Photon polarization oscillations

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Quantum corrections to the Maxwell equations induced by light-by-light (LbL) scattering can significantly modify the propagation of light in vacuum. Studying the Heisenberg-Euler Lagrangian, it can be shown that, in some configurations, the polarization of plane monochromatic waves oscillates periodically between different helicity states, due to LbL scattering. We discuss the physical implications of this finding, and the possibility of measuring this effect in optical experiments.

Keywords: Nonlinear optics in vacuum, light-by-light scattering, multiscale perturbative approach.

Despite the fact that the equations of the classical electromagnetic field are linear, quantum corrections due to photon-photon scattering introduce nonlinear effects in vacuum. The quantum corrections due to photon-photon scattering were calculated a long time ago by Heisenberg and Euler [1], and extensively studied by other authors [2–5]. The effective Lagrangian of the electromagnetic field, obtained retaining only one electron loop corrections, is [5]

$$L = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \epsilon^2 \left[(F_{\mu\nu}F^{\mu\nu})^2 - \frac{7}{16}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2 \right] , \tag{1}$$

where $F^{\mu\nu} = A^{\mu,\nu} - A^{\nu,\mu}$ is the electromagnetic field,[a] A^μ is the electromagnetic four-potential, $\tilde{F}^{\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$, and $\epsilon^2 = \alpha^2(\hbar/m_e c)^3/90 m_e c^2$, $\alpha = e^2/4\pi\epsilon_0\hbar c \simeq 1/137$ is the fine structure constant, ϵ_0 the dielectric permeability of vacuum, and m_e the electron mass. The Lagrangian (1) is accurate when it is possible to neglect other quantum effects. For instance, for low energetic photons of energies $E_\gamma \ll m_e c^2$, particles creation is inhibited, and the photon-photon scattering is the only process inducing quantum correction to the Maxwell equations.

The terms $\propto \epsilon^2$ in the Lagrangian (1) take into account photon-photon scattering, and induce cubic corrections in the equations for the four-potential A^μ. Since

[a] In this paper we use the covariant formalism, so that the zeroth coordinate is defined as $x^0 = ct$.

$\epsilon^2 \simeq 4 \times 10^{-31} m^3/J$, so that $\epsilon^2 F_{\mu\nu}F^{\mu\nu}$ is extremely small in realistic laboratory conditions, such corrections are usually negligible with high accuracy.

However, in many physical situations, tiny perturbations produce huge effects on a system, due to the action of hidden resonances. Indeed, resonances can be used to amplify the effect of extremely small perturbations. Exploiting this idea, in Refs. 6, 7 it has been shown that the dynamics described by the Lagrangian (1) is unstable for some configurations of the electromagnetic field, due to resonances.

For instance, considering two plane counterpropagating electromagnetic waves in vacuum, the nonlinear terms in (1) generate resonant (or secular) corrections in the equations of the electromagnetic field. Introducing a slow time variable, the secular terms can be treated in a multiscale scheme. In fact, the amplitudes of the two counterpropagating waves satisfy a system of nonlinear coupled ordinary differential equations in the slow time. The analysis of this system shows that, for some initial conditions, the effect of photon-photon scattering is unexpectedly relevant, and consists of a continuous oscillation in the polarization of the two beams between different helicity states.

Without loss of generality, we use the Lorentz gauge $\partial_\alpha A^\alpha = 0$; and we express the polarization vectors in terms of left and right polarizations $\hat{e}_L = (1, i, 0)/\sqrt{2}$ and $\hat{e}_R = (1, -i, 0)/\sqrt{2}$. Let us express the four-potential A of the classical electromagnetic field in the form

$$A = a^\alpha + b^\alpha + c.c., \quad \text{with} \quad a = \left(a_L \hat{e}^L + a_R \hat{e}^R\right) e^{ikx}, \, b = \left(b_L \hat{e}^L + b_R \hat{e}^R\right) e^{ihx}, \tag{2}$$

where c.c. stands for complex conjugate. The wave vectors in (2) are given by

$$k = (k_0, 0, 0, k_3), \quad h = (h_0, 0, 0, h_3), \tag{3}$$

with $k_0/k_3 = -h_0/h_3 = 1$, so that the two waves a and b are counterpropagating.

When nonlinearities are neglected, (1) reduces to the Maxwell Lagrangian, and (2) is a solution when polarization vectors are constant. When nonlinearities are considered, the solutions for the Lagrangian (1) are still in the form (2), but with the polarization vectors depending on a slow time

$$y^0 \equiv \epsilon^2 x^0. \tag{4}$$

The dependence of the polarization vectors from the slow time is given by the following system

$$\begin{aligned}
& i\partial_{y^0} a_L + 16 k_0 h_0^2 \left(-3\, a_L \left(|b_L|^2 + |b_R|^2\right) + 22\, a_R b_L \bar{b}_R\right) = 0 \\
& i\partial_{y^0} a_R + 16 k_0 h_0^2 \left(-3\, a_R \left(|b_L|^2 + |b_R|^2\right) + 22\, a_L b_R \bar{b}_L\right) = 0 \\
& i\partial_{y^0} b_L + 16 k_0^2 h_0 \left(-3\, b_L \left(|a_L|^2 + |a_R|^2\right) + 22\, b_R a_L \bar{a}_R\right) = 0 \\
& i\partial_{y^0} b_R + 16 k_0^2 h_0 \left(-3\, b_R \left(|a_L|^2 + |a_R|^2\right) + 22\, b_L a_R \bar{a}_L\right) = 0 \,.
\end{aligned} \tag{5}$$

that has been obtained by a multi-scale perturbative expansion of the equations of motion[6,7]. We recall that the multiscale approach is useful when the dynamics evolves on widely different scales. In this case, the time dependence of the electromagnetic field is split into fast and slow time variables x^0 and y^0.

Let us study (5) in detail. It is quite immediate to recognize that the energy densities $< \rho_a >= k_0^2 \left(|a_L|^2 + |a_R|^2\right)$ and $< \rho_b >= h_0^2 \left(|b_L|^2 + |b_R|^2\right)$ are constant. Therefore, the intensities of the two plane waves a^μ and b^μ are conserved separately. Furthermore, the spin conservation implies that the quantity $S = k_0 \left(|a_L|^2 - |a_R|^2\right) + h_0 \left(|b_L|^2 - |b_R|^2\right)$ is also constant. Exploiting these relations, the system can be simplified and then integrated, see Refs. 6, 7. However, to understand the dynamics under study, it is sufficient to solve (5) numerically.

We choose the initial conditions in such a way that, at least one of the products $a_L\, a_R$ or $b_L\, b_R$ is nonzero at the initial time. Numerical solutions show that, in such case, the polarizations of the two counterpropagating waves change periodically. In Fig. 1, we plot $|a_L|^2/|a_L^0|^2 + |a_R^0|^2$ and $|a_R|^2/|a_L^0|^2 + |a_R^0|^2$ for the solution of (5) with realistic initial values of the electromagnetic potential, that is $|a_L^0|^2 = 10^3 J/m$, $a_R^0 = 0$, $|b_L^0|^2 = |b_R^0|^2 = 10^3 J/m$, $k_0 = h_0 = 10^7\, m^{-1}$, corresponding to a laser of intensity $I \simeq 10^{23}\, W/cm^2$ and wavelenght $\lambda \sim 1\, \mu m$. We see that $|a_R|$ is initially zero, but it grows to $|a_R| = |a_L^0|$, while $|a_L|$ goes from $|a_L^0|$ to zero. Thus, the a beam, initially in the left-handed polarization, switches to the right-handed polarization. It remains in this state most of the time, until it returns to its initial left-handed configuration. The variation of the b beam is depicted in Fig. 2, where we plot $|b_L|^2/|b_L^0|^2 + |b_R^0|^2$ and $|b_R|^2/|b_L^0|^2 + |b_R^0|^2$ for the same initial values. The beam b is initially circularly polarized, since $|b_L^0|^2 = |b_R^0|^2$, but it rapidly goes to a left-handed configuration with $|b_R| = 0$ and $|b_L|^2 = 2|b_L^0|^2$. It remains in this state most of the time, until it returns to its initial state. This dynamics is repeated periodically.

Thus, the beam a oscillates between left and right polarizations, while the beam b switches periodically between linear and right-handed helicities. The period of such polarization oscillations is $\Delta y^0 \simeq 10^{-26}\, m^4/J$ in the slow time y^0. This value is in good agreement with theoretical estimations $\Delta y^0 \sim inf \left\{1/k_0 h_0^2 |b_0|^2, 1/k_0^2 h_0 |a_0|^2\right\}$ obtained in Refs. 6, 7. The corresponding period in the physical time $x^0 = t$ is $T = \Delta y^0/\epsilon^2\, c \simeq 10^{-4}\, s$.

Finally, we consider the possibility of observing the polarization oscillations in optical experiments. The search for signatures of the photon-photon scattering in optics is in progress[8–32]. We can estimate the time of recurrence of the polarization oscillations for light beams produced in petawatt class lasers, which will be available in the near future. The intensities attainable in these lasers reach $I \sim 10^{23}\, W/cm^2$ [33,34], giving a recurrence time $T_i \sim 4 \times 10^2\, (\lambda/m)\, s$, where λ/m is the laser wavelength in meters (we used $k \sim h \sim 2\pi/\lambda$ and $k^2 a^2 \sim k^2 b^2 \sim< \rho >\sim I/c$). Therefore, for realistic lasers with $\lambda \sim 1\, \mu m$, observation times can be of the order of $10^{-3}\, s$; to be compared with those estimated

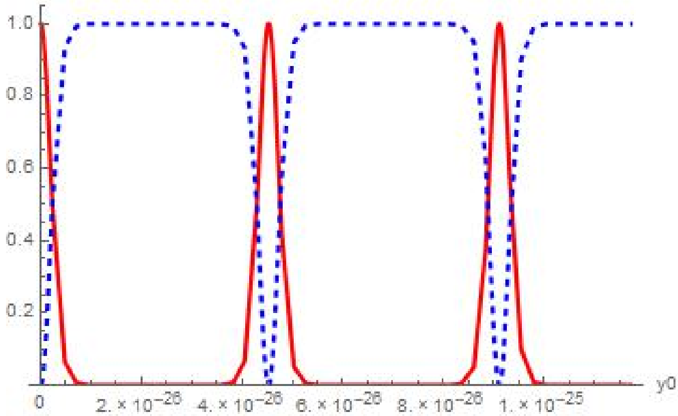

Fig. 1. We plot the evolution of $|a_L|^2/|a_L^0|^2+|a_R^0|^2$ (solid red line) and $|a_R|^2/|a_L^0|^2+|a_R^0|^2$ (dashed blue line) against y^0 (in units of m^4/J) for $|a_L^0|^2 = 10^3 J/m$, $a_R^0 = 0$, $|b_L^0|^2 = |b_R^0|^2 = 10^3 J/m$, $k_0 = h_0 = 10^7\, m^{-1}$.

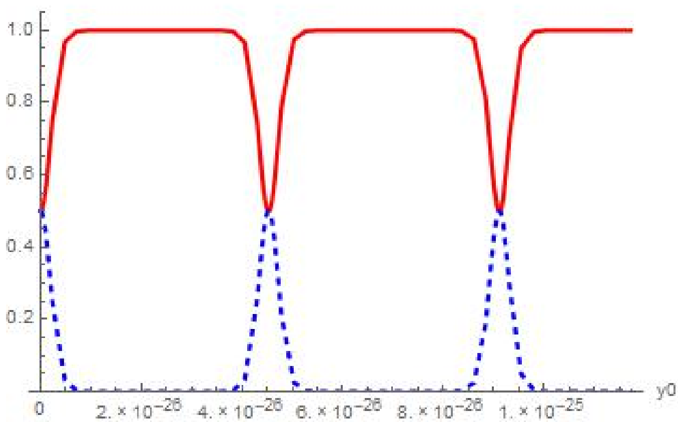

Fig. 2. We plot the evolution of $|b_L|^2/|b_L^0|^2+|b_R^0|^2$ (solid red line) and $|b_R|^2/|b_L^0|^2+|b_R^0|^2$ (dashed blue line) against y^0 (in units of m^4/J) for $|a_L^0|^2 = 10^3 J/m$, $a_R^0 = 0$, $|b_L^0|^2 = |b_R^0|^2 = 10^3 J/m$, $k_0 = h_0 = 10^7\, m^{-1}$.

in Ref. 14. This lets us hope to be able to observe polarization oscillations in two counterpropagating petawatt laser beams.

In conclusion, the extremely weak photon-photon interaction might be responsible for surprisingly strong deviations from the free dynamics of electromagnetic waves. In the case of two counterpropagating laser beams, the evolution of the electromagnetic waves can change dramatically with respect to the linear Maxwell equations, entailing slow oscillations of the polarizations of the beams.

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