perp|$.

By taking into account the Collins effect, one can write the differential cross-section of $e^+e^- \to \pi^+\pi^- X$ process: [26]

$$\begin{aligned} &\frac{d\sigma(e^+e^-\to\pi^+\pi^-X)}{dz_1dz_2d\Omega d^2\boldsymbol{q}_T} = \frac{3\alpha^2}{Q^2}z_1^2z_2^2\{A(y)\mathcal{F}[D\overline{D}] \\ &+ B(y)\mathcal{F}[(2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T-\boldsymbol{k}_T\cdot\boldsymbol{p}_T)\frac{H_1^\perp\overline{H}_1^\perp}{M_\pi^2}]\}, \end{aligned} \tag{18}$$

where $\boldsymbol{k}_T$ and $\boldsymbol{p}_T$ are the transverse momentum of $\pi^+$ and $\pi^-$ respectively, and the convolution is defined as follows [27]:

$$\begin{aligned} \mathcal{F}[\omega X\overline{X}] \equiv &\sum_{q,\bar{q}} e_q^2 \int d^2\boldsymbol{k}_T d^2\boldsymbol{p}_T \delta^2(\boldsymbol{p}_T+\boldsymbol{k}_T-\boldsymbol{q}_T) \\ &\times \omega(\boldsymbol{k}_T,\boldsymbol{p}_T)X(z_1,z_1^2\boldsymbol{k}_T^2)\overline{X}(z_2,z_2^2\boldsymbol{p}_T^2). \end{aligned} \tag{19}$$

One can see that the convolution of two Collins function gives rise to a $\cos 2\phi$ asymmetry, which is the main origin of the azimuthal asymmetry in the small $q_T$ region. In Eq. (18) we have not included the digluon radiation correction effect discussed in the previous section.

In literature, the Collins fragmentation function $\Delta^N D_{\pi/q^\uparrow}(z,p_\perp)$ is usually parameterized in the following form [19,20]:

$$\Delta^N D_{\pi/q^\uparrow}(z, p_\perp; Q^2) = \tilde{\Delta}^N D_{\pi/q^\uparrow}(z, Q^2) h(p_\perp) \frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi \langle p_\perp^2\rangle}, \quad (20)$$

where $\tilde{\Delta}^N D_{\pi/q^\uparrow}(z, Q^2)$ is the z-dependent part of the Collins function for $\pi$ meson, which can be written, in order to easily implement the proper positivity bounds, as

$$\tilde{\Delta}^N D_{\pi/q^\uparrow}(z, Q_0^2) = 2\mathcal{N}_q^C(z, Q_0^2) D_{\pi/q}(z, Q_0^2), \quad (21)$$

where $Q_0^2$ is the initial scale. The function $h(p_\perp)$ is defined as [20]:

$$h(p_\perp) = \sqrt{2e}\frac{p_\perp}{M_C} e^{-p_\perp^2/M_C^2}. \quad (22)$$

This allows a possible modification of the $p_\perp$ Gaussian width of the Collins function with respect to the unpolarized fragmentation functions.

For the parameters $\mathcal{N}_q^C$ in Eq. (21), according to the definition in literature [19,20], the fragmentation process of favored and disfavored can be distinguished, and the favored and disfavored functions are parameterized as [28]:

$$\mathcal{N}_{fav}^C(z) = N_{fav}^C z^\gamma (1-z)^\delta \frac{(\gamma+\delta)^{\gamma+\delta}}{\gamma^\gamma \delta^\delta}, \quad (23)$$

$$\mathcal{N}_{dis}^C(z) = N_{dis}^C, \quad (24)$$

respectively. The values of $N_{fav}^C$, $N_{dis}^C$, $\gamma$, $\delta$ and $M_C^2$ are extracted [28] from fitting the asymmetry data measured by the BELLE and BABAR Collaborations [17,19,26,27]

So far, we have obtained the contribution of the Collins effect to azimuthal asymmetry in $e^+e^- \to \pi^+\pi^- X$ in the GJ frame. A natural question that follows is how to combine the contribution of double gluon radiation correction with the contributions of double gluon radiation corrections. We note that a similar situation in semi-inclusive DIS recently has been solved, and the solution is that the two contributions can be directly added [29], which gives the total formula for the asymmetry that is correct up to power suppressed corrections (of order $q_T^2/Q^2$ in the low $q_T$ region and of order $M^2/q_T^2$ in the high $Q_T$ region), since the denominators of two asymmetry expressions coincide in $M^2 \ll q_T^2 \ll Q^2$. In the next section, we will also adopt this approach.

## 4. Numerical results of $\cos 2\phi$ azimuthal asymmetry in $e^+e^- \to \pi^+\pi^- X$ process

In this section, we will use the formula derived in the previous two sections to calculate the transverse momentum dependence $\cos 2\phi$ azimuthal asymmetry in $e^+e^- \to \pi^+\pi^- X$ process. Before calculating the azimuth asymmetry, we first consider the differential cross section of the transverse momentum component in $e^+e^- \to \pi^+\pi^- X$ process, which can be expressed in the following general form [30]:

$$\begin{aligned} \frac{dN}{d\Omega} &\equiv \left(\frac{d\sigma}{dz_1 dz_2 d^2\boldsymbol{q}_T}\right)^{-1} \frac{d\sigma}{dz_1 dz_2 d\Omega d^2\boldsymbol{q}_T} \\ &= F_1(1+\cos^2\theta) + F_2(1-3\cos^2\theta) + F_3\cos\theta \\ &+ F_4 \sin 2\theta \cos\phi + F_5 \sin^2\theta \cos 2\phi + F_6 \sin\theta\cos\phi \\ &+ F_7 \sin 2\theta \sin\phi + F_8 \sin^2\theta \sin 2\phi + F_9 \sin\theta \sin\phi, \end{aligned} \quad (25)$$

where the function $F_i$ is dependent on the momentum fraction $z_h = 2P_h \cdot q/Q^2$ and on $\boldsymbol{q}_T^2 \equiv Q^2$, $P_h$ denotes the momentum of the outgoing hadrons (here are $\pi^+$ and $\pi^-$), and $\boldsymbol{q}_T$ denotes the relative transverse momentum between $\pi^+$ and $\pi^-$. The polar and azimuth angles $\theta, \phi$ in Eq. (25) are given in lepton centrum system, with different definitions in different CM systems.

The coefficients $F_3, F_6$ and $F_9$ in Eq. (25), which are related to the parity-violating parameter $a$ [7,8], should vanish, as analyzed in Ref. [7]. Furthermore, from symmetry consideration, the values of the three coefficients $F_7, F_8$ and $F_9$, which are all odd functions of $\phi_1$, should approach zero [7]. From the analysis, Eq. (25) can be simplified as:

$$\begin{aligned} \frac{dN}{d\Omega} &\equiv \left(\frac{d\sigma}{dz_1 dz_2 d^2\boldsymbol{q}_T}\right)^{-1} \frac{d\sigma}{dz_1 dz_2 d\Omega d^2\boldsymbol{q}_T} \\ &= F_1(1+\cos^2\theta) + F_2(1-3\cos^2\theta) \\ &+ F_4 \sin 2\theta\cos\phi + F_5\sin^2\theta\cos 2\phi. \end{aligned} \quad (26)$$

In order to obtain the expression of the $\cos 2\phi$ azimuthal asymmetry in $e^+e^- \to \pi^+\pi^- X$ process, we first turn to the contribution of the radiation correction to the differential cross section shown in Eq. (12). In this case, the coefficients $F_1, F_2, F_4$ and $F_5$ in Eq. (26) satisfy the following equation:

$$F_1^{\mathrm{RC}} = \frac{3}{16\pi}, \quad (27)$$

$$F_2^{\mathrm{RC}} = \frac{3}{16\pi}\frac{1-\lambda_{GJ}}{\lambda_{GJ}+3}, \quad (28)$$

$$F_4^{\mathrm{RC}} = \mu_{GJ}, \quad (29)$$

$$F_5^{\mathrm{RC}} = \frac{\nu_{GJ}}{2}, \quad (30)$$

where the superscript "RC" denotes that the contribution is from the radiation correction. Thus, the coefficient of $\cos 2\phi$ in the last term in Eq. (12) is the $\cos 2\phi$ azimuthal asymmetry contributed by the radiation correction:

$$A_{\mathrm{RC}}^{\cos 2\phi} = \sin^2\theta \frac{3}{4\pi}\frac{Q^2 q_T^2}{2Q^4 + 4Q^2 q_T^2 + 2q_T^4}\cos 2\phi_1, l \quad (31)$$

where $\theta$, $\phi_1$ is defined as in Fig. 3.

Secondly, if only the Collins effect is taken into account, the angular differential cross section has the form [11]

$$\begin{aligned} \frac{dN}{d\Omega} &\equiv \left(\frac{d\sigma}{dz_1 dz_2 d^2\boldsymbol{q}_T}\right)^{-1} \frac{d\sigma}{dz_1 dz_2 d\Omega d^2\boldsymbol{q}_T} \\ &= \frac{3}{16\pi}\{(1+\cos^2\theta)\,\mathcal{F}[D\overline{D}] + \sin^2\theta\ \cos 2\phi \\ &\mathcal{F}[(2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\ \hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T - \boldsymbol{k}_T\cdot\boldsymbol{p}_T)\frac{H_1^\perp \overline{H}_1^\perp}{M_\pi^2}]\}/\mathcal{F}[D\overline{D}], \end{aligned} \quad (32)$$

where the polar and azimuthal angle $\theta, \phi$ are defined as in Fig. (3). In this case, the coefficients $F_1$ and $F_5$ in Eq. (26) satisfy the following equation:

$$F_1^{\mathrm{Collins}} = \frac{3}{16\pi}\mathcal{F}[D\overline{D}],$$

$$F_5^{\mathrm{Collins}} = \frac{3}{16\pi}\mathcal{F}[(2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\ \hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T - \boldsymbol{k}_T\cdot\boldsymbol{p}_T)\frac{H_1^\perp \overline{H}_1^\perp}{M_\pi^2}]\}/\mathcal{F}[D\overline{D}], \quad (33)$$

where the superscript "Collins" denotes the result coming from the Collins effect. Also $F_2$ and $F_4$ are equal to zero at the tree level [11].

Then the coefficient of $\cos 2\phi$ in the second term in Eq. (32) is the $\cos 2\phi$ azimuthal asymmetry contributed by the Collins effect:

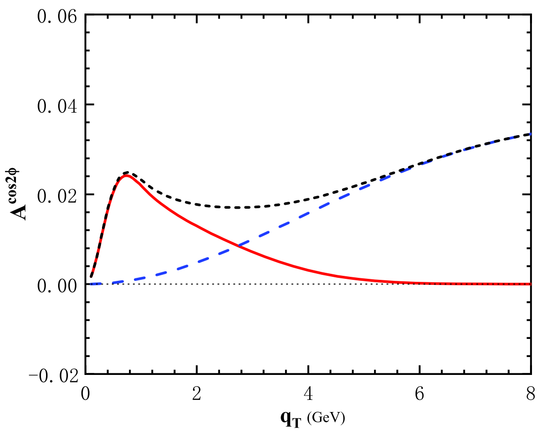


**Fig. 4.** The $\cos 2\phi$ azimuthal asymmetry vs $\boldsymbol{q}_T$ in process $e^+e^- \to \pi^+\pi^- X$ when $Q^2 = 112$ GeV. The dotted curve shows the asymmetry from the acoplanarity aroused by the di-gluon radiation. The solid curve shows the asymmetry from the Collins function. The dashed curve depicts the sum of two contributions.

$$A_{\text{Collins}}^{\cos 2\phi} = \sin^2\theta$$
$$\times \mathcal{F}[(2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\,\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T - \boldsymbol{k}_T\cdot\boldsymbol{p}_T)\frac{H_1^\perp \overline{H}_1^\perp}{M_\pi^2}]/\mathcal{F}[D\overline{D}]. \quad (34)$$

Here, the expressions of the fragmentation function $D$ and $H_1^\perp$ are shown in Eq. (17), Eq. (35) and Eq. (20), and we have applied the substitution: $p_{\perp 1} = -z_1\boldsymbol{k}_T$ and $p_{\perp 2} = -z_2\boldsymbol{p}_T$. It is worth noting that the Collins fragmentation function $H_1^\perp$, as analyzed in the section 2, is divided into favored $(u \to \pi^+, \bar{u} \to \pi^-, d \to \pi^-, \bar{d} \to \pi^+)$ and disfavored $(u \to \pi^-, \bar{u} \to \pi^+, d \to \pi^+, \bar{d} \to \pi^-)$ components [19,20].

Using the formula defined in Eq. (31) and Eq. (34), we calculate the $\cos 2\phi$ azimuthal asymmetry in $e^+e^- \to \pi^+\pi^- X$ process at $Q^2 = 112$ GeV that may be accessed at Belle and BaBar. For the Collins function, we apply the parametrization in Ref. [28], as shown in the previous section. For the unpolarized TMD fragmentation functions, we adopt a Gaussian form for the transverse momentum dependence [28]:

$$D_{\pi/q}(z, p_\perp) = D_{\pi/q}(z)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\,\langle p_\perp^2\rangle}, \quad (35)$$

where $D_{\pi/q}(z)$ is the unpolarized fragmentation function, for which we use the leading order set of de Florian, Sassot and Stratmann (DSS) FF [31]. The Gaussian width in the formula was chosen as [32]:

$$\left\langle p_\perp^2\right\rangle = 0.12\ \text{GeV}^2. \quad (36)$$

In Fig. 4, we plot the $\cos 2\phi$ azimuthal asymmetry vs $\boldsymbol{q}_T$ in the process $e^+e^- \to \pi^+\pi^- X$ at $Q = 10.58$ GeV, where the solid red line represents the contribution of the Collins effect, the blue dotted line represents the contribution of the radiation correction effect, and the black dotted line depicts the total contributions of two effects. We integrate the angle $\theta$ in Eq. (34) and (31) in the region $[0, \pi]$ and $z_1, z_2$ in the region [0.2, 0.9]. As the mean value of $\cos\phi_1$ is unknown, in this calculation we adopt the Drell-Yan parametrization 0.77 for $\langle\cos 2\phi_1\rangle$ [8]. The asymmetry is about several percent. We note that choosing a different value for $\langle\cos 2\phi_1\rangle$ may change the asymmetry quantitatively, however will not change the main result qualitatively. The results show that at small $Q_T$ region, the Collins effect dominates. The asymmetry from the Collins effect peaks at $q_T = 0.7$ GeV and then decreases with increasing $q_T$. In the large $q_T$ region the Collins effect almost vanishes. On the contrary, acoplanarity effect originated from the digluon radiation dominates in the large $q_T$ region ($q_T/Q > 0.5$) and is negligible in the small $q_T$ region. In the intermediate region the two contributions are comparable. This is similar to the case of the $\cos 2\phi$ asymmetry in the Drell-Yan process [8]. Our result indicates that it could be safe to directly extract the Collins fragmentation functions using the experimental data of $e^+e^- \to \pi^+\pi^- X$ in the small $q_T$ region ($q_T/Q < 0.1 \sim 0.2$) in the GJ frame, without considering the radiation correction effect. In addition, the data on the $\cos 2\phi$ asymmetry in the large $q_T$ region can be applied to study the radiation correction effect as well as extract the parameter describing the acoplanarity effect, i.e., the mean value of $\cos 2\phi_1$.

## 5. Summary

In this letter, we have studied the contributions of the acoplanarity due to the digluon radiation correction effect as well as the Collins effect to the azimuthal asymmetry in $e^+e^- \to \pi^+\pi^- X$ process. We have chosen to calculate the asymmetry in a frame equivalent to the Gottfried-Jackson frame, which is more convenient in comparing theoretical calculations with current experimental measurement. We have applied a recent parametrization on the Collins function, and we have also adopted the Drell-Yan parametrization for the mean value of $\cos 2\phi_1$. We have performed calculation at $\sqrt{s} = 10.58$ GeV, accessible at Belle and BaBar Collaborations. We find that in the region $q_T \ll Q$ region, the asymmetry is dominated by the Collins effect, which peaks at $q_T = 0.7$ GeV and then decreases with increasing $q_T$. On the contrary, the acoplanarity effect aroused from the digluon radiation dominates in the large $q_T$ region ($q_T/Q > 0.5$) and is negligible in the small $q_T$ region. In the intermediate region the two contributions are comparable. Our study may provide a better understanding on the azimuthal asymmetry in $e^+e^- \to h_1h_2X$ process.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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