# SIGNAL PROCESSING FOR RADIATION DETECTORS

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## ABSTRACT

In this series of lectures some fundamental aspects of signal processing for various particle and photon detectors will be discussed. The principal topics will be:

- Charge collection and signal formation in detectors.
- Physical sources of noise and fundamentals of amplification, low noise devices and preamplifiers.
- Noise and filtering. Amplitude, time and waveform measurements.
- Position sensing methods.
- Signal processing for semiconductor detectors, drift and time projection chambers, and for ionization chamber calorimeters.
- Signal processing circuits, role of hybrid and monolithic technology.

Examples illustrating joint optimization of the detector and the signal processing will be presented.

## OUTLINE OF LECTURES

Lecture: Time:

1. Signal formation in detectors.

   Noise: Origin and Properties.

2. Physical sources of noise.

   Charge amplification and related noise.

   Filtering; amplitude, time and waveform measurements.

   Some circuit considerations.

3. Position sensing methods:

   Delay line sensing, charge division,
   centroid finding methods.

4. Drift chambers and proportional detectors for very high counting rates.

   TPCs and "pad" detectors.

   Semiconductor detectors.

   Ionization chamber calorimeters.

# Lectures on Signal Processing for Radiation Detectors

1. Signal formation in detectors.

   Noise: Origin and Properties.

2. Physical sources of noise.

   Charge amplification and related noise.

   Filtering; amplitude, time and waveform measurements.

   Some circuit considerations.

3. Position sensing methods

   (Charge division, delay line sensing, centroid finding methods).

4. Drift chambers and proportional detectors for very high counting rates.

   TPC's and "pad" detectors.

   Semiconductor detectors.

   Ionization chamber calorimeters.

References

# Motivation :

## Detectors of interest :

1. Drift and proportional chambers.
2. TPC's and "pad" detectors.
3. Semiconductor " .
4. Ionization chamber calorimeters.
5. Special detectors for neutrino, proton decay experiments.

## Devices, circuits and technology

1. Amplifying devices.
2. Circuit configurations.
3. Technology, hybrids and monolithics.
4. Detector design (electrodes).

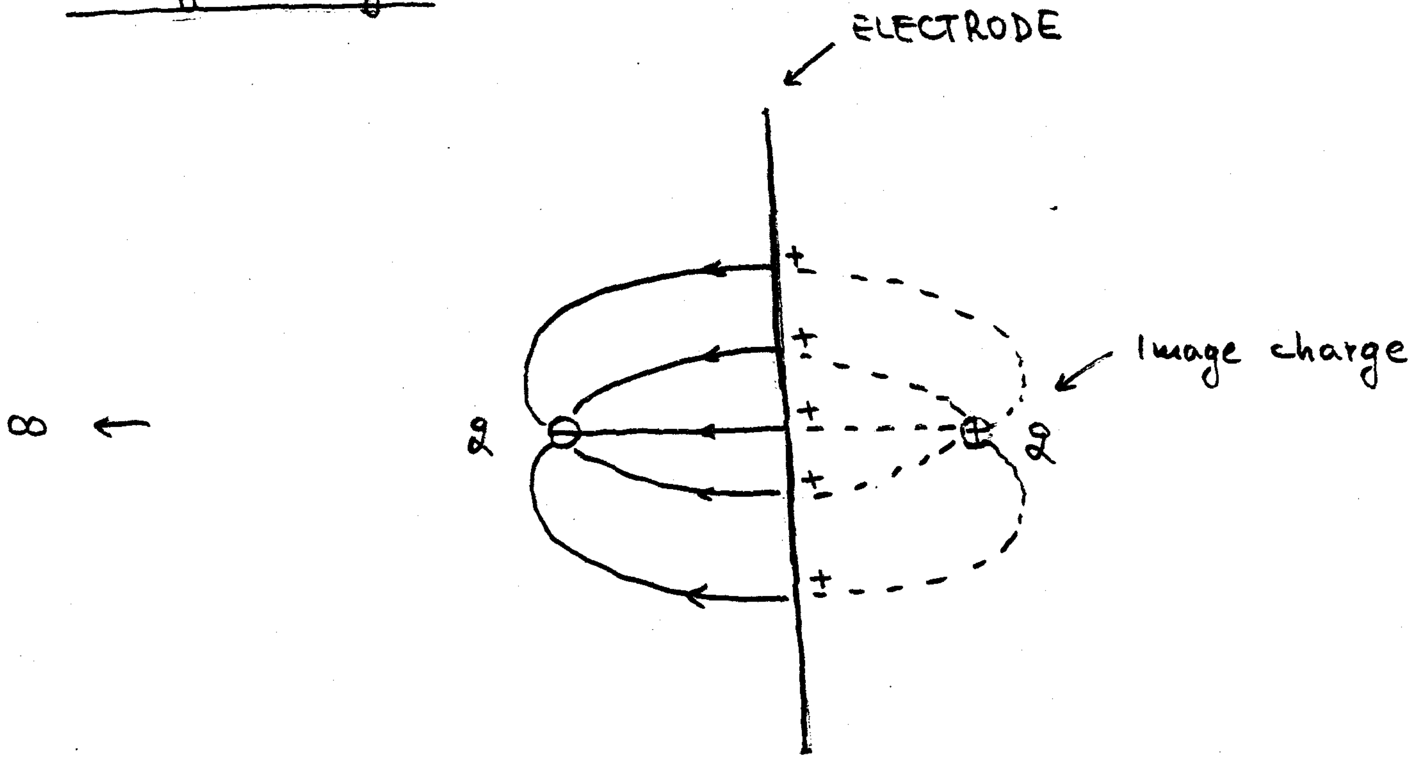
## Basics

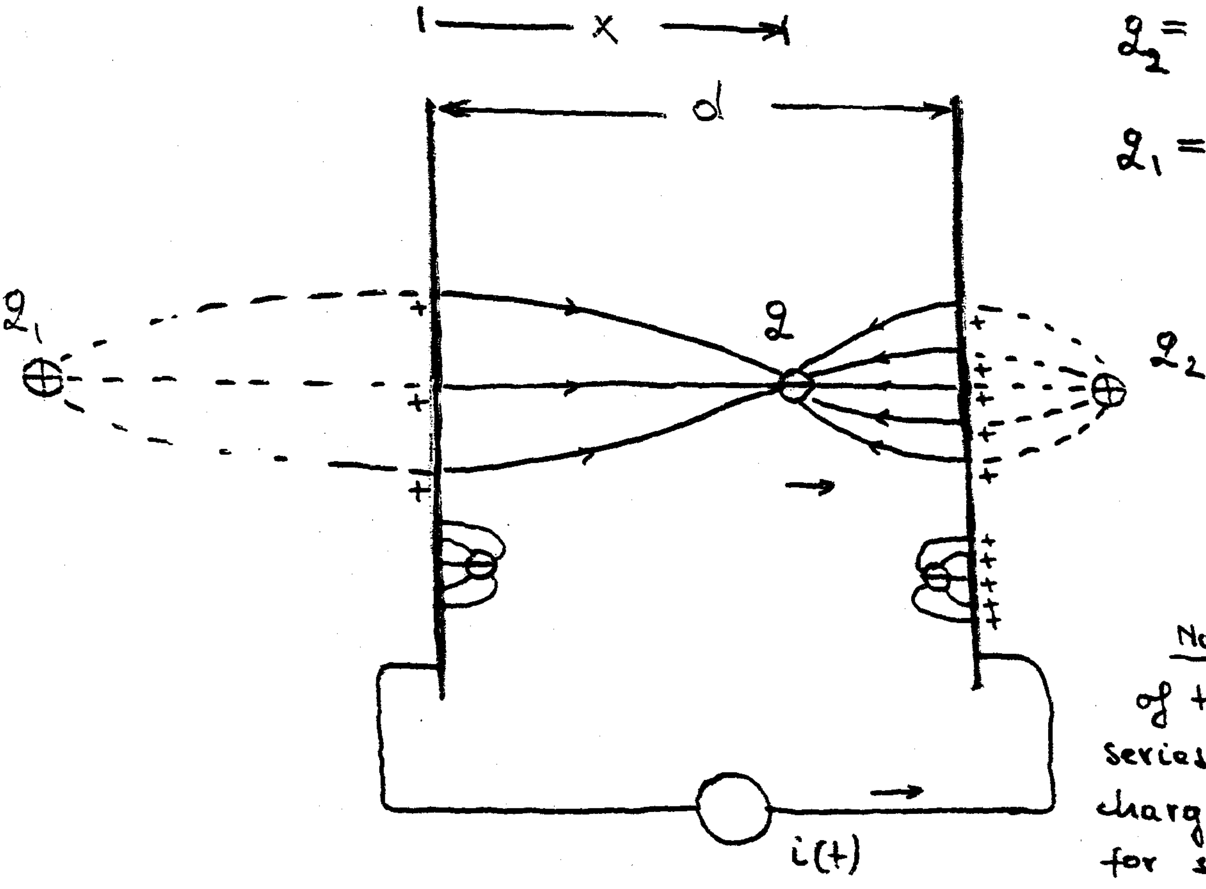
1. Signal formation.
2. Physical origins of noise.
3. Amplitude, time and waveform measurements, (i.e., energy, time and position), filtering.
4. Detector-amplifier charge transfer.
5. Position sensing methods.

LECTURE 1.

# SIGNAL FORMATION IN DETECTORS

# Image Charge



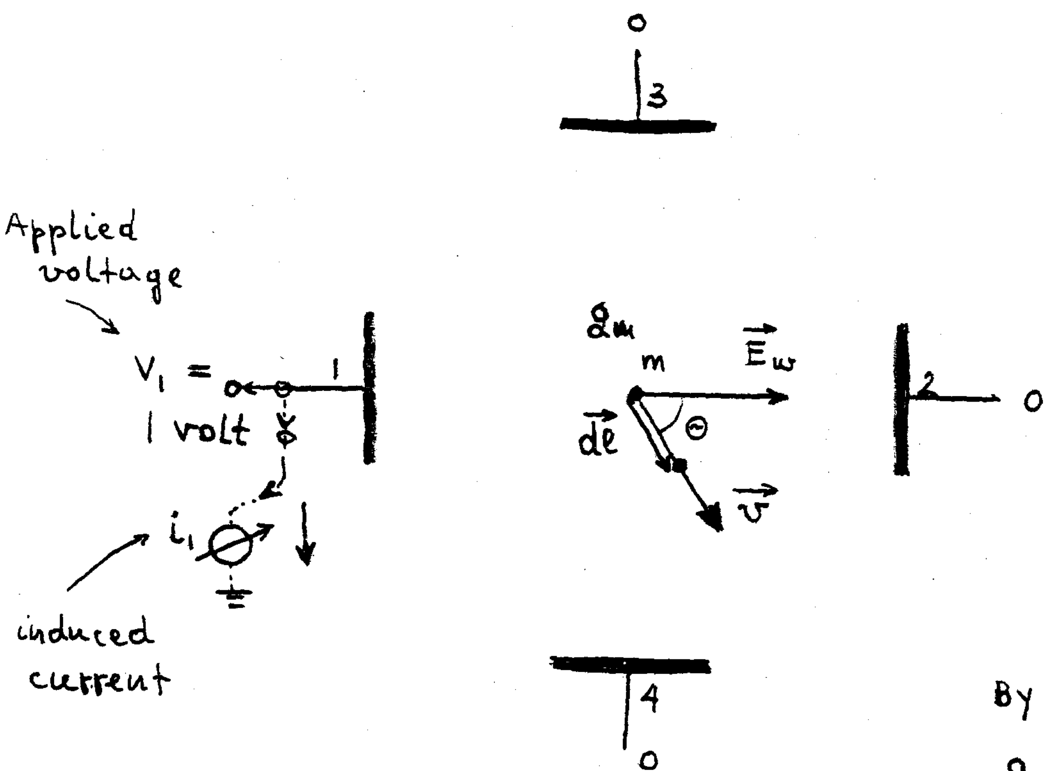

$$Q_2 = \frac{x}{d} Q$$

$$Q_1 = \frac{d-x}{d} Q$$

Note: only two of the infinite series of image charges are shown for simplicity.

# Current induced by the motion of charge

$v \ll c$




By reciprocity:

$$q_m V_m = Q_1 V_1$$

Normalize $V_1 = 1$ volt $\longrightarrow$ $Q_1 = q_m V_m$

$$i_1 = \frac{dQ_1}{dt} = \frac{d(q_m V_m)}{dt}$$

$q_m =$ const. $\qquad i_1 = q_m \frac{dV_m}{dt}\frac{\vec{dl}}{\vec{dl}} \qquad \frac{\vec{dl}}{dt} = \vec{v}$

<u>Weighting field</u> $\vec{E}_w$ (for 1 volt on electrode 1):

$$\frac{dV_m}{dl} = -E_w \cos\Theta \qquad E_w[1/\text{cm}]$$

current : $\boxed{i_1 = -q_m \vec{E}_w \vec{v}}$

charge coordinates

1. Find weighting field $\vec{E}_w(x,y,z)$
2. " the charge velocity $\vec{v}(x,y,z)$

$\Big\} \rightarrow i(x,y,z)$

3. " $x(t)$, $y(t)$, $z(t)$ $\longrightarrow$ $i(t)$

For general solution with floating electrodes see NIM 193 (1982) 651

# Induced signal in planar electrode geometry

$$\underline{i = +q\,\vec{E}_w\cdot\vec{v}}$$

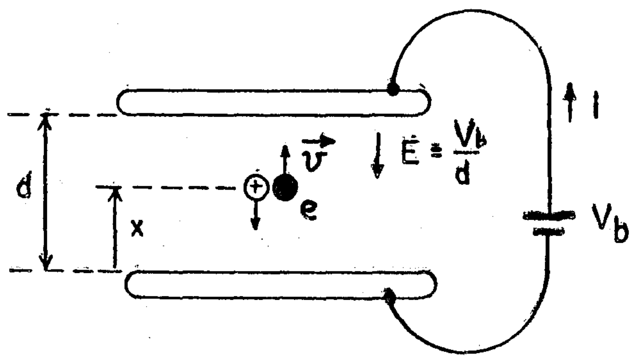
$E_w$ = "weighting" field , E = real field

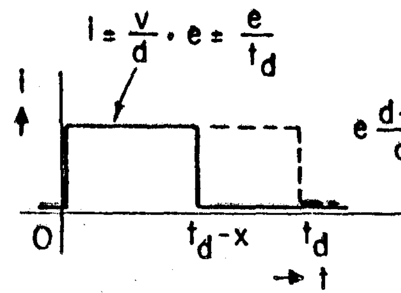
$$\vec{E}_w = \frac{1}{d}\ ;\quad \vec{v} = -v\ ;\quad q = -e$$

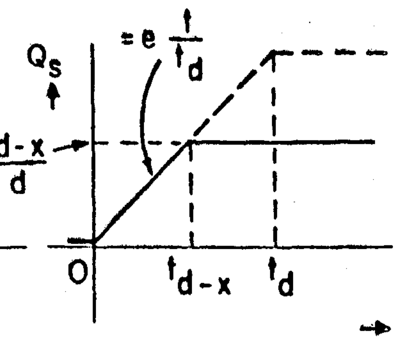
CURRENT CHARGE

(a)

single carrier ⊖ e



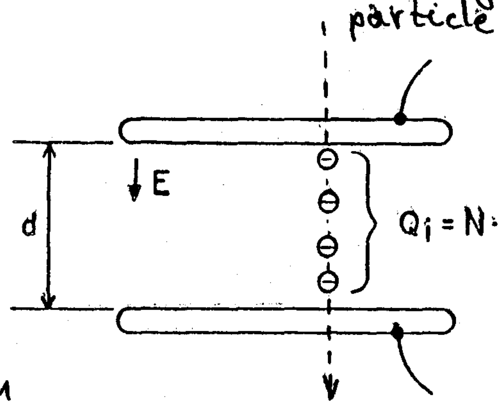



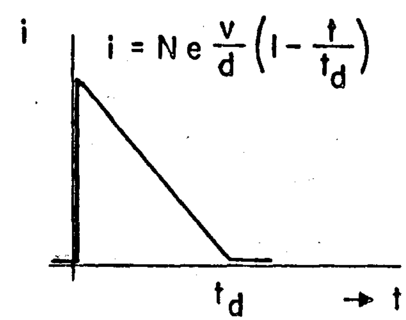



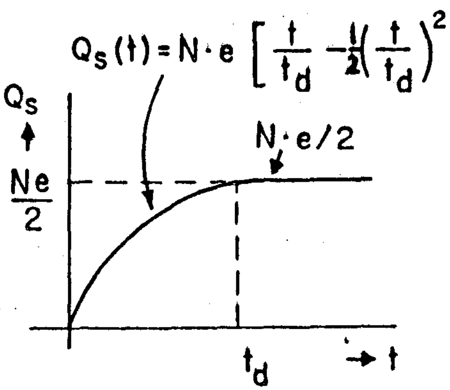

$t_d$ = transit time

(b)

continuous ionization

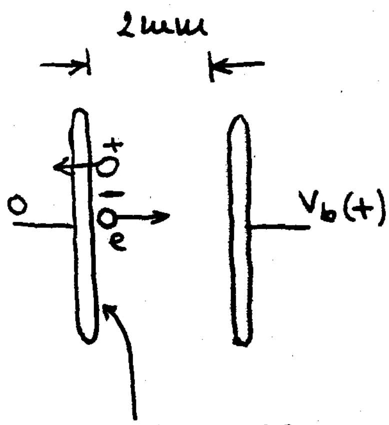


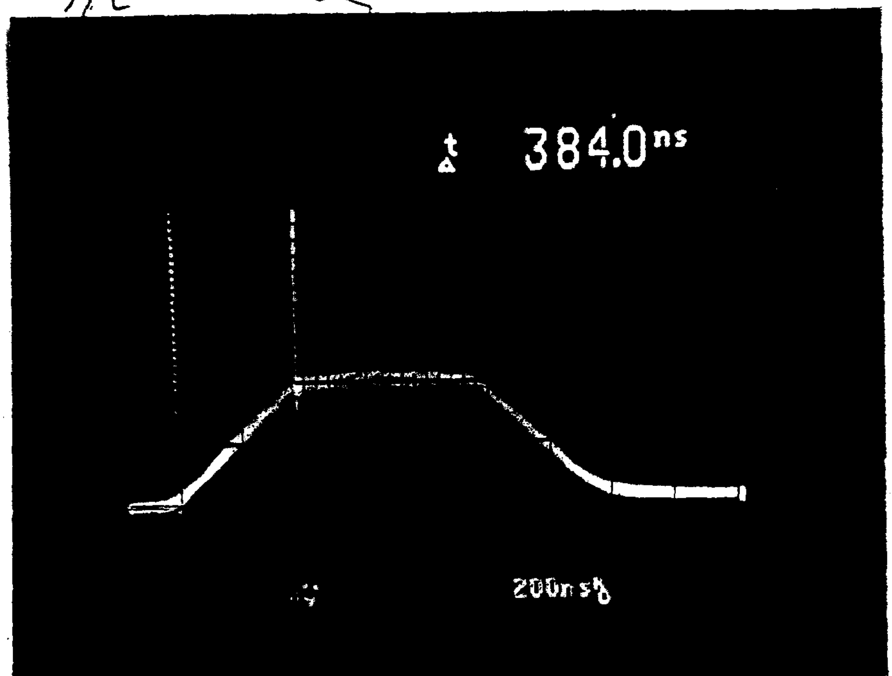

If N is very large, fluctuations in pulse shape are small (e.g. , liquids, solids) in calorimeters)

## Electron transport in LA



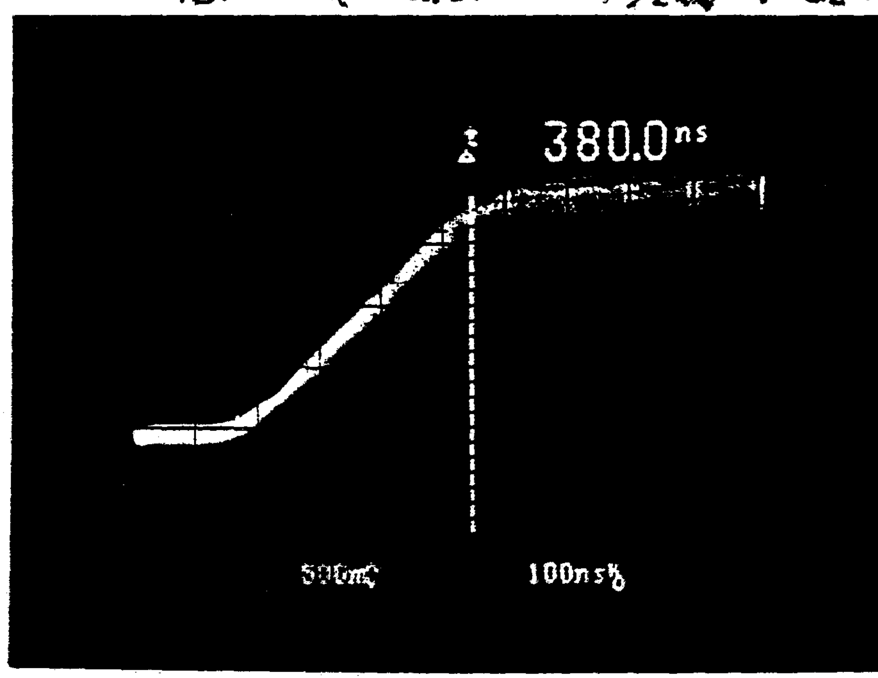

5.5 MeV α- source

$A_L$ α 3-12-85




5 KV/2mm

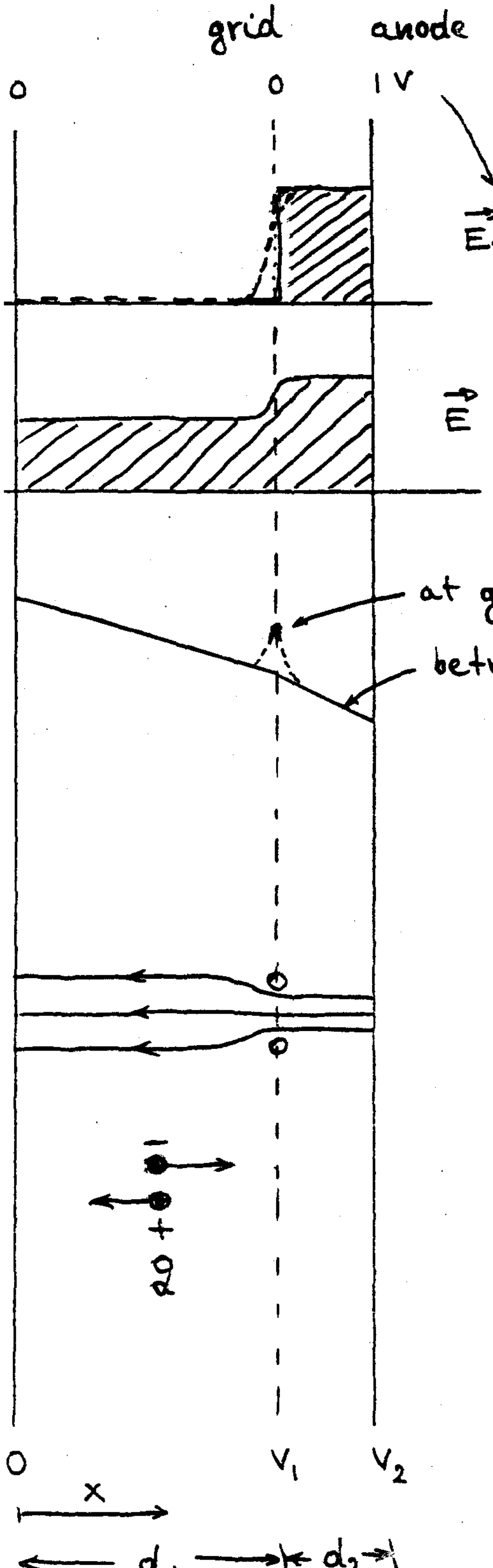
$A_L$. α Rise time 5 kV/2mm 4-22-85




Charge vs time

Drift time: ~ 190 nsec/mm

# Ionization Chamber with Frisch Grid



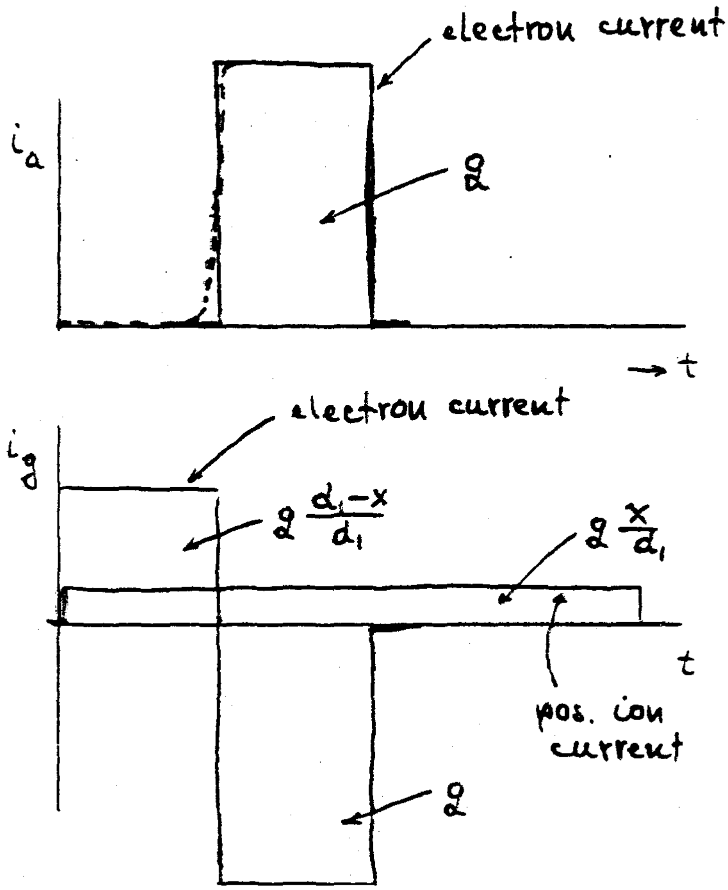

$\vec{E}_w$ weighting field

$\vec{E}$ "true" (applied) field

$\vec{v} = \mu \vec{E}$

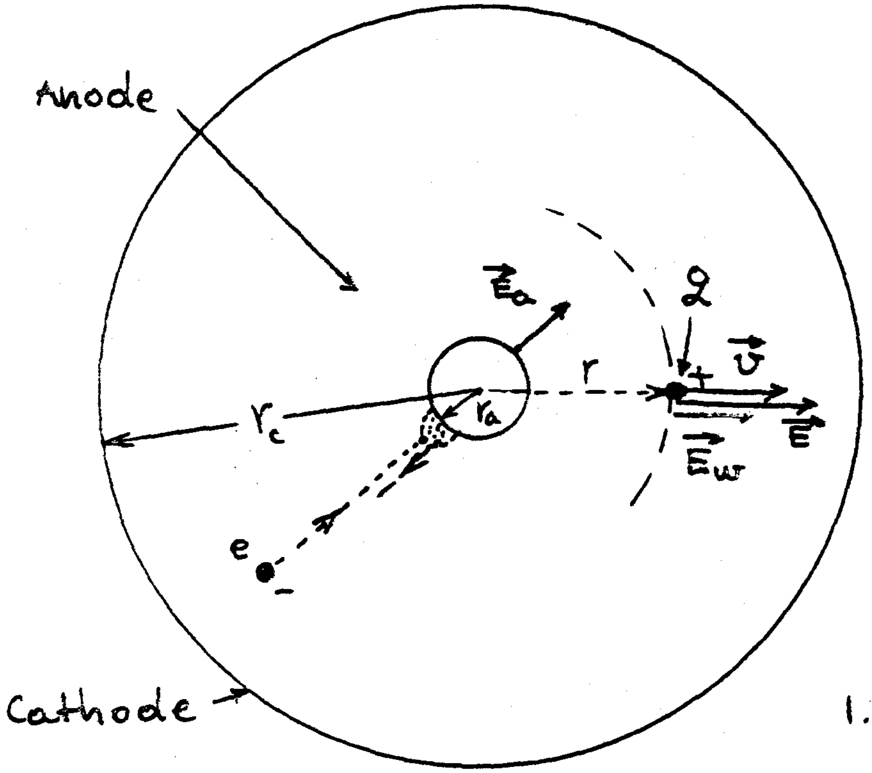
Potential (energy) for electrons

$$\underline{i = -q \vec{E}_w \vec{v}}$$




$$\frac{V_1}{d} < \frac{V_2 - V_1}{d_2}$$

# Proportional Detector, Cylindrical Geometry




$\vec{v}$ = drift velocity $= \mu \vec{E}$ (positive ions)

$\vec{E}_w$ = weighting field

$\vec{E}$ = "real" applied field

$\mu$ = mobility

$$E = \frac{V}{\ln \frac{r_c}{r_a}} \cdot \frac{1}{r} = E_a \frac{r_a}{r}$$

1. $$E_w = \frac{1}{\ln \frac{r_c}{r_a}} \frac{1}{r}$$

2. $v(r) = ?$ $\longrightarrow$ $$v = \frac{dr}{dt} = \mu E_a \frac{r_a}{r}$$

$$\underline{i(r) = Q \vec{E}_w \cdot \vec{v}} = Q \frac{\mu E_a}{\ln \frac{r_c}{r_a}} \frac{r_a}{r^2}$$

3. $r(t) = ?$ $\longrightarrow$ $r^2 = r_a^2 + 2\mu E_a r_a t$ $\qquad r \propto t^{1/2}$

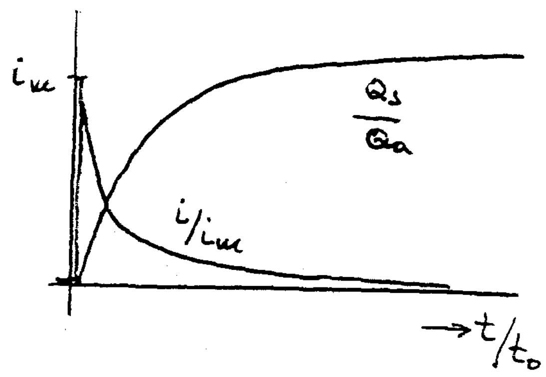
$$\frac{i(t)}{i_m} = \frac{1}{1 + t/t_0} \qquad t_0 = \frac{r_a}{2\mu E_a}$$

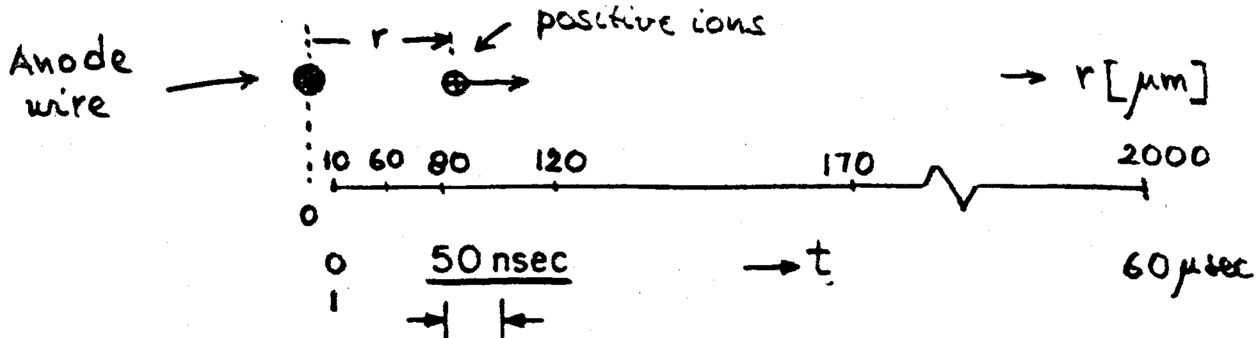
$$i_m = \frac{Q}{2 t_0 \ln \frac{r_c}{r_a}}$$

Observed (signal) charge from an avalanche $Q_a$:

$$\frac{Q_s}{Q_a} = \frac{1}{2 \ln \frac{r_c}{r_a}} \ln(1 + t/t_0)$$

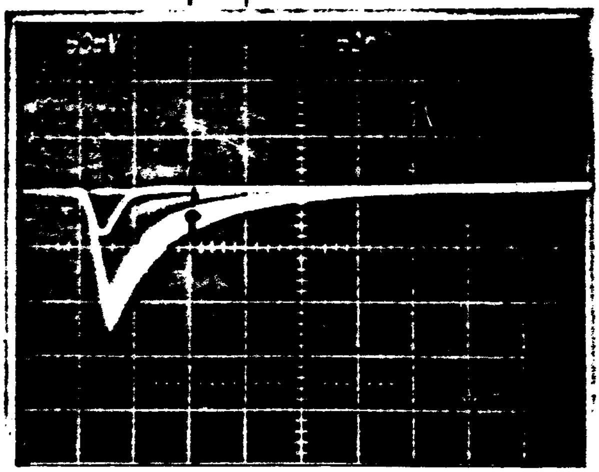
$$= \frac{\ln \frac{r}{r_a}}{\ln \frac{r_c}{r_a}}$$






Anode "current"

(a)



$$\frac{i}{i_m} = \frac{1}{1 + t/t_0}$$

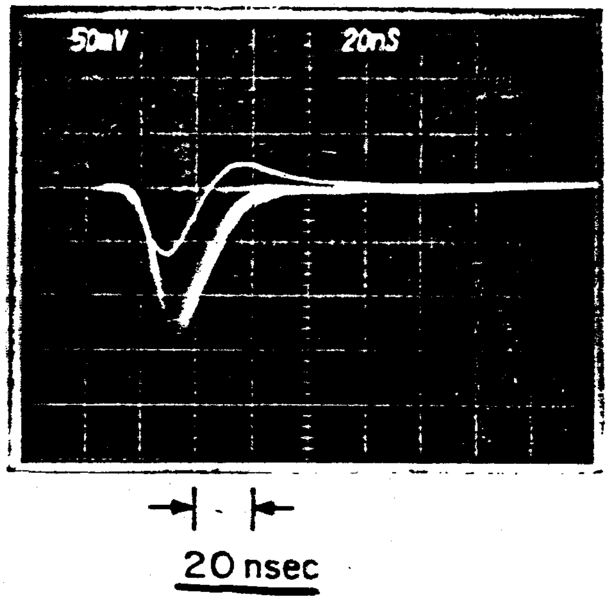
$$t_0 = \frac{r_a}{2\mu_+ E_a}$$

$\sim$ 3-4 nsec in Xe

$$i_m = \frac{Q_m}{2 t_0 \ln \frac{r_c}{r_a}}$$

With tail cancellation

(b)




$$Q(t) = \frac{Q_m}{2 \ln \frac{r_c}{r_a}} \ln(1 + t/t_0)$$

$t_0 \simeq 1.5$ nsec in Ar

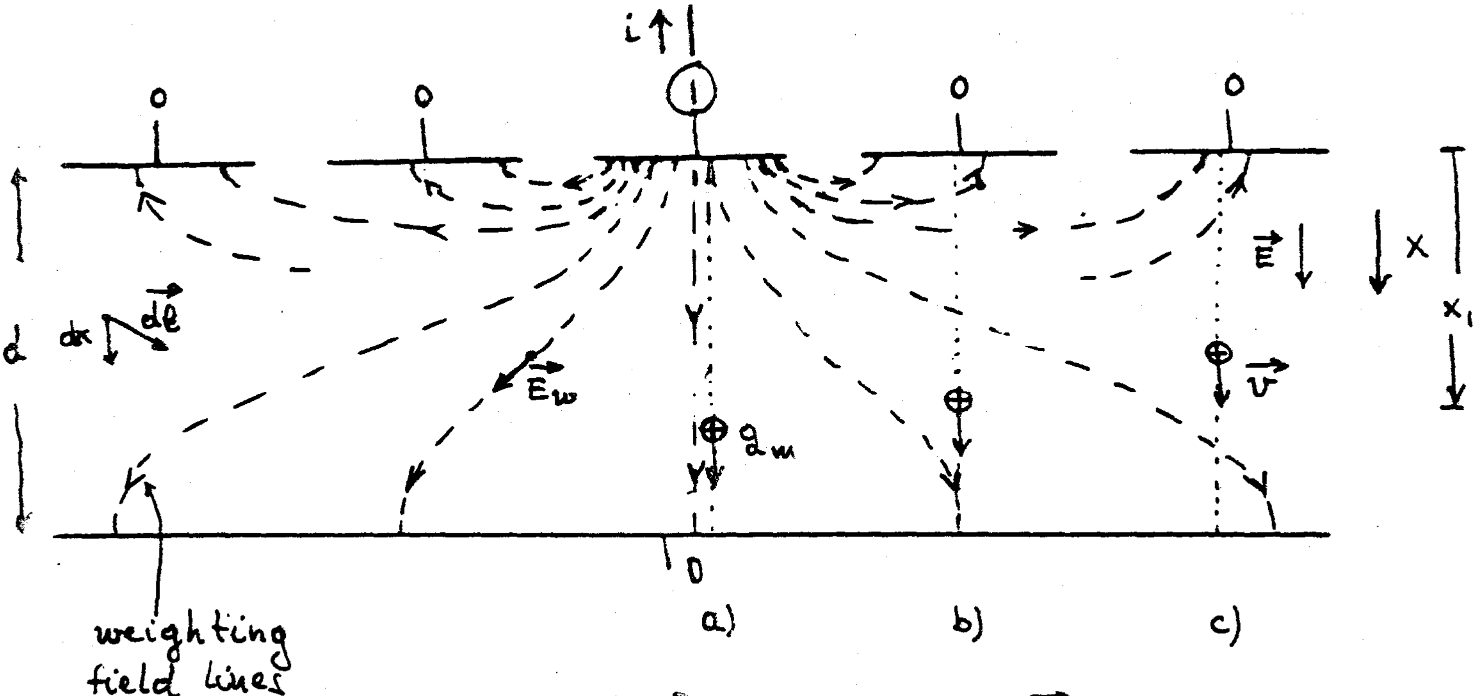
$r_a = 10\,\mu m$

| $t$ | 20 nsec | 1 μsec |
|---|---|---|
| $\frac{Q(t)}{Q_m}$ | $\sim 0.2$ | $\sim 0.55$ |

$$\frac{Q(20\,\text{nsec})}{Q(1\,\mu\text{sec})} \simeq \frac{1}{3}$$

CHARGE COLLECTION IN GAS PROPORTIONAL DETECTORS

# Strip electrodes (semiconductor detectors, parallel plate chambers, etc.)

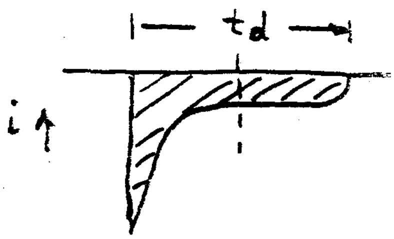



$$i = -q_m \vec{E}_w \vec{v} = -q_m \vec{E}_w \frac{\vec{dx}}{dt}$$

(pos. current into electrode)

$$Q = \int i\,dt = -q_m \int \vec{E}_w \vec{dx}$$

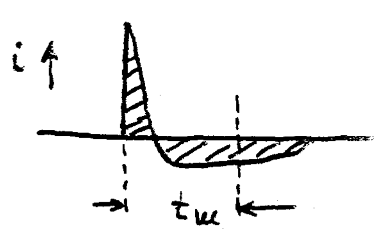
a)




$Q = -q_m$ , since $\int_0^d \vec{E}_w \vec{dx} = 1$

(in general $\int \vec{E}_w \cdot \vec{dl}$)

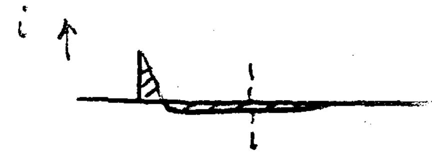
b)




$\int_0^d \vec{E}_w \vec{dx} = 0 \rightarrow Q = 0$ for $\underbrace{t_m > t_d}_{\text{waveform not important}}$

$Q \neq 0$ for $\underbrace{t_m < t_d}_{\text{waveform important}}$

c)




$\int_0^d \vec{E}_w \vec{dx} = 0$

$\left| \int_0^{x_1} \vec{E}_w \vec{dx} \right| > 0$

$t_m$ = measurement (observation time)

# REFERENCES FOR SIGNAL FORMATION IN DETECTORS

1. RAMO, P.I.R.E. 27(1939)584.

2. IONIZATION CHAMBERS AND COUNTERS, D. H. WILKINSON, CAMBRIDGE UNIVERSITY PRESS, 1950.

3. G. CAVALLERI, G. FABRI, E. GATTI, AND V. SVELTO:

3A ON THE INDUCED CHARGE IN SEMICONDUCTOR DETECTORS, NUCL. INSTR. & METH. 21(1963)177.

3B EXTENSION OF RAMO'S THEOREM AS APPLIED TO INDUCED CHARGE IN SEMICONDUCTOR DETECTORS, NUCL. INSTR. & METH. 92(1971)137.

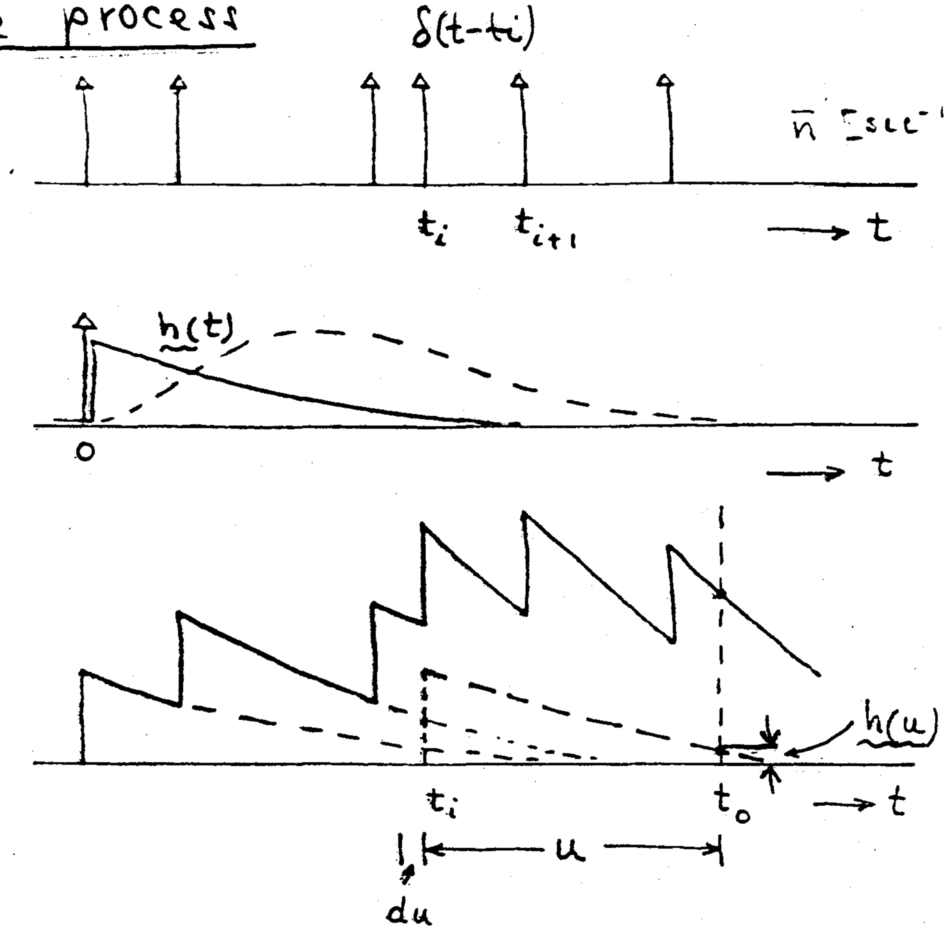
3C SIGNAL EVALUATION IN MULTIELECTRODE RADIATION DETECTORS BY MEANS OF A TIME DEPENDENT WEIGHTING VECTOR, E. GATTI, G. PADOVINI, AND V. RADEKA, NUCL. INSTR. & METH. 193(1982)651.

4. A. H. WALENTA, LEFT-RIGHT ASSIGNMENT IN DRIFT CHAMBERS AND MWPCS USING INDUCED SIGNALS, NUCL. INSTR. & METH. 151(1978)461.

# NOISE :

## Origin and Properties

## Noise process




POISSON sequence

$*$

Impulse response

$\downarrow$

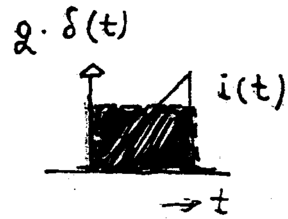
Output

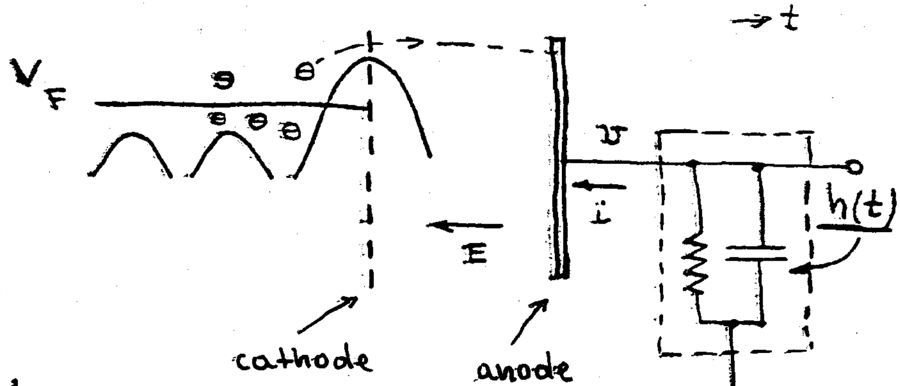
$$v_i = q\, h(t_o - t_i)$$

$$\overline{v^2} = q^2 \sum_{i=1}^{N} h^2(t_o - t_i)$$

$$d(\overline{v^2}) = q^2 h^2(u)\ \bar{n}\, du$$

$$\overline{v^2} = \sigma^2 = \underbrace{\bar{n}\, q^2}_{\text{NOISE PROCESS}} \underbrace{\int_0^{T,\infty} h^2(u)\, du}_{\text{PHYSICAL SYSTEM}}$$

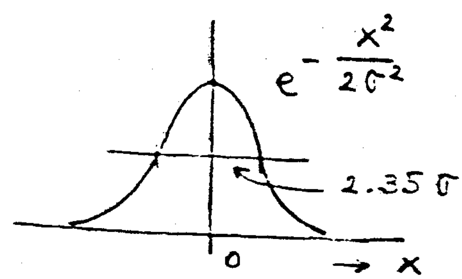



Campbell's theorem

$\overline{2kTR}$

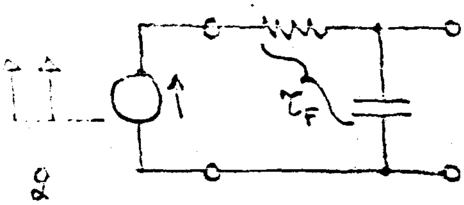
$$\sigma^2 = \frac{1}{T}\int_0^T x^2(t)\, dt$$



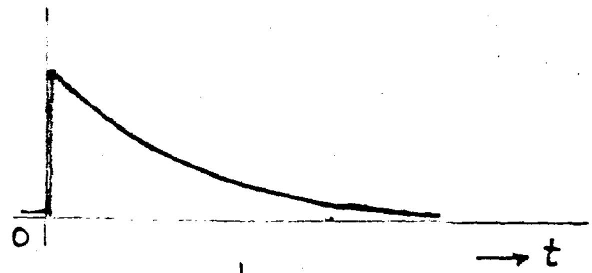

"Central limit theorem" → gaussian

# Power spectrum, correlation, total power and bandwidth

$h(t)$, $H(\omega)$



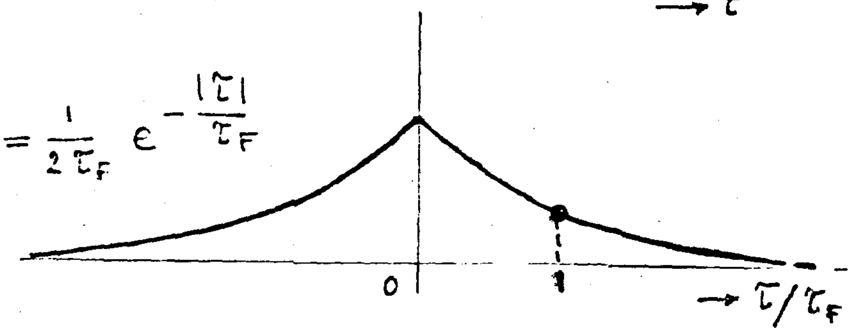

$h(t) = \frac{1}{\tau_F} e^{-\frac{t}{\tau_F}}$



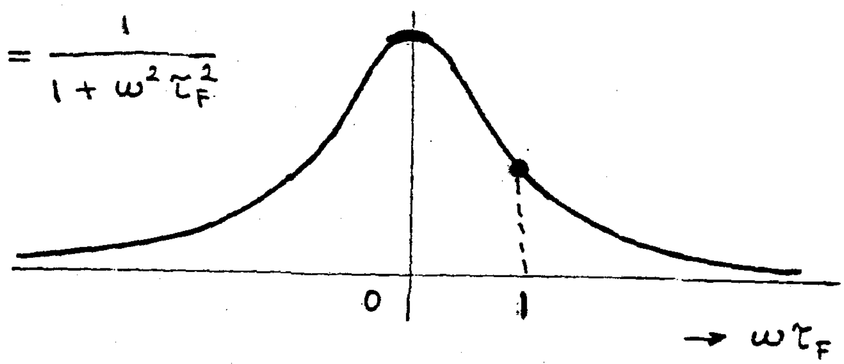

$w_0 = \bar{n} q^2$

Relaxation process

$K_h(\tau) = \frac{1}{2\tau_F} e^{-\frac{|\tau|}{\tau_F}}$




$|H(\omega)|^2 = \frac{1}{1+\omega^2\tau_F^2}$




$K(0) = \sigma^2 = \bar{n} q^2 \frac{1}{2\tau_F}$

$\tau_F \to 0 \quad \sigma^2 \to \infty$

$\omega^2 \tau_F^2 \gg 1$

$w(\omega) = w_0 |H(\omega)|^2 \propto \frac{1}{\omega^2}$

$\underline{\omega_h \cdot \tau_F = 1}$

WIENER – KHINTCHINE Theorem: $\quad W(\omega) \overset{\mathcal{F}}{\longleftrightarrow} K(\tau)$

$$W(f) = 2\int_0^\infty K(\tau)\cos(2\pi f\tau)\,d\tau$$

$$K(\tau) = 4\int_0^\infty W(f)\cos(2\pi f\tau)\,df$$

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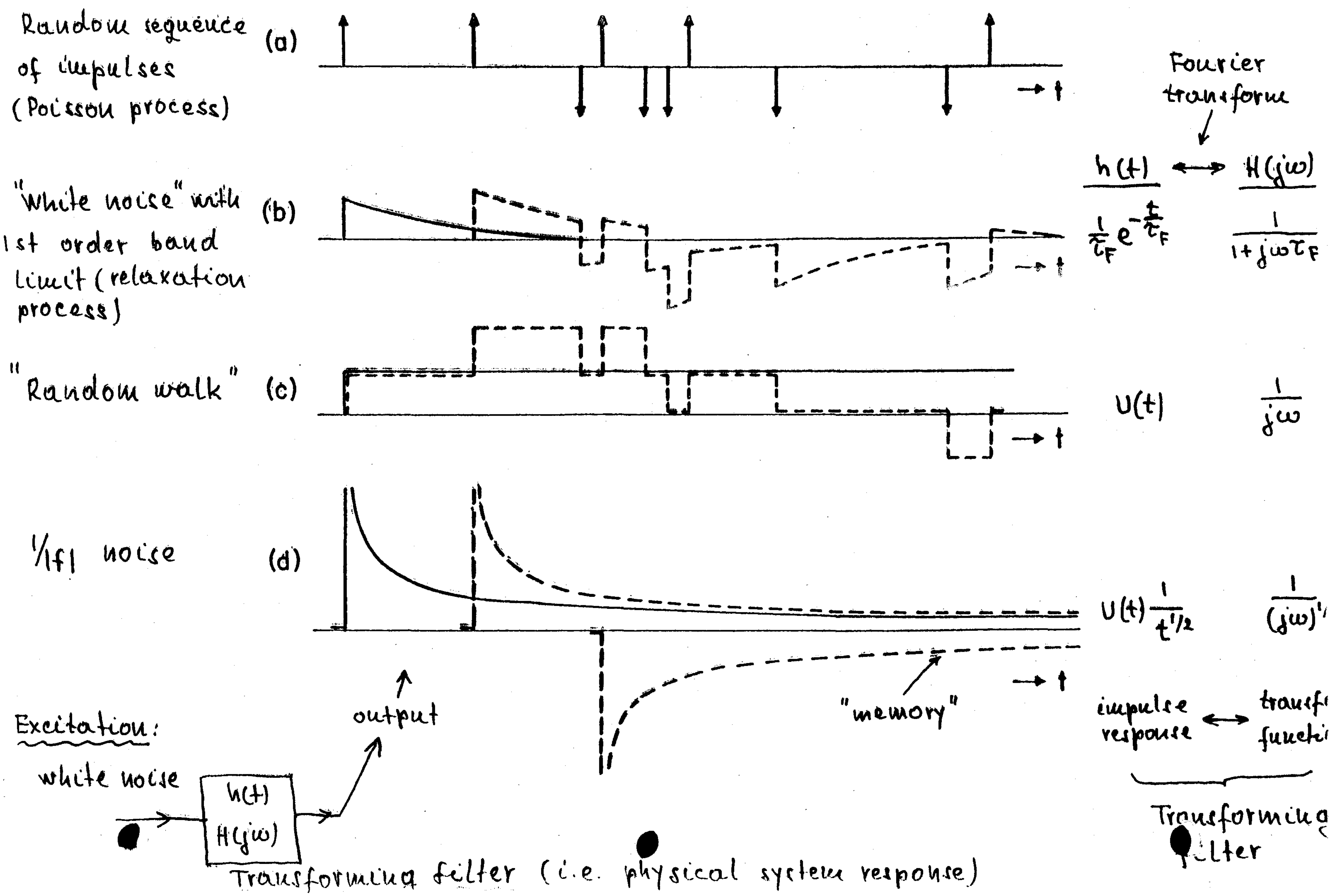
For an introduction see the book:

M.J. Buckingham, Noise in Electronic Devices and Systems, ELLIS HORWOOD Ltd., HALSTED PRESS 1983

# A MODEL FOR GENERATION OF NOISE SPECTRA

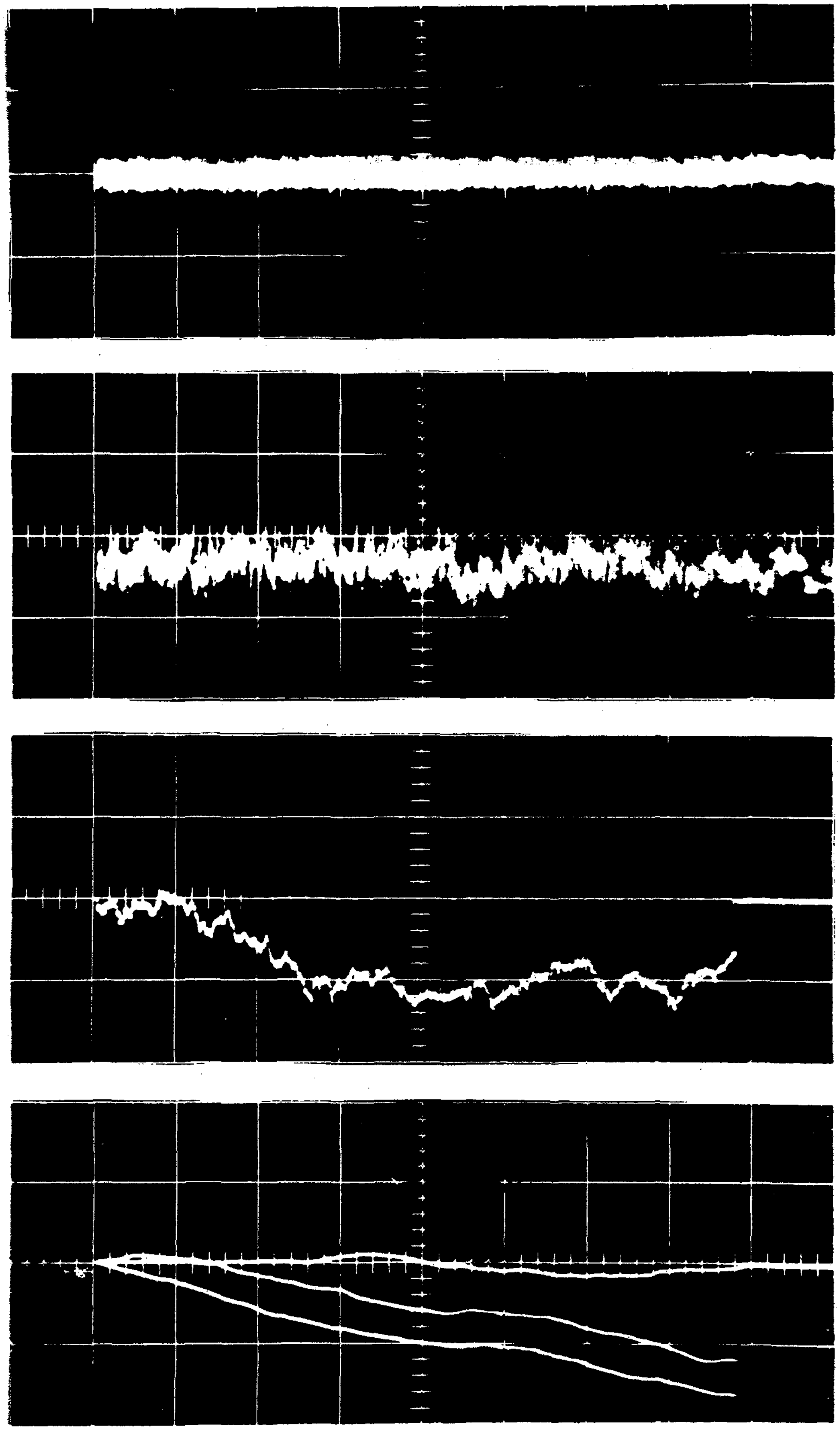


$|f|^0$ "white noise"

$|f|^{-1}$ "$1/f$ noise"

$|f|^{-2}$ "random walk"

$|f|^{-3}$



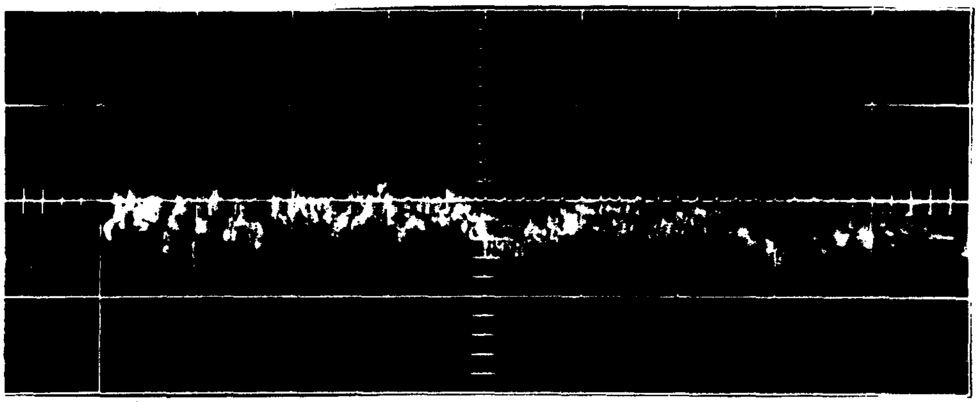
samples of noise waveforms

## "Self-similarity" of $1/|f|$ noise (scaling invariance)

(a)

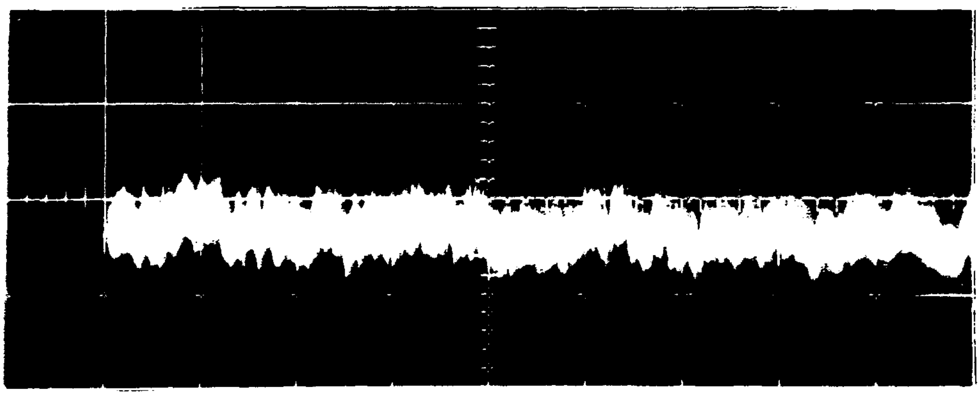
10 µsec/div.

(b)



50 µsec/div

(c)

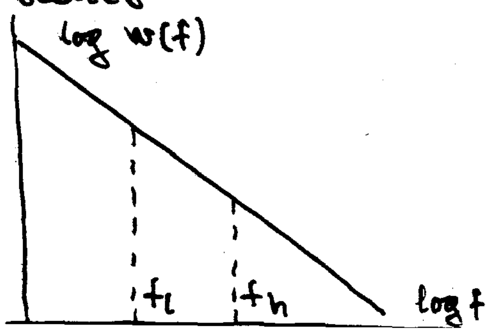


500 µsec/div

$1/|f|$ noise – different time scales

$$\sigma^2(f_h, f_l) = \int_{f_l}^{f_h} \frac{1}{|f|}\, df = \ln \frac{f_h}{f_l}$$

For $\frac{f_h}{f_l} = \text{const} \rightarrow \underline{\sigma^2(f_h, f_l) = \text{const}}$



# VARIANCE, i.e. MEASUREMENT ERROR DUE TO NOISE

## FREQUENCY DOMAIN:

$w_\alpha(f) = |f|^\alpha$ noise spectral density (normalized to $w_{oo}$)

Variance:

$$\sigma^2(f_h, f_\ell) = \int_{f_\ell}^{f_h} |f|^\alpha df = \frac{1}{1+\alpha}\left[f_h^{1+\alpha} - f_\ell^{1+\alpha}\right] \quad \text{FOR } \alpha \neq -1$$

$$\sigma^2(f_h, f_\ell) = \ln \frac{f_h}{f_\ell} \quad \text{FOR } \alpha = -1,$$

$1/|f|$ NOISE

FOR $1/f^2$ $\alpha = -2$, $\sigma^2(\infty, f_\ell) = \frac{1}{f_\ell}$

FOR $\alpha \geq -1$ HIGH FREQUENCY DIVERGENT

$\alpha \leq -1$ LOW -"- -"-

## TIME DOMAIN:

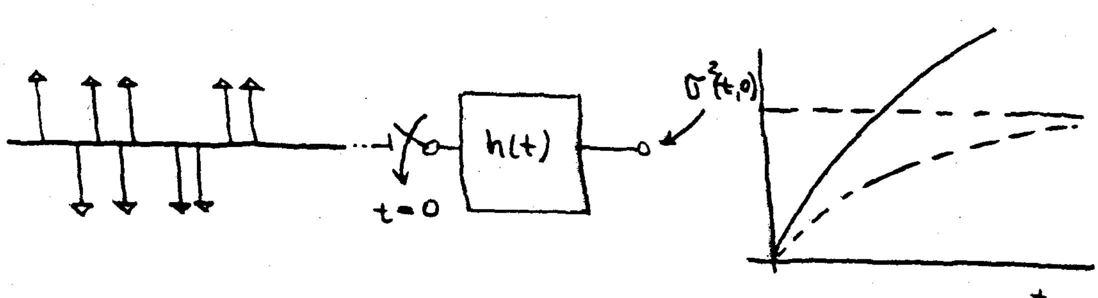
$$\sigma^2(0,t) = \int_\delta^t h^2(t)\,dt$$

Transforming filter:

$$h(t) = t^{-\frac{\alpha}{2}-1}$$

$\delta \to 0$

| | $|f|^\alpha$ |
|---|---|
| $= \ln \frac{t}{\delta}$ | $1/|f|$ NOISE |
| $t$ | $1/f^2$ |
| $t^2$ | $1/f^3$ |
| $t^3$ | $1/f^4$ |




For more detail see:
IEEE Trans. Nucl. Sci., NS-16
Oct. 1969, p. 17.

## NOISE VARIANCE AS A FUNCTION OF MEASUREMENT (FILTER) TIME FOR POWER LAW SPECTRA

$f_h / f_\ell = \text{const.}$ $\quad \tau_F \propto \dfrac{1}{f_{h,\ell}}$ filter parameter, i.e., measurement time

Spectrum - variance relation:

$$|f|^{\alpha} \longrightarrow \sigma^2(\tau_F) = K_\alpha \, \tau_F^{-1-\alpha} \qquad K_\alpha(\alpha, T, C, \cdots)$$

Spectrum:

$$W(f) = \cdots + W_1 |f|^1 + W_0 f^0 + W_{-1} |f|^{-1} + W_{-2} f^{-2} + \cdots$$

Variance:

$$\sigma^2(\tau) = \cdots + K_1 \tau_F^{-2} + \underbrace{K_0 \tau_F^{-1} + K_{-1} \tau_F^{0} + K_{-2} \tau_F^{1}}_{\text{WN} \qquad 1/|f| \qquad 1/f^2} + \cdots$$

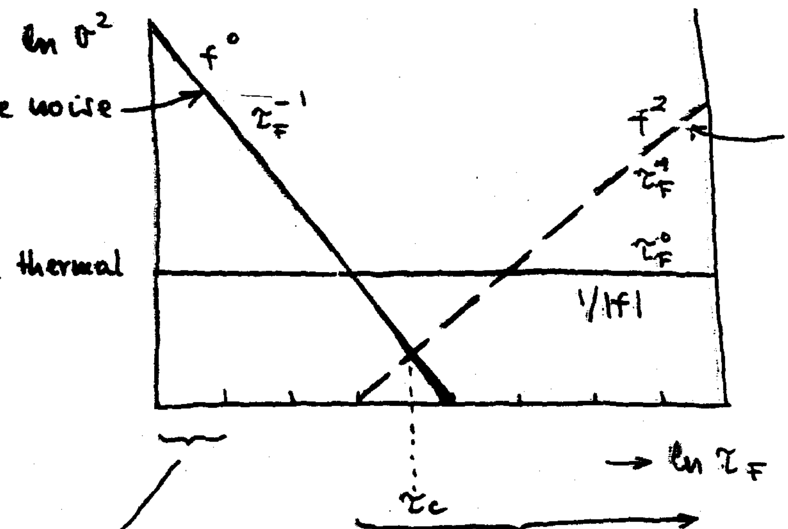
"white noise" (WN)

$1/f^2$: "random walk"

$W_{-3/2} |f|^{-3/2}$

$K_{-3/2} \tau_F^{1/2}$

fractional exponents




"series" white noise in charge amplifiers (FET channel thermal noise)

random walk, "parallel" white noise in charge amplifiers (leakage current and resistor noise)

FAST TIMING

AMPLITUDE MEASUREMENTS, MEASUREMENTS WITH ARBITRARY $\tau$ (NMR, EPR)

$\tau_c$ = noise "corner" time constant

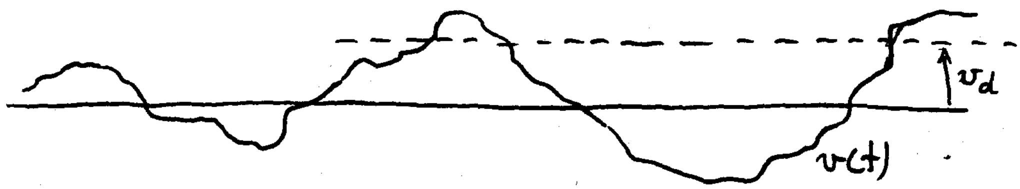
# LOW FREQUENCY DIVERGENCE OF $1/f^2$ NOISE AND OF $1/|f|$ NOISE

| $\alpha = -2$<br>$1/f^2$<br>$\sigma^2(\infty, f_\ell) = \frac{1}{f_\ell}$<br>$\sigma^2(t,0) = t$ | | $\alpha = -1$<br>$1/\lvert f\rvert$<br>$\sigma^2(f_h, f_\ell) = \ln \frac{f_h}{f_\ell}$<br>$\sigma^2(t,0) = \ln \frac{t}{\delta}$ | $\delta = 10^{-6}$ sec | $1/f^2$<br>$\alpha = -2$<br>$\sigma(t,0) = t^{1/2}$ |
|---|---|---|---|---|
| [sec] | [ ~ ] | $\log_{10}(t/\delta)$ | $[\log_{10}(t/\delta)]^{1/2}$ | $t$ |
| $10^{-6}$ | (1 $\mu$ sec) | 0 | 0 | 1 |
| 1 | | 6 | 2.45 | $10^3$ |
| $10^5$ | (~ 1 DAY) | 11 | 3.3 | $3 \times 10^5$ |
| $10^9$ | (~ 30 YEARS) | 15 | 3.9 | $3 \times 10^7$ |
| $10^{17}$ | (≈ AGE OF UNIVERSE) | 23 | 4.8<br>(rms NOISE) | $\infty$ |

Random walk, $1/f^2$ noise runs out of dynamic range for space or energy variables, except in special cases, i.e. phase of oscillators.

There is <u>no</u> divergence problem with $1/f$ noise. It has been observed (in MOS transistors, for example) down to $10^{-5}$ Hz.

# ZERO-CROSSING STATISTICS OF NOISE




The noise is described by: $W(\omega)$ – spectral density

$W(\omega) \xleftrightarrow{\mathcal{F}} K(\tau)$ $\quad K(\tau)$ – autocorrelation function

$p(v)$ – gaussian

Then: The frequency of positive zero crossings:

$$n_{zc+} = \frac{1}{2}\left[-\frac{K''(0)}{K(0)}\right]^{1/2} = \frac{1}{2}\left[-\frac{\int_0^\infty \omega^2 W(\omega)\,d\omega}{\int_0^\infty W(\omega)\,d\omega}\right]^{1/2}$$

The number of level crossings:

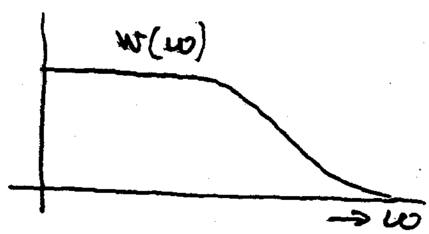
$$n(v_d)_+ = n_{zc+}\exp\left(-\frac{v_d^2}{2K(0)}\right)$$

$K(0) = \sigma^2$

$\sigma$ = rms noise

<u>Example:</u> 2-nd order high frequency cutoff
( <u>2 RC</u> integrations)

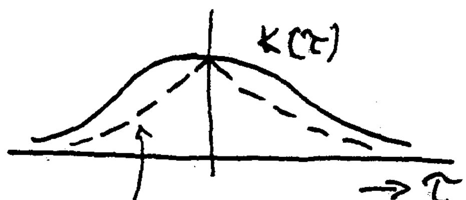
$$n_{zc+} = \frac{1}{2\pi(\tau_1\tau_2)^{1/2}}$$




<u>for $\tau_1$ or $\tau_2 \rightarrow 0$</u> $\quad$ <u>$n_{zc} \rightarrow \infty$</u> !

for $\tau_1 = \tau_2 = 5$ nsec

$n_{zc} \approx 30\,\text{MHz}$




<u>Level crossing rate:</u>

| $\frac{v_d}{\sigma}$ | $n_{lc}/n_{zc}$ |
|---|---|
| 2 | $1.4\times10^{-1}$ |
| 3 | $1.1\times10^{-2}$ |
| 4 | $3.3\times10^{-4}$ |
| 5 | $4\times10^{-6}$ |

1st order cutoff ($\tau_1 = 0$) not sufficient. Higher order cutoff necessary.

# HIGH FREQUENCY LIMIT OF NOISE

1. Thermal (Johnson) noise in resistors

$$\overline{v^2} = 4kTR\Delta f \cdot \frac{\frac{hf}{kT}}{\exp\left(\frac{hf}{kT}\right) - 1}$$

$h = 6.63 \times 10^{-34}$ Joules · sec
$k = 1.38 \times 10^{-23}$ Joules

For $\frac{hf}{kT} \ll 1 \quad \longrightarrow \quad \overline{v^2} = 4kTR\Delta f$

For $\frac{hf}{kT} = 1 \quad \rightarrow \quad f_h \cong \frac{kT}{h}$ , at $T = 300°K$

$f_h \approx 6 \times 10^{12}$ Hz

At $T = 0.3\,°K \quad \longrightarrow \quad f_h \approx 6$ GHz
$T = 3 \times 10^{-3}\,°K \quad \rightarrow \quad f_h \approx 60$ MHz

2. Shot noise (detector leakage current, transistor collector and base current, FET gate leakage current, etc.)

$$\overline{i_n^2} = 2\,\bar{n}\,\overline{g^2}\,\Delta f$$

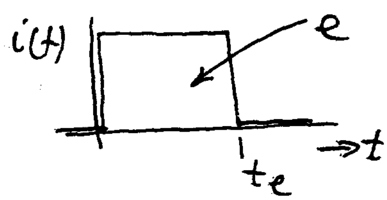
$\bar{n}\,\overline{g^2}$ "hides" physical properties of the source!

$\bar{n}$ = average rate of impulses
$\overline{g^2}$ = mean charge/impulse

$$\overline{i_n^2} = 2eI_o\Delta f$$ for electrons, $I_o$ = mean current

High frequency limit is determined by the electron transit time:

Induced current:




$$f_h \approx \frac{1}{t_e}$$ this is usually lower than for resistors (thermal noise)

# REFERENCES FOR MAXIMUM LIKELIHOOD METHODS AND STATISTICAL TREATMENT OF SIGNALS AND NOISE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