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The irreducible mass of Christodoulou-Ruffini-Hawking mass formula

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We reveal three new discoveries in black hole physics previously unexplored in the Hawking era. These results are based on the remarkable 1971 discovery of the irreducible mass of the black hole by Christodoulou and Ruffini, and subsequently confirmed by Hawking.

1. The Horizon Mass Theorem shows that the mass at the event horizon of any black hole: neutral, charged, or rotating, depends only on twice its irreducible mass observed at infinity.
2. The External Energy Conjecture proposes that the electrostatic and rotational energy of a general black hole exist completely outside the horizon due to the nature of the irreducible mass.
3. The Moment of Inertia Property shows that every Kerr black hole has a moment of inertia. When the rotation stops, there is an irreducible moment of inertia as a result of the irreducible mass.

Thus after 50 years, the irreducible mass has gained a new and profound significance. No longer is it just a limiting value in energy extraction, it can also determine black hole dynamics and structure. What is believed to be a black hole is a physical body with an extended structure. Astrophysical black holes are likely to be massive compact objects from which light cannot escape.

Keywords: Black holes; quasi-local energy; irreducible mass; Kerr metric; moment of inertia.

1. 50th Anniversary 1971–2021

This article is to celebrate the 50th anniversary of the discovery of the mass-energy formula of a Kerr-Newman black hole in 1971 by introducing three new results recently found in black hole physics. Surprisingly, these results all invove the concept of the irreducible mass.

First, congratulations to Demetrios Christodoulou and Remo Ruffini for their remarkable discovery of the *irreducible mass*[1] of the black hole and confirmed by Hawking,[2] one of the most important concepts in black hole physics.

This year is also the 50th anniversary of the renormalization of Yang-Mills theory[3] in 1971. Congratulations to Gerard 't Hooft and the late Martinus Veltman for their elucidation of the quantum structure of electroweak interactions, one of the great achievements in 20th Century physics.

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The irreducible mass formula discovered by Christodoulou and Ruffin in 1971 is the following:[1]

$$M^2 = \left(M_{irr} + \frac{Q^2}{4GM_{irr}}\right)^2 + \frac{J^2c^2}{4G^2M_{irr}^2}. \tag{1}$$

Here M is the total mass of the Kerr-Newman black hole, M_{irr} is the irreducible mass; Q is the electric charge and J is the angular momentum. All quantities are reckoned according to the distant observer. When Q and J are zero, the irreducible mass is the mass of a Schwarzschild black hole. 50 years later, the irreducible mass has gained unexpected new and profound significance besides energy extraction. It can also determine black hole dynamics and structure.

It is especially appropriate to explain the many definitions of a black hole in physics.[4] The mathematical black hole in general relativity has a singularity hidden by a horizon. However, neither singularity nor horizon has been observed. Compact objects like the one at the center of our galaxy are also called black holes in common usuage, even though their nature is still unknown. This is pointed out in the 2020 Nobel Prize in Physics citation. To Roger Penrose, the citation is 'for the discovery that black hole formation is a robust prediction of the general theory of relativity'. To Reinhard Genzel and Andrea Ghez, the citatation is 'for the discovery of a supermassive compact object at the center of our galaxy'. The term black hole is avoided. Strictly speaking, the black hole has not been discovered, but only a black hole-like object has been observed in astyrophysics. The Laplace 'dark star' introduced the concept of the black hole as a massive body from which light cannot escape due to its strong gravity.

Between 1965–1985, several important theorems on classical black holes were gradually discovered. They are known as:

(1) Singularity Theorem (1965),[5]
(2) Area Non-decrease Theorem (1972),[6]
(3) Uniqueness Theorem (1975),[7]
(4) Positive Energy Theorem (1983).[8]

These theorems have been well discussed for many years in general relativity and accepted as basic properties of the classical black hole. In recent years, three new results on black holes previously unexplored in the Hawking era are found. They were developed using the quasi-local energy approach and angular momentum consideration. Remarkably, they all contain the irreducible mass of Christodoulou and Ruffini. They are:

(5) Horizon Mass Theorem (2005),[9]
(6) External Energy Conjecture (2017),[10]
(7) Moment of Inertia Property (2018).[11]

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These results are derived completely within general relativity and therefore legitimate. They add new properties to the classical black hole with potential to resolve several long-standing paradoxes in black hole physics.

2. Horizon Mass Theorem

Theorem. *For neutral and charged black holes, the horizon mass is always twice the irreducible mass observed at infinity.*

For rotating black holes, the horizon mass is found to be extremely close to twice the irreducible mass for all rotations. It is conjectured that a rigorous proof will eventually show that the horizon mass is exactly twice the irreducible mass.

In notation, it is simply

$$M(r_+) = 2M_{irr} \tag{2}$$

where r_+ is the horizon radius of the black hole. The theorem relates the mass of a black hole observed at the event horizon to its irreducible mass observed at infinity. The irreducible mass does not contain electrostatic and rotational energy. The Horizon Mass Theorem is the final outcome of quasi-local mass applied to black holes.

The quasi-local energy is one of the most important concepts in general relativity after decades of searching for a consistent definition of gravitational energy. It was finally obtained in 1993. The Brown and York expression[12] for quasi-local energy is given in terms of the total mean curvature of a surface bounding a volume for a gravitational system in four-dimensional spacetime. The total energy E , including binding energy, is given in the form of an integral,

$$E = \frac{c^4}{8\pi G} \int_{^2B} d^2x \sqrt{\sigma}(k - k^0), \tag{3}$$

where σ is the determinant of the metric defined on the two-dimensional surface ^2B ; k is the trace of the extrinsic curvature of the surface, and k_0 , the trace of the curvature of a reference space. For asymptotically flat reference spacetime, k^0 is taken to be zero. The expression in Eq.(3) is the basis for establishing the Horizon Mass Theorem.

For a Schwarzschild black hole, the total energy contained in a sphere enclosing the black hole at a coordinate distance r is calculated,[12–14]

$$E(r) = \frac{rc^4}{G} \left[1 - \sqrt{1 - \frac{2GM}{rc^2}} \right]. \tag{4}$$

At the Schwarzschild radius, $r = r_+ = 2GM/c^2$, the above equation reduces to

$$E(r) = \left(\frac{2GM}{c^2} \right) \frac{c^4}{G} = 2Mc^2, \tag{5}$$

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giving the first case of the Horizon Mass Theorem in Schwarzschild spacetime, i.e. $M(r_+) = 2M$.

For a Reissner-Nordström black hole enclosed within a radius at coordinate r, the total energy calculated is now [9],

$$E(r) = \frac{rc^4}{G}\left[1 - \sqrt{1 - \frac{2GM}{rc^2} + \frac{GQ^2}{r^2c^4}}\right]. \tag{6}$$

Here M is the total mass of the black hole including electrostatic energy observed at infinity, and Q is the electric charge.

The irreducible mass is the final mass of the black hole when its charge is neutralized by adding oppositely charged particles, extracting energy from the black hole. It is defined as in electrostatics,

$$M = M_{irr} + \frac{Q^2}{4GM_{irr}}. \tag{7}$$

Inverting the equation to solve for M_{irr}, we find

$$M_{irr} = \frac{M}{2} + \frac{M}{2}\sqrt{1 - \frac{Q^2}{GM^2}}. \tag{8}$$

The horizon radius of a Reissner-Nordström black hole is known to be

$$r_+ = \frac{GM}{c^2} + \frac{GM}{c^2}\sqrt{1 - \frac{Q^2}{GM^2}}. \tag{9}$$

Combining Eq.(6) and Eq.(9), we find the mass contained within the horizon to be

$$\frac{E(r_+)}{c^2} = M(r_+) = M + M\sqrt{1 - \frac{Q^2}{GM^2}}, \tag{10}$$

i.e. $M(r_+) = 2M_{irr}$. This is the second case of the Horizon Mass Theorem in Reissner-Nordström spacetime. It is seen that the horizon mass of the charged black hole depends only on the energy of the black hole when it is neutral.

We proceed next to the case for a slowly rotating black hole with mass M and angular momentum J . The total energy contained within a sphere of radius r can only be given by an approximate expression. This is due to the complexity of the Kerr metric, and more importantly, due to the fact that the Kerr metric only has axial symmetry instead of spherical symmetry. At the horizon, it is found that,[15]

$$E(r) = \frac{rc^4}{G}\left[1 - \sqrt{1 - \frac{2GM}{rc^2} + \frac{\alpha^2}{r^2}}\right] + \frac{\alpha^2c^4}{6rG}\left[2 + \frac{2GM}{rc^2} + \left(1 + \frac{2GM}{rc^2}\right)\sqrt{1 - \frac{2GM}{rc^2} + \frac{\alpha^2}{r^2}}\right] + \cdots \tag{11}$$

where $\alpha = J/Mc$ is the angular momentun length paraneter. The leading term of the expression is similar to the energy expression in the Reissner-Nordström case,

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suggesting that the mass at the horizon is twice the irreducible mass. The next term depends on α^2, adding a small contribution to the leading term.

The irreducible mass of a Kerr black hole is the final mass when its rotational energy is completely extracted by adding external particles, such as in the Penrose process.[16] It is given in the form

$$M_{irr}^2 = \frac{M^2}{2} + \frac{M^2}{2}\sqrt{1 - \frac{J^2c^2}{G^2M^4}}. \tag{12}$$

The horizon radius in this case is,

$$r_+ = \frac{GM}{c^2} + \frac{GM}{c^2}\sqrt{1 - \frac{J^2c^2}{G^2M^4}}. \tag{13}$$

An approximate relation for the horizon energy is therefore found,

$$E(r_+) \approx 2M_{irr} + O(\alpha^2). \tag{14}$$

The conclusion is that there is very little rotational energy insiide the Kerr black hole.

It is natural to extend the quasi-local energy investigation to include higher rotations, and logically, all rotations. However, a severe challenge appeared at this stage and progress on black hole rotation in this approach stopped. The calculations became extremely difficult to perform. No analytical expression or numerical evaluation could achieve an *exact* expression for the horizon mass of the Kerr black hole. An analysis of the horizon mass in the teleparallel equivalent formulation of general relativity[17] reveals that it is strikingly close to twice the irreducible mass $2M_{irr}$ for all range of the parameter $0 \leq \alpha < GM/c^2$. The tiny discrepancy is likely due to evaluating method and describing the spherical horizon region in a system with intrinsic axial symmetry. A general principle based on equipartition of energy at the horizon also suggests the horizon mass result for the Kerr black hole[18] by invoking one-half of the horizon mass for compensating the negative gravitational potential energy and the other half for supplying the irreducible mass.

We give a heuristic argument for the Horizon Mass Theorem with the area concept of a black hole.[9] It has been known from the Kerr metric that the area at the event horizon of a Kerr black hole for all rotations is[1]

$$A = 4\pi(r_+^2 + \alpha^2) = \frac{16\pi G^2 M_{irr}^2}{c^4}, \tag{15}$$

and the area of a Schwarzschild black hole of mass M_S and radius R_S is

$$A = 4\pi R_S^2 = 4\pi\left(\frac{2GM_S}{c^2}\right)^2 = \frac{16\pi G^2 M_S^2}{c^4}. \tag{16}$$

The two areas can be related by invoking Hawking's Area Non-decrease Theorem in the energy extraction process. The theorem asserts that the area of a Kerr black hole is the same as the area of the final Schwarzschild black hole when rotational energy is extracted in a smooth and reversible process. Since the horizon mass of the

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Schwarzschild black hole is proven to be twice its asymptotic mass M_S, the horizon mass of the Kerr black hole in this process is therefore $2M_{irr}$. The result applies to all rotations. It is believed that a rigorous mathematical proof will eventually show that the horizon mass is *exactly* twice the irreducible mass.

The Horizon Mass Theorem is crucial for understanding processes occuring near the horizon, such as the merging of two black holes,[19] and quantum emission of Hawking radiation.[20]

3. External Energy Conjecture

Proposition. *The electrostatic energy and rotational energy of a general black hole exist completely outside the horizon.*

The conjecture is a direct consequence of the irreducible mass in the Horizon Mass Theorem.

By definition, the irreducible mass does not contain rotational energy or electrostatic energy. A rotating black hole does not have rotational energy inside the horizon; therefore rotational energy exists outside the surface. Similarly, an electrically charged black hole does not have electrostatic energy inside. Electrostatic energy exists only outside, like that of a conductor. When quantum particles carrying electric charges and spins reach the black hole, they are forbidden to enter inside. They can only stay outside or at the surface. Since all matter particles in Nature are quantum particles, this makes the interior of the black hole completely hollow. Classical particles do not exist in Nature; they are a tool in classical mechanics.

We may generalize the External Energy Conjecture to include other energies of a black hole and introduce a new paradigm.[11]

External Energy Paradigm:

> *All energies of a black hole are external quantities. They include: constituent mass, gravitational energy, electrostatic energy, magnetic energy, rotational energy, heat energy, etc.*

The validity of this paradigm will be demonstrated in the next section in which the moment of inertia of a black hole is presented.

4. Moment of Inertia Property

Statement. *A black hole with an angular momentum and an angular velocity at the event horizon has a moment of inertia given by:*

$$\textit{coefficient} \times \textit{Kerr mass} \times (\textit{ergosphere radius})^2 \ .$$

When rotation stops, there is an irreducible moment of inertia given by:

$$\textit{irreducible mass} \times (\textit{Schwarzschild radius})^2 \ .$$

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The result of this statement is derived solely from the Kerr metric and is therefore a *bona fide* property of general relativity. Every black hole has a moment of inertia, even when it is not rotating. Moment of inertia indicates structure of the black hole.

The Kerr metric[21] discovered in 1963 ushered in a new epoch in general relativity and in astrophysics. It is absolutely indispensable for the study of rotating black holes. We demonstrate the existence of moment of inertia uniquely from the Kerr metric, using angular momentum and angular velocity consideration. We present the Kerr metric in an *explicit* form of the Boyer-Lindquist coordinates[22] (t, r, θ, ϕ) so that the metric coefficients can be readily extracted for calculation. It contains two constants $\alpha = J/Mc$ and $m = GM/c^2$ for the stationary case,

$$\begin{aligned} ds^2 = & \left(\frac{r^2 + \alpha^2 cos^2\theta - 2mr}{r^2 + \alpha^2 cos^2\theta}\right) c^2 dt^2 + \left(\frac{4m\alpha r sin^2\theta}{r^2 + \alpha^2 cos^2\theta}\right) cd\phi dt \\ & - \frac{[(r^2+\alpha^2)(r^2+\alpha^2 cos^2\theta) + 2mr\alpha^2 sin^2\theta]}{r^2 + \alpha^2 cos^2\theta} sin^2\theta d\phi^2 \\ & -(r^2 + \alpha^2 cos^2\theta)\left(d\theta^2 + \frac{dr^2}{r^2 + \alpha^2 - 2mr}\right). \end{aligned} \tag{17}$$

The Kerr spacetime rotates with different angular velocities at different locations. The angular velocity at a point is defined as the change in azimuthal angle ϕ with respect to the change in coordinate time t. It can be expressed in terms of the metric coefficients as

$$\Omega = \frac{d\phi}{dt} = -\frac{g_{t\phi}}{g_{\phi\phi}}. \tag{18}$$

where $g_{t\phi} = g_{\phi t}$. At the equatorial region, $\theta = 90°$, the angular velocity expression at a distance r can be written as

$$\Omega = \frac{2m\alpha rc}{(r^2 + \alpha^2)r^2 + 2mr\alpha^2}. \tag{19}$$

Further simplification can be achieved at the event horizon $r = r_+$, using the identity $r_+^2 + \alpha^2 = 2mr_+$, i.e.

$$\Omega_+ = \frac{\alpha c}{r_+^2 + \alpha^2}. \tag{20}$$

In terms of actual physical quantities, we have an exact algebraic relation for the angular velocity of the Kerr black hole,

$$\Omega_+(J) = \frac{\frac{J}{M}}{\frac{2G^2M^2}{c^4}\left[1 + \sqrt{1 - \frac{J^2c^2}{G^2M^4}}\right]}. \tag{21}$$

Given an angular momentun J and a Kerr mass M determined by a distant observer, the angular velocity at the event horizon can be obtained in radians/sec.

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Equivalently, we can express the angular momentum J in terms of the angular velocity Ω through the moment of inertia $I(\Omega)$ in the form

$$J(\Omega) = I(\Omega) \cdot \Omega. \tag{22}$$

We find, from Eq.(21), after substantial algebra,

$$J(\Omega_+) = \frac{M\left(\frac{2GM}{c^2}\right)^2}{1+\left(\frac{2GM}{c^2}\right)^2 \frac{\Omega_+^2}{c^2}} \cdot \Omega_+. \tag{23}$$

The crucial step here is to extract a radius factor in the above relation. It is recognized that the factor $2GM/c^2$ is the ergosphere radius at the equator, since M is the Kerr mass which includes rotational energy. Accordingly, the moment of inertia of the Kerr black hole is

$$I(\Omega_+) = \frac{M\left(\frac{2GM}{c^2}\right)^2}{1+\left(\frac{2GM}{c^2}\right)^2 \frac{\Omega_+^2}{c^2}}. \tag{24}$$

It is further recognized that the entire denominator in the above expression becomes a dimensionless number and acts as a numerical coefficient. The structure of the Kerr black hole is therefore,

coefficient $\times$ *Kerr mass* $\times$ *(ergosphere radius)*^{2}.

As angular momentum is continually reduced in the energy extraction process, a slowly rotating black hole is formed. In the static limit, the quantity $\Omega_+^2 \to 0$ first, the Kerr mass becomes the irreducible mass, $M \to M_{irr}$ and the coefficient becomes exactly equal to 1. The moment of inertia of a Schwarzschild black hole is derived. It is the limiting value of the moment of inertia of the Kerr black hole in Eq.(24), given by

$$I = M_S R_S^2. \tag{25}$$

There is an irreducible moment of inertia of the Kerr black hole as a result of the irreducible mass. It is the rotatonal analogue of the rest mass of a moving body $E = mc^2$.

A further observation that leads quickly to the irreducible moment of inertia is by considering the angular momentum definition in the Kerr metric $J = M\alpha c$ and the angular velocity Ω_+ at the horizon found in Eq.(20). We find,

$$I = \frac{J}{\Omega_+} = M\left(r_+^2 + \alpha^2\right) = \sqrt{M_{irr}^2 + M_{rot}^2}\left(r_+^2 + \alpha^2\right). \tag{26}$$

As the rotational parameter $\alpha \to 0$, the rotational mass $M_{rot} \to 0$; while the horizon radius $r_+ \to R_S$. In the limit, $I = M_{irr} R_S^2 = M_S R_S^2$. If one directly puts $J = 0$ and

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$\Omega_+ = 0$ in the definition $I = J/\Omega_+$, one would get an undefined result 0/0. The static limit of the Kerr black hole hole has the structure,

$$\textit{irreducible mass} \times (\textit{Schwarzschild radius})^2.$$

A Schwarzschild black hole does not have an axis of rotation. Introducing an axis destroys its spherical symmetry. The center is therefore the point of symmetry. The moment of inertia $I = M_S R_S^2$ is a statement about the mass distribution of a body with respect to the center in that the total mass is to be located at the Schwarzschild radius. A natural interpretation is that the static black hole is a hollow massive shell. This would go against the Equivalence Principle and the Singularity Theorem. It is possible that the static black hole resulting from the energy extraction process of a Kerr black hole is fundamentally different from the original Schwarzschild black hole. The static black hole derived from a Kerr black hole may be a quasi-black hole. A quasi-black hole has the same exterior spacetime as that of the Schwarzschild black hole but without the conceptual difficulties associated with the latter.[23] Singularity does not exist because particles are forbidden to cross the horizon by the very presence of the moment of inertia. The black hole firewall is such a scenario.[24] The quasi-black hole would also provide a physical surface where electric charges can stay instead of hovering without cause in the case of the charged black hole. In addition, if the surface area is identified as the entropy of a black hole according to Bekenstein[25] and Hawking,[6] then it is logical to expect that all mass of the quasi-black hole is at the surface. The moment of inertia is a new property of the Kerr black hole.

5. Epilogue

After 50 years, the irreducible mass has progressed from the gedanken Penrose process to quantum energy extraction in gamma-ray bursts and active galactic nuclei emissions.[26] Instead of being a cold and inert body, the real black hole is a highly active object. The irreducible mass further plays a central role in newly discovered black hole properties. These results may resolve some of the long-standing paradoxes in black hole physics.

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