

# CPT and Lorentz Invariance: Their Relation and Violation

**Anca Tureanu**

Department of Physics, University of Helsinki, P.O. Box 64, 00014 Helsinki

E-mail: [anca.tureanu@helsinki.fi](mailto:anca.tureanu@helsinki.fi)

**Abstract.** The CPT theorem in quantum field theory, its validity, breaking and consequences are reviewed. One can show that for CPT theorem to hold, Lorentz invariance is not always needed. Also one can have CPT violation, while there is Lorentz invariance. Field theoretical examples for both cases are given and mass differences between particle-antiparticle are discussed as well. Unambiguous tests of CPT violation, unrelated to the breaking of Lorentz invariance, are suggested.

## 1. Introduction

Lorentz symmetry and the  $CPT$  invariance are two of the most fundamental symmetries of Nature, whose violation has not yet been observed. While the Lorentz invariance is a continuous symmetry of space-time, the  $CPT$  involves the discrete space- and time-inversions,  $P$ ,  $T$ , and the charge conjugation operation on the fields,  $C$ . Although the individual symmetries,  $C$ ,  $P$  and  $T$  have been observed to be violated in various interactions, their combined product,  $CPT$ , remarkably remains still as an exact symmetry. The first proof of  $CPT$  theorem was given by Lüders and Pauli [1,2] based on the Hamiltonian formulation of quantum field theory, which involves locality of the interaction, Lorentz invariance and Hermiticity of the Hamiltonian. Later on the theorem was proven by Jost [3] (see also [4–6]) within the axiomatic formulation of quantum field theory without reference to any specific form of interaction. This proof of  $CPT$  theorem relaxes the requirement of locality or *local commutativity* condition to the so-called *weak local commutativity*. Lorentz symmetry has been an essential ingredient of the proof, both in the Hamiltonian and in the axiomatic proofs.

A simple phenomenological classification of possible  $C$ ,  $P$ ,  $T$ ,  $CP$ ,  $PT$ ,  $TC$  and  $CPT$ -violating effects is presented in [7]. For consequences of  $CPT$  and their experimental tests, as well as some theoretical considerations on the possibilities of violation of Lorentz invariance and  $CPT$  in the known interactions, we refer to [8–13] and references therein.

It is important to clarify the relation between the  $CPT$  and Lorentz invariance and in particular to see whether the violation of any of them implies the violation of the other. This issue has recently become a topical one due to the growing phenomenological importance of  $CPT$  violating scenarios, namely in neutrino physics as well as its cosmological and astrophysical consequences. Indeed, the relation between the  $CPT$  and Lorentz invariance has acquired a prominent place in nowadays particle physics with the attempts of explaining in a unified manner the contradictory results, "anomalies", in the interpretation of various neutrino physics experiments, without enlarging the neutrino sector. The idea was first suggested by Murayama



and Yanagida [14] in the form of different masses for neutrino and antineutrino, based on phenomenological considerations. This proposal was formalized as a *CPT*-violating quantum field theory with a mass difference between neutrino and antineutrino in [15, 16]. The issue was taken up in relation with the Lorentz symmetry by Greenberg [17], the conclusion of Greenberg's analysis being that *CPT* violation implies violation of Lorentz invariance. In the following, we shall show that this claim is in general not valid. Moreover, a Lorentz-invariant model with mass splitting between particle and antiparticle has also been proposed.

## 2. Lorentz-invariance violating but *CPT*-invariant quantum field theories

During the last decade, we have learned that the violation of Lorentz invariance does not necessarily lead to the violation of the *CPT* theorem. The example comes from the quantum field theory on noncommutative space-time (NC QFT) with the canonical, Heisenberg-like, commutation relations for coordinate operators:

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}, \quad (1)$$

with  $\theta^{\mu\nu}$  an antisymmetric constant matrix.

In this case, by the nature of the above noncommutativity parameter  $\theta^{\mu\nu}$  being a constant but not a tensor, Lorentz invariance is broken, but not the *CPT* symmetry [18–21]. Translational invariance is valid. In addition to the Lorentz invariance violation, such NC QFTs are nonlocal in the noncommuting coordinates. However, the Lorentz symmetry violation is of a very particular form, and invariance under the stability group of the matrix  $\theta^{\mu\nu}$  is preserved under the so-called residual symmetry  $SO(2) \times SO(1, 1)$ . This reduced symmetry is enough to prove the *CPT* theorem only for the scalar fields (for which the *C* operation is a simple Hermitian conjugation) on the noncommutative space-time (1) [19]. A full proof of the *CPT* theorem in Lorentz-violating noncommutative quantum field theory, however, could be achieved [20] only by using the twisted Poincaré symmetry [22, 23] which these theories possess. The twisted Poincaré invariance is a deformation of the Poincaré symmetry, considered as a Hopf algebra, a concept coming from the theory of quantum groups [24], as compared with the Lie algebra. The irreducible representations of twisted Poincaré are identical to those of the usual Poincaré algebra, i.e. labeled by the mass and spin of the particles. Therefore, the meaning of the charge conjugation has survived intact in the noncommutative quantum field theories. While parity and time reversal symmetries can be defined with any concept of space and time, the notion of charge conjugation has meaning only in the framework of Lorentz symmetry. Antiparticles are a consequence of special relativity. Particle and antiparticle are in the same irreducible representation of the Poincaré group. The *CPT* theorem is thus strongly connected to the Poincaré group representations, and not so much to the Lorentz symmetry, as the validity of the *CPT* theorem in the noncommutative space-time shows.

## 3. *CPT*-violating but Lorentz-invariant nonlocal model

In [25] was proposed a class of models which preserve Lorentz invariance while breaking the *CPT* symmetry through a (nonlocal) interaction. The latter attitude is taken as responsible for the violations of a symmetry, based on our experience that all the discrete, *C*, *P* and *T* invariances, as well as other symmetries, are broken in our description of Nature by means of interaction. We also know that nonlocal field theories appear, in general, as effective field theories of a larger theory.

Consider a field theory with the nonlocal interaction Hamiltonian of the type

$$\mathcal{H}_{int}(x) = \lambda \int d^4y \phi^*(x)\phi(x)\phi^*(x)\theta(x_0 - y_0)\theta((x - y)^2)\phi(y) + h.c., \quad (2)$$

where  $\lambda$  is a coupling constant with dimension appropriate for the Hamiltonian density,  $\phi(x)$  is a Lorentz-scalar field and  $\theta$  is the Heaviside step function, with values 0 or 1, for its negative and positive argument, respectively. The combination  $\theta(x_0 - y_0)\theta((x - y)^2)$  in (2) ensures the Lorentz invariance, i.e. invariance under the proper orthochronous Lorentz transformations, since the order of the times  $x_0$  and  $y_0$  remains unchanged for time-like intervals, while for space-like distances the interaction vanishes. Also, the same combination makes the nonlocal interaction causal at the tree level, which dictates that there is no interaction when the fields are separated by space-like distances and thus there is a maximum speed of  $c = 1$  for the propagation of information.

On the other hand, it is clear that  $C$  and  $P$  invariance are trivially satisfied in (2), while  $T$  invariance is broken due to the presence of  $\theta(x_0 - y_0)$  in the integrand.

One can always insert into the Hamiltonian (2), without changing its symmetry properties, a weight function or form-factor  $F((x - y)^2)$ , for instance of a Gaussian type:

$$F = \exp\left(-\frac{(x - y)^2}{l^2}\right), \quad (3)$$

with  $l$  being a nonlocality length in the considered theory. Such a weight function would smear out the interaction and would guarantee the desired behaviour of the integrand in (2); in the limit of fundamental length  $l \rightarrow 0$  in (3), the Hamiltonian (2) would correspond to a local,  $CPT$ - and Lorentz-invariant theory. The nonlocality length  $l$  could be looked upon as being a characteristic parameter relating the effective field theory to its parent one, for instance the radius of a compactified dimension when the parent theory is a higher-dimensional one. Furthermore, with such a weight function, the interaction vanishes at infinite  $(x - y)^2$  separations and thus one can envisage the existence of in- and out-fields.

There exists a whole class of such  $CPT$ -violating, Lorentz invariant field theories involving different, scalar, spinor or higher-spin interacting fields [25]. Typical simplest examples are:

$$\mathcal{H}_{int}(x) = \lambda \int d^4y \phi_1^*(x)\phi_1(x)\theta(x_0 - y_0)\theta((x - y)^2)\phi_2(y) + h.c., \quad (4)$$

$$\mathcal{H}_{int}(x) = \lambda \int d^4y \bar{\psi}(x)\psi(x)\theta(x_0 - y_0)\theta((x - y)^2)\phi(y) + h.c., \quad (5)$$

$$\mathcal{H}_{int}(x) = \lambda \int d^4y \phi(x)\theta(x_0 - y_0)\theta((x - y)^2)\phi^2(y) + h.c. \quad (6)$$

The above Hamiltonians are nonlocal in time, and thus we expect difficulties in defining a unitary  $S$ -matrix in general. In addition, Marnelius [26] showed the breakdown of energy-momentum conservation in this class of theories if one naively applies canonical quantization. We suspect that this difficulty noted by Marnelius is associated with the absence of the well-defined canonical momentum and canonical quantization in theories nonlocal in time. To avoid this and related difficulties with canonical quantization, we use the path integral which is based on Schwinger's action principle, as explained in detail below. This path integral is quite general and has an advantage when applied to the analysis of a theory nonlocal in time: we do not start with the notion of canonical momentum, which is not generally defined in theories nonlocal in time. Instead, the canonical structure, should it exist, is fully extracted later from Green's functions defined in the path integral formalism by the Bjorken–Johnson–Low (BJL) prescription. The path integral thus defined is manifestly Lorentz-invariant and, in fact, Poincaré invariant. The energy-momentum conservation is thus ensured although the canonically quantized energy-momentum operator does not exist in general. Even the possible presence of Lorentz anomaly, i.e., quantum breaking of Lorentz symmetry, is reliably detected by the path integral.

#### 4. Lagrangian model of fermion mass splitting

It has been shown [27] that, by including the  $CPT$ -violation only in the interaction term of a Lorentz-invariant Lagrangian, the equality of masses of particle and antiparticle remains valid. The question arises whether a Lorentz-invariant model can be constructed in which the masses of particle and antiparticle split. Obviously, besides  $CPT$  violation, such a model has necessarily to feature  $C$  and  $CP$  violation, since otherwise the antiparticle can always be defined by an operation containing  $C$  and ensuring the equality of masses.

In the present nonlocal formulation, we have a new possibility which is absent in a smooth nonlocal extension of the  $CPT$ -even local field theory. The term  $i\mu\bar{\psi}(x)\psi(y)$  (to be precise,  $i\mu\bar{\psi}(x)\psi(x)$ ) with a real  $\mu$  does not appear in the local Lagrangian since it is canceled by its Hermitian conjugate. Also this term is  $CPT$ -odd. But in the present nonlocal theory one can consider the Hermitian combination

$$\int d^4x d^4y [\theta(x^0 - y^0) - \theta(y^0 - x^0)] \delta((x - y)^2 - l^2) [i\mu\bar{\psi}(x)\psi(y)], \quad (7)$$

which is non-vanishing. Under  $CPT$ , we have  $i\mu\bar{\psi}(x)\psi(y) \rightarrow -i\mu\bar{\psi}(-y)\psi(-x)$ . By performing the change of integration variables  $-x \rightarrow y$  and  $-y \rightarrow x$ , this combination is confirmed to be  $CPT = -1$ . In fact, we have the following transformation property of the operator part

$$\begin{aligned} C &: i\mu\bar{\psi}(x)\psi(y) \rightarrow i\mu\bar{\psi}(y)\psi(x), \\ P &: i\mu\bar{\psi}(x^0, \vec{x})\psi(y^0, \vec{y}) \rightarrow i\mu\bar{\psi}(x^0, -\vec{x})\psi(y^0, -\vec{y}), \\ T &: i\mu\bar{\psi}(x^0, \vec{x})\psi(y^0, \vec{y}) \rightarrow -i\mu\bar{\psi}(-x^0, \vec{x})\psi(-y^0, \vec{y}), \end{aligned} \quad (8)$$

and thus the overall transformation property is  $C = -1$ ,  $P = 1$ ,  $T = 1$ . Namely,  $C = CP = CPT = -1$ .

It is thus interesting to examine a new action, proposed in [28],

$$\begin{aligned} S &= \int d^4x \{ \bar{\psi}(x) i\gamma^\mu \partial_\mu \psi(x) - m\bar{\psi}(x)\psi(x) \\ &\quad - \int d^4y [\theta(x^0 - y^0) - \theta(y^0 - x^0)] \delta((x - y)^2 - l^2) [i\mu\bar{\psi}(x)\psi(y)] \}, \end{aligned} \quad (9)$$

which is Lorentz invariant and Hermitian. For the real parameter  $\mu$ , the third term has  $C = CP = CPT = -1$  and no symmetry to ensure the equality of particle and antiparticle masses.

The Dirac equation is replaced by

$$\begin{aligned} i\gamma^\mu \partial_\mu \psi(x) &= m\psi(x) \\ + i\mu \int d^4y [\theta(x^0 - y^0) - \theta(y^0 - x^0)] \delta((x - y)^2 - l^2) \psi(y). \end{aligned} \quad (10)$$

By inserting an ansatz for the possible solution

$$\psi(x) = e^{-ipx} U(p), \quad (11)$$

we have

$$\begin{aligned} \not{p}U(p) &= mU(p) \\ + i\mu \int d^4y [\theta(x^0 - y^0) - \theta(y^0 - x^0)] \\ &\times \delta((x - y)^2 - l^2) e^{-ip(y-x)} U(p) \\ &= mU(p) + i\mu [f_+(p) - f_-(p)] U(p), \end{aligned} \quad (12)$$

where  $f_{\pm}(p)$  is a Lorentz invariant form factor defined by

$$f_{\pm}(p) = \int d^4 z_1 e^{\pm i p z_1} \theta(z_1^0) \delta((z_1)^2 - l^2), \quad (13)$$

which are inequivalent for time-like  $p$  due to the factor  $\theta(z_1^0)$ . For time-like momentum  $p$ , one may choose a suitable Lorentz frame such that  $\vec{p} = 0$  and

$$f_{\pm}(p^0) = 2\pi \int_0^{\infty} dz \frac{z^2 e^{\pm i p^0 \sqrt{z^2 + l^2}}}{\sqrt{z^2 + l^2}}, \quad (14)$$

and for the space-like momentum  $p$  one may choose a suitable Lorentz frame such that  $p^0 = 0$  and

$$f_{\pm}(\vec{p}) = \frac{2\pi}{|p|^2} \int_0^{\infty} dz z \frac{\sin z}{\sqrt{z^2 + (|p|l)^2}}, \quad (15)$$

which is analogous to the Fourier transform of the Coulomb potential and real. The expression  $f_{\pm}(p)$  is mathematically related to the formula of the two-point Wightman function (for a free scalar field), which suggests that  $f_{\pm}(p)$  is mathematically well-defined for  $p \neq 0$  at least in the sense of distribution.

The (off-shell) propagator is defined by

$$\int d^4 x e^{i p(x-y)} \langle T^* \psi(x) \bar{\psi}(y) \rangle = \frac{i}{\not{p} - m + i\epsilon - i\mu[f_+(p) - f_-(p)]}, \quad (16)$$

which is manifestly Lorentz covariant. Note that we use the  $T^*$ -product for the path integral in accord with Schwinger's action principle, which is based on the equation of motion (10) with a source term added:

$$\langle 0, +\infty | 0, -\infty \rangle_J = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp i \left\{ S + \int d^4 x \mathcal{L}_J \right\}, \quad (17)$$

where the action  $S$  is given in (9) and the source term is  $\mathcal{L}_J = \bar{\psi}(x)\eta(x) + \bar{\eta}(x)\psi(x)$ . The  $T^*$ -product is quite different from the canonical  $T$ -product in the present nonlocal theory, and in fact the canonical quantization is not defined in the present theory. It is however important to note that the  $T^*$ -product can reproduce all the results of the  $T$ -product, if the  $T$ -product is well-defined, by means of the Bjorken–Johnson–Low prescription [29]. In the present example, the presence of the sine-function in the denominator of the correlation function complicates this procedure, which is an indication of the absence of the canonical quantization of (9). We also emphasize that the analysis of the mass-splitting can be performed in terms of the exact solution of the (modified) free Dirac equation (10), which also defines the propagator in the present path integral prescription.

The propagator (16) is also an exact propagator for (9) in the sense of the propagator theory of relativistic quantum mechanics, and thus it could describe the particle and antiparticle propagation if one understands the antiparticle with negative energy propagating backward in time. However, if one attempts to describe the particle and antiparticle propagation with definite masses by pole approximation, for example, then the off-shell Lorentz covariance of the propagator (16) is lost.

For the space-like  $p$ , the extra term with  $\mu$  in the denominator of the propagator (16) vanishes since  $f_+(p) = f_-(p)$  for  $p = (0, \vec{p})$ , as can be easily seen from (15). Thus the propagator has poles only at the time-like momentum, and in this sense the present Hermitian action (9) does

not allow a tachyon. By assuming a time-like  $p$ , we go to the frame where  $\vec{p} = 0$ . Then the eigenvalue equation is given by

$$p_0 = \gamma_0 \{m + i\mu[f_+(p_0) - f_-(p_0)]\}, \quad (18)$$

namely,

$$p_0 = \gamma_0 \left[ m - 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0 \sqrt{z^2 + l^2}]}{\sqrt{z^2 + l^2}} \right], \quad (19)$$

where we used the explicit formula in (14). The solution  $p_0$  of this equation (19) determines the possible mass eigenvalues.

This eigenvalue equation under  $p_0 \rightarrow -p_0$  becomes:

$$-p_0 = \gamma_0 \left[ m + 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0 \sqrt{z^2 + l^2}]}{\sqrt{z^2 + l^2}} \right]. \quad (20)$$

By sandwiching this equation by  $\gamma_5$ , which is regarded as  $CPT$  operation, we have

$$-p_0 = \gamma_0 \left[ -m - 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0 \sqrt{z^2 + l^2}]}{\sqrt{z^2 + l^2}} \right], \quad (21)$$

i.e.,

$$p_0 = \gamma_0 \left[ m + 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0 \sqrt{z^2 + l^2}]}{\sqrt{z^2 + l^2}} \right], \quad (22)$$

which is not identical to the original equation in (19). In other words, if  $p_0$  is the solution of the original equation,  $-p_0$  cannot be the solution of the original equation except for  $\mu = 0$ . The last term in the Lagrangian (9) with  $C = CP = CPT = -1$  splits the particle and antiparticle masses.

As a crude estimate of the mass splitting, one may assume  $\mu \ll m$  and solve these equations iteratively. If the particle mass for (19) is chosen at

$$p_0 \simeq m - 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[m\sqrt{z^2 + l^2}]}{\sqrt{z^2 + l^2}}, \quad (23)$$

then the antiparticle mass for (22) is estimated at

$$p_0 \simeq m + 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[m\sqrt{z^2 + l^2}]}{\sqrt{z^2 + l^2}}. \quad (24)$$

Thus, the simple Lorentz invariant  $CPT$  violating Lagrangian model in (9), which produces the splitting of particle and antiparticle masses. The simple Lagrangian model will provide a useful theoretical laboratory when one investigates Lorentz invariant  $CPT$  violation effects.

The idea has been extended to the originally posed problem of the possible splitting of neutrino and antineutrino masses [30], assuming the neutrino masses to be predominantly Dirac-type in the Standard Model and without spoiling the  $SU(2)_L \times U(1)$  gauge symmetry.

An attempt has been made [31] to incorporate the electromagnetic interaction in a Lorentz invariant but  $CPT$  violating nonlocal model with particle-antiparticle mass-splitting, which

is regarded as a modified QED. The gauge invariance is maintained by the Schwinger non-integrable phase factor but the electromagnetic interaction breaks  $C$ ,  $CP$  and  $CPT$  symmetries. To introduce the electromagnetic interaction in (9), we consider the simplest scheme (a modified QED):

$$\begin{aligned}
 S = & \int d^4x \left\{ \bar{\psi}(x) i\gamma^\mu D_\mu \psi(x) - m\bar{\psi}(x)\psi(x) \right. \\
 & - \int d^4y [\theta(x^0 - y^0) - \theta(y^0 - x^0)] \delta((x - y)^2 - l^2) i\mu \bar{\psi}(x) \exp \left[ ie \int_y^x A_\mu(z) dz^\mu \right] \psi(y) \left. \right\} \\
 & - \frac{1}{4} \int d^4x F_{\mu\nu}(x) F^{\mu\nu}(x),
 \end{aligned} \tag{25}$$

with

$$D_\mu = \partial_\mu - ieA_\mu(x). \tag{26}$$

We added the Schwinger non-integrable phase factor,

$$\exp \left[ ie \int_y^x A_\mu(z) dz^\mu \right], \tag{27}$$

to make the nonlocal term gauge invariant. This action is invariant under the gauge transformations

$$\begin{aligned}
 \psi(x) & \rightarrow e^{i\alpha(x)} \psi(x), \\
 A_\mu(x) & \rightarrow A_\mu(x) + \frac{1}{e} \partial_\mu \alpha(x),
 \end{aligned} \tag{28}$$

and the  $C$ ,  $CP$  and  $CPT$  transformation properties of each term in the action (25) are the same as in the theory without electromagnetic couplings.

It is natural to consider the non-integrable phase factor in (25) as an independent dynamical entity rather than a given external factor. In fact, Y. Nambu emphasized in many occasions the non-integrable phase factor as a manifestation of string-like objects appearing in the theory.

The proposal of [31] is to replace the non-integrable phase factor in (25) by a first quantized very massive particle propagation defined by the covariant path integral

$$\begin{aligned}
 & \exp \left[ ie \int_y^x A_\mu(z) dz^\mu \right] \delta_{\alpha,\beta} \Rightarrow \\
 & \int \mathcal{D}z^\mu \exp \left\{ i \int_y^x \frac{1}{2} \left[ (\dot{z}^\mu)^2 + M^2 \right] d\tau + ie \int_y^x A_\mu(z) \frac{dz^\mu}{d\tau} d\tau \right\} \delta_{\alpha,\beta},
 \end{aligned} \tag{29}$$

where the factor  $\delta_{\alpha,\beta}$  contracts the spinor indices, an analogue of the Chan-Paton factor in string theory. In this way, the non-integrable phase factor becomes more dynamical and the flow of the charge is visualized in Feynman diagrams, although the second quantized particle and the first quantized particle appear in a mixed manner in Feynman diagrams. This use of a semi-static massive particle for the non-integrable phase factor is common in lattice gauge theory.

The full set of Ward–Takahashi identities, i.e. the relations among different Green’s functions for the presented modified QED, can be derived formally in the path integral quantization, but the exact current becomes more involved due to the presence of the non-integrable phase factor in the full action [31]. An analysis of higher order effects in the electromagnetic coupling in the presence of the non-integrable phase factor even in the lowest order of the  $CPT$ -violation parameter  $\mu$  is an interesting subject of future study.

## 5. Conclusion

Besides the fact that both  $CPT$  and Lorentz invariance are two most fundamental symmetries in physics, whose violations have not been hitherto observed, the relation between the two symmetries and their possible breaking are of considerable theoretical and experimental interest. Recent MINOS neutrino experiments with their favoured interpretation through a mass difference for muon neutrino and antineutrino have revived interest in  $CPT$  violation and its possible implication on Lorentz invariance breaking [14, 32–34].

It is an interesting question whether the  $CPT$  violation the models described above [25, 27, 28, 30, 31] could be a long distance effective description of some modified structure of space-time at short distances, for example.

A treatment of chiral gauge theory in the present scheme, which breaks  $C$  invariance, is in general a difficult but important future task to complete the analysis of the induced  $CPT$  violation in the present scheme. However, relaxing the quadratic  $CPT$ -violating term in the Lagrangian used in the present work as in (9), the question remains as of which symmetry is responsible for the equality of the masses of particle and antiparticle. By taking  $CPT$  violation to be due to only interaction (when  $C$  and  $CP$  are also violated), one can show that the equality of masses persists [35]. Thus, we can infer that as long as the quadratic part in the Lagrangian is not altered, the equality of the masses of particle and antiparticle is due to Lorentz invariance rather than to  $CPT$  [28, 35]. A further clarification of the basic mechanism which can ensure equal masses to the particle and anti-particle in the absence of  $C$ ,  $CP$  and  $CPT$  symmetries is an interesting remaining issue. At the same time, if there exists a  $CPT$  violating interaction in Nature, for composite particles such as  $p - \bar{p}$ ,  $\pi^+ - \pi^-$ ,  $K^0 - \bar{K}^0$ , etc., as well as for bound states, such as hydrogen-antihydrogen, there are differences in their masses, due to different bound-state energies, in their total widths (life-times) and magnetic moments. Therefore, the experimental measurements on the latter characteristics of composite particles or bound systems would reveal the existence of a  $CPT$ -violating theory, although the theory is Lorentz invariant.

As for practical implications of  $CPT$  breaking in the modified QED [31], the search for the mass splitting of particle and antiparticle, just as the search for the neutrino antineutrino mass splitting in oscillation experiments [36], is interesting [37]. In the atomic transitions of the matter or antimatter systems, the frequency differences caused by the small mass difference between the electron and positron such as in (23) and (24) will be important. Other possibilities are to look for the possible small  $C$  and  $CP$  breaking in electromagnetic interactions other than those caused by weak interactions.

One problem for further study is whether the mechanism of inducing mass splitting between particle and antiparticle can be used to generate the baryon asymmetry of the Universe in equilibrium (with  $CPT$  violation, one of the three Sakharov conditions can be ignored) and what does such a case imply for the parameters of the model.

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