

Foundational Issues Relating Spacetime, Matter, and Quantum Mechanics

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Abstract. This article proposes the following themes: 1. Space time must be discrete at the micro level. 2. Holonomy is central to any foundational approach to relating spacetime and quantum mechanics. 3. The gravitational field equations should be trace free: gravity is essentially a conformal theory. 4. Times passes; past, present, and future are fundamentally different. 5. Causation is not only bottom-up: contextual effects occur, associated with symmetry breaking 6. Theories must adequately take account of the quantum measurement issue 7. Penrose's entropy issue is a real issue for cosmology that must be taken into account.

This paper I do not present here a coherent model of quantum gravity as a rival to any of the standard theories (for a survey see [64]). Rather I present some fundamental issues that I believe should be taken into account in any such eventual theory relating gravity and quantum physics. The sections are,

- (i) space time should be discrete at the micro level (Section 1);
- (ii) holonomy should be central to any foundational approach (Section 2);
- (iii) the gravitational equations should be trace free: gravity is essentially a conformal theory (Section 3);
- (iv) times passes; past, present, and future are fundamentally different (Section 4);
- (v) contextual effects occur, with associated symmetry breaking (Section 5);
- (vi) quantum gravity must adequately take account of the quantum measurement issue via contextual wave function collapse (Section 6);
- (vii) cosmology must take seriously Penrose's entropy issue (Section 7).

This will lead to a growing block universe based on a discrete substrate and with loss of information as time progresses. The gravitational equations will be based in holonomy in a suitable fibre bundle over a discrete foundation. The equations will be relational: they will be essentially conformal except for a matter term that will break the conformal symmetry and give scale to the whole. They will turn off gravity at very high densities so as to solve Penrose's entropy problem.



1. Space time discrete at the micro level

The basic viewpoint here is that no infinities should occur in a physical theory [41], so at its foundation spacetime should emerge from a discrete structure. The point to remember is that, contrary to the way infinity is treated in many physics papers, infinity is *not* just a very large number; it is in fact an unattainable state rather than a number of any kind. It will therefore never occur in the real universe, as stated by David Hilbert [55]:

“The infinite is nowhere to be found in reality, no matter what experiences, observations, and knowledge are appealed to.”

I adopt this statement, which I call *Hilbert’s Golden Rule*, as a foundational philosophical principle underlying physics applications.

Physics of infinity The distinction between mathematical infinity and physical effective infinity is set out in [41]. As far as potential physical infinities are concerned, there are two essentially different kinds:

- $\infty_{(VL)}$: ‘infinity’ being used as a placeholder for a very large number
- $\infty_{(ESS)}$: essential infinity, where the paradoxical nature of infinity (such as ‘Hilbert’s Hotel’ [54]) come into play.

The paradoxical nature of essential infinity is represented by the relations

$$\forall a, b, (i) \infty_{(ESS)} + a = \infty_{(ESS)}, (ii) b \times \infty_{(ESS)} = \infty_{(ESS)}. \quad (1)$$

No finite number can satisfy both these relations, so they can be taken together as an operational characterisation of infinity. It is precisely because infinity satisfies the relations (1) that it cannot occur in physical reality; in essence, it fails to obey conservation laws. Apart from some claims about multiverses, this does not appear to play a role in physics. Similarly, for “zero”, the dual of “infinity”, we can distinguish two physical kinds:

- $0_{(VS)}$: ‘zero’ being used as a placeholder for a very small number
- $0_{(ESS)}$: essential zero, dual to $\infty_{(ESS)}$.

This is characterised by the relations

$$\forall a, b, (i) 0_{(ESS)} + a = a, (ii) b \times 0_{(ESS)} = 0_{(ESS)}. \quad (2)$$

The first of these characterises zero as unphysical: in essence, it fails to obey the action - reaction rule, because it affects everything by (ii), but is affected by nothing, by (i). if it were to exist in some material sense, it would have no physical effect. Thus dual to the very large number $\infty_{(ESS)}$ one can determine a very small number $0_{(VS)}$ as an effective zero in various physical contexts.

Consequence The immediate application is that there is then an unacceptable infinity in standard physics views of spacetime: They assume there is an uncountable infinity of physically distinct points existing between my fingers when I hold them 10cm apart. This contradicts the principle just set out. This points to the need for discreteness at the foundations of spacetime (hence no such uncountable infinity of physical points characterised by local separation), Consequently spacetime must be discrete at the micro level, with an emergent spacetime continuum view emerging via coarse graining. Thus one needs something like causal set theory [23] [53] [90] but with extra structure in addition to the conformal (Section 3), or spin foam structure [72] [11]. Note that one needs discreteness at the foundations, not as an outcome, as is the case in Loop Quantum Gravity.

Basic statement In the *Feynman Lectures on Physics* [45], Richard Feynman said the most important statement one can make about physics is that “Matter is made of atoms” :

“If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words? I believe it is the atomic hypothesis (or the atomic fact, or whatever you wish to call it) that all things are made of atoms little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another. In that one sentence, you will see, there is an enormous amount of information about the world, if just a little imagination and thinking are applied.” ([45]:1-2).

In parallel, one can propose that Hilbert’s statement above (interpreted in this context) is:

“Spacetime is made of atoms”

- an extended quantum principle! Loop Quantum Gravity [75] suggests this is indeed the case.

2. Holonomy central to any foundational approach

The proposal here is the foundation of physics should a geometric approach based in parallel transport in a suitable fibre bundle over space-time, with holonomy (variation in the effect of parallel transport round different closed loops through a point p) determining the physical outcome. That holonomy should be a foundational aspect of theory is indicated by the importance of parallel transport in Yang-Mills theories [92] and the widespread applications of Berry Phase [13] [9]. That it is a key effect in quantum physics *per se* is indicated by the Aharonov-Bohm effect [3], which has been confirmed by various experiments, e.g. [82] [83] [67] [12]. Indeed one can even claim it is the core of the Feynman path integral method [47] which with a suitable definition of parallel transport of phase, essentially involves summing of holonomy over many closed paths¹.

Yang-Mills The basis of the standard model of particle physics [73] [4] is a gauge theory [51] [81] based on a non-abelian symmetry group [92]. There is a covariant derivative, with a vector potential as the connection, and the field given by commutators of the derivatives. The Lagrangian gives equations of motion which are equivalent to Jacobi identities, and path integrals lead to Wilson loop variables [89] [49] [81] which use holonomy to determine the outcome.

Combine with discrete spacetime The previous sections suggests this holonomy should be reformulated as a theory of fundamental physics, including gravity,² in a discrete spacetime context. This is obviously favourable to Wilson Loop variables and so Loop Quantum Gravity; however as pointed out above, that theory is not actually based in a discrete pre-spacetime. What is needed is something like spin foam evolution of a spin network [11], involving parallel transport in a suitable bundle over discrete structures; something like like causal set theory [16] [23] [53], but the conformal symmetry has to be broken to get scale.

3. The Vacuum Energy Problem: Trace-free Gravitational equations?

A major issue for present day theory is the problem of vacuum energy. Estimates of the Quantum Field Theory (‘QFT’) vacuum energy density suggest a huge value for that energy, discrepant with General Relativity if that vacuum gravitates [86]. It would be an effective cosmological

¹ A forward path γ_1 from p_1 to p_2 and a reverse path γ_2 from p_2 to p_1 form a closed curve $\gamma = \gamma_2 \circ \gamma_1$ based in p_2

² But not necessarily a unification of all four forces in a single framework, as in String/M theory. Indeed from a General Relativity perspective that is a questionable goal, as at a fundamental level, gravity is not a force. It is an effective result of space-time curvature [52].

constant at least 80 orders of magnitude larger than indicated by the Planck data [1] [2]. This is a major problem for basic physics, because there seems to be a flagrant contradiction when we combine two of our best tested physics theories (General Relativity and QFT) in the obvious way.

One way out suggested by a number of prominent physicists is that this is solved by existence of a multiverse together with anthropic selection. I will not discuss that option here, because (i) it seems a massive violation of Ockham's Razor (the word "infinity" is thrown around with gay abandon in these discussions, in contrast to the view presented in Section 1 of this paper), (ii) the underlying proposed physics is not well established or tested, and (iii) its status as a scientific theory is disputable [42]. Rather I will look for a solution in terms of querying what the true gravitational field equations are.

3.1. Trace-free equations

The key insight is that the vacuum does not gravitate if we use the trace-free Einstein equations ("TFE") plus separate conservation equations for matter [44], giving a theory essentially equivalent to what is often labelled as "unimodular gravity" [85] [48] [84]. This then solves profound contradiction arising between QFT and EFE if we join them in the obvious way: then the vacuum does not gravitate.

Taking the trace-free part Starting with the standard Einstein Field Equations ('EFE')

$$R_{ab} - (1/2)Rg_{ab} + \Lambda g_{ab} = \kappa T_{ab} \quad (3)$$

(10 equations), these equations identically implies energy-momentum conservation:

$$T^{ab}_{;b} = 0 \quad (4)$$

and the vacuum energy Λ_{vac} gravitates if we regard it as an effective fluid with equation of state

$$\{\rho_{vac} = -p_{vac} = \Lambda_{vac}\} \Rightarrow \{\nabla_a \rho_{vac} = 0, \rho_{vac} + p_{vac} = 0\} \quad (5)$$

where the first equality on the right follows from (4). Instead, we take the trace free part of (3):

$$R_{<ab>} - (1/2)Rg_{<ab>} + \Lambda g_{<ab>} = \kappa T_{<ab>}, \quad (6)$$

where $<, >$ just means trace free part, and use these as our gravitational equations. Explicitly, because $g_{<ab>} = 0$, (6) is the Trace Free Equations ('TFE'):

$$R_{<ab>} = \kappa T_{<ab>} \Leftrightarrow R_{ab} - (1/4)Rg_{ab} = \kappa(T_{ab} - (1/4)Tg_{ab}) \quad (7)$$

(9 equations), which no longer implies energy-momentum conservation (4); we have to assume that separately. The key point is that the energy density ρ_{vac} no longer contributes to spacetime curvature via the TFE (7), because of (5). Taking the trace of (7) to get the scalar part of the gravitational equations gives the identity $0 = 0$: the vacuum energy (5) no longer gravitates, because (7) has no scalar part. Taking the divergence of (7) and using (4), on integration one recovers (3), because the divergence of the left hand side of (3) vanishes identically; but now Λ is a constant of integration that has nothing to do with vacuum energy (it is unrelated to $T := T^a_a$). It can take any value whatever.

The options This possibility was pointed out already by Weinberg in [86], however he did not pursue it further. Indeed Einstein tried this in 1919 [29], but used a wrong form, because at that time it was not known how to handle symmetries of tensor equations consistently. There are four possibilities:

$$G_{ab} = \kappa T_{ab}, \quad (8)$$

$$G_{<ab>} = \kappa T_{ab}, \quad (9)$$

$$G_{ab} = \kappa T_{<ab>}, \quad (10)$$

$$G_{<ab>} = \kappa T_{<ab>}. \quad (11)$$

Only the first and last are acceptable (the second and third have different index symmetries on the left and right; Einstein tried the second). The first is the standard EFE; the last is the TFE that solves the GR-QFT incompatibility. It works out acceptably in the cosmological context, and in particular as regards inflation (see [34]), essentially because what matters in inflation is not the absolute value of the inflaton potential $V(\phi)$, but the difference $\Delta V := V(\phi_i) - V(\phi_f)$ between the values of the potential at the start and end of inflation.

This is related to unimodular gravity [48], [84],[85] where either general covariance is replaced by restricted covariance subject to $g := \det\{g_{ab}\} = \text{const}$, or one considers equivalence classes of solutions as g is varied (it is no longer a dynamical variable). One can obtain variational principles for such theories, as discussed e.g. in [7].

Quantum field theory version This is classical. What does a QFT version of gravity say? Various authors have discussed this, including Feynman, Deser, Weinberg, and Zee, but they have all in one way or another assumed that the gravitational equations imply conservation (4), which then necessarily leads to the standard equations (3). However because the graviton h_{ab} is a symmetric trace free tensor:

$$h_{ab} = h_{(ab)}, \quad h^a_a = 0 \Rightarrow h_{ab} = h_{<ab>} \quad (12)$$

when one regards gravitation as a field theory, this should also give the trace free version 7 of the gravitational equations. Specifically, one should assume energy-momentum conservation (4) separate from gravity equations and then add a term to the Lagrangian of the form.

$$L = T^{ab}h_{ab} = T^{ab}h_{<ab>} = T^{<ab>}h_{<ab>} \quad (13)$$

where (12) has been used at the second equality, and the third is a simple identity. Thus the trace T of T^{ab} does not occur in these equations, so it has no ability to affect space-time curvature. One should therefore necessarily get the trace-free equations (7) from this procedure. These therefore have a good claim to be the correct equations. More sophisticated analyses (e.g. [85]) confirm this view.

3.2. Is gravity essentially a conformal theory? The Weyl tensor as variable

If the determinant g is not a dynamical variable, one has essentially a conformal theory of gravity. This is related to the possibility of using the Weyl tensor C_{abcd} (considered as the free gravitational field, in analogy to the electromagnetic field F_{ab}) as the key gravitational variable rather than the metric. Then the Bianchi Identities act as dynamical equations (analogous to Maxwell's equations) for the Weyl tensor, with the Einstein Field Equations 3 determining the matter source terms in those equations. This relates to the idea that gravity is theory of 3-d or

4-d conformal degree of freedom, whereas matter with mass is not: it has timelike characteristics, as opposed to the null characteristics of the gravitational field [52].

A summary of these proposals, going back particularly to Petrov, Pirani, Ehlers, and Trümper,³ is given in [30], [59], and [40]. Relative to an observer's 4-velocity $u^a : u^a u_a = -1$, the Weyl tensor is split into 3-dimensional electric and magnetic parts:

$$E_{ac} = C_{abcd} u^b u^d = E_{<ac>}, \quad E^a_a = 0, \quad E_{ab} u^b = 0, \quad (14)$$

$$H_{ac} = {}^*C_{abcd} u^b u^d = H_{<ac>}, \quad H^a_a = 0, \quad H_{ab} u^b = 0, \quad (15)$$

each a spin-2 field. The Bianchi identities $R_{ab[cd;e]} = 0$ must be satisfied, and in 4 dimensions they are equivalent to

$$\nabla^d C_{abcd} = \nabla_{[a} (R_{b]c} + (1/6) R g_{b]c}). \quad (16)$$

Using the EFE (3) to algebraically substitute for R_{bc} in terms of T_{bc} , one obtains firstly the constraints (where D^b is the spatial covariant derivative orthogonal to u^a),

$$D^b E_{ab} = -3\omega^b H_{ab} + \frac{1}{3} D_a \rho + [\sigma, H]_a \quad (17)$$

$$D^b H_{ab} = 3\omega^b E_{ab} + (\rho + p)\omega_a - [\sigma, E]_a, \quad (18)$$

in obvious analogy with the divergence Maxwell equations, and second the evolution equations:

$$\frac{dE_{<ab>}}{dt} - \text{curl} H_{ab} = -\theta E_{ab} + 3\sigma_{c<a} E_{b>}^c - \omega^c \epsilon_{cd(a} E_{b)}^d + 2a^c \epsilon_{cd(a} H_{b)}^d - \frac{1}{2}(\rho + p)\sigma_{ab} \quad (19)$$

$$\frac{dH_{<ab>}}{dt} + \text{curl} E_{ab} = -\theta H_{ab} + 3\sigma_{c<a} H_{b>}^c - \omega^c \epsilon_{cd(a} H_{b)}^d - 2a^c \epsilon_{cd(a} E_{b)}^d - \frac{1}{2}(\rho + p)\sigma_{ab} \quad (20)$$

again in analogy with the evolutionary Maxwell equations. This agrees in essence with the TFE, because in the vacuum case, $(\rho + p) = 0$, $D^\rho = 0$) so the matter does not gravitate in the sense of entering as a source term in these equations for the free gravitational field C_{abcd} . This suggests that one in going to quantum gravity, one should quantize the Weyl tensor (or its potential: some connection terms) rather than a 'graviton' h_{ab} as mentioned above.

Weyl-squared theories How does this relate to conformal theories using the square of the Weyl tensor in the Lagrangian? [60] Suppose that in analogy to the way there is a term $S_{em} = -\alpha \int d^4x \sqrt{-g} F^{ab} F_{ab}$ in QED to represent the electromagnetic field, we replace the gravitational term $S_{grav} = -\alpha_{grav} \int d^4x \sqrt{-g} R$ in the Lagrangian by

$$S_{grav} = -\alpha_G \int d^4x \sqrt{-g} C^{abcd} C_{abcd} \quad (21)$$

Then one obtains for the gravitational sector on its own the Bach equation

$$2\nabla^a \nabla^d C_{abcd} + C^d_{abc} R^a_d = 0 \quad (22)$$

This is a field equation for C_{abcd} , which is subject to the identity (16). Matter terms have to be added in to get the full theory [60]. One can rewrite the action in terms of Ricci tensor variables because the Gauss-Bonnet invariant $\tilde{G} := \int d^4x \sqrt{-g} (R_{abcd} R^{abcd} - 4R_{ab} R^{ab} + R^2)$ vanishes for space-times topologically equivalent to Euclidean space.⁴ The key point is that although these are higher derivative theories ([19]:§4.2), they reduce to the GR equations in the Ricci flat case ($R_{ab} = 0$) and the conformally flat case ($C_{abcd} = 0$) and so include as solutions both the Schwarzschild solution and the FLRW solution [60]. However, in common with other theories, the Newtonian limit is tricky (see Appendix A). Perturbation solutions in cosmology can be found [61] [8] I do not pursue this further here, but note it as a promising line to investigate.

³ Trautmann, Bondi, Sachs, Newman, and Penrose developed the null version of these equations

⁴ More generally, it is given by the Euler characteristic $\chi(M)$ of the manifold.

3.3. The deep issue: The origin of scale

the deep underlying issue is that one can claim that all physics measurements are relative measurements, so that one should only use dimensionless variables in basic equations (see e.g. the literature on ‘Shape Dynamics’, and [79]). If so, where does scale come from? It has to be via broken symmetries [10]. At one level, the answer is that the Higgs term breaks the conformal symmetry of the Standard Model of particle physics, and so provides a scale for the matter term in the EFE through the matter source terms in (17)-(20). At a deeper level, this does not answer the problem, and still needs resolution.

4. Times passes; past, present, and future are fundamentally different.

In contrast to many claims in the literature, my view is that spacetime should such that time passes, in accord with both everyday experience and the laws of macroscopic physics and physical chemistry. Thus spacetime should be an *Evolving Block Universe* (‘EBU’) [33], [39], where a genuine passage of time takes place at both macro and micro levels because the future boundary of spacetime is continually advancing. This is essentially taken for granted in most cosmological analyses. Thus for example, the age of the universe was measured by the Planck Satellite team in 2015 as $t_0 = (13.813 + T_0) \times 10^9$ years where $|T_0| < 0.038$. In 2018, it is clearly 3 years older: the age of the universe is $t_0^* = t_0 + 3$. It is taken for granted by the Planck team that the Age of the Universe is a meaningful concept. This requires an EBU view of spacetime.

4.1. Flow of time: Evolving spacetime

Consider a massive object with two computer controlled rocket engines that move it right or left. Let the computer determine the outcome on the basis of measurements of decay products of excited atoms. Then because of the foundational nature of quantum uncertainty [32], as conclusively proved by the 2-slit experiment and by the decay of radioactive atoms [46], the outcome is unpredictable in principle. If the object is massive enough, it curves spacetime. The future spacetime structure is therefore not determinable or predictable from current data

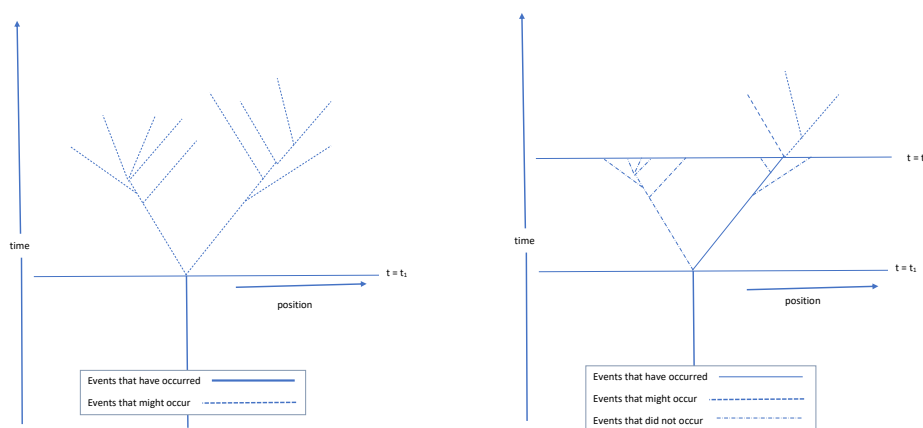


Figure 1. The change from uncertainty to certainty: the present is where the indefinite future changes to the determined past.

The evolving block universe grows with time: the determined past is there and fixed, the indeterminate future is not yet there and still has to be fixed (Figure 1). In this case, it is not true that the future is determined at the present. The present $t = t_0$ at any instant is where the change from possibility to definiteness takes place. It is crucially different from the past and future, and indeed separates them. At any time t_0 , The future $\{\mathcal{F}\} : t > t_0$ does not exist in the

same sense as the past $\{\mathcal{P}\} : t < t_0$ or the present $\{\mathcal{T}_0\} : t = t_0$, because it is not yet clear what will occur there. The determinate region grows with time. Spacetime itself is growing, because its future boundary $t = t_0$ is continually extending (this is what distinguishes it from the past boundary of spacetime at the initial big-bang at time $t = 0$).

The Direction of Time This leads to the existence of the global Direction of Time, determined by the Universe as a whole. In the expanding universe, the start is a fixed initial boundary to spacetime. The present is an ever changing future boundary to spacetime: as time passes it moves to the future, in the direction indicated by the Direction of Time. At each instant spacetime exists from the start up to the present, but not to the future (because of quantum randomness, it is not yet determined what will happen in the future, so it is not a spacetime domain with definite properties), as indicated in Figure 2).

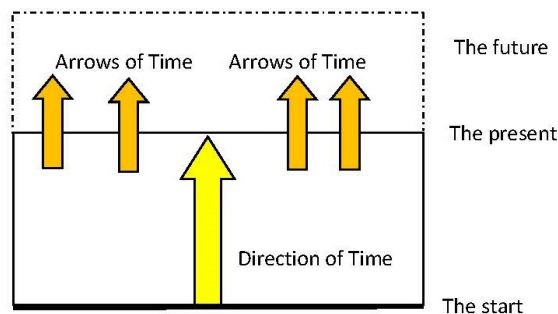


Figure 2. *The global Direction of Time is set by the expansion of the universe since its start. Local Arrows of Time are determined by local physical properties (Section 5.3).*

The time surfaces The standard objection to this proposal is that it will not work because according to Special Relativity, the surface of simultaneity $\{t = t_0\}$ depends on the motion of the observer, and so is not well defined. The response is that the big news is that Special Relativity does not describe the real universe: General Relativity does! [52]. In any realistic cosmological spacetime, there is a preferred family of observers whose motion is determined by the curved spacetime geometry as the timelike eigenvector⁵ of the Ricci tensor [30] [40]. What happens physically at macro scales is determined along timelike world lines⁶ rather than on spacelike surfaces (simultaneity is irrelevant to physical interactions!)

Thus the time surfaces in the EBU are secondary to uniquely defined timelike world lines (eigenlines of the stress tensor T_{ab}). To determine the time surfaces, start at the beginning of time $t = t_0$ (here, I assume the universe had a start), and measure proper time τ along fundamental world lines from $t = 0$ to the present event $t = 0$. Then the surfaces $\{\tau = \text{const}\}$ are the surfaces of constant proper times since the start of the universe that determine the present transition time $\tau = \tau_0$ at each instant as time evolves along the fundamental world lines. these are uniquely defined, as there are no vacuum regions in a realistic model of the universe (the Cosmic background Radiation is non-zero everywhere) Thus the view is that so change happens on preferred surfaces, secondary to the timelike world lines, that need not even be spacelike.

⁵ For realistic matter, this will be unique.

⁶ The average motion of radiation takes place on timelike world lines: it is an almost perfect fluid with $p = \rho/3$.

The ADM formalism To express this in the ADM formalism [6], choose a family of spacelike surfaces with lapse function $N(x^i)$, metric $g_{ij}(x^k)$, and second fundamental form $\pi_{ij}(x^k)$, together with a family of timelike world lines with shift vector $N_i(x^j)$. The field equations for $g_{ij}(x^k)$ are as follows (where 3-dimensional quantities have the prefix (3)): four constraint equations

$$^{(3)}R + \pi^2 - \pi^{ij}\pi_{ij} = 16\pi\rho_H, \quad (23)$$

$$R^\mu := -2\pi^{\mu j}{}_{|j} = 16\pi T_0^\mu, \quad (24)$$

where $|j$ represents the covariant derivative in the 3-surfaces, and twelve evolution equations

$$\partial g_{ij}/\partial t = 2Ng^{1/2}(\pi_{ij} - \frac{1}{2}g_{ij}\pi) + N_{i|j} + N_{j|i}, \quad (25)$$

$$\partial \pi_{ij}/\partial t = -Ng^{-1/2}(^{(3)}R_{ij} - \frac{1}{2}g_{ij}^{(3)}R) + F_{ij}(\pi, N, N_{|i}) + 16\pi^{(3)}T_{ij}. \quad (26)$$

We choose a unique gauge by specialising the time surfaces and flow lines in that formalism to those defined above.

1. We choose the flow lines to be Ricci Eigenlines:

$$T_0^\mu = 0 \Rightarrow R^\mu = 2\pi^{\mu j}{}_{|j} = 0 \quad (27)$$

which algebraically determines the shift vector $N_i(x^j)$, thereby solving the constraint equations (24);

2. We determine the lapse function $N(x^i)$ by the condition that the time parameter t measures proper time along the fundamental flow lines:

$$ds^2 = -d\tau^2 \Rightarrow N^2 = 1 + N^i N_i \quad (28)$$

These conditions together uniquely determine the lapse and shift.

What determines how the matter evolves? Equations of state for the matter terms $\{\rho_H, T_0^\mu, ^{(3)}T_{ij}\}$ in (23), (24), (26) must be added, and the matter conservation equations (4) added to the set of equations to be satisfied. Then the evolution is determined:

ADM Theorem: *Given the equations of state and dynamical equations for the matter, equations (25), (26) determine the time evolution of the metric in terms of proper time along the fundamental flow lines from initial data at some time t_0 ; if they are initially satisfied at time $\{t = t_0\}$, the constraints (23), (24) are conserved because of energy-momentum conservation (4).*

The development of spacetime with time takes place just as is the case for other physical fields, with the relevant time parameter being proper time along the fundamental flow lines. There is no problem with either the existence or the rate of flow of time. The spacetime develops accordingly via (25), (26). Time passes at the rate of 1 second per second [63], as determined by the metric tensor locally at each event. In this evolving block universe, the present is where the future changes to the past, which takes place along timelike worldlines [39]. This construction automatically provides chronology protection, because a space-time singularity will occur if these eigenlines intersect.

Predictability Do these equations mean the spacetime development is uniquely determined to the future and the past from initial data? That all depends on the equations of state of the matter content. For a perfect fluid with barotropic equation of state $p = p(\rho)$, the answer is yes. However one can have an explicitly time dependent equation of state:

$$p = 1/3 g^{ij} {}^{(3)}T_{ij}, \quad \pi_{ij} = {}^{(3)}T_{ij} - p g_{ij} = F(\tau) \Pi_{ij}(0) \quad (29)$$

where $F(\tau)$ represents local dynamics, which could possibly involve random processes generated via quantum uncertainty (indeed this is the mechanism that underlies the possibility of Figure 1 above). So equations (25), (26) determine the time evolution of the spacetime, but do not guarantee predictability; this is what underlies the uncertainty of evolution depicted in Figure 1. If quantum unpredictability gets amplified to macro scales, the spacetime evolution is intrinsically undetermined till it happens (as during the generation of seed inhomogeneities in the inflationary era in the very early universe [66]).

The quantum gravity case This discussion has looked at physics occurring in an evolving classical spacetime, but with quantum fields in that spacetime. One should require the same kind of evolving spacetime structure to occur in a viable theory of quantum gravity. That seems to be the case for example for both spinfoam [72] [11], representing a growing spacetime, and causal set theory [23] [90] [91].

5. Contextual effects

Because gravity is the dominant long range force (as a consequence of gravitational mass always being positive), so larger scale effects can accumulate to influence smaller scales in crucial ways, and can even diverge.⁷ Thus top-down (contextual) effects occur ([36]: Chapter 6).

5.1. The traditional contextual effects

The traditional such top-down effects (i.e. those discussed before 1965) are [78] [17] [31]

- *Mach's Principle*: what is the origin of inertia? Newton's bucket experiment suggests it is related to the most distant matter in the universe.
- *Olber's paradox*: why is the night sky dark? A simple calculation in a static universe suggests the energy density of radiation received from all the stars in the universe should diverge (each shell with the same $\Delta R := R_2 - R_1$ adds the same amount of radiation).
- *The arrow of time*: how does a direction of time arise out of underlying time-symmetric physics? Boltzmann's derivation of the H-theorem from kinetic theory does not in fact explain the Second Law of Thermodynamics $dS/dt > 0$, because that derivation remains true if one sets $t \rightarrow t' := -t$, thus it also shows that $dS/dt' > 0$.

The issue of boundary conditions at infinity influencing local conditions led to Einstein's and Wheeler's preference for spatially closed universes, where infinity does not occur [88] (in concordance with Section 1). Olber's paradox is now resolved by our Hot Big Bang models [26] [74] which explain why the temperature of the night sky is $2.7^\circ K$. Mach's principle is unresolved. The arrow of time remains a key issue, discussed below (Section 5.3).

⁷ Which is why Newton never succeeded in deriving a cosmological theory based in summation of gravitational forces acting on a particle in an infinite universe: this is the spatial version of Olber's paradox.

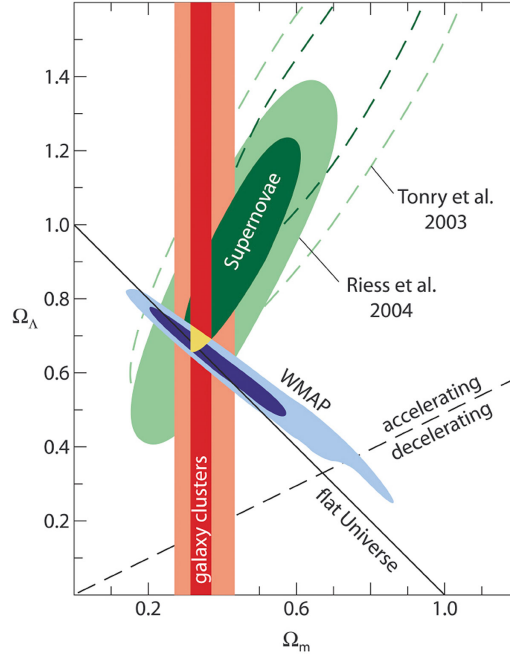


Figure 3. Testing model by standard candles (supernovae) versus by contextually shaped outcomes of structure formation in the expanding universe (galaxy clusters), and their effect on CMB anisotropies as measured by WMAP. The latter two top-down effects give stronger constraints on the cosmological matter density parameter Ω_m and cosmological constant parameter Ω_Λ than the direct measurements of cosmological parameters via the supernovae. Credit: ESO

5.2. Emergence of structure: cosmology and astrophysics

The theme that the flow of causation is not only bottom up, from smaller to larger scales, but it is also top down [36], now has other important manifestations in the standard cosmology that has developed since the 1960s [22] [26] [74]. Specifically

- *Nucleosynthesis* in the early universe depends on the cosmological rate of expansion $a(t)$ at that time, which is determined by the total cosmological density $\rho(t)$ of matter and radiation then (a global parameter);
- *Structure formation* in the later universe is similar: gravitational instability develops structure (density inhomogeneities $\delta\rho$) in a way that also depends on the background expansion rate $a(t)$, which again depends on the cosmological model parameters $\rho(t), \Lambda$; for example in the pressure-free case, the comoving fractional density parameter \mathcal{D} grows according to [38]

$$\ddot{\mathcal{D}} + \frac{2}{3}\theta\dot{\mathcal{D}} - \frac{1}{2}\kappa\rho\mathcal{D} = 0 \quad (30)$$

where θ is the cosmological rate of expansion, determined by the Friedmann equation, and ρ is the cosmological density of matter.⁸

- Because these processes depend on the background cosmology, observing their outcomes is the most sensitive way of determining the background model parameters. Thus the

⁸ Thus in a static universe $a(t) = a_0$, $\delta\rho$ grows exponentially, while in a universe expanding as a power law, $\delta\rho$ grows as a power law.

perturbation equations determine the power spectrum $P(k)$ of matter density fluctuations [22], which then determine the angular power spectrum $\mathcal{C}(l)$ of CMB anisotropies [77] [26]. Measuring these matter and radiation power spectra provides the best test of cosmological parameters [1] [2], see Figure 3.

Thus the fact that the cosmology provides the context for local existence determines the nature of what can exist! This applies also to the vexed question of why the cosmos is of such a nature as to allow life to exist (the *Anthropic* issue: [35]:§6.1), which I will not pursue further here.

5.3. The direction of time

Micro physics is time symmetric, except for extremely weak time-symmetry breaking by weak interactions, which does not affect everyday life at the present time.⁹ How can a difference emerge between the future and the past, on the basis of time symmetric underlying microphysics?

- How does macro physics know the direction of time?
- Why does time flow the same direction everywhere?

There is no basis for such a determination in microphysics alone: because of the symmetry $T : \{t \rightarrow -t\}$ in that underlying foundation, the H-theorem derived from it (in the classical case by Boltzmann, and in the QFT case by Weinberg [87]) applies equally in both directions of time (this has to be so, because of the T symmetry of those derivations)¹⁰. Microphysics cannot provide a foundation for the second law of thermodynamics with a unique arrow of time. In an EBU (section 4.1), the arrows of time arises fundamentally because the future does not yet exist: a global asymmetry in the physics context, which affects local physics in a top-down way.

Historical effects In the evolving block universe (Section 4), the past exists and is developing to the future, which does not yet exist. It is this asymmetry that leads to the arrows of time associated with historical effects. One can be influenced at the present time from many causes lying in our past, as they have already taken place and their influence can thereafter be felt today. One cannot be physically influenced by causes coming from the future, for they have not yet come into being. This is the rationale for saying the past exists but the future does not: if something can influence you, it exists. The past affects today as regards:

- Existence of elements (C,N,O, ...) on Earth, which arose in the past through primordial and stellar nucleosynthesis
- Arrival of particles (cosmic rays) due to events which took place in the past.
- Arrival of radiation from the past, both from nearby sources (e.g. the Sun) and the distance past (e.g. the Cosmic Black Body Radiation).

We have no such effects (at the macro scale) causing effects here and now due to future events; indeed they do not yet exist, as what they will be is not yet determined, due to quantum uncertainty (Section 4.1). In summary: the future can't affect today, because it is not yet definite what it is: future possibilities exist, not specific outcomes. That is a core feature of quantum theory (Section 6).

Local Arrows of time This broken symmetry then affects microphysics. The global direction of time leads to local *Arrows of Time*, which include quantum, electrodynamic, gravitational, thermodynamic, biological, and psychological arrows of time in the local physical context. Most important are

⁹ It might have played a role in the emergence of the matter-antimatter asymmetry in the early universe.

¹⁰ Weinberg's derivation does not include the aspects of the weak force that break time symmetry.

- The *Second Law of Thermodynamics*, crucially affecting macrophysics, physical chemistry, and biology [28]. This arrow occurs only because of special initial conditions in the early universe (a *Past Condition* [5] [70]): the universe must start off in a low entropy (smooth) state, else entropy cannot increase. This condition is required in order that the thermodynamic arrow of time be well defined and agree with the Direction of Time.¹¹
- The *electrodynamic arrow of time*: Electromagnetic radiation is received after it is sent. From a QED viewpoint, this is because the Feynman propagator can only be integrated over the past, as the future spacetime domain does not yet exist in an EBU. Advanced and retarded Feynman propagators are not on an equal footing, because of the cosmological context. The same will apply to gravitational radiation.
- A *quantum arrow of time*, discussed in the next section.

The other arrows of time (biological, psychological) will follow from these. They are all determined in a top-down way by the EBU context, together with the Past Condition.

6. Theories must adequately take account of the quantum measurement issue

It is crucial for physical theories to face the measurement problem for quantum physics, and for foundational theories to take some stance on this issue as regards quantum gravity. The key point is that we only get specific results from quantum physics, via the Born rule, if some form of collapse of the wave function to an eigenstate occurs; otherwise the probabilities represented by the wave function ψ are never instantiated, hence it has no physical meaning. In brief, an ensemble of events, to which one can apply statistics, only exists if individual events occur.

One particular area where this is relevant in cosmology is the issue of how inflationary perturbations become classical, which is ignored by almost everyone except Sudarsky and colleagues (e.g. [58]). Until this is resolved, there is a major lacuna in current cosmology.

Measurement Problem The quantum measurement process is, according to the standard view¹² ([46], [32] and references there) that the quantum to classical transition is associated with quantum *events*: collapse of the wave function to an eigenstate, which happens in a contextual way [25] (e.g. what is measured in an experiment depends on what apparatus is used and when it is turned on). If a measurement of an observable A takes place at time $t = t_*$, initially the wave function $\psi(x)$ is a linear combination of eigenfunctions $u_n(x)$ of the operator \tilde{A} that represents A : for $t < t_*$, the wave function is

$$\psi_1(x) = \sum_n c_n u_n(x). \quad (31)$$

But immediately after the measurement has taken place, the wave function is an eigenfunction of the operator \tilde{A} :

$$\psi_2(x) = a_N u_N(x) \quad (32)$$

for some specific value N . The data for $t < t_*$ do not determine the index N ; they just determine a probability $p_n = |c_n|^2$ for each outcome n (this is the Born rule). One can think of this process as due to the probabilistic time-irreversible collapse of the wave function; it cannot occur as a result of the unitary Schrödinger eqn. This process is irreversible: information is lost, because knowing N for $t > t_*$ does not determine the c_n for $t < t_*$. This irreversible nature of physics and loss of information at the micro scale implies an irreversible nature of physics, and loss of information at the macro level when micro uncertainty gets amplified to macro levels [71].

¹¹ Inflation does not solve this issue: see [70] and Section 7.

¹² There is a group of quantum cosmologists who vociferously defend the Everett ('many worlds') interpretation, but they are a very small fraction of physicists overall who work with quantum theory. The pilot wave theory also has strong proponents, but again they are a small minority of quantum theorists.

Contextual Wavefunction Collapse But the question is how does such a collapse process take place? In “Contextual Wavefunction Collapse: An integrated theory of quantum measurement” [25], Barbara Drossel and I propose one can solve the quantum measurement problem via the top-down effect of local physical context, with no extra *ad hoc* term introduced in the Schrödinger equation. This proposal is based in two previous papers: “On the limits of quantum theory: Contextuality and the quantum-classical cut” [32], which considers under what conditions one may expect quantum physics to describe a physical system as a whole (its component parts at the lowest levels will of course always be described by quantum theory), and “Ten reasons why a thermalized system cannot be described by a many particle wave function” [24].

These papers together make a strong case that there is no meaningful “Wave Function of the Universe” in any realistic sense (remember that quantum cosmology operates in the context of ‘mini-superspace’, in which all physical degrees of freedom except a few are ignored), and if there is, there is no reason to believe it will obey a unitary equation (*inter alia*, the universe contains many thermalised systems). On the contrary, physics is not causally complete at the quantum level: events take place, time passes, and information is lost at this level too.

Information loss? There is no law of information conservation in quantum physics, provided one takes measurement events into account (it was emphasized above that if we do not, the wavefunction has no meaning). On the contrary: information is lost at every “event”, and in particular at every measurement. This is associated with the passage of time and the global direction of time, as discussed in [25]. The real world does not evolve in a unitary fashion! [32]. This may relate firstly to the “information loss paradox” as regards black holes (it may well be that one can regard the emission of Hawking radiation as involving wave function collapse and hence loss of information), and the issue of how quantum perturbations in inflation become classical.

7. Penrose entropy issue

The fine tuned initial state While it is often stated that inflation solves the flatness and horizon problems in the early universe, that is not in fact the case, as Penrose has pointed out in various writings that are summarised in Chapter 3 of *Fashion, Faith, and Fantasy* [70]. The essential point is that the maximum entropy of a given amount of matter is attained by collapsing it into a black hole; the most probable early universe state is a plethora of black holes. By contrast, “the Big Bang was an event of extraordinarily low entropy ... the gravitational degrees of freedom were completely suppressed” Penrose estimates the extraordinary precision that was involved in setting the initial state of the universe as $10^{10^{123}}$. This situation is not made clear by standard inflationary studies because they consider only perturbed Robertson-Walker geometries. It is not solved by inflation: rather inflation needs special initial conditions, not dominated by the large number of black holes that would correspond to maximum entropy, in order to start.

There is a counter view held by Kleban and colleagues [27] [56] [20] [21] that this is incorrect: inhomogeneous initial conditions will lead to inflation as usually claimed, and the Penrose argument is incorrect because the maximum entropy state is that given by de Sitter spacetime (because of the entropy associated with its cosmological event horizon.) I do not concur. While those models are certainly a great improvement over starting with perturbed FLRW models, they are not inhomogeneous enough to include the kind of multi-black hole situation envisaged by Penrose; and the (constant) entropy associated with that cosmological horizon, if it has any physical meaning at all, is only associated with observers in the very far future of the universe - long long after the present day, and even further to the future of the pre-inflationary situation that is Penrose’ concern. It has no relevance to physical effects at those early time.

Gravitational entropy The issue here is the nature of gravitational entropy in general, not only in the case of black holes. The real gravitational entropy problem is the following: start off a gas in a very large container in an inhomogeneous state. Without gravity, the gas will relax to its most probable state of uniformity. Turn on Newtonian gravity, and gravitational instability will result in a non-uniform final state with matter clustered into stars. Turn on general relativity, and the final state will be a non-uniform state of matter clustered into stars and black holes ([70]:258). In the expanding universe, the latter is what indeed happens during the “dark ages” when the first stars form and black holes are not dominant, if they exist at all at that time. How this is compatible with the Second Law of Thermodynamics, which tell us the matter should tend to a uniform state? The only answer has to be that we must associate gravitational entropy in a general spacetime with the free gravitational field, that is, with the Weyl tensor (as implied by Penrose in [68], [69]:765-768).

Penrose has not given a specific formula for the gravitational entropy of a general spacetime (as opposed to a black hole, where everyone agrees on the Bekenstein-Hawking definition). However Clifton *et al* [18] propose such a formula for gravitational entropy based on the Bel-Robinson tensor: a square of the Weyl tensor that is the effective super-energy-momentum tensor of the free gravitational field. The detailed form of this entropy proposal in the Coulomb-like case shows that gravitational entropy increases as structure formation occurs in the expanding universe. This allows overall entropy (matter and gravity) to increase as structure forms, which is not true if one does not take this gravitational entropy into account. It is also in keeping with Penrose’s Weyl curvature hypothesis [70] that the Weyl tensor must be zero in the limit at the initial singularity, which is a form of Past Condition to allow local thermodynamics to function the way it does (Section 5.3).

This gravitational entropy vanishes in the de Sitter universe; that spacetime therefore cannot be a state of maximum entropy, as claimed by Kleban. The same is true for any other definition of gravitational entropy based in the Weyl tensor. The most probable (highest entropy) initial gravity state is highly inhomogeneous, as claimed by Penrose.

Turning off gravity at very high densities Penrose proposes to solve this problem of how to avoid massive initial inhomogeneity by invoking a Conformal Cyclic Cosmology [70] This is a creative idea, but the mechanisms involved in realizing such a proposal are not all clear.

By contrast, consider a mechanism proposed by Greene *et al* [50]: suppose the strength of the gravitational force dies away at very high energies. In that case the collapse to black holes that causes the problems identified by Penrose will not occur: thermal processes will be able to overcome gravitational forces in the very early universe and prevent collapse to blackholes; the most probable state will be smooth. The gravitational degrees of freedom will not be suppressed, they will simply not dominate the degrees of freedom of ordinary matter as in the standard case considered by Penrose. Greene *et al* state [50], in response to the issue of collapse leading to highly inhomogeneous situations [68][69],

Our scenario does pass one crucial test, avoiding the double standard. Often natural-seeming scenarios, such as a ball rolling down a hill and coming to a stop due to friction, become completely unnatural viewed in time reverse. In our scenario, the time reverse is completely natural: gravity is strong and the universe starts in a clumpy state. Then at some point gravity turns off. The clumps start to dissolve into a smooth state. The state at all times is natural with respect to the laws operating at those times. Passing the double standard test requires that the universe spend a sufficient period of time in the weak gravity phase, otherwise there will not be sufficient time (in the time reversed sense) for a generic initial clumpy state to smooth out. In our model, we accomplish this by tuning the parameter h in the Lagrangian. The initial conditions are allowed to be generic.

What is then particularly intriguing is that one can have a similar scenario with the Higgs as the inflaton [80]. This is highly desirable, as this is the only case in which the inflaton is related to testable and indeed tested particle physics [43].

Appendix A. Conformal gravity, local sources, and alternative theories of gravity

Mannheim and Kazanas [62] investigated the Newtonian limit of conformal gravity in and obtained the following exact equation of motion:¹³

$$\frac{3}{B(r)}(W_0^0 - W_r^r) = B'''' + \frac{4B'''}{r} = \frac{(rB)'''}{r} = \nabla^4 B = \frac{3}{4\alpha_g B(r)}(T_0^0 - T_r^r) = f(r) \quad (\text{A.1})$$

where $B(r) = -g_{00}(r)$. For a source localized to a region $r < r_0$ the solution is

$$B(r) = 1 - \frac{2\beta}{r} + \gamma r \quad (\text{A.2})$$

where

$$\gamma = -\frac{1}{2} \int_0^{r_0} dr' r'^2 f(r'), \quad 2\beta = \frac{1}{6} \int_0^{r_0} dr' r'^4 f(r') \quad (\text{A.3})$$

They pointed out that if the source was a delta function then the coefficient β would be zero. Thus it had to have a more singular structure that would hold at the level of the microscopic sources in a star. Thus the source that gives the Newton term has to be more singular than the source that one uses in standard gravity. They also pointed out that if $f(r)$ was uniform, one would obtain $\beta \sim r_0^2 \gamma$, and that could be ruled out by solar system measurements. Thus the source would have to be far from uniform.

Now in a conformal invariant theory, all mass has to be generated by dynamical symmetry breaking fields, and without them sources could not even localize into sources at rest. Thus the energy-momentum tensor of the source must contain dynamical fields and could not be approximated by a kinematic perfect fluid in which $\rho \gg p$ (a condition that would anyway violate the tracelessness of a conformal invariant energy-momentum tensor). In consequence one cannot infer the sign of the β integral by just giving it the same sign as the T_{00} term is given in GR. (In the conformal theory T_{rr} is just as big as T_{00} anyway.)

Alternative gravity Now one may or may not feel that this result rules out this particular theory. But the issue here is bigger than that. From Lovelock's Theorem [57], as nicely characterised by Clifton *et al* [19], the only alternative gravity options are that one can

- (i) add other fields in addition to the metric tensor,
- (ii) add higher than second order derivatives of the metric,
- (iii) consider other than four spacetime dimensions,
- (iv) consider non-locality, such as arises through the inverse d'Alembertian,
- (v) propose emergent gravity, where the field equations do not arise from an action.

Any alternative theory of gravity (i) or (ii) that adds other fields or leads to higher derivative equations will face similar issues: they are not confined to this particular theory.

¹³ I thank Prof Mannheim for the following note

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