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Topcolor: Top Quark Condensation in a Gauge Extension of the Standard Model

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Abstract

We describe a color embedding, $SU(3)_c \rightarrow SU(3)_1 \times SU(3)_2$ that is designed to break electroweak symmetries through top quark condensation. The minimal scheme of ref. [1] can be recovered by taking the scale of embedding large and fine-tuning the ratio of coupling constants. Models of this kind allow computation of the strength of higher dimension operators which are found to be small. Perhaps fine-tuning can ultimately be avoided in this scheme.

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The symmetry breaking of the standard model may be a dynamical mechanism involving conventional quark or lepton condensates [1,2,3]. In preliminary experiments with this idea one implements a BCS or Nambu–Jona-Lasinio (NJL) mechanism in which a new fundamental interaction associated with a high energy scale, M, involving principally the top quark, is postulated [1]:

$$\sim \frac{g^2}{M^2}(\bar{\psi}_L^a t_{Ra})_i(\bar{t}_{Rb}\psi^b)^i \tag{1}$$

where (a, b) are color indices and (i) is an $SU(2)_L$ index (to avoid inherent fine–tuning this can be generalized to a fourth generation scheme with Majorana neutrino interactions to elevate the fourth neutrino species, or placed in a supersymmetric context [3]). With sufficiently large g^2 the interaction of eq.(1) can trigger the formation of a low energy condensate, $\langle\bar{t}t\rangle$, with the requisite quantum numbers to break $SU(2) \times U(1) \to U(1)$ in the usual way. We are able to derive accurate predictions for m_{top} and for a composite Higgs boson mass, m_{Higgs}, in a fine–tuned version of this scheme. These follow from the renormalization group and "compositeness boundary conditions" [1, 4].

This approach has been discussed [5] and recently criticized [6] on the basis of the possible occurence of other large coefficient, higher dimension operators that might arbitrarily modify the compositeness conditions. One of the results of the present paper is to strengthen the claim that compositeness treated in the manner of ref.[1] is reasonable, and that the effects of higher dimension operators, calculable in the present class of models, are negligible. We will not propose a solution to the fine-tuning problem in the minimal scheme when $m_t \ll M$. However, we briefly consider applying this scheme with $M \sim f_\pi \sim 240$ GeV, at the end of the paper.

The interaction of eq.(1) is only an effective description of a more fundamental theory. Observe that a Fierz rearrangement of the interaction leads to:

$$(\bar{\psi}_L^a t_{Ra})_i (\bar{t}_{Rb}\psi^b)^i \rightarrow -(\bar{\psi}_{iL}\gamma_\mu \frac{\lambda^A}{2}\psi_L^i)(\bar{t}_R\gamma^\mu \frac{\lambda^A}{2} t_R) + O(1/N) \qquad (2)$$

where $N = 3$ is the number of colors. This suggests that the new theory may be a gauge theory leading to a current–current form of the effective Lagrangian. We further note that: (i) the gauge theory must be broken at a scale of order M; (ii) it is strongly coupled at the breaking scale; (iii) it involves the color degrees of freedom of the top quark (or fourth generation fermions) in a manner analogous to QCD. The relevant models will involve the embedding of QCD into some large group G which is sensitive to the flavor structure of the standard model [7].[1]

Let us construct a first version of such a theory. We presently seek a gauge interaction which leads to a term as in eq.(2) but which, like minimal technicolor, will leave the light quarks and leptons massless. A subsequent extension of the theory is required to give masses and mixing angles to light fermions, and we do not address this potentially problematic issue at present. Therefore, consider a minimal extension of the standard model such that at scales $\mu \gg M$, we have $U(1) \times SU(2)_L \times SU(3)_1 \times SU(3)_2$. We assign the usual light quark and lepton fields to representations under

[1] If $U(1)$ can have a nontrivial UV fixed point then a different class of models may be possible along the lines discussed by Giudice and Raby, [8], in which top carries an extra, strong $U(1)$ charge. This does not seem to be favored by large-N, since the Fierz rearrangement of a $U(1)$ current–current interaction will have a term as in eq.(1) suppressed by $1/N$.

$(SU(2)_L, SU(3)_1, SU(3)_2)$ as follows:

$$\begin{array}{rcl} (u,\ d)_L;\qquad (c,\ s)_L & \rightarrow & (2,3,1) \\ (\nu_e,\ e)_L;\qquad (\nu_\mu,\ \mu)_L;\qquad (\nu_\tau,\ \tau)_L & \rightarrow & (2,1,1) \\ u_R,\ d_R,\ c_R,\ s_R,\ b_R & \rightarrow & (1,3,1) \\ e_R,\ \mu_R,\ \tau_R,\ (\nu_{iR}) & \rightarrow & (1,1,1) \end{array} \tag{3}$$

while the top quark is assigned:

$$(t,\ b)_L \rightarrow\ (2,1,3);\qquad t_R \rightarrow\ (1,1,3) \tag{4}$$

This assignment is not anomaly free, but we can minimally realize all anomaly cancellations provided we introduce the following electroweak singlet quark:

$$Q_R \rightarrow (1,1,3);\qquad Q_L \rightarrow (1,3,1) \tag{5}$$

Both Q_R and Q_L have weak hypercharge $Y = -2/3$, hence electric charge $Q = -1/3$.

Since we wish to break the symmetry $SU(3)_1 \times SU(3)_2 \rightarrow SU(3)_c$ at the scale M, we introduce a scalar (Higgs) field $\Phi^a_{b'}$ which transforms as $(1,3,\bar{3})$. By a general choice of the Φ potential a VEV develops of the form: $\langle\Phi\rangle = \mathrm{diag}(M,\ M,\ M)$. This VEV breaks $SU(3)_1 \times SU(3)_2$ to the massless QCD gauge group $SU(3)_c$ with gluons, A^A_μ and a residual global $SU(3)'$ with degenerate, massive gauge bosons ("colorons") B^A_μ.[2] Q must be given a large enough Dirac mass so that it does not further influence

[2] It should be noted, however, that with the given the quantum numbers of $\bar{Q}Q$ there is an

the dynamical symmetry breaking. We invoke a Higgs-Yukawa coupling of the Φ field to the combination $\bar{Q}_L Q_R$. Thus, if we take:

$$\kappa \Phi_a^{b'} \bar{Q}_L^a Q_{Rb'} + h.c. \tag{6}$$

then Q receives a mass of κM. A not-too-restrictive lower bound on κ will be estimated below. Let the coupling constants of $SU(3)_1 \times SU(3)_2$ be respectively h_1 and h_2. Then the gluon (A_μ^A) and coloron (B_μ^A) fields are defined by:

$$\begin{aligned} A_{1\mu}^A &= \cos\theta A_\mu^A - \sin\theta B_\mu^A \\ A_{2\mu}^A &= \sin\theta A_\mu^A + \cos\theta B_\mu^A \end{aligned} \tag{7}$$

where:

$$h_1 \cos\theta = g_3; \qquad h_2 \sin\theta = g_3; \tag{8}$$

and thus:

$$\tan\theta = h_1/h_2; \qquad \frac{1}{g_3^2} = \frac{1}{h_1^2} + \frac{1}{h_2^2} \tag{9}$$

where g_3 is the QCD coupling constant at M. In what follows we envision $h_2 \gg h_1$ and thus $\cot\theta \gg 1$ to select the top quark direction for condensation. The mass of

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intriguing possibility that in extensions of this scheme the $\langle \bar{Q} Q \rangle$ condensate may form dynamically to break $SU(3)_1 \times SU(3)_2 \to SU(3)_c$, so that an explicit Φ field may not be required. For example, if we assign instead $(c,\ s)_L \to (2,1,3)$, we find that anomaly cancellation requires the Q be a triplet with $Y = 0$. Gauging this triplet with yet another, unbroken $SU(3)_3$ allows a QCD-like chiral condensate of the form $\langle \bar{Q} Q \rangle$ which is $(1,\bar{3},3)$, and the symmetry breaks as described here. This model leads to a low energy two-Higgs doublet scheme.

the degenerate octet of colorons is given by:

$$M_B = \left(\sqrt{h_1^2 + h_2^2}\right) M = \left(\frac{g_3}{\sin\theta\cos\theta}\right) M \tag{10}$$

The $SU(3)_c$ current will be the usual QCD current for all quarks while the $SU(3)'$ current (multiplied by its coupling strength) takes the form:

$$\begin{aligned} h_\mu^A &= g_3 \cot\theta \left(\bar{t}\gamma_\mu \frac{\lambda^A}{2} t + \bar{b}_L \gamma_\mu \frac{\lambda^A}{2} b_L + \bar{Q}_R \gamma_\mu \frac{\lambda^A}{2} Q_R\right) \\ &\quad + g_3 \tan\theta \left(\bar{b}_R \gamma_\mu \frac{\lambda^A}{2} b_R + \bar{Q}_L \gamma_\mu \frac{\lambda^A}{2} Q_L + \sum_i \bar{q}_i \gamma_\mu \frac{\lambda^A}{2} q_i\right) \end{aligned} \tag{11}$$

where the sum extends over all first and second generation quarks. If $h_2 \gg h_1$ the dominant coloron mediated interaction takes the form of eq.(1) provided we identify:

$$\frac{g^2}{M^2} \equiv \frac{g_3^2 \cot^2\theta}{M_B^2} \tag{12}$$

Let us now ask what condition on θ implies dynamical symmetry breaking through the formation of a top condensate. The scale at which the four fermion interaction softens to a gauge boson exchange is given by the mass of the coloron M_B, and we will treat the effective interaction as a four-fermion form at all scales $\mu \ll M_B$, and we will impose a cut-off Λ on loops in the gap equation of $\Lambda = M_B$ (the cut-off Λ can be rigorously identified with M_B upon comparison of the NJL gap loop with a full coloron exchange loop, or upon comparison of the NJL fermion bubble with the full box diagram, as shown below). This is an analytical approximation, in that we can in principle solve for the exact, large-N, $m_t(q^2)$. The results we estimate below for the finite corrections to the compositeness conditions depend upon this approximation.

but we believe that there are no further large corrections that arise if a full $m_t(q^2)$ expression is used there. Therefore, in this approximation the gap equation can be written for the spontaneous formation of the top-condensate [1]:

$$m_t = m_t \frac{g_3^2 N \cot^2\theta}{8\pi^2 M_B^2}\left[M_B^2 - m_t^2 \log(M_B^2/m_t^2)\right] \tag{13}$$

The existence of the condensate implies:

$$\frac{g_3^2 N \cot^2\theta}{8\pi^2} > 1 \qquad \text{or} \qquad \frac{N}{2\pi}\alpha_3(M_B)\cot^2\theta \geq 1 \tag{14}$$

where $\alpha_3 = g_3^2/4\pi$ is the QCD coupling.

On scales below the M_B we expect that the analysis of ref.[1] holds. If $M_B \gg M_W$ then to have an acceptable top mass we must fine-tune θ so that $\frac{N}{2\pi}\alpha_3(M_B)\cot^2\theta \approx 1$ to a high precision.

It is also crucial that the spectator Q be sufficiently heavy so that a $\bar{\psi}Q$ condensate *does not form* (the unbroken custodial $SU(2)_R$ leads otherwise to potential problems with extra unwanted Goldstone bosons, and a vacuum alignment problem which may ultimately break $U(1)_{EM}$). The gap equation for an induced $\mu\bar{\psi}_L Q_R$ term is, in the presence of $M_Q \bar{Q}_L Q_R$ for $\mu \ll M_Q$:

$$\mu = \mu \frac{g_3^2 N \cot^2\theta}{8\pi^2 M_B^2}\left[M_B^2 - M_Q^2 \log(M_B^2/M_Q^2)\right] \tag{15}$$

With fine-tuning, hence the approximate equality of the condition of eq.(14) in force such that eq.(13) has the nontrivial solution, eq.(15) will then have no solution other than $\mu = 0$ for $M_Q \gg m_t$ or $\kappa \gg m_t/(M_B \log(M_B/m_t))$. Note that κ need not be large to enforce the decoupling of Q.

Here we have neglected the small rotation of the condensate into the other flavor channels. Light fermion masses are not generated at this stage, since we have only spontaneously broken one chiral symmetry. The resulting low energy theory is effectively a strongly coupled (Higgs–Yukawa and quartic Higgs couplings) standard model near the scale M.

The model has a prediction for the top quark mass in terms of the electroweak scale [1] (as well as a prediction for the mass of the composite scalar Higgs boson). In the present case this prediction will be slightly modified relative to the NJL or RG results of ref.[1] and we wish to estimate the size of the correction. In the present letter we will only briefly describe the full analysis.

The top mass prediction is controlled by the induced wave-function normalization constant, Z_H, for the composite Higgs boson [1]. Since the Higgs boson is dynamically generated by the theory, Z_H is determined and in principle calculable. In the NJL model it follows from the fermion loop diagram of Fig.(1). The full loop takes the form, with a momentum cut-off of Λ:

$$Loop = \frac{i\beta N}{8\pi^2 M_B^4}\int_0^1 dx\ \left[\Lambda^2 + x(1-x)s(3\ln(\Lambda^2/\Delta^2) - 2)\right] \tag{16}$$

where:

$$\beta = g_3^4 \cot^4\theta; \qquad \Delta^2 = m_t^2 - x(1-x)s; \qquad s = (p_1+p_2)^2. \tag{17}$$

The first term (Λ^2) contributes to the mass of the Higgs boson, but actually cancels against the tree diagram when the gap equation is implemented [1]. $Z_{H\,NJL}$ follows from the s dependent term in the loop:

$$
\begin{aligned}
Z_{H\,NJL} &= \left(s\text{-dependent term}\right)/(i\beta/M_B^4) \\
&= \frac{N}{8\pi^2}\int_0^1 dx\, \left[x(1-x)(3\ln(\Lambda^2/\Delta^2)-2)\right]. \qquad (18)
\end{aligned}
$$

Now, in ref.[1] the definition of Z_H that is ultimately employed is given by the block spin renormalization group, which differs from the full NJL result above by a constant:

$$Z_{H\,RG} = \frac{N}{16\pi^2}\ln(\Lambda^2/\mu^2) \qquad (19)$$

Here μ is the renormalization group scale at which Z_H is evaluated, and we should identify $s = -\mu^2$ in comparing eq.(18) and eq.(19). From eq.(19) the compositeness condition, $Z_H \to 0$ as $\mu \to \Lambda$ is inferred. This, in turn, implies that the Higgs–Yukawa coupling constant for the top quark behaves as $g_t(\mu) \to \infty$ at the composite scale, which leads to the infrared RG fixed point prediction for m_{top} in the theory [1,4]. We see, however, that there are constant contributions that differ even between the NJL result for $Z_{H\,NJL}$ and the RG result $Z_{H\,RG}$:

$$Z_{H\,NJL} = Z_{H\,RG} + \left(\frac{1}{2}\right)\frac{N}{8\pi^2} \qquad (20)$$

where we use:

$$\frac{1}{2} = -\int_0^1 dx\; 3x(1-x)\ln(x(1-x)) - 1/3 \qquad (21)$$

This small constant correction is negligible when the logarithm becomes large, *i.e.*, when the theory is fine–tuned.

How large is the analogous constant correction in the topcolor model? The leading large-N contribution to the composite Higgs boson comes from the full planar box diagram of Figure (2). One might estimate the constant correction by computing the box to leading order in μ^2/M_B^2. The full box diagram is both s and t dependent. It can be projected onto an $l = 0$ partial wave, corresponding to the Higgs boson $\bar{t}t$ boundstate, by integrating over:

$$A_{l=0} = \frac{1}{\pi}\int_0^{\pi} d\theta A(\theta) = -\frac{1}{\pi}\int_0^{-s} \frac{dt}{\sqrt{-ts-t^2}} A(s,t) \tag{22}$$

This leads to the following expression for the full box diagram:

$$\begin{aligned} Box|_{l=0} &= \frac{i\beta N}{8\pi^2}\int_0^1 dx \left[\frac{1}{M_B^2} + O(m_t^2/M_B^4) + \right. \\ &+ \left. \frac{s}{M_B^4}\left\{x(1-x)(3\ln(M_B^2/\Delta^2) - 6 + 1/2) - 1/24\right\}\right] \end{aligned} \tag{23}$$

We perform the x integrals to obtain in the limit $s \gg m_t^2$:

$$Box|_{l=0} = \frac{i\beta N}{8\pi^2 M_B^4}\left[M_B^2 + s\left\{\frac{1}{2}\ln(M_B^2/\mu^2) - 1/8\right\}\right] \tag{24}$$

From this result we see that (1) Λ is rigorously identified with M_B (as opposed, say to $\frac{1}{2}M_B$) and (2) the full topcolor result for $Z_{H\,box}$ is:

$$Z_{H\,box} = \frac{iN}{16\pi^2}\left\{\ln(M_B^2/\mu^2) - 1/4\right\} \tag{25}$$

Therefore, we find a very small correction to the RG result:

$$Z_{H\,box} = Z_{H\,RG} - \left(\frac{1}{8}\right)\frac{N}{8\pi^2} \tag{26}$$

The effect of a constant correction to Z_H in the operator notation is associated with "higher dimension operators" as considered in refs.[5, 6]:

$$L = L_0 + \chi \frac{G}{M_B^2} (D^\mu \bar{\psi}_L^a D_\mu t_{Ra})_i (\bar{t}_{Rb} \psi^b)^i + \; h.c. \tag{27}$$

and we can ask what effect these have on the behavior of Z_H in large-N. One finds [5]:

$$Z_H = Z_{H\;RG} + \frac{N}{8\pi^2}\left(-\chi + \chi^2/8\right) \tag{28}$$

If we identify the constant correction obtained for the box diagram in the topcolor scheme with the operator result we obtain:

$$\chi \approx -\frac{1}{8} \tag{29}$$

(Of course, the box diagram sums the full infinite series of higher dimension operator corrections and should not strictly be associated with χ).

In the fine-tuned scenario (where the logarithm becomes large due to the long running over the desert) this has negligible effect on the predictions of BHL for m_t (or m_H), shifting the low energy predictions by $< 1.0\%$ as seen in Fig.(2). In fact, we are surprised at how small the result for χ is in this scheme, as we might have expected $\chi \sim 1$. We thus believe that the issue raised in ref. [6] is not relevant to realistic theories leading to an interaction as in eq.(2). The Higgs boson produced at low energies in this model is certainly dynamically generated, since the high energy theory is an asymptotically free gauge theory which contains no fundamental Higgs, and yet the analysis of [1] applies (though one may always object to the fine-tuning).

The higher dimension operator coefficients required in [6] to mutilate the predictions of the analysis of [1] are typically ~ 10, or two orders of magnitude greater than our result, and seem to us to be unnaturally large. However, we note that it is possible that the interaction of eq.(1) is generated directly at the high energy scale, *e.g.*, as in $SU(5)$ where the $\{5\}$ Higgs can be taken to have strong coupling to the $\{10\} \times \{10\}$ fermion combination, but a large mass $\sim 10^{15}$ GeV. In this case the compositeness conditions are not implementable, *i.e.*, $Z_H \rightarrow 1$, corresponding to $|\chi| \sim 8\pi^2/N \sim 10$ as we match onto the fundamental Higgs boson in the theory, and the objection of ref.[6] is then valid. Clearly in this case we never have the composite interpretation of the low energy Higgs boson.

The key issue being raised here is the viability of a theory that is strongly coupled to drive formation of $\left\langle \bar{Q}Q \right\rangle$, yet *does not confine because it is spontaneously broken.* We have limited experience with this mode of a gauge theory, though there is evidently nothing in principle wrong with the possibility as the large–N analysis indicates. In fact, this possibility is similar to that invoked in recent walking technicolor schemes [9]. This demands some reinforcement of intuition by lattice gauge theory analysis. While we believe we can treat the theory in a large–N approximation, we have $N = 3$ in practice and there may be large corrections. We have shown there is no evidence that there are large corrections that modify the analysis of the fine–tuned case [1].

Ideally we would like to take $M \lesssim TeV$, and still have an acceptable electroweak breaking through the top quark. The large–N analysis would suggest that top becomes too heavy in this case [1]. Perhaps the full strong dynamics saves us here, giving either a conventional ρ–parameter result and lower top mass, or an incalculable top mass with strong compensating corrections to ρ (the large–N analysis of [1] says this is not the case). As $\Lambda \to 240$ GeV the effective Lagrangian description breaks down, the Higgs disappears from the low energy spectrum (much like the hadronic σ state), and the top is approximately confined by the $SU(3)_2$ interaction. We expect that then the top mass is effectively a constituent quark mass, $m_t \sim NM$. In such a scheme the B colorons will be produced, and presumably also the Φ and/or Q states near the 300 GeV scale.

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References

1. W. A. Bardeen, C. T. Hill, M. Lindner *Phys. Rev.* **D41**, 1647 (1990).

2. Y. Nambu, "BCS Mechansim, Quasi-Supersymmetry, and Fermion Mass Matrix," Talk presented at the Kasimirz Conference, EFI 88-39 (July 1988); "Quasi-Supersymmetry, Bootstrap Symmetry Breaking, and Fermion Masses," EFI 88-62 (August 1988); a version of this work appears in "*1988 International Workshop on New Trends in Strong Coupling Gauge Theories*," Nagoya, Japan, ed. Bando, Muta and Yamawaki; "Bootstrap Symmetry Breaking in Electroweak Unification," EFI Preprint, 89–08 (1989). V. A. Miransky, M. Tanabashi, K. Yamawaki, *Mod. Phys. Lett.* **A4**, 1043 (1989); *Phys. Lett.* **221B** 177 (1989); W. J. Marciano, *Phys. Rev. Lett.* **62**, 2793 (1989).

3. C. T. Hill, M. Luty, E. A. Paschos, *Phys. Rev.* **D43** 3011 (1991);
T. E. Clark, S. Love, W. A. Bardeen, *Phys. Lett.* **B237**, 235 (1990); M. Carena, T. E. Clark, C. E. M. Wagner, W. A. Bardeen, K. Sasaki, "Dynamical Symmetry Breaking and the Top Quark Mass in the Minimal Supersymmetric Standard Model," FERMILAB-PUB-91/96-T; PURD-TH-91-01 (1991).

4. C. T. Hill, *Phys. Rev.* **D24**, 691 (1981);
C. T. Hill, C. N. Leung, S. Rao, *Nucl. Phys.* **B262**, 517 (1985).

5. M. Suzuki, *Mod. Phys. Lett.* **A5**, 1205, (1990); see also W. A. Bardeen, "Electroweak Symmetry Breaking: Top Quark Condensates," Talk presented at the 5th Nishinomiya Yukawa Memorial Symposium, Nishinomiya City, Japan, Oct. 25, (1990).

6. A. Hasenfratz, P. Hasenfratz, K. Jansen, J. Kuti, Y. Shen, "The Equivalence of the Top Quark Condensate and the Elementary Higgs Field," UCSD/PTH 91-06 (1991).

7. See, *e.g.*, P. Frampton and S. L. Glashow, *Phys. Lett.* **190B** 157 (1987).

8. G. Giudice and S. Raby, "A New Paradigm for the Revival of Technicolor Theories," OSU Preprint-DOE-ER-01545-550, UT Preprint-UTTG-02-91 (1991).

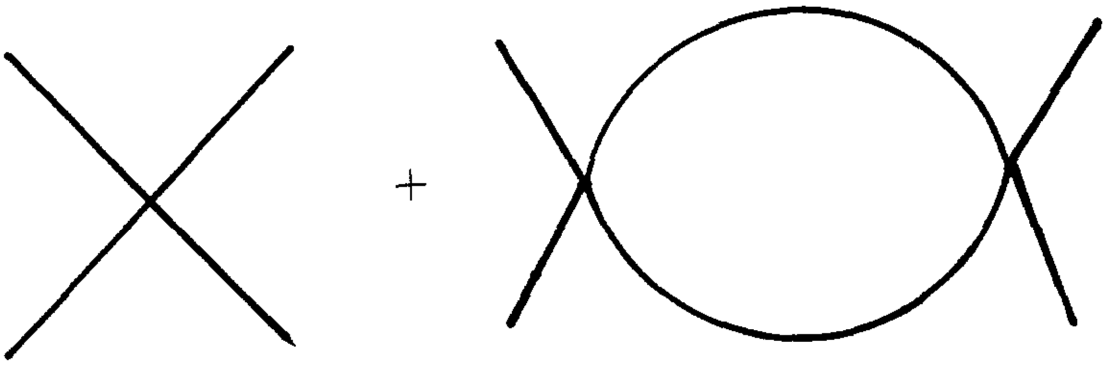
9. T. Appelquist, M. Einhorn, T. Takeuchi, L. C. R. Wijewardhana, *Phys. Lett.* **B220** 223 (1989).

Figure Captions

Figure 1: The large-N contribution to $Z_{H\,NJL}$ in the NJL model.

Figure 2: The leading large-N contribution to $Z_{H\,box}$ in the topcolor model.

Figure 3: The weak dependence of low energy predictions upon the parameter (higher dimension operator coefficient) χ, where $\chi \approx -1/8$ in the topcolor model (from W. Bardeen, [5]) The solid (dashed) lines are top mass (Higgs mass) predictions in pure NJL, and the full renormalization group approximations.



(1a) (1b)

Fig. (1)

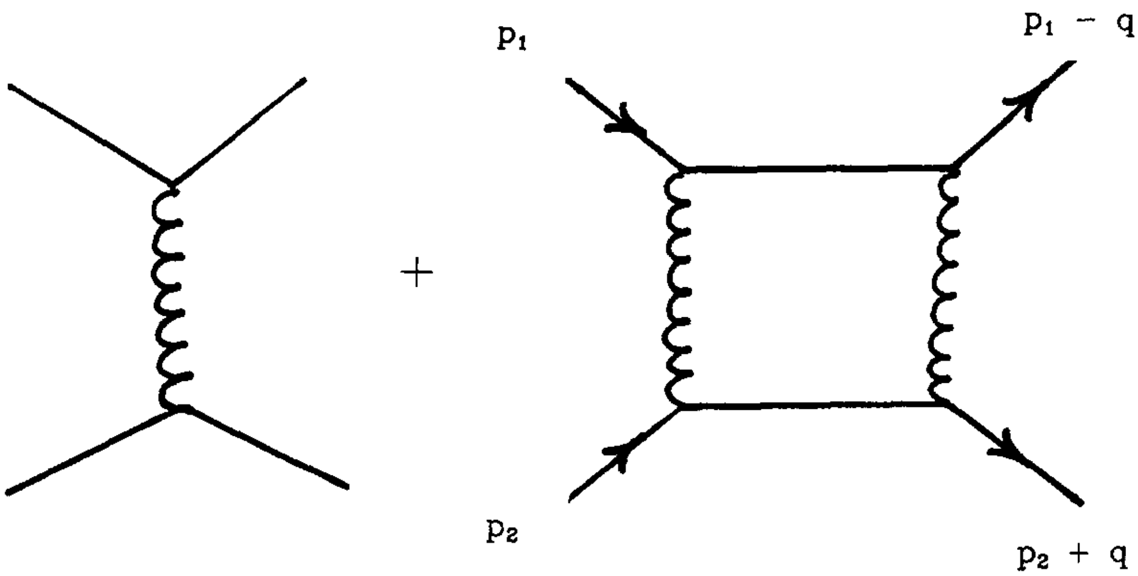


Fig. (2)

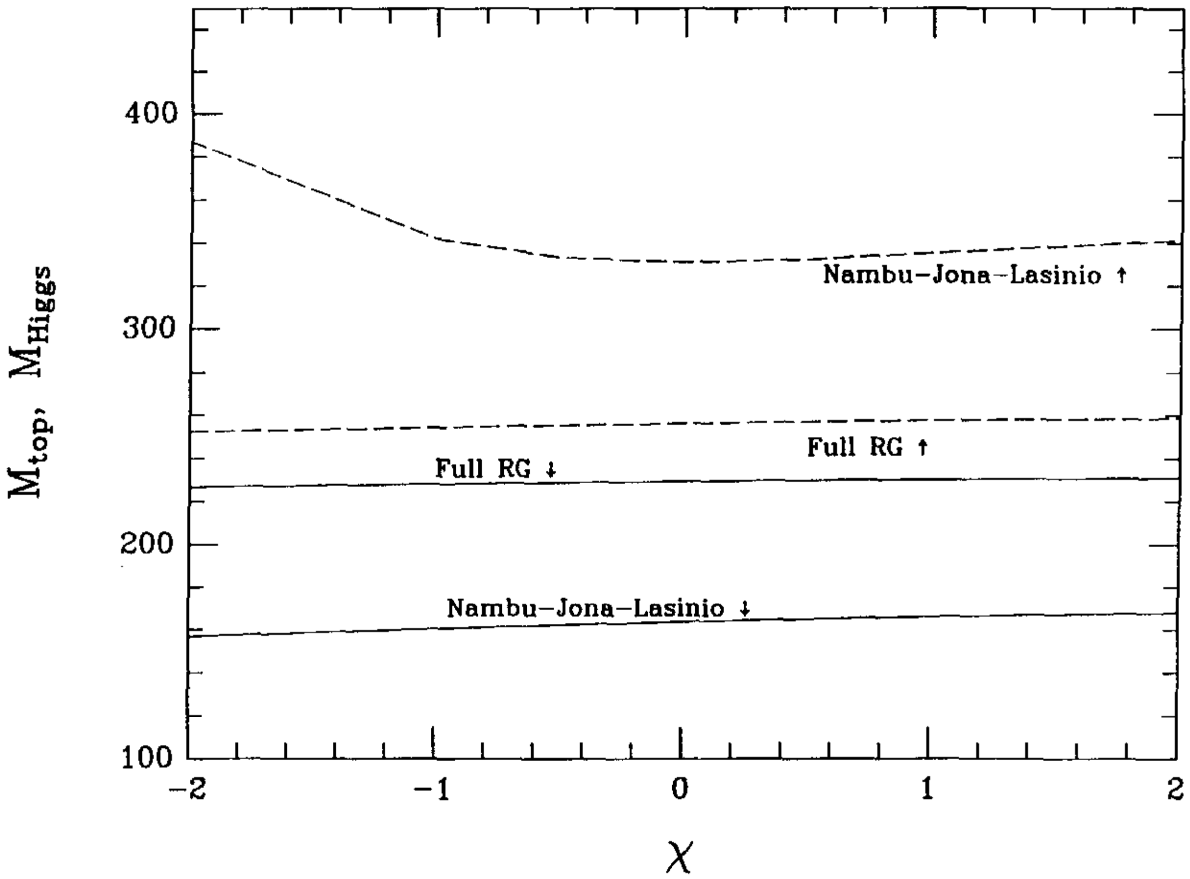

400
300
200
100
Nambu–Jona–Lasinio ↑
Full RG ↑
Full RG ↓
Nambu–Jona–Lasinio ↓
−2
−1
0
1
2
M_{top}, M_{Higgs}
χ

Fig. (3)