Improved estimate of the cross section for inverse beta decay

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Abstract. Inverse beta decay (IBD) is the dominant mechanism of antineutrino scattering at energies below a few tens of MeV. Its cross section is currently considered to be known with uncertainty of a fraction of percent. Here I point out that in the existing cross-section calculations the vector part of the hadronic current is not conserved, although its conservation is invoked to express the vector form factors by their electromagnetic counterparts. I obtain the IBD cross section in the most general case, with six contributing form factors, and then use theoretical arguments and experimental constraints to reduce their number. Ensuring conservation of the vector current leads to the results which converges to previous calculations at energies of several MeV but is appreciably lower near the reaction threshold. These findings suggest that the current estimate of the flux of geologically produced antineutrinos may be underestimated. The proposed search for light sterile neutrinos using a ^{144}Ce–^{144}Pr source is predicted to collect a lower event rate and to observe a spectral distortion independent of the distance from the source. In reactor-neutrino experiments, the predicted event rate is reduced—diminishing the size of the reported anomaly—and the positron spectra are altered.

The process of inverse beta decay (IBD), $\bar{\nu}_e + p \to e^+ + n$, is by far the most important mechanism of interaction of low-energy antineutrinos, with the corresponding cross section exceeding those for scattering off electrons and nuclei by a few orders of magnitude at energies below ∼20 MeV [1]. As a consequence, its accurate estimate is essential for the areas of physics as seemingly distant as supernova explosions, the energy budget of the Earth, neutrino oscillations, and nuclear nonproliferation.

At energies of tens of MeV, the calculation of the IBD cross section by Llewellyn Smith [2]—widely employed at higher energies—is not suitable because it neglects the difference between the neutron and proton masses. As a remedy, Vogel and Beacom [3] obtained a low-energy approximation accounting for this difference. As this approximation becomes inaccurate at energies above ∼20 MeV, Strumia and Vissani [4] performed fully relativistic calculations.

The IBD cross section is currently considered to be known with uncertainty of a fraction of percent. However, as I have pointed out [5], the vector part of the hadronic current employed in [3, 4] is not conserved, although its conservation is invoked to express the vector form factors by their electromagnetic counterparts. I have proposed to remove this theoretical inconsistency by using an appropriate matrix representation of the current and observed that this procedure sizably changes the description of the IBD process near the threshold, reducing the total cross section and increasing the directionality of the produced positrons. Here I obtain the expression for the IBD cross section [5] in a different way, inspired by the comment of Carlo Giunti [6].

In section 1, the IBD cross section is calculated in the most general case of six nonvanishing form factors. As described in section 2, currently it would not be possible to determine these functions from experimental data alone and, therefore, for practical reasons they need to be constrained using also theoretical arguments, such as conservation of the vector current. In section 3, the IBD cross-section estimate [5] is analyzed in the context of the positron-production directionality and low-energy applications such as geoneutrinos and reactor antineutrinos. Section 4 states the conclusions.

1. General considerations

To obtain the IDB cross section, recall that the corresponding matrix element can be accurately calculated within the Fermi theory,

$$\mathcal{M} \propto J_\mu^{\mathrm{lept}}\, J^\mu_{\mathrm{hadr}}, \tag{1}$$

as an interaction between the leptonic and hadronic currents,

$$J_\mu^{\mathrm{lept}} = \bar{u}_e \gamma_\mu (1+\gamma_5) u_{\bar\nu} \quad \text{and} \quad J^\mu_{\mathrm{hadr}} = \bar{u}_n (V^\mu + A^\mu) u_p, \tag{2}$$

with the Dirac spinors $u_{\bar\nu} = u_{\bar\nu}(k,\lambda)$, $u_e = u_e(k',\lambda')$, $u_p = u_p(p,s)$, and $u_n = u_n(p',s')$ describing the electron antineutrino, positron, proton, and neutron, respectively. In the most general case [7, 8], the Lorentz invariance requires the axial and vector parts of the hadronic current to read

$$A^\mu = \gamma^\mu \gamma_5 F_A + i\sigma^{\mu\kappa}\gamma_5 \frac{q_\kappa}{2M} F_T + \frac{q^\mu}{M}\gamma_5 F_P, \tag{3}$$

$$V^\mu = \gamma^\mu F_1 + i\sigma^{\mu\kappa} \frac{q_\kappa}{2M} F_2 + \frac{q^\mu}{M} F_3. \tag{4}$$

Note that the form factors F_X are functions of four-momentum transfer squared q^2, with $q = k - k' = p' - p$, and may depend on both the neutron and proton masses, M_n and M_p, or—equivalently—on the average nucleon mass $M = \frac{1}{2}(M_n + M_p)$ and the nucleon mass difference $\Delta = M_n - M_p$.

Making use of the identities

$$\bar{u}_n \left[i\sigma^{\mu\kappa}\gamma_5 q_\kappa\right] u_p = \bar{u}_n \left[(M_n - M_p)\gamma^\mu\gamma_5 - (p+p')^\mu \gamma_5\right] u_p, \tag{5}$$

$$\bar{u}_n \left[i\sigma^{\mu\kappa} q_\kappa\right] u_p = \bar{u}_n \left[(M_n + M_p)\gamma^\mu - (p+p')^\mu\right] u_p, \tag{6}$$

one can simplify expressions (3) and (4) to

$$A^\mu = \gamma^\mu\gamma_5 \left(F_A + \frac{\Delta}{2M} F_T\right) - \frac{(p+p')^\mu}{2M}\gamma_5 F_T + \frac{q^\mu}{M}\gamma_5 F_P, \tag{7}$$

$$V^\mu = \gamma^\mu (F_1 + F_2) - \frac{(p+p')^\mu}{2M} F_2 + \frac{q^\mu}{M} F_3. \tag{8}$$

Standard calculations lead to the differential cross section as a function of q^2 that can be cast in the form [2, 4]

$$\frac{d\sigma^{\mathrm{tree}}}{dq^2} = \frac{(G_F \cos\theta_C)^2}{8\pi M_p^2 E_\nu^2}\left[M^4 A + M^2 B(s-u) + C(s-u)^2\right], \tag{9}$$

where $s-u=4M_pE_\nu+q^2-m^2-2M\Delta$, E_ν being the neutrino energy, G_F and θ_C denote the Fermi constant and the Cabibbo angle, and

$$\begin{aligned}
A = {} & 4(\tau+\mu)\Big\{(\tau-\mu)\left(G_M^2+G_A^2\right)-G_E^2+G_A^2+4\mu F_P(\tau F_P-F_A)+(4\mu F_3^2-\tau F_T^2)(1+\tau)\Big\}\\
& -8\mu\frac{\Delta}{M}\left[G_M\left(G_A+\mu F_3\right)+F_1F_3(1+\tau)+\mu F_TF_P\right]+4\frac{\Delta}{M}F_AF_T\left(\tau^2+\tau\mu+\tau\right)\\
& +\frac{\Delta^2}{M^2}\left[\tau G_M^2-(1+\tau)F_1^2-F_A(F_A+4\mu F_P)-\mu F_T^2\right]\\
& +\frac{\Delta^2}{M^2}(\tau+\mu)\left[G_M^2-(1+\tau)F_2^2-F_A^2+4\mu F_P^2\right],\\
B = {} & 4\tau G_MG_A-4\mu\left[G_EF_3+\frac{1}{2}F_T\left(G_A-2\tau F_P\right)\right]-\mu\frac{\Delta}{M}\left[G_MF_2+2F_AF_P\right],\\
C = {} & \frac{1}{4}\left[F_1^2+F_A^2+\tau(F_2^2+F_T^2)\right],
\end{aligned} \tag{10}$$

with the shorthand notations $G_E=F_1-\tau F_2$, $G_M=F_1+F_2$, $G_A=F_A+\frac{\Delta}{2M}F_T$, $\tau=-q^2/4M^2$ and $\mu=m^2/4M^2$.

2. How can we constrain the form factors?

While equations (9) and (10) are valid in the most general case, their practical applications are very limited, because currently it is not possible to determine all the form factors from experimental data alone. As a consequence, we need to use theoretical arguments to reduce the number of these unknown functions.

Appearing in the axial part of the hadronic current (7), the pseudoscalar form factor F_P can be related to the axial form factor F_A as

$$F_P=\frac{2M^2}{m_\pi^2-q^2}F_A \quad \text{with} \quad F_A=\frac{g_A}{(1-q^2/M_A^2)^2} \tag{11}$$

where m_π is the pion mass [8]. The axial coupling constant g_A is extracted from neutron beta-decay measurements, and the axial mass M_A is determined predominantly from neutrino scattering off deuteron [9]. As available experimental data constrain the second class axial form factor to $F_T/F_A=-0.15\pm0.17$ [10], the corresponding contribution to the cross section is between -0.024% and $+0.002\%$ for $E_\nu\leq 20$ MeV, and F_T can safely be neglected. In this way we reduce the number of independent form factors in the axial current to one.

Due to the isospin invariance of strong interactions, the vector currents $\bar{u}_nV^\mu u_p$ and $\bar{u}_p\gamma_0V^{\mu\dagger}\gamma_0u_n$ together with the isovector electromagnetic current form a triplet of conserved currents [11, 12], and the vector form factors F_i $(i=1,2)$ are related to the electromagnetic form factors of proton and neutron. For the vector current (4) to be conserved, however, the following expression needs to vanish identically,

$$q_\mu\bar{u}_nV^\mu u_p=\bar{u}_n\left[(M_n-M_p)F_1+\frac{q^2}{M}F_3\right]u_p=0 \quad \text{and therefore} \quad F_3=-\frac{\Delta M}{q^2}F_1. \tag{12}$$

When the nucleon-mass difference Δ can be neglected, conservation of the vector current (CVC) requires $F_3\equiv 0$. However, when Δ plays an important role, nonvanishing F_3 is necessary to ensure CVC. Note that *corrected for effects of isospin-symmetry breaking*, the world data for superallowed $0^+\to0^+$ nuclear beta decays confirm the CVC hypothesis with great precision, at

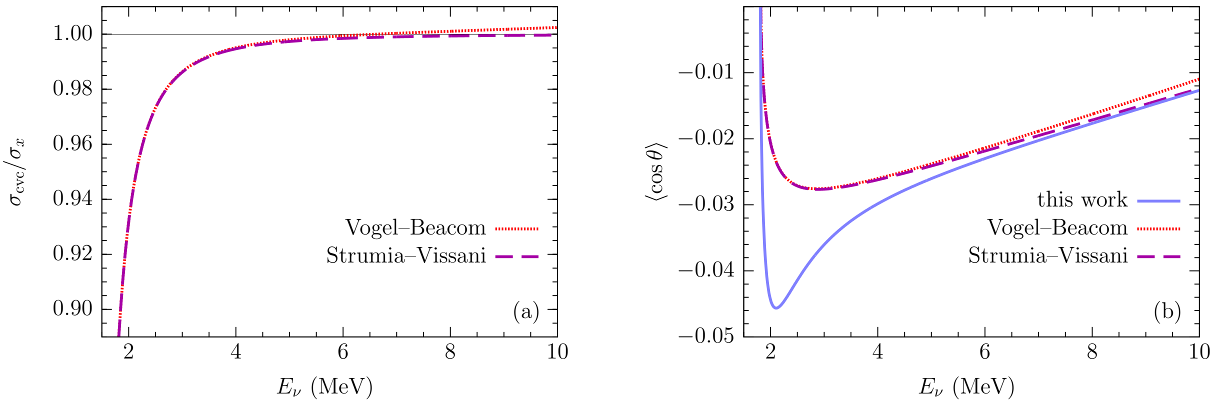

Figure 1. (a) Reduction of the total IBD cross section due to ensured conservation of the vector current, as a function of antineutrino energy. The estimate [5] is divided by the results of Vogel and Becom [3] and of Strumia and Vissani [4]. (b) Comparison of the average cosine of the positron production angle calculated within these three approaches.

the level of 10^{-4} [13]. The standard cross-section estimates [3, 4] assume vanishing F_3, despite of relying on CVC to express the vector form factors.

To remove this inconsistency one can use the vector current [5]

$$V^\mu_{\rm cvc} \;=\; \gamma^\mu\left(F_1+F_2\right)-\frac{(p+p')^\mu}{2M}F_2-q^\mu\frac{\Delta}{q^2}F_1, \tag{13}$$

which reduces to the standard expression in the limit $\Delta\to 0$, but is conserved for $\Delta\neq 0$. From now on, I denote the cross section calculated using the vector current (13) as $\sigma_{\rm cvc}$. It corresponds to setting $F_3=-\frac{\Delta M}{q^2}F_1$ in equations (9) and (10).

3. Results

Playing an important role only at low absolute values of q^2, the CVC restoring procedure (13) affects the IBD cross section in an appreciable manner solely at low neutrino energies. This feature is illustrated in figure 1(a), showing the ratio of $\sigma_{\rm cvc}$ to the calculations of Vogel and Beacom [3] and those of Strumia and Vissani [4]. In all the cases, the same treatment of the radiative corrections has been applied. While at $E_\nu = 2$ MeV, the cross section $\sigma_{\rm cvc}$ is lower than the results [3, 4] by as much as $\sim$6.8%, this effect reduces to $\sim$0.5% at 4 MeV. Note that at higher energies the difference between the cross sections [3] and [4] gradually becomes visible but remains below 0.5% for $E_\nu \leq 13$ MeV.

As the kinematic region of low $|q^2|$ corresponds to high $\cos\theta$, with θ being the positron's production angle, the observed reduction of the cross section at low $|q^2|$ translates into a decrease of the average value of $\cos\theta$, shown in figure 1(b). The manifest increase of the directionality at energies $E_\nu\sim$ 2–3 MeV, resulting predominantly from the last term in the B factor (10), may be relevant, e.g., for spatially mapping geoneutrinos [14].

The IBD cross section is generally considered to be subject to low uncertainties and, therefore, its CVC-related reduction may have important consequences. For example, in the context of a determination of the geoneutrino flux, I find that $\sigma_{\rm cvc}$ leads to the ^{232}Th and ^{238}U components higher by 6.1 and 3.7%, respectively, than the estimates based on the cross sections [3, 4]. Those values are calculated using the spectra [15] and refer to the KamLAND site.

Recently, it has been proposed to search for light sterile neutrinos using a ^{144}Ce–^{144}Pr $\bar\nu_e$ source [16]. Using the flux [17], I predict an overall 3% reduction of the event rate with respect to simulations employing the cross sections [3, 4], and a spectral distortion that depends on energy but not on the distance from the source. As such an experiment is currently underway [18], these predictions can be tested within a 1-year time frame.

The reevaluation of the antineutrino spectra emitted by nuclear reactors [19] has recently lead to the conclusion that the event rates observed in past reactor experiments underestimate the predicted rates by $5.7 \pm 2.3\%$ [20]. Combining the contributions from the individual isotopes [19, 21] according to the weights [22], I estimate that the CVC-related reduction of the cross section lowers the predicted rate by 0.9%, reducing the reactor anomaly.

Moreover, as the antineutrino energy is closely related to the prompt energy of the produced positron, $E_{\text{prompt}} \simeq E_\nu - 0.78$ MeV, the results of Fig. 1(a) corresponding to the low-E_{prompt} region can be expected to bring into better agreement the predictions and the prompt energy spectra measured in near detectors of ongoing reactor experiments [22, 23].

4. Conclusions

The description of inverse beta decay at the kinematics corresponding to low $|q^2|$ turns out to be very sensitive to conservation of the vector part of the weak current. I have considered a prescription to restore it, which sizably lowers the total cross section for energies in the vicinity of the reaction threshold. These findings may soon be verified by an experiment employing a ^{144}Ce–^{144}Pr source to search for light sterile neutrinos. Should they be confirmed, the deviations from the standard description would be of particular relevance for an estimate of the geoneutrino flux and would lead to a reduction of the reactor anomaly, affecting also the predicted positron spectra in reactor-neutrino experiments.

As a final remark I would like to note that the process of neutron decay involves the same physical mechanisms as inverse beta decay and, therefore, such data should also be used to verify the discussed procedure to restore conservation of the vector current.

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