COSMOLOGICAL RELAXATION OF THE ELECTROWEAK SCALE

J.R. ESPINOSA

ICREA, Institució Catalana de Recerca i Estudis Avançats, Barcelona, Spain

IFAE, Universitat Autònoma de Barcelona 08193 Bellaterra, Barcelona, Spain

Recently, a new mechanism to generate a naturally small electroweak scale has been proposed, the so-called "relaxion". It exploits the coupling of the Higgs to an axion-like field and a long era in the early universe where the axion unchains a dynamical screening of the Higgs mass. I present a new realization of this idea with the new feature that it leaves no signs of new physics up to a rather large scale, 10^9 GeV, except for two very light and weakly coupled axion-like states. One of these scalars can be a viable Dark Matter candidate. Such a cosmological Higgs-axion interplay could be tested with a number of experimental strategies.

1 The Relaxion Idea

The common lore states that natural solutions to the hierachy problem of the electroweak (EW) scale require new particles and interactions at or below the TeV scale, with supersymmetry and composite Higgs as the two main examples. In fact, the naturalness argument is the main (only?) argument to expect new physics at the LHC. Roughly a year ago, a solution to the hierarchy problem that challenges this common lore was proposed: the relaxion mechanism [1]. The idea is to promote the Higgs mass term in the potential to a field-dependent quantity

$$V(h) = \frac{1}{2}m_H^2(\phi)h^2 + ... = \frac{1}{2}(-\Lambda^2 + g\phi\Lambda)h^2 + ... \ , \tag{1}$$

with the quadratic cutoff term Λ^2 not required to cancel by any symmetry reason. The field ϕ, the relaxion, is then supposed to roll during cosmological evolution eventually stopping at some value ϕ_0 such that $m_H^2(\phi_0) \sim m_{EW}^2 \ll \Lambda^2$, solving in a dynamical way the hierarchy problem.

Figure 1 shows schematically the shape of the relaxion potential in the simplest realization of this idea [1], based on the following three terms of the $h-\phi$ scalar potential:

$$V = -\frac{1}{2}\left(\Lambda^2 - g\Lambda\phi\right)h^2 + \Lambda^3 g\phi + \epsilon\Lambda_c^3 h\cos(\phi/f) + \cdots . \tag{2}$$

I have already mentioned above the first term. For ϕ larger (smaller) than some critical value $\phi_c = \Lambda/g$, the Higgs mass term is positive (negative) and the EW symmetry is unbroken (broken), $\langle h\rangle = 0$, ($\langle h\rangle \neq 0$). The second term provides a non-zero slope for ϕ to scan its field range

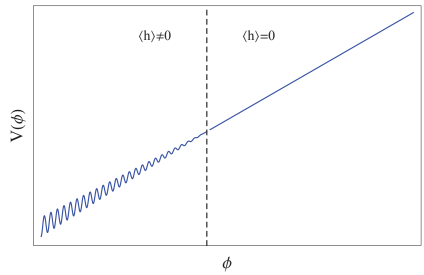

Figure 1 – *Relaxion potential. The critical value for EW breaking is marked by the dashed line.*

(which has to be of order ϕ_c). We will see below that this slope is required to be very small. The third term is crucial as it provides the feedback from EW breaking that raises barriers in ϕ that stop the relaxion close to the critical ϕ_c. We then assume that ϕ starts, at the beginning of inflation, with $\phi \gtrsim \phi_c$, and slow-rolls until it passes ϕ_c, at which point the Higgs mass becomes tachyonic and a nonzero $\langle h \rangle$ turns on and raises the barriers in the third term of the potential (2). The stopping value for ϕ and h is determined by the balance between the overall slope of the ϕ potential and that of the barriers, which grows with higher h, resulting in

$$v \equiv \langle h \rangle \simeq \frac{gf\Lambda^3}{\epsilon \Lambda_c^3} \ . \tag{3}$$

This formula determines the EW scale in terms of fundamental parameters. It is possible to have $\langle h \rangle \ll \Lambda$, and therefore solve the hierarchy problem, by choosing $g \ll 1$. The small value of g is technically natural as it is the spurion that breaks the symmetry $\phi \to \phi + 2\pi f$. Moreover, the shape of the potential (2) should be radiatively stable.

One necessary ingredient for this mechanism to work is some kind of friction that avoids the field ϕ overshooting the range of vacua for which $\langle h \rangle$ is of EW size. The simplest possibility for this is to invoke inflation to provide a slow-roll evolution of the relaxion. Usually, the number of e-folds required for the mechanism to be natural is quite large and the inflationary sector is the less satisfactory part of this mechanism. Although some alternatives have been proposed [2] there is room for improvement in the model-building of this sector.

What is the origin of the potential barriers in (2)? The simplest model proposed in [1] identifies ϕ with the QCD axion. The barriers then correspond to the axionic potential generated by instanton effects,

$$V(\phi) = (m_u + m_d)\langle q\bar{q} \rangle \cos(\phi/f) \ , \tag{4}$$

where $m_{u,d}$ are the up and down quark masses and $\langle q\bar{q} \rangle \sim \Lambda_{QCD}^3$ is the QCD quark-condensate. Comparing with (2) we therefore have $\Lambda_c \sim \Lambda_{QCD}$ and $\epsilon \sim y_u$, where y_u is the up-quark Yukawa coupling. This is a very appealing model that could explaing the EW hierarchy with $g \sim m_u \Lambda_{QCD}^3/(f\Lambda^3) \ll 1$. For instance, for a cutoff $\Lambda \sim 10^7$ GeV and $f \sim 10^9$ GeV, one needs $g \sim 10^{-35}$. [a] Unfortunately, the model also predicts the wrong value for the QCD theta angle [of $O(1)$ due to the nonzero slope of the axion potential!]. Possible solutions to this problem were also discussed in Ref. [1]. One possibility is to break the link between Λ_c and Λ_{QCD} assuming a non-QCD strong gauge sector to generate the barriers. This requires Λ_c below the TeV scale (as the physics that generates $\Lambda_c h$ breaks the EW symmetry) and this introduces the coincidence problem of why Λ_c is close to the EW scale.

[a] Even though this tiny value is technically natural, some people seem uncomfortable with such small numbers. One should perhaps remember that (non-perturbative) baryon number violation in the SM is suppressed by factors of order $e^{-2\pi/\alpha_w} \sim 10^{-81}$.

2 Double Scanning Model

Here I would like to discuss an alternative idea, proposed in Ref. [3], that takes $\Lambda_c = \Lambda$ and assumes the barrier to be generated as $\epsilon\Lambda^2|H|^2\cos(\phi/f)$ without breaking the EW symmetry. This idea runs into an immediate difficulty: the potential shape is not radiatively stable. Indeed, just by closing H in a loop, the term $\epsilon\Lambda_c^4\cos(\phi/f)$ is induced at one loop and such term produces everywhere barriers that would stop the ϕ evolution before the Higgs is turned on. The problem with such large cutoff correction to the barrier height should be reminiscent of the large cutoff corrections to the Higgs mass. We get around it precisely in the same manner, by advocating a field dependent barrier height with an additional scalar field that will also scan.

The crucial new ingredient of this proposal, with respect to Ref. [1], is therefore a second scanning field, σ. The potential, up to order ϵ, g_σ and g, reads

$$V(\phi,\sigma,H) = \Lambda^3\left(g\phi + g_\sigma\sigma\right) - \Lambda^2\left(\alpha - \frac{g\phi}{\Lambda}\right)|H|^2 + \lambda|H|^4 + A(\phi,\sigma,H)\cos\left(\phi/f\right)\,, \tag{5}$$

where the barrier height is given by

$$A(\phi,\sigma,H) \equiv \epsilon\Lambda^4\left(\beta + c_\phi\frac{g\phi}{\Lambda} - c_\sigma\frac{g_\sigma\,\sigma}{\Lambda} + \frac{|H|^2}{\Lambda^2}\right)\,, \tag{6}$$

and we take $0 < g, g_\sigma, \epsilon \ll 1$, and $\alpha, \beta, c_\phi, c_\sigma$ are positive coefficients of $O(1)$. A partial UV completion of this model that reproduces this field dependence of A can be found in Ref. [3] (long arXiv version).

From the above equations we see that ϕ scans the Higgs mass as before, while σ scans $A(\phi,\sigma,H)$, the overall amplitude of the oscillating term. The dependence of $A(\phi,\sigma,H)$ on σ and H is crucial for the double scanning mechanism to work, and the other terms in Eq. (6) are added as they are generated radiatively (by H loops). The potential shape given in Eq. (5) is radiatively stable provided $\epsilon \lesssim v^2/\Lambda^2$.

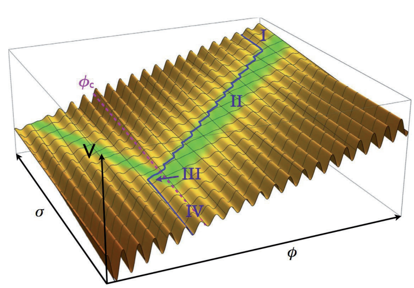

Figure 2 – *Scalar $V(\phi,\sigma)$ potential. The band without barriers is in green while the barriers getting high(er) are dark(er) brown. The blue line shows a possible slow-roll cosmological trajectory of the fields during inflation.*

As in the original relaxion models [1], inflation is assumed to provide the friction needed for the fields to slow-roll and reach the desired minimum with $v \ll \Lambda$. The evolution of σ is quite simple: for $\epsilon \ll 1$, it simply rolls down in time $\sigma(t) = \sigma_0 - g_\sigma\Lambda^3 t/(3H_I)$. The cosmological evolution of ϕ passes through four different stages, depicted in Figs. 2 and 3:

I) We assume $\phi \gtrsim \Lambda/g$ and $\sigma \gtrsim \Lambda/g_\sigma$ at the beginning of inflation, so that $m_H^2(\phi) > 0$ (so that the Higgs field is zero) and $|A|$ is of order $\epsilon\Lambda^4$. The field ϕ is stuck in one of the minima separated by the barriers due to the $A\cos(\phi/f)$ term in the potential.

II) With σ rolling down, the barrier height A gets smaller and smaller. Eventually the slope of the barrier walls is smaller than the overall slope along the ϕ direction, [for ϕ_* such that

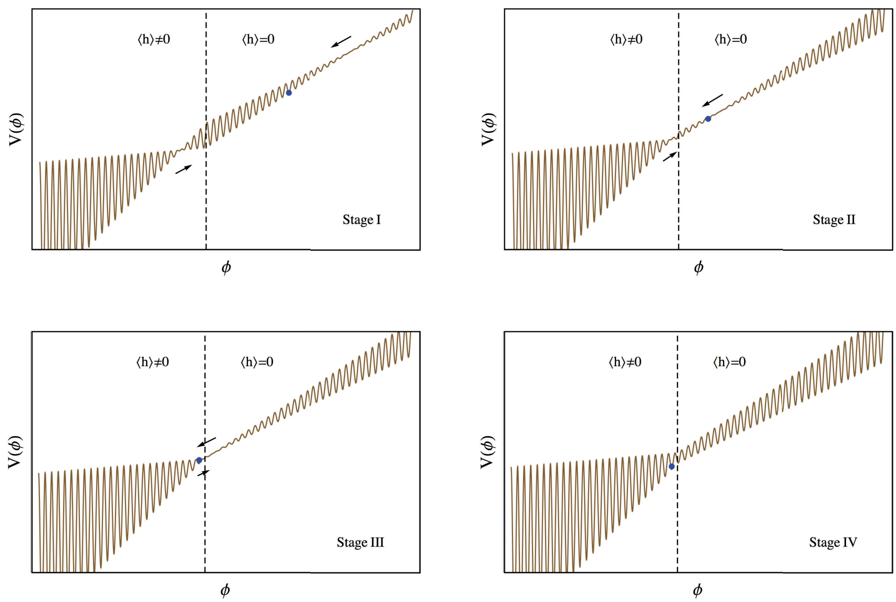

Figure 3 – *Sketch of the four stages in the evolution of ϕ, marked by the blue dot, in the time-dependent effective potential for ϕ obtained after integrating out σ and H but corresponding to the same potential as in Fig. 2.*

$A(\phi_*, \sigma, h(\phi_*)) \lesssim gf\Lambda^3$, green band in Fig. 2] and ϕ starts to roll down too. For $g_\sigma \lesssim g$, ϕ tracks σ: $\phi(t) \simeq \text{const.} + c_\sigma g_\sigma \sigma(t)/(c_\phi g)$, corresponding to $A \approx 0$.

III) At some point ϕ reaches the critical value $\phi_c \equiv \alpha\Lambda/g$ and $m_H^2(\phi)$ becomes negative and turns on H. This induces a positive contribution to A, that, for certain generic values of the parameters of Eq. (5), bends the direction of the green-band as shown in Fig. 2. As a result, the field ϕ moves out of the smooth green-band.

IV) Out of the smooth region of the potential, ϕ gets stuck in another minimum from $A\cos(\phi/f)$. Meanwhile, σ has continued its evolution towards its minimum, making A larger.[b]

Fig. 3 illustrates the ϕ evolution just described, showing four snapshots (corresponding to the four stages I-IV) of the time-dependent potential $V(\phi) \equiv V(\phi, \sigma(t), h(\phi))$, obtained by integrating out σ and h. In stages I and II, one sees two $A \approx 0$ regions moving towards each other. These regions merge at stage III near the critical ϕ_c and disappear at stage IV.

It is worth noting that the mechanism just described works independently of the value of the relaxion field, ϕ_i, at the begining of inflation t_i, as long as $\phi_c < \phi_i < \phi_*(t_i)$, which is a natural and sizable range of the available field space. The cosmological evolution described above is purely classical. Quantum fluctuations give corrections, but do not spoil the solution of the hierarchy problem, see Ref.[3] for more details.

[b]This picture brings to mind an analogy from Geology. Early geologists puzzled about large rocks that differed in composition from the one typical of the area in which they were found. This "naturalness problem" was eventually solved as a result of standard geological history: such rocks, known nowadays as glacial erratics, were transported by ancient glaciers over hundreds of kilometers. In our case, ϕ plays the role of glacial erratic and σ of glacier and the apparently unnatural smallness of the electroweak scale is the result of the workings of a "cosmological glacier".

3 Parameter Constraints

For this model to provide a natural solution to the hierarchy problem, a number of conditions must be satisfied:

1) The potential shape should be radiatively stable. Quantum corrections generate potential terms like $\epsilon^2\Lambda^4\cos^2(\phi/f)$ or $\epsilon^2\Lambda^3 g\phi\cos^2(\phi/f)$ whose amplitudes cannot be cancelled by σ simultaneously to $A\cos(\phi/f)$. These are dangerous as they could give a barrier to ϕ at values above the critical ϕ_c. However, they are subdominant to the Higgs barrier of Eq. (5) if $\epsilon \lesssim v^2/\Lambda^2$. This condition also ensures that the contribution to the Higgs mass coming from $\epsilon\Lambda^2|H|^2\cos(\phi/f)$ is at most of electroweak size and does not spoil the tracking behaviour.

2) ϕ gets trapped by the Higgs barrier. The feedback from a nonzero Higgs field should be responsible for stopping the rolloing of ϕ. This condition gives the electroweak scale in terms of microscopic parameters as: $v^2 \simeq g\Lambda f/\epsilon$.

In addition, two quantities crucial for the cosmological evolution of this model – H_I, the Hubble rate during inflation, and N_e, the number of e-folds – are also constrained:

3) Inflation is independent of the ϕ and σ evolution. For the typical energy density carried by ϕ and σ to remain smaller than the inflation scale, we need $\Lambda^2/M_P \lesssim H_I$ with $M_P \simeq 2.4\times10^{18}$ GeV. In addition, the two fields ϕ and σ should be slowly-rolling during inflation, which requires $g_\sigma\Lambda, g\Lambda \lesssim H_I$.

4) Classical roll dominates over quantum jumps. During inflation light fields are subject to quantum fluctuations of typical size H_I. This jittery motion remains smaller than the classical field roll provided $H_I^3 \lesssim g_\sigma\Lambda^3$.

5) Inflation lasts long enough for scanning. The range scanned by ϕ and σ during the inflationary epoch should be of the order of (or larger than) Λ/g and Λ/g_σ respectively. This requires a long enough period of inflation: $N_e \gtrsim H_I^2/(g_\sigma^2\Lambda^2)$.

Combining the previous parameter constraints, we find that the couplings g_σ and g are bounded to the interval $\Lambda^3/M_P^3 \lesssim g_\sigma \lesssim g \lesssim v^4/(f\Lambda^3)$. As f cannot be much smaller than Λ [the scale at which the $\cos(\phi/f)$ term is generated] we get an upper bound on the cut-off of our model:

$$\Lambda \lesssim (v^4 M_P^3)^{1/7} \simeq 2\times10^9\,\text{GeV}\ . \tag{7}$$

Fig. 4 illustrates the constraints above for the particular choice $\Lambda = f$ and $g_\sigma/g = 0.1$. Notice that the number of e-folds and the excursion of ϕ during inflation $\Delta\phi/M_P$ are in general exponentially large. However, for small values of the cutoff scale and the upper range of g, one has N_e, $\Delta\phi/M_P \sim O(1)$.

4 Signatures

4.1 Collider Signals

The new-physics/cutoff scale of the model can be as high as $\Lambda \sim 10^9$ GeV, and we do not expect new states around the weak scale. Only the two scalars σ and ϕ are lighter than the weak scale. These scalars are very weakly-coupled to the SM particles and can have phenomenological impact through astrophysical and cosmological effects only.

After ϕ reaches the EW minimum, $A(\phi,\sigma,H)\sim\epsilon\Lambda^4$. The mass of ϕ is then determined by the $A\cos(\phi/f)$ potential term as $m_\phi^2 \sim \epsilon\Lambda^4/f^2 \sim g\Lambda^5/(fv^2) \lesssim v^2$. For the σ field, higher-order terms in $g_\sigma\sigma/\Lambda$, not shown for simplicity in Eq. (5), give it a mass of order $m_\sigma^2 \sim g_\sigma^2\Lambda^2 \ll m_\phi^2$. Contours of constant m_ϕ and m_σ are shown in Fig. 4.

These two scalar fields interact with SM particles mainly through mass mixing with the Higgs. The relevant mixing angles are $\theta_{\phi h} \sim g\Lambda v/m_h^2$ and $\theta_{\sigma\phi} \sim g_\sigma f v^2/\Lambda^3$ while $\theta_{\sigma h}$ is the maximal value between $\theta_{\sigma\phi}\theta_{\phi h}$ and $g^2/(16\pi^2)[g_\sigma\Lambda^7/(f^2v^3m_h^2)]$. Both ϕ and σ decay through their mixing with the Higgs, with widths given by $\Gamma_\phi \sim \theta_{\phi h}^2\Gamma_h(m_\phi)$ and $\Gamma_\sigma \sim \theta_{\sigma h}^2\Gamma_h(m_\sigma)$, where $\Gamma_h(m_i)$ is the SM Higgs width evaluated at $m_h = m_i$.[4]

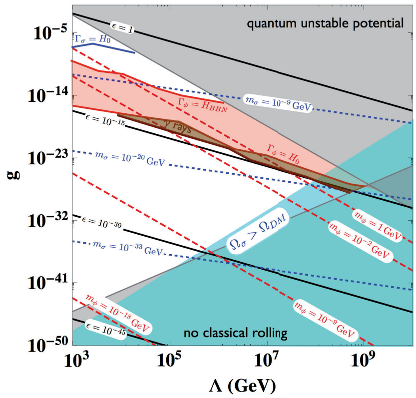

Figure 4 – *Parameter space of the double-scanning model able to solve the hierarchy problem through the cosmological evolution of the fields ϕ and σ. The plot assumes $\Lambda = f$ and $g_\sigma = 0.1g$.*

The scalar potential (5) also produces ϕ-Higgs interactions: $\phi\phi hh$ of order $\epsilon\Lambda^2/f^2$ and $\phi\phi h$ of order $\epsilon v\Lambda^2/f^2$. Such interactions are important for the thermal production of ϕ.

4.2 Cosmological Implications

For cosmological implications, the small decay widths of ϕ and σ must be compared with today's Hubble constant, H_0 (for cosmological stability) and with H_{BBN} [for potential trouble with Big Bang Nucleosynthesis (BBN)]. In a sizeable part of the parameter space, see Fig. 4, ϕ is cosmologically unstable ($\Gamma_\phi > H_0$), but sufficiently long-lived to decay after BBN [$\Gamma_\phi < H_{BBN} \equiv H(T = 1\,\text{MeV})$]. That region of the parameter space can then be constrained by cosmology. On the other hand, σ is cosmologically stable in most of the relevant parameter space – it decays within the age of the universe only in a small corner of parameter space.

Abundances of ϕ and σ from Vacuum Misalignment. After inflation and reheating the fields ϕ and σ generically end up displaced from their minima. Eventually they will fall to these minima and will oscillate around them if their lifetimes are large enough. The energy density stored in those oscillations scales like cold dark matter (DM) with the potential to overclose the universe or dissociate light elements (if the decay happens during or after BBN). More concretely, we expect that during inflation σ slow-rolled to its global minimum, located somewhere in its $\sim \Lambda/g_\sigma$ range. This needs a number of e-folds similar to the value estimated for enough ϕ scanning. Due to its quantum jittery motion during inflation, σ has an average displacement from the minimum at the end of inflation $(\Delta\sigma)_i \sim \sqrt{N_e}H_I$. This corresponds to an energy density of the order $\rho_i^\sigma \sim m_\sigma^2(\Delta\sigma)_i^2 \sim H_I^4$. The energy density stored in σ oscillations today, relative to the critical energy density, is then $\Omega_\sigma \gtrsim (H_I M_P/\Lambda^2)^4\,(4\times10^{-28}/g_\sigma)^{3/2}\times(\Lambda/10^8\,\text{GeV})^{13/2}$. The bound to avoid universe overclosure turns into a lower bound for g_σ as a function of Λ, shown in Fig. 4 in the case $H_I = \Lambda^2/M_P$.

Interestingly, σ is a good DM candidate in some regions of the allowed parameter space, reaching to large Λ. For some values of m_σ, there are other cosmological constraints. For instance, for $\Omega_\sigma \gtrsim \Omega_{DM}/20$, the mass range $10^{-32}\,\text{eV} \lesssim m_\sigma \lesssim 10^{-25.5}\,\text{eV}$ is excluded by structure formation [5], while masses around $m_\sigma \sim 10^{-11}\,\text{eV}$ may be constrained by Black Hole superradiance [6]. For the particular case $m_\sigma \sim 10^{-24}\,\text{eV}$, σ can be searched for by the SKA pulsar timing array experiment [7].

Thermal Production of ϕ. Concerning ϕ, its initial energy density from its displacement due to quantum spreading is at most $\rho_i^\phi \sim H_I^4$, and, since $m_\phi \gg m_\sigma$ and then $T_{osc}^\phi \gg T_{osc}^\sigma$, it gives today a negligible effect. However, one should also consider the possible thermal production of ϕ. This arises mainly from the $\phi\phi hh$-coupling discussed above, that leads to double-production from the thermal bath via $hh \to \phi\phi$. At $T \gtrsim m_h$, this double-production cross-section goes like $\langle\sigma_A \mathrm{v}\rangle \sim \epsilon^2(\Lambda^4/f^4)/T^2$. So, ϕ can reach thermal equilibrium only for T in the interval $[m_h, \epsilon^2 M_P(\Lambda/f)^4]$ (in which the ϕ production rate is faster than the rate of expansion). This region corresponds roughly to the area above the $\Gamma_\phi = H_{BBN}$ line of Fig. 4 and we conclude that in most of the parameter space ϕ never thermalizes.

The number density of ϕ produced thermally is $Y_\phi(T) \sim 10^{-4}\epsilon^2\Lambda^4 M_P/(f^4 T)$, where $Y_\phi = n_\phi/s$ and s is the entropy per comoving volume. The ϕ production is maximal at $T \sim m_h$. In the parameter region where ϕ is cosmologically stable, the contribution of ϕ to DM today is $\Omega_\phi \sim m_\phi Y_\phi s_0/\rho_c$ (where s_0 is the present entropy density) and it varies from $\Omega_\phi \lesssim 10^{-4}$ along the line $\Gamma_\phi = H_0$ to $\Omega_\phi \lesssim 10^{-10}$ for $\Gamma_\phi \simeq 10^{-10} H_0$.

Constraints from BBN and Gamma-Ray Observations. The region in parameter space in which ϕ is not cosmologically stable and decays after BBN can be problematic if the decay of ϕ injects into the thermal bath an energy per baryon $E_{p.b} \gtrsim O(\mathrm{MeV})$ as this would distort the light element abundances. Since $E_{p.b} \sim m_\phi Y_\phi n_\gamma/n_b$, this results in the bound $m_\phi Y_\phi \lesssim 10^{-12}\,\mathrm{GeV}$ (sensitively weakened depending on the precise value of the lifetime [8]). Moreover, the Cosmic Microwave Background (CMB) constrains lifetimes $\sim [10^{10} - 10^{13}]\,\mathrm{s}$ for $E_{p.b}$ down to $O(\mathrm{eV})$. Therefore, it is expected that most of the region of parameter space delimited by the lines $\Gamma_\phi = H_{BBN}$ and $\Gamma_\phi = H_0$ in Fig. 4 is excluded.

On the other hand, if the ϕ lifetime is larger than the age of the universe, there are strong constraints from decays generating a distortion in the galactic and extra-galactic diffuse X-ray or gamma-ray backgrounds. In particular, sub-GeV DM decaying into photons should satisfy $\tau_{DM} \gtrsim 10^{27}\,\mathrm{s}$.[9] Since the gamma-ray flux scales as $d\Phi_\gamma/dE \propto Y_\phi\Gamma_\phi$, we can translate this bound into $\tau_\phi > 10^{27}\,\mathrm{s} \times \Omega_\phi/\Omega_{DM}$, and this excludes the thin brown band of Fig. 4.

However, the cosmological constraints derived above can be evaded if the temperature of the universe never reaches m_h, in which case the thermal production of ϕ is suppressed.

5 Conclusions and Outlook

The relaxion idea proposed in Ref. [1] represents the last twist in the long fruitful history of interplay between particle physics and cosmology. In the past, particle physics has been a crucial ingredient in the understanding of the universe cosmological history. If this new idea (or some variant) turns out to be realized in nature, then cosmology would be a key ingredient for the understanding of key parameters of particle physics. In the original formulation, the size of the electroweak scale is an accident of the early dynamical evolution of the relaxion field and this offers a brand new class of solutions to the hierarchy problem.

In this talk I have focused on a sequel[3] to the original proposal, in which the SM can be made natural up to a cutoff of order 10^9 GeV without requiring visible new-physics at present (or far future) colliders. The model is an extension of the original one with two axion-like states ϕ and σ. Its dynamical cosmological evolution and interplay with the Higgs field leads to a naturally small electroweak scale. The only new-physics in this model consists of these two scalars, which in most of parameter space are very light and weakly coupled to SM particles. The model signatures are therefore to be found not at high energy colliders but rather in dedicated searches in the sub-GeV regime or through cosmological signals.

Interestingly, σ could be a good dark matter candidate. On the ther hand, ϕ cannot contribute to more than $\Omega_\phi \lesssim 10^{-10}$. For this maximum value, it might be detectable in gamma-ray observations from its late decay. Part of the parameter space of this model is testable by observations of the diffuse gamma-ray background, black hole superradiance and even pulsar timing arrays. In addition, there is a rather rich BBN and CMB phenomenology which motivates a more thorough study.

Concerning the relaxion paradigm in a broader context, there remain many open issues. Current models have unpleasant features, specially in the inflationary sector, which requires to provide rather extreme values of the number of e-folds. This is certainly a place where there is room for improvement in model building. Alternative mechanisms to provide friction (to slow down the field evolution) would also be welcome. It is also an open question how high the cutoff could be pushed up as well as possible ultraviolet completions and applications to other naturalness problems (e.g. related to supersymmetry breaking or the cosmological constant) as well as the origin and justification of the relaxion potential. For recent work of interest along some of these lines, the reader is directed to Refs. [11,12].

Acknowledgments

I thank Christophe Grojean, Giuliano Panico, Alex Pomarol, Oriol Pujolas and Geraldine Servant for a most enjoyable collaboration on this project and the Moriond2016 organizers for their kind invitation and for providing a very fruitful conference atmosphere. I acknowledge support by the Spanish Ministry MEC under grants FPA2014-55613-P, FPA2013-44773-P, by the Generalitat de Catalunya grant 2014-SGR-1450 and by the Severo Ochoa excellence program of MINECO (grant SO-2012-0234).

References

1. P. W. Graham, D. E. Kaplan and S. Rajendran, Phys. Rev. Lett. **115** (2015) no.22, 221801 [hep-ph/1504.07551].
2. E. Hardy, JHEP **1511** (2015) 077 [hep-ph/1507.07525].
3. J. R. Espinosa, C. Grojean, G. Panico, A. Pomarol, O. Pujolas and G. Servant, Phys. Rev. Lett. **115** (2015) no.25, 251803 [hep-ph/1506.09217].
4. F. Bezrukov and D. Gorbunov, JHEP **1005**, 010 (2010) [hep-ph/0912.0390].
5. R. Hlozek, D. Grin, D. J. E. Marsh and P. G. Ferreira, Phys. Rev. D **91**, no. 10, 103512 (2015) [astro-ph.CO/1410.2896].
6. A. Arvanitaki, M. Baryakhtar and X. Huang, Phys. Rev. D **91**, 084011 (2015) [hep-ph/1411.2263].
7. A. Khmelnitsky and V. Rubakov, JCAP **1402**, 019 (2014) [astro-ph.CO/1309.5888].
8. R. H. Cyburt, J. R. Ellis, B. D. Fields and K. A. Olive, Phys. Rev. D **67**, 103521 (2003) [astro-ph/0211258].
9. R. Essig, E. Kuflik, S. D. McDermott, T. Volansky and K. M. Zurek, JHEP **1311**, 193 (2013) [hep-ph/1309.4091].
10. J. R. Espinosa, G. F. Giudice and A. Riotto, JCAP **0805** (2008) 002 [hep-ph/0710.2484]; A. Hook, J. Kearney, B. Shakya and K. M. Zurek, JHEP **1501**, 061 (2015) [hep-ph/1404.5953]; J. R. Espinosa et al., [hep-ph/1505.04825].
11. R. S. Gupta, Z. Komargodski, G. Perez and L. Ubaldi, JHEP **1602** (2016) 166 [hep-ph/1509.00047]; K. Choi and S. H. Im, JHEP **1601** (2016) 149 [hep-ph/1511.00132]; D. E. Kaplan and R. Rattazzi, Phys. Rev. D **93** (2016) no.8, 085007 [hep-ph/1511.01827]; L. E. Ibanez, M. Montero, A. Uranga and I. Valenzuela, JHEP **1604** (2016) 020 [hep-th/1512.00025]; A. Hebecker, F. Rompineve and A. Westphal, JHEP **1604** (2016) 157 [hep-th/1512.03768].
12. B. Batell, G. F. Giudice and M. McCullough, JHEP **1512** (2015) 162 [hep-ph/1509.00834]; J. L. Evans, T. Gherghetta, N. Nagata and Z. Thomas, [hep-ph/1602.04812].