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Special Issue

Selected Papers from "Physics beyond the Standard Model in Leptonic & Hadronic Processes and Relevant Computing Tools"

Edited by

Prof. Dr. Theocharis Kosmas, Prof. Dr. Mario E. Gómez, Dr. Odysseas Kosmas and Dr. Alessandro Spatafora

MDPI

https://doi.org/10.3390/particles7030040 *particles*

Article

Neutron Star–Dark Matter Admixed Objects in the Mass Gap Region

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Abstract: To this day, the nature of dark matter (DM) remains elusive despite all our efforts. This type of matter has not been directly observed, so we infer its gravitational effect. Since galaxies and supermassive objects like these are most likely to contain DM, we assume that dense objects such as neutron stars (NSs) are also likely to host DM. The NS is considered the best natural laboratory for testing theories and collecting observational data. We mainly focus on two types of DM particles, fermions and bosons, with a mass range of [0.01–1.5] GeV and repulsive interactions of about [10^{-4}–10^{-1}] MeV $^{-1}$. Using a two-fluid model to solve the TOV equations, we find stable configurations that span hundreds of kilometers and weigh tens or even hundreds of solar masses. To visualize results, we think of a giant invisible compact DM object and the NS in the center as the core, the only visible part. Stability criteria are met for these configurations, so collapsing into a black hole is unlikely. We go further and use this work for smaller formations that exist inside the mysterious Mass Gap. We also find stable configurations of 3–4 solar masses, with NS-DM mixing capable of describing the mass gap. Regardless, the present theoretical prediction, if combined with corresponding observations, could shed light on the existence of DM and even more on its fundamental properties.

Keywords: neutron star; dark matter; two-fluid model; two-fluid stability; mass gap

Citation: Vikiaris, M. Neutron Star–Dark Matter Admixed Objects in the Mass Gap Region. *Particles* **2024**, *7*, 692–702. https://doi.org/10.3390/particles7030040

Academic Editors: Theocharis Kosmas, Mario E. Gómez, Odysseas Kosmas and Alessandro Spatafora

Received: 29 May 2024
Revised: 26 July 2024
Accepted: 2 August 2024
Published: 8 August 2024

1. Introduction

The mystery of dark matter (DM) remains unsolved and continues to challenge the world of astrophysics and cosmology. We assume that DM does not self-annihilate and that it is captured and exists in the gravitational field of compact and supermassive astronomical objects.

There are extensive observational data on NSs [1–3]. If this type of object exists and DM can clump sufficiently and coexist with nuclear matter, then this mixing could affect the measurable properties of NSs (mass, radius, tidal polarizability, etc.). Therefore, many works have proposed that DM assembles into NSs by studying how its accumulation changes the structure of the star and its properties or the tidal polarization in NS binaries [4–49]. In our work, we focus on studying the NS-DM mixture by considering these entities as two separate fluids that do not interact with each other and find out the properties of this compact object by solving the Tolman–Oppenheimer–Volkov (TOV) multi-fluid equations [50–52].

This work is an extension of [53], and here, a more detailed research in the region of the mass gap is studied. In our studies, we use the TOV equations to obtain the basic NS properties, and by checking the stability criteria of the two fluids, we find the stability regions in the MR diagram that indicate the existence of a composite NS-DM object of very large dimensions and immense size in mass but, at the same time, compact and not collapsing into a black hole. This object is hundreds of kilometers long and has a mass of tens, or even hundreds, of solar masses, with NS at the center as the core.

The mass gap region is of great importance in astrophysics because there seems to be a gap in the mass between the heaviest neutron star ($\approx 2.5 M_{\odot}$; for more information see Ref. [54]) and the lightest black hole ($5M_{\odot}$). Despite our thorough research in this region, we cannot clearly identify objects that could fill this gap. Ultra-light black holes or binary mergers are some of the answers for this mystery but are not yet generally accepted [55–58].

To ensure that this object is stable and does not collapse into a black hole, the properties of the DM particle require the mass of the DM particle to be $m_x < 1$ GeV and the interaction to be relatively strong, with the relationship being that the smaller the mass and/or the greater the interaction, the larger the resultant radius and mass ($R_{max} - M_{max}$).

In the present study, we use repulsive self-interaction between DM particles. This is fundamental because this type of interaction is responsible for the very large accumulation of DM and leads to the formation of these compact objects (for a relevant discussion, see [59]). By using the two-fluid model, we aim to obtain useful information about the properties of this compact object. All this work is carried out under strict conditions set by the two-fluid stability criteria.

The study is structured as follows: in Section 2, we describe the two-fluid model with a subsection for the equations of state (EoSs) for the two fluids; in Section 3, we discuss the stability criteria; in Section 4, we present and discuss the results of our studies; and in Section 5, we summarize the work with the concluding remarks.

2. The Two-Fluid Model

In our work, we study the admixture of NS and DM as follows: we consider the two types of matter as two separate fluids that coexist in each other's gravitational field. This means that there is no interaction between the two types of particles, other than gravitational interactions. The first fluid is composed of NS matter (mainly neutrons and some protons and electrons), while the second fluid is composed of non-destructive DM particles. Naturally, the two TOV equations now become four, each set for a specific fluid. Therefore, we solve the four TOV equations simultaneously. These equations are the following [31,50,51,60]:

$$\begin{aligned}\frac{dP_{\rm NS}(r)}{dr} &= -\frac{G\mathcal{E}_{\rm NS}(r)M(r)}{c^2r^2}\left(1+\frac{P_{\rm NS}(r)}{\mathcal{E}_{\rm NS}(r)}\right)\\ &\times\left(1+\frac{4\pi P(r)r^3}{M(r)c^2}\right)\left(1-\frac{2GM(r)}{c^2r}\right)^{-1}\end{aligned} \tag{1}$$

$$\frac{dM_{\rm NS}(r)}{dr} = \frac{4\pi r^2}{c^2}\mathcal{E}_{\rm NS}(r) \tag{2}$$

$$\begin{aligned}\frac{dP_{\rm DM}(r)}{dr} &= -\frac{G\mathcal{E}_{\rm DM}(r)M(r)}{c^2r^2}\left(1+\frac{P_{\rm DM}(r)}{\mathcal{E}_{\rm DM}(r)}\right)\\ &\times\left(1+\frac{4\pi P(r)r^3}{M(r)c^2}\right)\left(1-\frac{2GM(r)}{c^2r}\right)^{-1}\end{aligned} \tag{3}$$

$$\frac{dM_{\rm DM}(r)}{dr} = \frac{4\pi r^2}{c^2}\mathcal{E}_{\rm DM}(r) \tag{4}$$

where also $M(r) = M_{\rm NS}(r) + M_{\rm DM}(r)$, and $P(r) = P_{\rm NS}(r) + P_{\rm DM}(r)$ (the subscripts NS and DM stand for the neutron star and dark matter, respectively).

NS and DM Equation of State

For the NS matter, we use the EoS derived by Akmal et al. (APR) [61] because it complies with the maximum masses measured today ([54,62–64] pulsar observations for the possible maximum mass) and some astrophysical constraints for radii [65]. It is worth mentioning that the whole work is mostly affected by the DM EoS, but we wanted to test a reliable and realistic EoS for the NS too.

For fermionic DM, we consider the particles to be relativistic fermions which interact with each other through a repulsive force. This EoS according to [66] is the following:

$$\begin{aligned}
\mathcal{E}_{\rm DM}(n_\chi) &= \frac{(m_\chi c^2)^4}{(\hbar c)^3 8\pi^2}\Big[x\sqrt{1+x^2}(1+2x^2) \\
&- \ln(x+\sqrt{1+x^2})\Big] + \frac{y^2}{2}(\hbar c)^3 n_\chi^2, \\
P_{\rm DM}(n_\chi) &= \frac{(m_\chi c^2)^4}{(\hbar c)^3 8\pi^2}\Big[x\sqrt{1+x^2}(2x^2/3-1) \\
&+ \ln(x+\sqrt{1+x^2})\Big] + \frac{y^2}{2}(\hbar c)^3 n_\chi^2
\end{aligned} \tag{5}$$

Whereas for bosonic DM, we use the EoS derived by [35,67] and which is the following:

$$\begin{aligned}
\mathcal{E}_{\rm DM}(n_\chi) &= m_\chi c^2 n_\chi + \frac{u^2}{2}(\hbar c)^3 n_\chi^2, \\
P_{\rm DM}(n_\chi) &= \frac{u^2}{2}(\hbar c)^3 n_\chi^2
\end{aligned} \tag{6}$$

The total energy density and total pressure are given if we simply add the two corresponding energy densities and pressures, respectively [61].

3. Stability

The stability of such objects can be studied by considering small radial perturbations of the equilibrium configuration by solving the Strum–Liouville eigenvalue equation and finding the eigenfrequencies [1]. These eigenfrequencies are real numbers [68], and a negative value leads to exponential growth in the radial perturbation and collapse of the star. Only when all the eigenfrequencies are positive will the star be stable [68,69]. A relevant study for pulsation equations has been developed recently in [70].

In this work, we follow a more simple approach, where we examine the behavior of the number of baryons and DM particles for a fixed total mass M of the object [52,71–76]. When the number of particles reaches an extreme value (maximum and minimum), we receive a pair of central pressures for baryonic and dark matter which will serve as the coordinates for the creation of the stability curve. Therefore, we need to solve the following equations [76]:

$$\begin{aligned}
\left(\frac{\partial N_b}{\partial P_c^{\rm NS}}\right)_{\rm M=const} &= \left(\frac{\partial N_\chi}{\partial P_c^{\rm NS}}\right)_{\rm M=const} = 0 \\
\left(\frac{\partial N_b}{\partial P_c^{\rm DM}}\right)_{\rm M=const} &= \left(\frac{\partial N_\chi}{\partial P_c^{\rm DM}}\right)_{\rm M=const} = 0
\end{aligned} \tag{7}$$

where N_b and N_x are the number of baryonic and dark matter particles, respectively [48]. The solution of these equations will provide us with the points at which these quantities are extremized and therefore will provide us with the central pressures of baryonic and dark matter and identify the regions of stability and instability. In our results, we show the curves of the number of dark matter and baryonic matter particles with respect to the baryonic central pressure (the dark matter central pressure follows the same pattern).

The stability is thus examined as follows [76]: beginning from a stable pure DM star, slowly increasing baryonic matter, and the points where N_x decreases while N_b increases are the stable regions. The other way around is the unstable region. Note that there can be fluctuations in the diagram, noting different regions of stability and instability. By obtaining enough points of stability and instability following the same approach, one can create the stability curve diagram.

4. Results and Discussion

What is important to note for a better understanding of the following figures and the research in total is that instead of the known curve on an M-R diagram, in the two-fluid model, we have a surface rather than a curve. This is because the number of free parameters has changed to three: DM particle mass (m_x), the strength of the interaction (y), and the fraction $(f) = P^c_{\mathrm{DM}}/P^c_{\mathrm{NS}}$ of the central densities. In the present study, we showcase three results for objects found in the mass gap region, a region of significant astrophysical interest [77], showing that this region can now be interpreted.

The corresponding figures show our results and findings on the subject. In Figure 1a, we show the known M-R diagram for the compact object. The black dotted line corresponds to a pure dark star, while the blue line corresponds to the admixed object. The first figure depicts the M-R diagram of a compact object and a pure dark star with a dark matter particle of 200 MeV mass and an interaction of 0.001 (the interaction is given in terms of MeV^{-1}). The red cross corresponds to the configuration, the stability of which we choose to check. The properties of this object are $M = 3.56M_\odot$, $R = 291.51$ km, and a compactness of $C = 0.018$, and the fraction of dark matter in terms of total matter used to derive this diagram is $f = 3.03 * 10^{-2}$. The notation of the red cross in the upper right shows the properties of the neutron star that is in the center of the compact object and has a radius of $R = 11.9$ km and $M = 1.4M_\odot$.

Also, it is worth mentioning that even if the APR EoS reaches a maximum mass of $2.16M_\odot$, the strong repulsion that we introduce can result in compact objects of a higher mass and reach $3.56M_\odot$ even with small DM fractions of $f = 3.03 * 10^{-2}$. The repulsive interactions are mainly responsible for the huge compact objects that we examine in this research.

Figure 1b depicts the dependence of the number of dark matter and baryonic matter particles with respect to the baryonic central pressure, that is derived following the procedure mentioned above. The black line corresponds to the number of dark matter particles, and the green line corresponds to the baryonic matter particles. The red crossed points indicate the same configuration as the above Figure 1a. These are the lines that indicate the stability. For the stability, we work as such: start from a pure dark start, check where we obtain a dark star that is inside the mass gap region, slowly add baryonic matter, and check the stability of the object. The configuration, the stability of which we choose to work with, is obtained for the initial conditions of central baryonic pressure $P^c_{\mathrm{NS}} = 74$ and central dark matter pressure $P^c_{\mathrm{DM}} = 2.3149$.

As we slowly increase the baryonic matter central pressure for our initial conditions of solving the TOV equations, it is clear that baryonic matter should increase, while dark matter should decrease. The continued lines indicate this case, where the configurations received in these regions are stable. Whenever the opposite happens, dark matter increases and baryonic matter decreases; the objects are unstable, and there we have the dotted lines. These are the points that we have local minima and maxima as the solution of Equation (7) suggests.

This diagram is not the stability curve but a stability discrimination for one particular configuration. The same curves can be found for other configurations too. The stability curve can be created if for the same constant mass, one receives the values of P^c_{DM} and P^c_{NS} where we have local maxima and minima and plot them on a P^c_{DM}-P^c_{NS} diagram. This is one of our future works.

Figure 2a,b are exactly the same as Figure 1a,b, but now the dark matter particle properties are $m_x = 500$ MeV and $y = 0.01$, and the corresponding compact object's parameters are $M = 3.24M_\odot$, $R = 35.22$ km, and $C = 0.135$ with a fraction of $f = 4.462 * 10^{-1}$. The NS in the center has a mass of $M = 0.77M_\odot$ and $R = 9.28$ km. The configuration, the stability of which we choose to work with, is obtained for the initial conditions of central baryonic pressure $P^c_{\mathrm{NS}} = 100$ and central dark matter pressure $P^c_{\mathrm{DM}} = 86$.

Figure 3a,b are exactly the same as Figure 1a,b but now the dark matter particle properties are $m_x = 1500$ MeV and $y = 0.033$, and the corresponding compact object's parameters are $M = 3.036 M_\odot$, $R = 25.896$ km, and $C = 0.172$ with a fraction of $f = 5.49 * 10^{-1}$. The NS in the center has a mass of $M = 0.63 M_\odot$ and $R = 8.64$ km. The configuration, the stability of which we choose to work with, is obtained for the initial conditions of central baryonic pressure $P^c_{\rm NS} = 100$ and central dark matter pressure $P^c_{\rm DM} = 122.02$.

We have to note that for increasing mass, the interaction is increased, while for decreasing mass, the interaction is also decreased. This happens because we are using a repulsive interaction, meaning that it drives the total structure into achieving a bigger mass and radius. If we combine it with a small particle mass, this can lead to thousands of solar masses and thousands of km radius. If we want to achieve small structures in the scale of the mass gap, we have to control the total structure with the right combination of DM particle mass and interaction.

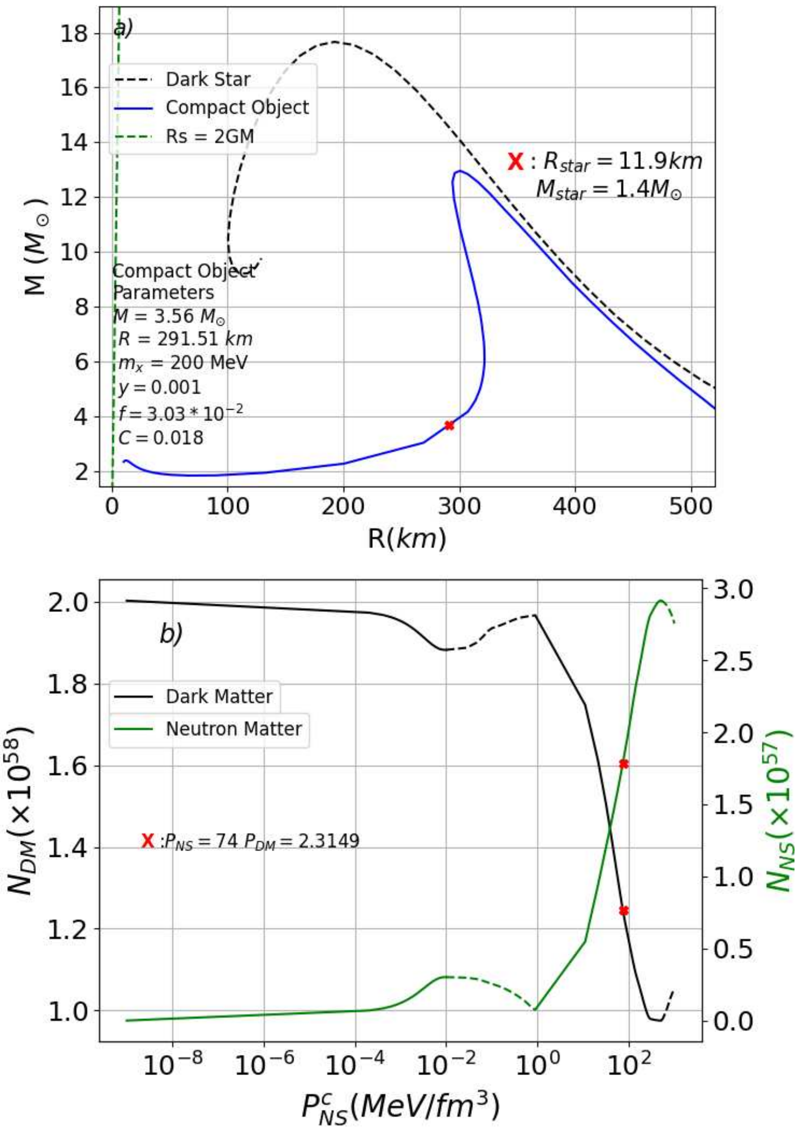

Figure 1. (**a**) The M-R diagram for the case of pure fermionic DM (dashed line) and compact object (solid line). The parameters of dark matter and the total structure are displayed on the figure. The red cross depicts the configuration of a DM object with a NS in its center. The red cross on the upper right shows the parameters of the NS in the center. The green dashed line represents the $R < Rs = 2GM$ line, where the radii reach the Schwarzschild radius. (**b**) The dependence of the number of DM particles $N_{\rm DM}$ and baryons $N_{\rm NS}$ on the pressure $P^c_{\rm NS}$ for equilibrium configurations of equal mass $M = 3.56\ M_\odot$. The solid lines indicate the stable region, while the dashed line indicates the unstable region. The red crosses point to the specific configuration whose stability is investigated.

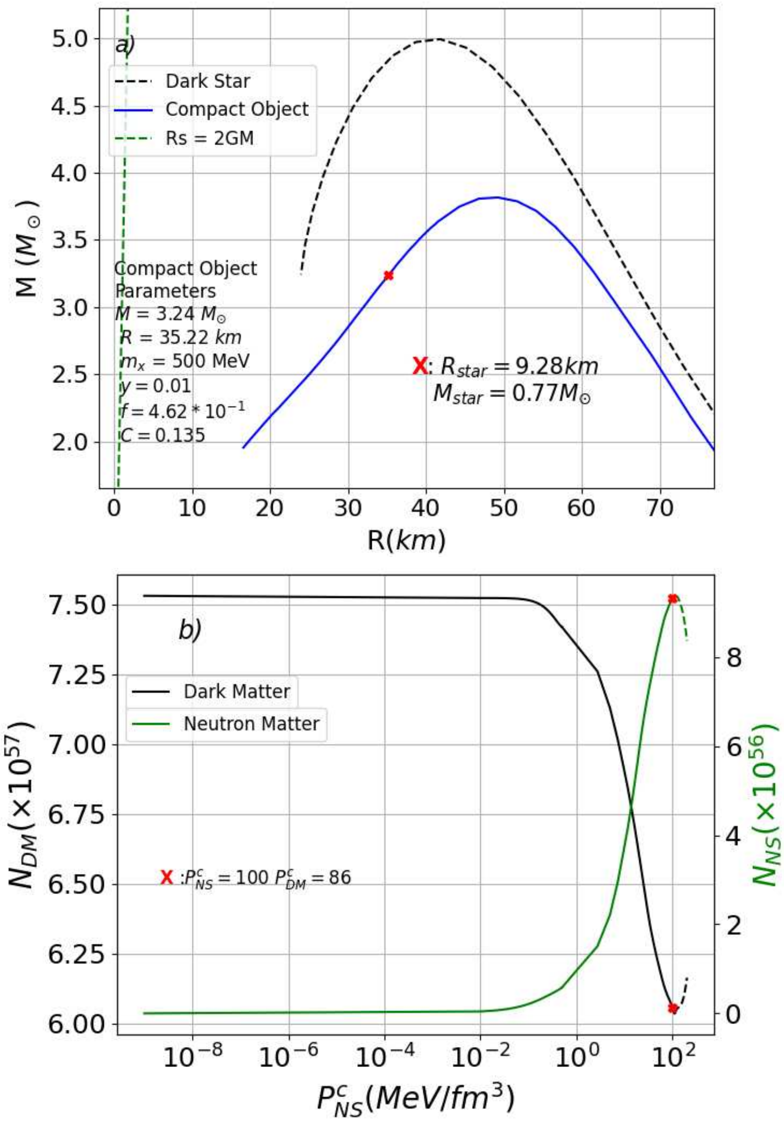

Figure 2. (**a**) The M-R diagram for the case of pure fermionic DM (dashed line) and compact object (solid line). The parameters of dark matter and the total structure are displayed on the figure. The red cross depicts the configuration of a DM object with a NS in its center. The red cross on the upper right shows the parameters of the NS in the center. The green dashed line represents the $R < Rs = 2GM$ line, where the radii reach the Schwarzschild radius. (**b**) The dependence of the number of DM particles N_{DM} and baryons N_{NS} on the pressure P^c_{NS} for equilibrium configurations of equal mass $M = 3.56\ M_\odot$. The solid lines indicate the stable region, while the dashed line indicates the unstable region. The red crosses point to the specific configuration whose stability is investigated.

The last two figures show that in the case of big particle masses and strong interactions, although we obtain a stable total structure that is within the desired mass gap, the neutron star is very small.

It is worth mentioning that most of the works performed to date have mainly focused on using a small percentage of DM, leading to small configurations with mass similar to NS, and a DM that either falls inside the core of the NS or that expands as a halo up to a few tens of kilometers. We work in the opposite way, having the DM be in charge, leading to huge objects of hundreds of solar masses and hundreds of kilometers long.

Also, it is worth pointing out that the stability of such objects using the two-fluid model differs a lot from the ordinary one-fluid stability, and it is a computationally difficult and time-consuming problem.

Lastly, we have made it clear that the previous results lead to the existence of such objects in the mass gap region. These stable configurations indicate the existence of regular neutron stars that have a different structure and gravitational behavior due to the DM component that puts the whole compact object inside the mass gap region.

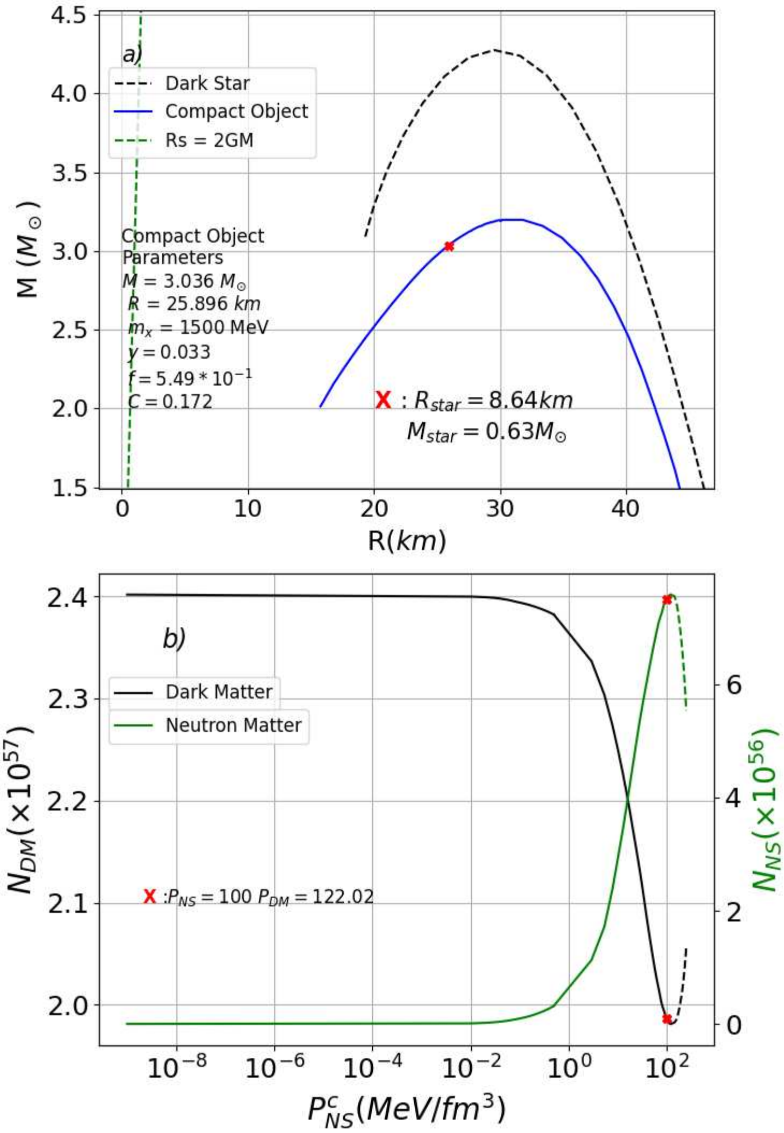

Figure 3. (**a**) The M-R diagram for the case of pure fermionic DM (dashed line) and compact object (solid line). The parameters of dark matter and the total structure are displayed on the figure. The red cross depicts the configuration of a DM object with a NS in its center. The red cross on the upper right shows the parameters of the NS in the center. The green dashed line represents the $R < Rs = 2GM$ line, where the radii reach the Schwarzschild radius. (**b**) The dependence of the number of DM particles N_{DM} and baryons N_{NS} on the pressure P^c_{NS} for equilibrium configurations of equal mass $M = 3.56\ M_{\odot}$. The solid lines indicate the stable region, while the dashed line indicates the unstable region. The red crosses point to the specific configuration whose stability is investigated.

5. Concluding Remarks

The main aim of this work was to prove the existence of supramassive compact objects and, with their help, explain the mystery of the mass gap (recent measurements inside the mass gap region [55–57]. We used self-interacting fermionic and bosonic DM and, unlike other works that have DM form halos in small percentages or even fall inside the NS, we focused mainly first on cases where DM dominates and creates a supramassive compact object where the NS is the core of it, and second on cases where we have DM and NS matter being somewhat equal to create compact objects with an NS-DM admixture that exist in the mass gap region.

Lastly, we have to make some propositions on how to locate these kind of objects. There are two most prominent methods that we suggest: (a) observing the gravitational lensing effect, where the spacetime is distorted by the gravitational field of these objects, and (b) studying possible mergers with the known detectors where we could have the merger of two such objects or a merger between such an object and another compact object, such as an NS or a black hole. The gravitational waves will provide us with valuable information about the structure of these objects [78]. The detection of gravitational waves is extremely important since we can measure the tidal deformability of NSs and the effects that DM has on this property from the binary merger events that we observe.

Furthermore, the size and dimensions of such objects seem to play an important role on the tidal polarizability parameter since the latter are extremely dependent on the mass and radius of the object. "Abnormal" measurements of this property could provide us with a strong "hint" that configurations observed with a mass that falls inside this gap (greater mass than the heaviest observed NS and lower mass than the lightest black hole) could in fact be such an object.

Another possibility of observing such an object is by studying a neutron star with the most stiff EoS we can make. The mass that this NS could provide us with is around $3.2M_\odot$ [79]. Any heavier object observed cannot possibly be a neutron star. That leaves us with the possibility of either a quark star or the compact object of an NS-DM admixture that we suggest.

Either way, further research and studies with astrophysical observations will shed light on this mystery.

Funding: This research received no external funding.

Data Availability Statement: Code available upon request.

Conflicts of Interest: The author declares no conflicts of interest.

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