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Hawking Radiation from the Boundary Scalar Field and the Information Loss Paradox

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Abstract: Hawking radiation is an essential property of the quantum black hole. It results in the information loss paradox and provides an important clue with regard to the unification of quantum mechanics and general relativity. In previous work, the boundary scalar fields on the horizon of black holes were used to determine the microstates of BTZ black holes and Kerr black holes. They account for Bekenstein–Hawking entropy. In this paper, we show that the Hawking radiation can also be derived from those scalar fields. Hawking radiation is a mixture of the thermal radiation of right- and left-moving sectors at different temperatures. Based on this result, for static BTZ black holes and Schwarzschild black holes, we propose a simple solution for the information loss paradox; i.e., the Hawking radiation is pure due to its entanglement between the left-moving sector and the right-moving sector. This entanglement may be detected in an analogue black hole in the near future.

Keywords: boundary scalar field; Hawking radiation; information loss paradox

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1. Introduction

One important contribution by Steven Hawking is the discovery of Hawking radiation [1,2]. A black hole can radiate with a black body temperature, T_H. The thermal nature of such radiation lies at the heart of the "information loss paradox" [3–5], which refers to how the evaporation of a black hole will break the unitary, or, in other words, how a pure state can evolve into a mixed state. There have been many proposals for its resolution. For example, recently, in [6], it was shown that the radiation fields represent a pure quantum state for black hole evaporation based on the moving mirror model. On the other hand, based on string theory, the Page curve was semi-classically reproduced by evaluating the island [7–9] in the quantum external surface or replica wormholes [10,11]. Since Hawking's seminal works, there have been many other derivations that confirm Hawking's results. For a review, see [12] and the references therein.

However, Hawking's analysis is semi-classical: the black hole spacetime is treated as a classical background. This implies that, if the black hole spacetime is treated as a quantum object, the thermal Hawking radiation may need modifications to contain information. Bekenstein [13–15] proposed that black holes play the same role in gravity as the atoms play in quantum mechanics. This analogy suggests that the area may have a discrete spectrum:

$$A_n = \alpha n \hbar, \quad n = 1, 2, 3, \cdots, \tag{1}$$

where α is a dimensionless constant. Two possible values of α are often used: $\alpha = 8\pi$ [14,16] and $\alpha = 4 \log k$ with $k = 2, 3, \cdots$ [17–19]. The discrete area spectrum implies the discrete energy spectrum, which changes the continuous Hawking radiation to a discrete line emission. Bekenstein and Mukhanov [18] later suggested that the radiation of a quantum

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Schwarzschild black hole of mass M should be at integer multiples of the fundamental frequency:

$$\omega_0 = \alpha T_H = \frac{\alpha}{8\pi M}. \tag{2}$$

In [20], the authors suggest that this special frequency may be observed from the gravitational wave signals.

In [21], we provide the microstates for Bañados–Teitelboim–Zanelli (BTZ) black holes and Kerr black holes from boundary scalar fields. Those microstates can account for the Bekenstein–Hawking entropy. Due to the compactness of the scalar fields, one can find that the area of the the Kerr black hole has a discrete spectrum (1) with $\alpha = 8\pi$. In this paper, we will show that those states can also provide the Hawking radiation with some modifications that are similar to Bekenstein and Mukhanov's suggestions. The Hawking radiation is a mixture of the thermal radiation of right- and left-moving sectors at different temperatures. Based on this result, for static BTZ black holes and Schwarzschild black holes, we propose a simple solution for the information loss paradox, which is that the Hawking radiation is pure due to its entanglement between the left-moving sector and right-moving sector.

The paper is organized as follows. In Section 2, the BTZ black hole is analyzed. In Section 3, the same method is applied to the Kerr black hole. In Section 4, we propose that the Hawking radiation is pure. Section 5 is the conclusion. In this paper, we set $G = 1$.

2. The BTZ Black Hole

In this section, we analyze the BTZ black hole in three dimensional spacetime.

2.1. The Entropy of the BTZ Black Hole

The metric of the BTZ black hole is [22]:

$$ds^2 = -N^2 dv^2 + 2dvdr + r^2(d\varphi + N^\varphi dv)^2, \tag{3}$$

where $N^2 = -8M + \frac{r^2}{l^2} + \frac{16J^2}{r^2}, N^\varphi = -\frac{4J}{r^2}$, l is the radius of the AdS spacetime, and (M, J) represents the mass and angular momentum of the black hole, respectively. The black hole has the event horizon at $r = r_+$ and the inner horizon at $r = r_-$ with

$$r_\pm^2 = 4Ml^2(1 \pm \sqrt{1 - \frac{J^2}{M^2 l^2}}). \tag{4}$$

The thermodynamic quantities for the BTZ black hole are

$$T_H = \frac{r_+^2 - r_-^2}{2\pi r_+ l^2}, \quad S_{BH} = \frac{2\pi r_+}{4}, \quad \Omega_H = \frac{r_-}{r_+ l}, \tag{5}$$

and satisfy the first law

$$dM = T_H dS + \Omega_H dJ. \tag{6}$$

The number distribution of Hawking radiation at infinity for the bosons is provided by

$$< N_m(\omega) >= \frac{1}{e^{\beta(\omega - m\Omega_H)} - 1}, \tag{7}$$

where $\beta = \frac{1}{T_H}$ is the inverse temperature, and m is the azimuthal angular momentum number. From this distribution, one can obtain Planck's thermal distribution for black holes.

Now, let us consider this distribution from the boundary scalar field on the horizon. Firstly, we will review some of the results in the previous work [21]. The scalar field has a mode expansion

$$\phi(v',\varphi) = \phi_0 + p_v v' + p_\varphi \varphi + \sqrt{\frac{8\pi}{lA}} \sum_{n\neq 0} \sqrt{\frac{1}{2\omega'_n}} [a_n e^{-i(\omega'_n v' - k_n \varphi)} + a_n^+ e^{i(\omega'_n v' - k_n \varphi)}], \quad (8)$$

where $v' = \frac{v}{\gamma} = \frac{r_+}{l} v, \omega'_n = \frac{|n|}{r_+}, k_n = n$, and $A = 2\pi r_+$ is the length of the circle. The scalar field $\phi(v',\varphi)$ can be considered as collectives of harmonic oscillators, and a general quantum state can be represented as $|p_v, p_\varphi; \{n_k\} >\sim (a_1^+)^{n_1} \cdots (a_k^+)^{n_k} |p_v, p_\varphi >$, where p_v, p_φ represents the zero mode parts, and $\{n_k\}$ represents the oscillating parts. The zero mode parts provide important information about the fractional charges of the quasi-particles [23]. The oscillating parts are associated with the entropy and the radiation. The Hamiltonian and angular momentum operators have the expression

$$\begin{aligned} \hat{H} &= \hat{H}_0 + \sum_{k\neq 0} \frac{|k|}{r_+} \hat{a}_k^+ \hat{a}_k = \hat{H}_0 + \sum_{k\neq 0} \varepsilon'_k \hat{n}_k, \\ \hat{J} &= \hat{J}_0 + \sum_{k\neq 0} k \hat{a}_k^+ \hat{a}_k = \hat{J}_0 + \sum_{k\neq 0} J_k \hat{n}_k, \quad k \in Z, \end{aligned} \quad (9)$$

where $\varepsilon'_k = \frac{|k|}{r_+}, J_k = k$ represents the energy and angular momentum for the energy level k, and $\hat{n}_k = \hat{a}_k^+ \hat{a}_k$ is the number operator. The Hawking radiation is observed at infinity, where the energy is red-shifted with

$$\omega \sim \frac{\partial}{\partial t} = \frac{\partial}{\partial v} = \frac{\partial}{\partial v'} \frac{1}{\gamma} \sim \frac{\omega'}{\gamma}. \quad (10)$$

We denote energy (temperature) as infinity un-primed, and that on the horizon with prime $'$. They can transform into each other with scalar-factor $\gamma = \frac{l}{r_+}$.

The calculation of the entropy for the BTZ black hole is as follows [23]. For the BTZ black hole with parameters (M, J), the oscillating parts satisfy the constraints:

$$\frac{1}{c} \sum \frac{|k|}{r_+} n_k = M\gamma, \quad \frac{1}{c} \sum k n_k = J, \quad n_k \in N^+, \quad c = 3l/2. \quad (11)$$

We denote the positive part $k > 0$ as the right-moving sector, and the negative part $k < 0$ as the left-moving sector. They satisfy

$$\frac{1}{c} \sum_{k>0} k n_k^R = \frac{1}{2}(M\gamma r_+ + J), \quad \frac{1}{c} \sum_{k<0} (-k) n_k^L = \frac{1}{2}(M\gamma r_+ - J). \quad (12)$$

The problem transforms into a mathematical problem: How many different sequences satisfy the above constraints? The solution is the famous Hardy–Ramanujan formula. The entropy for the right- and left-moving sector can be calculated by the Hardy–Ramanujan formula to obtain

$$\begin{aligned} S_R &= 2\pi \sqrt{c \frac{Ml + J}{12}} = \frac{\pi}{4}(r_+ + r_-), \\ S_L &= 2\pi \sqrt{c \frac{Ml - J}{12}} = \frac{\pi}{4}(r_+ - r_-). \end{aligned} \quad (13)$$

The total entropy is

$$S = S_R + S_L = \frac{2\pi r_+}{4}. \quad (14)$$

Thus, we obtain the entropy for the BTZ black holes.

2.2. The Hawking Radiation on the Horizon

Now, we consider the Hawking radiation on the horizon, which is the main goal of this paper. For the right- and left-moving sector, from Equation (12), we can associate them with the following energy:

$$E'_R = \frac{\gamma}{2}(M + \frac{J}{\gamma r_+}), \quad E'_L = \frac{\gamma}{2}(M - \frac{J}{\gamma r_+}). \tag{15}$$

Now, we can define the dimensional temperatures on the horizon for the right- and left-moving sectors using standard thermodynamics:

$$T'_R = (\frac{\partial E'_R}{\partial S_R})_V = \frac{r_+ + r_-}{2\pi l r_+} = \gamma\frac{\tilde{T}_R}{l} = \gamma T_R, \quad T'_L = (\frac{\partial E'_L}{\partial S_L})_V = \frac{r_+ - r_-}{2\pi l r_+} = \gamma\frac{\tilde{T}_L}{l} = \gamma T_L, \tag{16}$$

where $V = 2\pi r_+ c$ is the volume, and $\tilde{T}_{R/L} = \frac{r_+ \pm r_-}{2\pi l}$ represents dimensionless temperatures. It is easy to show that those temperatures satisfy the relation

$$\frac{2}{T_H} = \frac{1}{T_R} + \frac{1}{T_L}. \tag{17}$$

The constraints (12) can be written as

$$\sum_{k>0} \varepsilon'_k n^R_k = cE'_R, \quad \sum_{k<0} \varepsilon'_k n^L_k = cE'_L. \tag{18}$$

From statistical mechanics, it is well known that the most probable distribution that satisfies the above constraint is the Bose–Einstein distribution with zero chemical potential:

$$< n^R_k(\varepsilon'_k) >= \frac{1}{e^{\beta'_R \varepsilon'_k} - 1}, \quad < n^L_k(\varepsilon'_k) >= \frac{1}{e^{\beta'_L \varepsilon'_k} - 1}. \tag{19}$$

The radiation between energy level k, k' has the following energy and angular momentum spectrum:

$$\omega'_n = \frac{|n|}{r_+}, \quad m = n, \quad n \equiv k - k'. \tag{20}$$

Thus, for Hawking radiation at infinity, the real energy for radiation is

$$\omega_n = \frac{|n|}{r_+}\frac{1}{\gamma} = \frac{|n|}{l}. \tag{21}$$

They have constant frequency spacing and a minimal frequency:

$$\Delta\omega = \omega_0 = \frac{1}{l} = \omega_{min}. \tag{22}$$

We now provide the main result of this paper. The distribution (7) can be rewritten as follows:

$$\begin{aligned} < N_m(\omega_n) > &= \frac{1}{e^{\beta(\omega_n - m\Omega_H)} - 1} = \frac{1}{e^{\beta(\frac{|n|}{l} - n\frac{r_-}{r_+ l})} - 1} \\ &= \frac{1}{e^{\beta_R \omega_n} - 1}H(n) + \frac{1}{e^{\beta_L \omega_n} - 1}H(-n) =< n^R_n(\omega_n) > + < n^L_n(\omega_n) > \\ &= \frac{1}{e^{\beta'_R \omega'_n} - 1}H(n) + \frac{1}{e^{\beta'_L \omega'_n} - 1}H(-n) =< n^R_n(\omega'_n) > + < n^L_n(\omega'_n) >, \end{aligned} \tag{23}$$

where $H(n)$ is the Heaviside step function. That is to say, the Hawking radiation is a mixture of the thermal radiation (19) of the right- and left-moving sector at different temperatures. As stated, the thermal distribution is the most probable distribution, but not the only possible distribution.

With those temperatures, the entropy and energy can be rewritten as some suggesting forms:

$$S_{R/L} = \frac{\pi}{6} V T'_{R/L}, \quad E'_{R/L} = \frac{\pi}{12} V T'^{2}_{R/L}. \tag{24}$$

They are only the entropy and energy for phonon gas in a one-dimensional circle with the length $V = 2\pi r_+ c$.

Different from Bekenstein's suggestion, we now consider black holes as quantum many-body systems composed of many atoms. The thermal radiation of the right- and left-moving sector is not exactly a continuous distribution, since the energy has discrete levels and a minimal energy, as in Equation (22).

3. The Kerr Black Hole

The same method can be applied to Kerr black holes.

3.1. The Entropy of the Kerr Black Hole

The metric of a Kerr black hole can be written as [24]:

$$ds^2 = -(1 - \frac{2Mr}{\rho^2})dv^2 + 2dvdr - 2a\sin^2\theta dr d\varphi - \frac{4aMr\sin^2\theta}{\rho^2} dv d\varphi + \rho^2 d\theta^2 + \frac{\Sigma^2 \sin^2\theta}{\rho^2} d\varphi^2, \tag{25}$$

where $\rho^2 = r^2 + a^2\cos^2\theta, \Delta^2 = r^2 - 2Mr + a^2$, and $\Sigma^2 = (r^2 + a^2)\rho^2 + 2a^2 Mr \sin^2\theta$, with $(M, J = Ma)$ represent the mass and angular momentum of the Kerr black hole. The event horizon is localized at $r = r_+ = M + \sqrt{M^2 - a^2}$.

The thermodynamics quantities for the Kerr black hole with parameters (M, J) are:

$$T_H = \frac{r_+ - r_-}{8\pi r_+ M}, \quad S_{BH} = \frac{4\pi(r_+^2 + a^2)}{4}, \quad \Omega_H = \frac{a}{r_+^2 + a^2}. \tag{26}$$

Similar to the BTZ black hole case, the boundary can also support a massless scalar field [21]:

$$\phi(v', \theta, \varphi) = \phi_0 + p_v v' + p_\theta \ln(\cot\frac{\theta}{2}) + p_\varphi \varphi + \sqrt{\frac{2\pi}{MA}} \sum_{l \neq 0} \sum_{m=-l}^{m=l} \sqrt{\frac{1}{2\omega_l}} [a_{l,m} e^{-i\omega_l v'} Y_l^m(\theta, \varphi) + a_{l,m}^+ e^{i\omega_l v'} (Y_l^m)^*(\theta, \varphi)], \tag{27}$$

where $\omega_l^2 = \frac{l(l+1)}{r_+^2}$ and $Y_l^m(\theta, \varphi)$ are spherical harmonics, and $A = 4\pi r_+^2$. The scalar field $\phi(v', \theta, \varphi)$ can be considered collectives of harmonic oscillators, and a general quantum state can be represented as $|p_v, p_\varphi; \{n_{l,m}\} >$, where p_v, p_φ represents the zero mode part, and $\{n_{l,m}\}$ represents the oscillator part.

However, different from the BTZ black hole case, the scalar field of a Kerr black hole should have an interaction, and a special interaction results in a particular form for the Hamiltonian and angular momentum operator [23]:

$$\begin{aligned} \hat{H}_{full} &= \hat{H}_0 + \sum_m \frac{|m|}{r_+} \hat{n}_m = \hat{H}_0 + \sum_m \varepsilon'_m \hat{n}_m, \\ \hat{J} &= \hat{J}_0 + \sum_m m\hat{n}_m = \hat{J}_0 + \sum_m J_m \hat{n}_m, \quad m \neq 0, \end{aligned} \tag{28}$$

which have the same forms as those for the BTZ black hole, as shown in Equation (9).

The microscopic states of the Kerr black hole are represented by $|0,0;\{n_m\} >$ and satisfy the following constraints:

$$\sum_{m\neq 0}|m|n_m = \frac{cMr_+\gamma}{4}, \quad \sum_{m\neq 0} mn_m = \frac{cJ}{2}, \quad \gamma \equiv \frac{r_+^2 + a^2}{r_+^2}, \quad c = 12Mr_+. \tag{29}$$

We separate the sequence $\{m\}$ into the right-moving sector $\{m_+\}$, with all $m_+ > 0$, and the left-moving sector $\{m_-\}$, with all $m_- < 0$. The constraints then become

$$\sum_{m>0} mn_m^+ = \frac{\gamma}{4}(\frac{Mr_+}{2} + \frac{J}{\gamma}), \quad \sum_{m<0}(-m)n_m^- = \frac{\gamma}{4}(\frac{Mr_+}{2} - \frac{J}{\gamma}). \tag{30}$$

For the right- and left-moving sectors on the horizon, we can associate them with the following energy and entropy:

$$\begin{aligned} E_R' = \frac{\gamma}{4}(\frac{M}{2} + \frac{J}{\gamma r_+}) = \frac{\gamma}{8}(M+a), \quad E_L' = \frac{\gamma}{8}(M-a), \\ S_R = 2\pi\sqrt{c\frac{M^2+J}{24}} = \pi M(r_+ + a), \\ S_L = 2\pi\sqrt{c\frac{M^2-J}{24}} = \pi M(r_+ - a). \end{aligned} \tag{31}$$

The total entropy is

$$S = S_R + S_L = 2\pi M r_+, \tag{32}$$

which is the Bekenstein–Hawking entropy for the Kerr black hole.

3.2. The Hawking Radiation on the Horizon

In the following, we consider the radiation on the horizon. We can define the dimensional temperatures for the right- and left-moving sectors:

$$T_R' = (\frac{\partial E_R'}{\partial S_R})_V = \gamma\frac{r_+ + a}{8\pi M r_+} \equiv \gamma T_R = \frac{\tilde{T}_R}{r_+}, \quad T_L' = (\frac{\partial E_L'}{\partial S_L})_V = \gamma\frac{r_+ - a}{8\pi M r_+} \equiv \gamma T_L = \frac{\tilde{T}_L}{r_+}, \tag{33}$$

where $V = 2\pi r_+ c$ is the volume, and $\tilde{T}_{L/R} = \frac{1}{4\pi}(1 \pm \frac{a}{r_+})$ represents dimensionless temperatures. This volume is related to the geometrical volume [25,26] $V' = \frac{1}{3}A_H r_H = \frac{8\pi}{3}Mr_+^2$ by $V = 9V'$. It is easy to show that those temperatures satisfy the following relation:

$$\frac{2}{T_H} = \frac{1}{T_R} + \frac{1}{T_L}. \tag{34}$$

From the expression (28), one can see that the energy and the azimuthal angular momentum for the radiation between two levels k, k' are as follows:

$$\omega_m' = \frac{|m|}{r_+}, \quad m = m, \quad m \equiv k - k'. \tag{35}$$

Similar to the BTZ black hole case, the energy at infinity should be

$$\omega_m = \frac{|m|}{r_+}\frac{1}{\gamma} = \frac{|m|}{2M}. \tag{36}$$

They have constant frequency spacing and a minimal frequency:

$$\Delta\omega = \omega_0 = \frac{1}{2M} = \omega_{min}. \tag{37}$$

Now, we can rewrite the distribution as follows:

$$\begin{aligned}
< N_m(\omega_m) >= \frac{1}{e^{\beta(\omega_m - m\Omega_H)} - 1} &= \frac{1}{e^{\beta(\frac{|m|}{2M} - m\frac{a}{r_+^2 + a^2})} - 1} \\
= \frac{1}{e^{\beta_R \omega_m} - 1} H(m) + \frac{1}{e^{\beta_L \omega_m} - 1} H(-m) &=< n_m^R(\omega_m) > + < n_m^L(\omega_m) > \\
= \frac{1}{e^{\beta'_R \omega'_m} - 1} H(m) + \frac{1}{e^{\beta'_L \omega'_m} - 1} H(-m) &=< n_m^R(\omega'_m) > + < n_m^L(\omega'_m) > .
\end{aligned} \tag{38}$$

This is a new result. The Hawking radiation is also a mixture of the thermal radiation of the right- and left-moving sector at different temperatures, as in the BTZ black hole case. With those temperatures, the energy and entropy can be rewritten as

$$S_{R/L} = \frac{\pi}{6} V T'_{R/L}, \quad E'_{R/L} = \frac{\pi}{12} V T'^2_{R/L}. \tag{39}$$

They are the entropy and energy for phonon gas in a one-dimensional circle with length $V = 2\pi r_+ c$, similar to the BTZ black hole case. This suggests that the four-dimensional Kerr black hole might essentially be a $(1+1)$ object [27,28].

4. Hawking Radiation Is Pure

In the previous sections, we analyzed the Hawking radiation for the BTZ black hole and the Kerr black hole and obtained the main results of this paper (23) and (38). Both these results can be considered a mixture of the thermal radiation of the right- and left-moving sector at different temperatures. We will show that the entanglement between the left-moving sector and the right-moving sector will keep the final state pure. However, we first consider a similar situation: a key property of the ground state for a general quantum Hall state [29,30]. Separate the whole region into two parts, A and B, with a common boundary C. When the coupling between those two parts is ignored, there are chiral and anti-chiral edge states propagating along the boundary. Due to the entanglement between A and B, the reduced density matrix of the ground state for the right-moving sector is thermal. The ground state $|\Omega>$ for an abelian fractional quantum Hall state with the fixed topological sector is provided by the following [30]:

$$|\Omega >= \frac{1}{\sqrt{Z}} \sum_n e^{-\tau_0(H_R + H_L)} |k(n) >_R \otimes | - k(n) >_L . \tag{40}$$

Here, τ_0 is the extrapolation length that is determined by the energy gap. H_L and H_R denote the Hamiltonian for the left-moving and right-moving edge sectors. $k(n)$ denotes the momentum of the state. Notice that, due to the chirality, the right-moving (or left-moving) edge sector only contains excitations with positive (or negative) momentum. The reduced density matrix for the right-moving sector is

$$\rho_R = \frac{1}{Z} \sum_n e^{-4\tau_0 H_R} |k(n) >_R < k(n)|_R, \tag{41}$$

which is a thermal state.

For BTZ black holes, the horizon separates spacetime into two regions: the exterior and interior of the black hole. Similar to the quantum Hall states, the boundary modes on the horizon contain the left-moving sector and the right-moving sector. The entropy for the right- or left-moving sector is provided in (13). We consider such entropy as the entanglement entropy of Hawking radiation for the right- or left-moving sectors. To cause the Hawking radiation to be a pure state, a necessary condition is that the entanglement entropy for the two sectors should be equal, that is, $r_- = 0$. Thus, in the following, we will consider the static BTZ black holes.

The thermal radiation for the right-moving sector $< N^R(\omega_k) >$ corresponds to a thermal density matrix:

$$\rho_R = \frac{1}{Z}\sum_{k>0} e^{-\beta_R H_R}|k,k>_R< k,k|_R, \tag{42}$$

where the first k represents the energy, and the second k represents the angular momentum of the state.

The entanglement between the exterior and interior of the black hole leads to the entanglement between the left-moving edge state and right-moving edge state. Thus, similar to the fractional quantum Hall state case (40), the state for the Hawking radiation is pure:

$$|\Omega>= \frac{1}{\sqrt{Z}}\sum_{k>0} e^{-\beta_H(H_R+H_L)/2}|k,k>_R \otimes|k,-k>_L, \tag{43}$$

even though it appears as thermal. This state is an example of a maximally entangled state, and, for any right-moving state with angular momentum k, there is a left-moving edge state with opposite angular momentum $-k$. It is this entanglement that causes the state to be pure. This entanglement can be detected in principle, unlike the case for the Hartle–Hawking state for the eternal Schwarzschild black hole.

Similar to the static BTZ black hole, one can show that the Hawking radiation for Schwarzschild black holes is also pure.

5. Conclusions

In this paper, we derived the Hawking radiation from the boundary scalar fields. The Hawking radiation can be considered a superposition of the thermal radiation of the right- and left-moving sectors on the horizon at different temperatures $T'_{R/L}$. The entropy and energy of black holes have the same form as phonon gas in a one-dimensional circle, both for the BTZ black hole and the Kerr black hole. Based on these results, we propose a simple solution to the information loss paradox for static BTZ black holes and Schwarzschild black holes. The entanglement between the left-moving edge state (with angular momentum $k < 0$) and right-moving edge state (with angular momentum $k > 0$) is key to keeping the final state pure. Since the Hawking radiation is always pure, there is no information loss.

For Kerr black holes, the above procedure requires some modifications. One possibility is that a Kerr black hole first evolves into a Schwarzschild black hole caused by a spontaneous quantum superradiance emission effect. This Schwarzschild black hole can then radiate the Hawking radiation, which is always pure. However, the details require further investigation.

It is impossible to detect Hawking radiation for real black holes due to technical limitations. However, analogue Hawking radiation has been observed in an analogue black hole [31]. We hope that the entanglement between the left-moving and right-moving edge states (43) can also be observed in this type of experiment in the near future.

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