

Monte Carlo phase space integration for initial state radiation

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Abstract. Efficient phase space integration is important for some calculations in collider physics. Using the Altarelli-Parisi splitting functions as the underlying probability for a splitting, we developed a phase space calculation for initial state radiation, that relies on Monte Carlo integration.

1. Introduction

In perturbative approach of Quantum ChromoDynamics QCD, many interesting problems are still unsolved. One of the most important problems that has direct relevance for experimental data, and makes difficult to calculate the matrix elements, is the QCD radiation and high multiplicity during high energy hadronic collision.

To solve this problem, many studies focussed on regions where the emission of QCD radiation is enhanced, namely collinear parton splitting and soft (low-energy) gluon emission [1] - [5] using an appropriate numerical technique such as Monte Carlo, to make efficient distributions in region of large numbers of partons. Within this context lots of efforts were undertaken to improve the simulation results [6], [7].

Initial state radiation in Proton-Proton collision occurs when a constituent parton from each of the incoming hadrons starts at high energy emitting partons and losing energy [8] - [12]. This radiation arises because incoming colored particles can radiate before entering into the hard subprocess. The branching of these partons terminates when they collide to initiate the hard subprocess, in this case we would like to distribute the variables t , z and x according to the following probability distribution function

$$P_b(x; t, z, \varphi) = \sum_{a,c} f_{a \rightarrow bc}(t, z) \Delta_b(x; t, t_{\max}) \quad (1)$$

The functions $f_{a \rightarrow bc}(t, z)$ are the splitting functions of partons at energy t and momentum fraction z , $\Delta_b(x; t, t_{\max})$ is the *Sudakov* form factor: A given parton can only branch once, if it did not already branched, in other words the probability for a splitting not to occur, for a parton b starting from a branching vertex at the scale t_{\max} , down to a scale t is represented by Sudakov form factor

$$\Delta_b(x; t, t_{\max}) = \exp \left\{ - \int_t^{t_{\max}} dt' \int dz f_b(t', z) \right\} \quad (2)$$



In equation (1), a is the flavor of the initial particle before the emission, b the flavor after the emission, and c the flavor of the radiated particle. These indices a , b and c stand for partons, it could be quark, antiquark or gluon.

The splitting function $f_{a \rightarrow bc}(t, z)$ is written in the following form,

$$f_{a \rightarrow bc}(t, z) = \frac{\alpha_s}{2\pi} \frac{1}{t} P_{a \rightarrow bc}(z) \frac{1}{z} \frac{f_a(t, x')}{f_b(t, x)} \quad (3)$$

$f_a(t, x')$ and $f_b(t, x)$ are the parton distribution functions for the mother a and daughter b particles (respectively). To simplify, we will make the approximation that α_s is constant. The functions $P_{a \rightarrow bc}(z)$ are kernels for different possible splittings.

Unlike the splittings in final state radiation, which have been well studied [6], [7], in initial state radiation three additional splitting kernels will emerge due to the configuration of mother a , daughter b and emitted c particles, this makes the calculation even more challenging.

For example: There is a difference between the following two splittings $q \rightarrow qg$ and $q \rightarrow q\bar{q}$, in the first case the radiated particle is gluon, while in the second case is quark. Now we are dealing with seven splitting functions in initial state radiation instead of four in final state radiation, $P_{q \rightarrow qg}(z)$, $P_{q \rightarrow q\bar{q}}(z)$, $P_{\bar{q} \rightarrow \bar{q}g}(z)$, $P_{\bar{q} \rightarrow \bar{q}\bar{q}}(z)$, $P_{g \rightarrow q\bar{q}}(z)$, $P_{g \rightarrow \bar{q}q}(z)$, and $P_{g \rightarrow gg}(z)$.

We write the total splitting function for an emitted parton in initial state radiation:

$$f_b(t, z) = \frac{\alpha_s}{2\pi} \frac{1}{t} \sum_{a,c} P_{a \rightarrow bc}(z) \frac{1}{z} \frac{f_a(t, x')}{f_b(t, x)} \quad (4)$$

2. Splitting functions and probability distribution in initial state radiation

Based on facts mentioned above we write the probability of splitting for different partons in initial state radiation.

Quark probability of splitting

$$P_q(x; t, z) = [f_{q \rightarrow qg}(t, z) + f_{g \rightarrow q\bar{q}}(t, z)] \Delta_{q \rightarrow qg}(x; t, t_{\max}) \Delta_{g \rightarrow q\bar{q}}(x; t, t_{\max}) \quad (5)$$

Antiquark probability of splitting

$$P_{\bar{q}}(x; t, z) = [f_{\bar{q} \rightarrow \bar{q}g}(t, z) + f_{g \rightarrow \bar{q}q}(t, z)] \Delta_{\bar{q} \rightarrow \bar{q}g}(x; t, t_{\max}) \Delta_{g \rightarrow \bar{q}q}(x; t, t_{\max}) \quad (6)$$

Gluon probability of splitting

$$P_g(x; t, z) = [f_{g \rightarrow gg}(t, z) + f_{q \rightarrow gq}(t, z) + f_{\bar{q} \rightarrow g\bar{q}}(t, z)] \times \Delta_{g \rightarrow gg}(x; t, t_{\max}) \Delta_{q \rightarrow gq}(x; t, t_{\max}) \Delta_{\bar{q} \rightarrow g\bar{q}}(x; t, t_{\max}) \quad (7)$$

We could have used the ordinary veto algorithm introduced in Pythia [5], [6], [7] to distribute according to the exact form of $P(t, z)$ in final state radiation, however this is not feasible anymore for initial state radiation in Proton-Proton collisions. The probability distribution is quite complicated, since it does not only depend on energy t and momentum fraction z , it also depends on the value of momentum x , furthermore the probability distribution function is a product of splitting function and Sudakov form factor, both of which cannot be calculated analytically. Thus, we will use a grid version of the probability distribution function while integrating the probability of splitting $P(x; t, z)$ over all space, in this case we need to include the dependence on x into the grid.

2.1. Monte Carlo calculation and the grid version of the probability distribution

We are aiming to build a three dimensional grid for the probability $P(x; t, z)$, to simplify and make its usage more efficient, we will map the values of x , t and z respectively on the value of new variables r , s and v , so the region $x_{min} < x < x_{max}$ corresponds to the region $0 < r < 1$, the region $t_{min} < t < t_{max}$ corresponds to the region $0 < s < 1$ and finally the region $z_{min} < z < z_{max}$ corresponds to the region $0 < v < 1$.

This means, we will switch variables to build a smooth form for the function $P(s; v; r)$ before putting it on a grid, then, we will have a set of grids describing the probability distribution $P(s; v)$ for different values of r .

First we calculate the functional dependence of the variable r on x , the variable s dependence on t is the same for the four splittings, however, the variable v dependence on z is different for the four existing splitting functions, once we have done that, we write the algorithm for building the grid and determining the x , z , and t values for initial state radiation

1. For x values we determine the corresponding value of r , for which we want to calculate the Initial state Radiation.
2. For the value of r we calculate the corresponding two dimensional grid.
3. According to that grid we distribute the values of s and v .
4. Finally we convert these values back into x , t and z .

2.2. Method

Let's now apply the algorithm step by step: Integrating the distribution function over all space will give us unity, and this is applicable for all the splitting probabilities we defined.

For t variable It is difficult to distribute all the way to $t = 0$ because of singularity of the splitting function, we will choose to distribute according to $P(x, t, z)$ for $t > t_{IR}$, and according to a flat distribution for $t < t_{IR}$ using Sudakov form factor $\Delta(x; t_{IR}, t_{max})$.

Quark distribution

$$\int_{t_{IR}}^{t_{max}} \int_{z_{min}}^{z_{max}} dt dz P_q(x; t, z) = 1 - [\Delta_{q \rightarrow qg}(x; t_{IR}, t_{max}) \Delta_{g \rightarrow q\bar{q}}(x; t_{IR}, t_{max})] \quad (8)$$

AntiQuark distribution

$$\int_{t_{IR}}^{t_{max}} \int_{z_{min}}^{z_{max}} dt dz P_{\bar{q}}(x; t, z) = 1 - [\Delta_{\bar{q} \rightarrow \bar{q}g}(x; t_{IR}, t_{max}) \Delta_{g \rightarrow \bar{q}q}(x; t_{IR}, t_{max})] \quad (9)$$

Gluon distribution

$$\int_{t_{IR}}^{t_{max}} \int_{z_{min}}^{z_{max}} dt dz P_g(x; t, z) = 1 - [\Delta_{g \rightarrow gg}(x; t_{IR}, t_{max}) \Delta_{q \rightarrow gq}(x; t_{IR}, t_{max}) \times \Delta_{\bar{q} \rightarrow g\bar{q}}(x; t_{IR}, t_{max})] \quad (10)$$

The probability function $P(x, t, z)$ depends on the momenta of particles involved, we want to integrate it over a region in x, t, z space of volume V , the variance can be reduced by a change of variables that flattens the integrand, using the Jacobian $\text{Jac}(x(r), t(s, r), z(s, r, v))$.

This change of variables maps the phase space regions in the limits $x_{min} < x < x_{max}$, and $t_{IR} < t < t_{max}$, and $z_{min}(t) < z < z_{max}(t)$ respectively onto a unit cube space $0 < s < 1$, and $0 < v < 1$, and finally $0 < r < 1$, the distribution is flat using these new variables, this allows us to write the phase space volume as an n dimensional hypercube with volume 1.

According to these considerations we write the distribution for three types of partons in initial state radiation.

$$\int_{x_{\min}}^{x_{\max}} \int_{t_{IR}}^{t_{\max}} \int_{z_{\min}}^{z_{\max}} dx dt dz P_b(x; t, z) = \int_0^1 \int_0^1 \int_0^1 dr ds dv [Jac(x(r), t(s, r), z(s, r, v)) \times P_b(x(r), t(s, r), z(s, r, v))] \quad (11)$$

The function $P(x(r), t(s, r), z(s, r, v))$ has less variance than $P(x, t, z)$ itself, then the error will be reduced by distributing points uniformly in r, s, v space.

Now we define the grid function as

$$G_{ijk} = \frac{dx dt dz}{dr ds dv} (x_{ijk}, t_{ijk}, z_{ijk}), \quad (12)$$

The values of i and j are determined by which bin the value $s(t)$ and $v(t; z)$ fall in, k is determined by the value of $r(x)$ in phase space.

If we have n points $\{x_k, t_i, z_j, \text{ where } i, j, k = 1, \dots, n\}$ distributed randomly in the phase space volume V , the central limit theorem of statistics allows the calculation of mean value of $P(x, t, z)$ in these points as an estimator of the integral,

$$P_{grid}(t > t_{IR}) = \frac{1}{n_i n_j n_k} \sum_{ijk} G_{ijk} = 1 - \Delta(x; t_{IR}, t_{\max}) \quad (13)$$

Equation (13) must be the total probability for $t > t_{IR}$ in this case we select t and z by choosing the grid tile according to the definition, then calculate the exact values of t and z using a flat distribution within the tile.

We have calculated the grid version of the probability distribution of splittings, each grid is $[[10 \times 10] \times 10]$ dimensions, they represent the integration of the distribution function of the splitting of Altarelli-Parisi evolution equation, in the limits of the kinematics of the interaction (the hard subprocess).

In total we have created 31 grid considering the flavor and the color of the parton going into the hard subprocess, this number is reduced into eleven grids according to the flavor, if we don't consider the Top quark: five grids of probability distribution for quark, five grids of probability distribution for antiquark and one grid of probability distribution for gluon.

3. Conclusion

We tested our algorithm using different checking, all the tests were almost in perfect agreement with the method we encoded Monte Carlo for a set of splittings we have, we also introduced the calculation of the error using the distribution in the grid tiles.

A Monte Carlo method has been used to generate events according to QCD splitting function, and to compute the statistical uncertainty on the resulting kinematic distribution, since each point corresponds to a set of momentum for the particles involved, the main idea of this method is to develop a generator that produces one body phase space, starting from a given kinematics values, write one point phase space in order to create a recursive phase space generator that at each step adds one extra parton to phase space. The main power of this approach comes from choosing the distribution according to QCD splitting functions.

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4. References

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