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Generalized K-essence inflation in Jordan and Einstein frames

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Abstract

We here explore a generalized K-essence model which exhibits characteristics akin to ordinary matter. The inflationary framework proposed aims to unify old with chaotic inflation into a single scheme and it considers minimally and non-minimally coupled scenarios, adopting three classes of potentials, in both Jordan and Einstein frames. We show that, to obtain a suitable amount of particles obtained from vacuum energy conversion during inflation, mitigating the classical cosmological constant problem, large-field inflation and, particularly, the Starobinsky-like class of solutions appears the most suitable one.

Keywords: K-essence, inflation, cosmological constant

1. Introduction

Inflation is a theoretical scenario of great significance in addressing the main issues related to the standard Big Bang paradigm [1–5]. Despite its importance, there is presently no consensus toward the potential that definitively describes the inflationary epoch [6–11].

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Originally, inflation was described by incorporating a 'de Sitter' phase, which relied on a scalar field undergoing a first-order phase transition[6]. Within this scenario, the scalar field is initially trapped in a local minimum of the potential and then eventually reaches the true minimum of the potential.

This initial description, dubbed *old inflation*, was soon abandoned due to the challenges it faced in explaining the end of inflation and the resulting highly chaotic nature of the Universe. As an alternative, *new inflation* models were proposed[7] [14–16]. Subsequently, an improved version, called *chaotic inflation*, was introduced, which successfully addresses both the initial conditions and the exit problems [17]. Here, the combination of chaotic initial conditions and evolution of the scalar field naturally leads to the end of inflation without requiring any external mechanism, referred to as the *graceful exit*.

After these initial attempts, a wide variety of inflationary models have been proposed [18, 19]. Generally, single-field theories can be categorized into classes based on their properties, say (a) *small field models*: in these models, inflation is driven by a scalar field that evolves from small values to larger ones, moving towards the minimum of the potential [8, 9, 20]; (b) *large field models*: in these models, inflation is driven by a scalar field that evolves from large values to smaller ones, moving towards the minimum of the potential [6, 21, 22].

Nevertheless, according to the Planck satellite results [23], the Starobinsky potential seems to be the most promising inflationary framework [6]. Notably, one of the significant findings from the Planck satellite's Bayesian analysis on various potential models is that the quartic potential $V(\phi) = \frac{1}{4}\lambda\phi^4$, and more broadly power-law potentials, are strongly disfavored. However, including a *Yukawa-like* non-minimal coupling to curvature, R, leads to statistically significant improvements. In this regard, the concept of non-minimally coupled inflation has been extensively explored and clearly cannot be excluded *a priori* [24–33].

Thus, based on our current understanding, the following points can be made regarding the best models of inflation: (1) successful models transport vacuum energy, behaving as quasi-de Sitter phase and allowing a sufficient release of energy; (2) the Starobinsky potential can be obtained through a ϕ^4-potential non-minimally coupled, passing from the Jordan to Einstein frame[8]; (3) polynomial frameworks that involve non-minimal coupling are more effective in describing the stages of inflation, suggesting that such coupling may indeed exist; (4) the Higgs inflation model is mathematically equivalent to change the frame, specifically transitioning from the Jordan frame to Einstein frame, using the quartic term model [7].

Motivated by the above points, we here explore specific inflationary models that integrate a mechanism to counteract vacuum energy, triggered by a phase transition. In so doing, recalling that K-essence field had been introduced to explain late-time cosmic evolution [43], we here develop a generalized K-essence field that appears viable also to explain early inflation. We show that this generalized K-essence fluid exhibits characteristics resembling those of ordinary matter. In this context, our objective is to scrutinize the circumstances under which this fluid can provide a solution to the *classical* cosmological constant problem.

[6] In a first-order phase transition, there is a discontinuity in the first derivatives of the 'Gibbs enthalpy' during the transition, while in a second-order phase transition Gibbs enthalpy remains continuous in the first derivatives but discontinuous in the second derivatives. For a thermodynamic perspective of inflation see e.g. [12, 13].

[7] Here, the scalar field begins in a state of thermal equilibrium in the false vacuum and gradually rolls down into degenerate minima through a second-order transition to the true vacuum.

[8] The original formulation of the Starobinsky potential involves a quadratic extended gravity Lagrangian, $\mathcal{L} \sim R + \alpha R^2$. So far, no clear evidences have been found in favor of extended theories of gravity, but only stringent cosmological or gravitational limits have been put, see e.g. [34–42].

Particularly, the cosmological constant problem is, in fact, a multi facets question. Here, the quantity to scrutinize is the effective cosmological constant, Λ_{eff}, made up by (at least) two contributions:

$$\Lambda_{\text{eff}} \equiv \Lambda_B + \Lambda_{\text{vac}}\,. \tag{1}$$

In this respect, the generalised K-essence fluid will face the issue of removing the *classical contribution* to Λ_{eff}, provided by the value of the potential at its minimum and conventionally named Λ_{vac} in equation (1), that conversely would affect the value of the effective cosmological constant today, see e.g. [44].

In so doing, we consider the model introduced in [45] where inflation corresponds to a the metastable stage of a phase transition where the Universe speeds up by means of the vacuum energy itself and, accordingly, to ensure that inflation comes to an end, the corresponding potential can effectively counteract and cancel out the quantum fluctuations of vacuum energy, being responsible for the aforementioned *cancellation mechanism*.

Hence, such a cancellation mechanism *is not* able to directly explain the small value of the *bare cosmological constant*, Λ_B, of equation (1), i.e. the one we measure. However, if the minimum of the potential is erased, namely if the *classical* cosmological constant problem is somehow fixed, promising hints on how to predict Λ_B have been found in [46], where the bare contribution can be naturally fine-tuned through a geometric mechanism of vacuum energy cancellation. This effective dark fluid is also well-suited for describing small perturbations, where the Jeans length remains identically zero across all scales, see e.g. [47].

In view of this, we show that during the phase transition, it is possible to predict a strong cosmic speed up reinterpreted in terms of inflation. In this context, we study the corresponding inflationary dynamics that incorporates the above-quoted cancellation mechanism. Since during the transition the corresponding potential may change its form, we assume that it might be continuous at the end of inflation matching the generalised K-essence fluid evolution. In this respect, we present three distinct models, classified into two main categories. The first pertains to a potential that resemble the Starobinsky one, while the second concerns more general symmetry breaking potentials. Regarding the first category, we introduce a Starobinsky-like potential that varies from the pure Starobinsky potential due to certain constants. In the second category, we first present a W model, characterized by finite potential walls, and then a Ω model, showing infinite potential walls. In all these scenarios, we emphasize how to provide a graceful exit, unifying *de facto* the old with chaotic inflation under the same standards through our proposed potentials. We work out minimal and non-minimal couplings, where the latter is performed by virtue of an effective *Yukawa-like* term. Following this recipe, we discuss how the geometric contribution ensures that vacuum energy may be converted into particles, likely different from baryons. As a result, we thoroughly investigate the characteristics of these potentials in both the Jordan and Einstein frames. For each case, we calculate the slow roll parameters, the tensor-to-scalar ratio and the spectral index, in order to determine the compatibility of these models with observational data. We conclude that the small field class of potentials appears to be disfavored based on our analysis and also confirm that coupling the potential with curvature improves the quality of our overall results in analogy with the quartic potential. We demonstrate that the most prominent potential appears the Starobinsky-like, constructed in a different way that does not involve any generalization of Einstein's gravity. In this regard, we propose an alternative perspective to generate the potential itself involving directly the generalised K-essence picture. Moreover, based on our findings, we conjecture that the Jordan and Einstein frames may be interchangeable and that the description of our inflationary stages simply requires formulation in the most suitable frame. As final conclusion, since both frames yield consistent results in terms of inflationary dynamics, we speculate on how the abundance of generated particles could align with present observations, producing a bare cosmological constant as byproduct of our treatment.

The paper is structured as follows. In section 2, we start introducing the key features of the generalised K-essence field. Our novel potentials, as well as the dynamics of inflation within the generalised paradigm in a minimally coupled scenario, are explored in section 3. Results obtained with non-minimal coupling are presented in section 4. We conclude by summarizing with the main findings derived from our paradigms in relation to current observations and providing some perspectives in section 5.

2. Generalising K-essence

The aim of a generalised K-essence fluid is to establish an alternative model characterized by a vanishing sound speed. As zero sound speed is typically associated with components resembling dust, one can speculate that the physical properties of this fluid yield some sort of *matter with pressure* [45, 48]. In this context, particles expanding the standard model of particle physics, or even those connected to the Higgs boson itself, emerge as plausible candidates for describing the generalised K-essence field with dust-like characteristics, albeit exhibiting non-zero pressure.

In this regard the effective Lagrangian description for this approach reads

$$\mathcal{L} = K(X,\phi) + \lambda Y[X,\nu(\phi)] - V(\phi)\,, \tag{2}$$

where $K(X,\phi)$ is a generalized kinetic term written in terms of $X \equiv \frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi$, $V(\phi)$ is the potential that drives the dynamics and λ is a Lagrange multiplier which enforces the total energy constraint of the Universe.

The functions $K(X,\phi)$ and $Y[X,\nu(\phi)]$ are not predetermined and do not have, *a priori*, specified forms, but rather the entire Lagrangian relies on a generalised K-essence field ϕ, whereas the function $\nu(\phi)$ governs the specific inertial mass of the field itself [49].

Particularly, as $\nu(\phi)$ enters the Lagrange multiplier, it is constructed by considering all the possible forms of energy associated with the ϕ-fluid, playing the role of *chemical potential*. Hence, it bounds X, and consequently the generalized kinetic contribution, $K(X,\phi)$, to have precise energy imposed by the $\nu(\phi)$ strength. Phrasing it differently, λ constraints the kinetic energy with the potential term in $\nu(\phi)$ [45]. Without including it, there is not chance to obtain a generalised K-essence field from a fundamental Lagrangian, since the pressure would exhibit a kinetic contribution.

The term $\lambda Y[X,\nu(\phi)]$ shows its physical meaning computing the energy-momentum tensor, especially playing a key role in determining the pressure and energy density features that characterize the evolution of the generalised K-essence fluid [50].

To display this, one can wonder the strategy to obtain dust from a generalised K-essence field. Simply, one can imagine that the kinetic term is constrained to be exactly equal to the potential itself and so, implementing this requires a pure Lagrange multiplier term that, however, does not contribute to the dynamics of the system.

Indeed, the pressure might be identically vanishing on all solutions and so the energy follows geodesics.

To provide a matter-like fluid with non-zero pressure, instead, it is possible to generalize this treatment by adding some function of the generalised K-essence field itself, plus its derivatives, exactly as we performed in our proposed Lagrangian.

The Lagrangian is therefore modeled by a dust-like field where the Lagrange multiplier λ constrains the kinetic energy with the potential term in $\nu(\phi)$. As a physical byproduct of this recipe, we infer a fluid that presents a constant pressure, mimicking at the same time matter, but exhibiting a negative equation of state.

To better focus on this point, let us now introduce the corresponding effective 4-velocity $v_\alpha \equiv \partial_\alpha \phi/\sqrt{2X}$, obtaining the energy-momentum tensor

$$T_{\alpha\beta} = 2X\mathcal{L}_{,X} v_\alpha v_\beta - (K - V)\, g_{\alpha\beta}\,, \tag{3}$$

where the density and the pressure terms yield

$$\rho = 2X\mathcal{L}_{,X} - (K - V)\,, \tag{4a}$$
$$P = K - V. \tag{4b}$$

Particularly, if $K = K_0 = const$ and $\lambda = 0$, then $\mathcal{L}_{,X} = 0$ and $P/\rho = -1$ *always*, i.e. the generalised K-essence recipe is not applicable to a Lagrangian made by kinetic plus potential terms only. However, if $K = K_0 = const$ and $\lambda \neq 0$, then $\mathcal{L}_{,X} = \lambda Y_{,X}$ and, in fact, we require equation (4) to read

$$\rho = 2X\mathcal{L}_{,X} + \mathcal{V}(\phi)\,, \tag{5}$$
$$P = -\mathcal{V}(\phi)\,, \tag{6}$$

where by definition $\mathcal{V}(\phi) \equiv V - K$ and, specifically assuming a constant generalized kinetic term as above, it reads $\mathcal{V}(\phi) = V - K_0$.

In a homogeneous and isotropic Universe, the above relations are mainly simplified. An interesting feature that arises from simplifying equations (5) and (6) can be shown below,

$$\rho = 2\mathcal{L}_{,X}X + \mathcal{V}(\phi)\,, \tag{7}$$
$$P = -\mathcal{V}(\phi)\,. \tag{8}$$

In view of the above results, we can wonder which cases correspond to a constant generalized kinetic term.

To do so, we can remark that the standard thermodynamics of perfect fluid is here generally preserved. Our effective Lagrangian is therefore for non-dissipative fluids and, so, we can apply the shift symmetry on the field itself to preserve the cosmological principle. Moreover, we can recast the conservation of the energy-momentum tensor by virtue of the Carter-Lichnerowicz equations [51]

$$n\mathcal{W}_{\alpha\nu}v^\nu = nT\nabla_\alpha\sigma - \varsigma_\alpha \nabla^\nu n_\nu\,, \tag{9}$$

with $\mathcal{W}_{\alpha\nu} = \nabla_\nu\varsigma_\alpha - \nabla_\alpha\varsigma_\nu$ is the vorticity tensor, $\varsigma^\alpha = h/nv^\alpha$ the current of the enthalpy per particle, and $\sigma = s/n$ the entropy per particle.

Then, as the 4-velocity is the derivative of the field ϕ and $\nabla^\alpha n_\alpha = 0$, it is possible to show that

$$\mathcal{W}_{\alpha\nu} = 0 \quad \Rightarrow \quad \text{the fluid is irrotational}\,, \tag{10}$$
$$\nabla_\alpha\sigma = 0 \quad \Rightarrow \quad \text{the fluid is isentropic}\,, \tag{11}$$

that, studying linear perturbations as presented in [45], it is straightforward to show that K is constant in order to guarantee the Jeans length to vanish at all scales, mimicking *the facto* the dark fluid behavior, see e.g. [47], and guaranteeing structures to form at all scales.

Conversely, attempts to get the field modifying *directly* the energy-momentum tensor have been developed in [52], trying not to pass through the use of a Lagrange multiplier and by assuming

$$T_{\mu\nu} \to T_{\mu\nu} + V(\phi)\, g_{\mu\nu} - X g_{\mu\nu}\,. \tag{12}$$

While appealing, this prescription suffers from the thorny caveat of not having a Lagrangian description, appearing unphysical from a fundamental description.

The main differences between the K-essence and generalised K-essence fields consist in the consequences that the shift in the energy-momentum tensor implies on the sound speed, $c_s \equiv \sqrt{\frac{\partial P}{\partial \rho}}$. In particular reminding that P, equation (6), does not depend on X, we have

$$c_s = \frac{\partial P}{\partial \rho} = \frac{\partial P}{\partial X}\frac{\partial X}{\partial \rho} = \frac{\partial(-\mathcal{V}(\phi))}{\partial X}\frac{\partial X}{\partial\left(2X\mathcal{L}_{,X} + \mathcal{V}(\phi)\right)} = 0. \tag{13a}$$

The generalised K-essence behaves as matter with zero perturbations but with a non-vanishing equation of state, resembling dust. Specifically, dust is defined when the equation of state, along with pressure, vanishes, whereas matter can even exhibit pressure, typically constant to remove the cosmological constant contribution, see e.g. [45]. However, both cases are characterized by a vanishing sound speed, i.e. the perturbation speed is zero. Concerning perturbations in cosmology, this implies that the Jeans length is zero and the clustering can occur at all scales. Accordingly, if a given effective fluid, that is not barotropic as the generalised K-essence is, is identified by a Lagrangian providing a constant net pressure, the corresponding sound speed vanishes in analogy to the previous two cases. This case exhibits a matter-like behavior of the fluid, namely the effective Lagrangian behaves similarly to matter. Conversely, considering a standard scalar field Lagrangian with zero mass, the sound speed is one, i.e. it appears analogous to stiff matter, as well as the equation of state [48, 53]. Standard scalar field Lagrangian is plagued by this issue: the corresponding effective fluid clusterizes like relativistic matter, being in tension with recent observations of structures. Hereafter, we will refer to this property as a matter-like fluid. Notably, examples of such fluids can be found in the literature, often delving into the realm of *unified dark energy-dark matter models*, see e.g. [54–66], and specifically, the model characterized by a constant pressure, and varying density and equation of state, see e.g. [65]. Nevertheless, it appears evident that after the transition the Universe accelerates *because of the presence of a negative matter pressure*, that acts as *emergent cosmological constant*.

3. Minimally coupled inflation within the generalised K-essence picture

The inflationary dynamics is slightly modified by the presence of a generalised K-essence fluid. Particularly, it can be described starting from the continuity equation associated with the Lagrangian in equation (2)

$$\ddot{\phi} + \frac{3}{2}H\dot{\phi} + \frac{\mathcal{V}'(\phi)}{2\mathcal{L}_{,X}} = 0. \tag{14}$$

Here, we remark that the so-obtained equation represents a forced harmonic oscillator, where the force is provided by the term $\frac{\mathcal{V}'(\phi)}{2\mathcal{L}_{,X}}$, with the viscous part $\sim H\dot{\phi}$.

In addition, the first Friedmann equation of generalised K-essence is

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\rho = \frac{8\pi}{3M_{Pl}^2}\left(\mathcal{L}_{,X}\dot{\phi}^2 + \mathcal{V}(\phi)\right). \tag{15}$$

3.1. Slow roll in generalised K-essence scenarios

We have now all the ingredients to better understand the conditions under which a generalised K-essence field can drive an inflationary period. Thus, we consider a *slow roll* stage, where the potential energy of the inflaton dominates over its kinetic energy. Hence, the slow roll conditions in our case become

$$\mathcal{L}_{,X}\dot{\phi}^2 \ll \mathcal{V}(\phi)\,,\ \ddot{\phi} \ll \frac{3}{2}H\dot{\phi}, \tag{16}$$

implying that $H^2 \gg |\dot{H}| \simeq 0$. We emphasize that a constant H corresponds to a de Sitter expansion, consequently, a slowly rolling stage leads to a quasi-de Sitter phase [67], where the associated Friedmann and continuity equations yield respectively

$$H^2 \simeq \frac{8\pi}{3M_{Pl}^2}\mathcal{V}(\phi), \tag{17}$$

$$3H\dot{\phi} \simeq -\mathcal{V}'(\phi). \tag{18}$$

This approximation is encoded in the Hubble *slow roll parameters* defined as

$$\epsilon_H(\phi) \equiv \frac{M_{Pl}^2}{4\pi\,\mathcal{L}_{,X}}\left(\frac{H'(\phi)}{H(\phi)}\right)^2, \tag{19a}$$

$$\eta_H(\phi) \equiv \frac{M_{Pl}^2}{4\pi\,\mathcal{L}_{,X}}\frac{H''(\phi)}{H(\phi)}. \tag{19b}$$

The subscript 'H' refers to Hubble slow roll. Here, we made a more suitable choice of slow roll parameters than the widely-used *potential* slow roll parameters:

$$\epsilon_V \equiv -\frac{\dot{H}}{H^2} \simeq \frac{M_{Pl}^2}{16\pi\,\mathcal{L}_{,X}}\left(\frac{\mathcal{V}'}{\mathcal{V}}\right)^2, \tag{20a}$$

$$\eta_V \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} - \frac{\dot{H}}{H^2} \simeq \frac{M_{Pl}}{8\pi\,\mathcal{L}_{,X}}\left(\frac{\mathcal{V}''}{\mathcal{V}}\right). \tag{20b}$$

Commonly the latter relations appear overused in the literature, as due to their simplicity. They represent first-order approximated terms, being easier to handle from a computational viewpoint [68]. However, as the approximation starts to break down, say if $H^2 \gtrsim \dot{H}$, as due to the complexity of potentials, it is not convenient to work equations (20a) and (20b) out, but rather to employ equations (19a) and (19b).

We notice that the above definitions are intertwined, and, in fact, ϵ_V and η_V can be written in terms of the Hubble slow roll parameters by

$$\epsilon_V = \epsilon_H\left(\frac{3-\eta_H}{3-\epsilon_H}\right)^2, \tag{21a}$$

$$\eta_V = \sqrt{\frac{M_{Pl}^2}{4\pi\,\mathcal{L}_{,X}}\epsilon_H \frac{\eta_H'}{3-\epsilon_H}} + \left(\frac{3-\eta_H}{3-\epsilon_H}\right)(\epsilon_H + \eta_H)\,, \tag{21b}$$

where the first term in equation (21*b*) is higher order with respect to the second one, and so hereafter it will be neglected.

During the slow roll, the conditions for sustaining inflation occur when

$$\ddot{a} > 0 \iff \epsilon_H \ll 1, \tag{22}$$

thus inflation ends when $\epsilon_H = 1$. Setting $\epsilon_H = 1$, equations (21*a*) and (21*b*) can be expressed as follows:

$$\epsilon_V = \left(\frac{3-\eta_H}{2}\right)^2, \tag{23a}$$

$$\eta_V \simeq \left(\frac{3-\eta_H}{2}\right)(1+\eta_H)\,. \tag{23b}$$

Subsequently, it is straightforward to derive the expression for η_H and substitute it into equation (23*a*), leading to the conclusion that $\epsilon_H = 1$ as

$$\epsilon_V \simeq \left(1 + \sqrt{1 - \frac{\eta_V}{2}}\right)^2. \tag{24}$$

In this respect, equation (24) consists in a further order of approximation in the slow roll parameters, and it implies that $\epsilon_V \geqslant \epsilon_H = 1$.

Considering only the potential slow roll parameters, we drop out the subscript V and straightforwardly we compute the number of e-foldings between the beginning and end of inflation within the slow roll approximation.

If we denote the values of the inflaton field at the beginning and end of inflation as ϕ_i and ϕ_f respectively, the total number of e-foldings can be calculated as follows

$$N \equiv \int_{\tau_i}^{\tau_f} H d\tau \simeq -\frac{8\pi}{M_{Pl}^2}\int_{\phi_i}^{\phi_f} \frac{\mathcal{V}}{\mathcal{V}'} d\phi. \tag{25}$$

To have enough amount of inflation we require that $N \gtrsim 60$, as in the standard picture [69].

For the sake of completeness, it might be remarked that the aforementioned expression is just the first-order approximation of a more general expression, $\tilde{N}$, that by definition satisfies $\tilde{N} \leqslant N$ and requires a e-folding number $\tilde{N} \gtrsim 60$ to have sufficient inflation, see e.g. [70]. In this work, we have set the initial value for inflation as $\tilde{N} = 70$. Following this recipe, we here consider $\tilde{N}$ up to the second order

$$\tilde{N} \simeq -\sqrt{\frac{4\pi}{M_{Pl}^2}} \int_{\phi_i}^{\phi_f} \frac{1}{\sqrt{\epsilon(\phi)}} \left(1 - \frac{1}{3}\epsilon(\phi) - \frac{1}{3}\eta(\phi)\right) d\phi. \tag{26}$$

Hereafter, we may confuse N with $\tilde{N}$, without losing generality. Afterwards, as the slow roll ends, we expect that the inflaton field starts oscillating around the minimum of the potential, ϕ_0. This phenomenon, dubbed reheating [71], does not influence the *graceful exit* that depends instead on the chaotic form of the potential involved into equation (14).

Table 1. Three different bounds obtained with three different data sets, considering as background model the ΛCDM + r benchmark. Bounds provided by Planck mission, as reported in [23].

	r	n_s
Planck TT,TE,EE +lowEB+lensing	<0.11	0.9659 ± 0.0041
Planck TT,TE,EE +lowE+lensing+BK15	<0.061	0.9651 ± 0.0041
Planck TT,TE,EE +lowE+lensing+BK15+BAO	<0.063	0.9668 ± 0.0037

From the Planck mission data [23], we argue an experimental feedback regarding inflationary models and from the cosmic microwave background anisotropies and their power spectrum, we can extract information that can be encoded in two key parameters: the tensor-to-scalar ratio, denoted as r, and the spectral index, denoted as n_s [72, 73].

These parameters play a crucial role in characterizing the properties of inflationary models. In the slow roll regime, the tensor-to-scalar ratio and spectral index can be expressed as follows:

$$r = 16\epsilon^*, \tag{27a}$$

$$n_s = 1 + 2\eta^* - 6\epsilon^*, \tag{27b}$$

where the slow roll parameters, ϵ^* and η^*, are evaluated at the horizon crossing of the pivot scale, i.e. at the moment when the physical wavelength of fluctuations characterized by $k_* = 0.002\,\mathrm{Mpc}^{-1}$ reaches the Hubble horizon during the slow roll phase. The fluctuations at this stage are directly related to cosmic microwave background anisotropies. Typically, this horizon crossing occurs between the onset of the slow roll phase and an e-folding number up to approximately 45, when the slow roll parameters are still much smaller than 1.

The Planck mission has provided several bounds on these parameters, which vary depending on the chosen background model and the specific data set used. The corresponding constraints and results have been reported in table 1 for completeness.

By comparing the predicted values of r and n_s from inflationary models with the measured values derived from early time data, we can assess the viability and consistency of our inflationary scenarios.

3.2. *Inflationary potentials*

In our search for a novel, and likely more complicated form of the inflationary potential, we require that it reduces to the standard ϕ^4 potential after the transition. More precisely,

- the potential under consideration should ensure continuity in the energy budget of the Universe through a first-order phase transition, implying that its behavior before and after the transition should be proportional to ϕ^4;
- once the metastable phase ends, the Universe successfully escapes the transition and settles into the symmetry minimum, ensuring a *graceful exit* from the transition, i.e. from inflation;
- the potential should be compatible with current observations and the coupling constant χ should be associated with vacuum energy.

For the sake of completeness, the cancellation mechanism bears resemblance to the concept of *old inflation*, characterized by a phase transition. However, in our case the potential can escape the metastable phase with the graceful exit. Particles are thus produced as due to the coupling with the curvature [74, 75]. Accordingly, our picture enables particles to form even during inflation, being reinterpreted in terms of non-baryonic constituents, see e.g. [46, 76–78].

Thus, we start from the most general double exponential form [50, 75]

$$V(\phi) = V_0 + \frac{\chi\phi_0^4}{4}\left[\frac{e^{\beta\phi^d} - e^{\beta\phi_0^d}e^{\alpha\left(\phi^c - \phi_0^d\right)}}{1 - e^{\beta\phi_0^d - \alpha\phi_0^d}}\right]^2. \tag{28}$$

This proposal is general and contains all the information to construct a wide classes of possible potentials, encoded into the constants $V_0, \chi, \phi_0, \alpha, \beta, c, d$. It has several applications that span from theoretical up to solid state physics. For examples, it is possible to reobtain the Morse potential by invoking a benchmark scenario as in equation (28) [79]. In addition, we leave all these constants *a priori* free, fixing by physical motivations each of them, in order to realize the best choice to drive up the Universe to accelerate.

First, we would like to emphasize that our selection of V_0 and χ is guided by the findings in works such as [45, 50, 78]. Specifically, we expect that in the case of large field inflation, $V_0 \neq 0$, whereas for small-field inflation, $V_0 = 0$. The value of χ is then determined as a consequence of V_0, in order to satisfy energy conditions, as extensively detailed in the aforementioned papers. If we assume a vanishing offset, the potential is no longer suitable for the cancellation mechanism. Nevertheless, we analyze both cases to determine the viability of such an inflationary scenario. It is important to note that, the offset V_0 plays a key role in cosmological constant cancellation mechanisms [80–82]. Specifically, when it is non-vanishing, it provides a potential solution to address the classical cosmological constant problem within the framework of generalized K-essence, otherwise, it does not [45].

For example, if we handle $\beta = 0, \alpha < 0$ and $c = 1$, we obtain

$$V_S(\phi) = V_0 + \frac{\chi\phi_0^4}{4}\left[\frac{1 - e^{-|\alpha|(\phi-\phi_0)}}{1 - e^{|\alpha|\phi_0}}\right]^2, \tag{29}$$

reducing, through a simple shift over the field in terms of ϕ_0, and by virtue of the following further positions:

$$\Lambda_S^4 \simeq \frac{\chi\phi_0^4}{4}\left(1 - e^{|\alpha|\phi_0}\right)^{-2}, \qquad \alpha \simeq \sqrt{\frac{2}{3}}\frac{1}{M_{Pl}}. \tag{30}$$

to the Starobinsky model, written under the form,

$$V_S = \Lambda_S^4\left(1 - e^{-\sqrt{2/3}\phi/M_{Pl}}\right)^2, \tag{31}$$

where M_{Pl} is the Planck mass. Among the various inflationary models, the Starobinsky potential has been found to be the best-suited scenario according to current observational data. Our picture, however, is slightly different than a genuine Starobinsky potential, due to the coupling between α and Λ_S and then deserves further investigation.

We recall that the constant Λ depends on the choice of parameters within the corresponding potential. Thus, to clarify the notation, we consistently adopt the same label of the potential, or we explicitly redefine Λ whenever necessary.

Two more choices of parameters leading to symmetry breaking potentials are $\beta = 0, \alpha < 0$ and $c = 2$. This class of models provides two symmetric minima, resulting into a *W model* [50]

$$V_W(\phi) = V_0 + \frac{\chi\phi_0^4}{4}\left[\frac{1 - e^{-|\alpha|\left(\phi^2 - \phi_0^2\right)}}{1 - e^{|\alpha|\phi_0^2}}\right]^2. \tag{32}$$

This model presents finite walls in analogy to the more popular class of T-models [10]. Here, in principle, an apparent problem seems to occur, as the energy required to exit from the minimum is finite. However, the inflaton, during the accelerated phase, loses the initial energy because of the Hubble friction term, equation (14), that, at the same time, dampens the oscillations around the minimum at ϕ_0. In other words, to have a well-defined graceful exit, the friction should be enough to decrease the energy of the inflaton not to exceed the wall. Moreover, in order to reduce the complexity of the potential, the height of the walls is fixed as $V_W(0) = V_W(\pm\infty)$, i.e. $|\alpha| = \frac{\ln 2}{\phi_0^2}$.

Instead, by considering $\beta \neq 0$, and, for $\alpha < 0, |\alpha| = \beta$ and $c = d = 2$ we get the Ω potential

$$V_\Omega(\phi) = V_0 + \frac{\chi\phi_0^4}{4}e^{2|\alpha|\phi^2}\left[\frac{1 - e^{-2|\alpha|\left(\phi^2 - \phi_0^2\right)}}{1 - e^{2|\alpha|\phi_0^2}}\right]^2. \tag{33}$$

It also provides a symmetry breaking with two symmetric minima, but this potential has infinite walls. Let us highlight that the requirements $|\alpha| = \beta$ and $c = d$ are fundamental to avoid nonphysical or complex potentials.

The last two proposals involve chaotic potentials with a symmetry breaking functional form. The W model exhibits both small and large field behaviors due to its finite potential walls, while the latter exhibits small field behavior. In this context, inflation occurs during the metastable phase of a first-order phase transition. Consequently, the offset is $V_0 = 0$ for the Ω potential while the W potential can be studied with $V_0 = 0$ and $V_0 \neq 0$.

The conclusion of this phase transition is characterized by a chaotic graceful exit, where the inflaton field loses energy while it is oscillating around the minimum of the potential.

Hence, this picture provides a unification of old and new inflation through a chaotic scheme.

Summing up, we thus introduced two main classes of potentials:

- Starobinsky-like potentials, starting from a general double exponential potential, equation (28), within the generalised K-essence hypothesis, we derived a large field potential denoted as V_S, equation (31), that mimes the Starobinsky features [50].
- Symmetry breaking potentials, namely V_W and V_Ω, respectively equations (32) and (33). The latter displays infinite potential walls and a small field nature, while the former, due to its finite potential walls, can exhibit both small and large field behaviors.

We emphasize that the distinction between Starobinsky-like and symmetry-breaking potentials depends upon the selection of free parameters, notably on α. Hence, within the framework of the same general double exponential potential, α determines the general behaviour, while the remaining parameters govern the specific features of the recovered potentials. Consequently, the values of α can appear quite different for distinct potentials, as the corresponding physical characteristics of the potentials themselves often differ significantly from one another. The potential patterns and the corresponding choices of the parameters are shown respectively in figure 1 and in table 2.

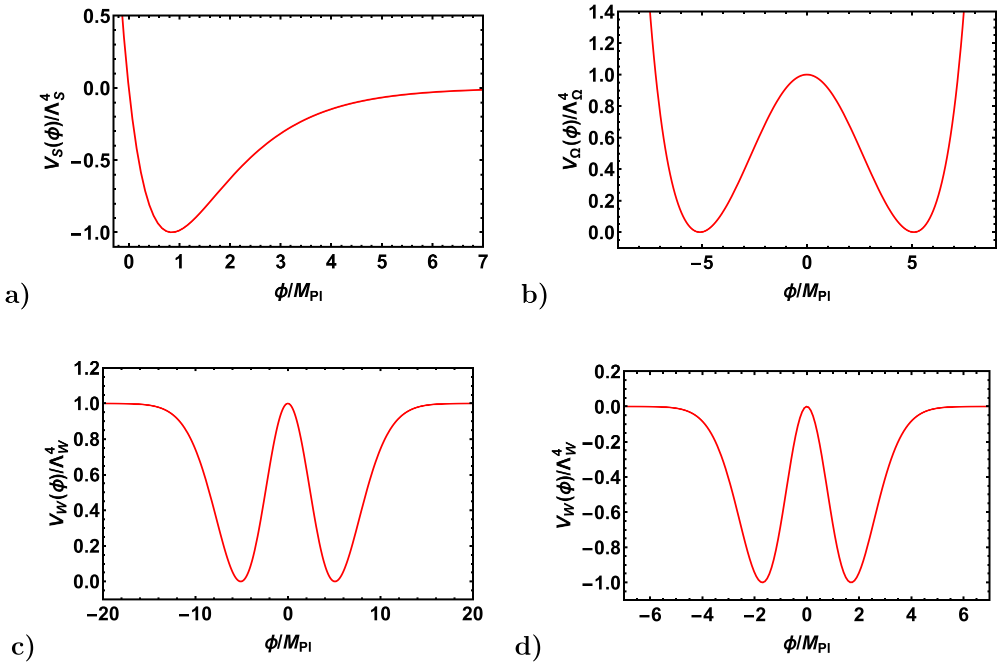

Figure 1. Inflationary potentials introduced in section 4: (a) $V_S(\phi)$, equation (31), with $\phi_0 = \sqrt{3/2}M_{Pl}\ln 2$, $\chi = 1.66$, $\alpha = \sqrt{2/3}/M_{Pl}$ and $V_0 = -\chi\phi_0^4/4$; (b) $V_\Omega(\phi)$, equation (33), with $\phi_0 = 6\sqrt{3/2}M_{Pl}\ln 2$, $\chi = 1.28 \cdot 10^{-3}$, $\alpha = \ln 2/\phi_0^2$ and $V_0 = 0$; (c) and (d) $V_W(\phi)$, equation (32), with respectively $\phi_0 = 6\sqrt{3/2}M_{Pl}\ln 2, 2\sqrt{3/2}M_{Pl}\ln 2$, $\chi = 1.28 \cdot 10^{-3}, 0.10$, $\alpha = \ln 2/\phi_0^2$ and $V_0 = 0, -\chi\phi_0^4/4$. In these plots we consider $\Lambda_\Omega^4 = \Lambda_W^4 = \frac{\chi\phi_0^4}{4}$. Further, the plots refer to as the potentials V without considering the kinetic term, namely $\mathcal{V} = V - K_0$.

Consequently, we examined the inflationary dynamics induced by these potentials in both minimally and non-minimally coupled scenarios.

3.3. Minimally coupled inflationary dynamics

Let us analyze the inflationary dynamics in the minimally coupled scenario, i.e. $\xi = 0$. In this respect we individuate the parameters that describe the inflationary dynamics, i.e. the slow roll parameters, (20*a*) and (20*b*), the e-folding number, equation (26), initial and final conditions, and finally, the evolution in the configuration space of the inflaton field. We emphasize that, while generally a potential is either large or small field, our W potential, equation (32), shows both the small and large behavior due to the finite potential walls. Further, the choice of free parameters is reported in table 2.

First, we compute the slow roll parameters, equations (20*a*) and (20*b*). As shown in figure 2, their initial value are far smaller than 1, however, as the inflationary phase concludes, they rapidly increase. In this way we identify the slow roll phase and we get the final value of the inflaton field, i.e. where equation (24) holds.

Table 2. Resuming table of the minimally coupled inflationary features addressed in this work. We present the chosen values for the free parameters of the potentials, the values obtained at the pivot scale for comparison with observational constraints, and, lastly, the energy values of the inflaton field.

$\xi=0$	V_S	V_W^{small}	V_W^{large}	V_Ω
V_0	$-\chi\phi_0^4/4$	0	$-\chi\phi_0^4/4$	0
χ	1.66	$1.28\cdot10^{-3}$	0.10	$1.28\cdot10^{-3}$
ϕ_0	$\sqrt{3/2}M_{Pl}\ln 2$	$6\sqrt{3/2}M_{Pl}\ln 2$	$2\sqrt{3/2}M_{Pl}\ln 2$	$6\sqrt{3/2}M_{Pl}\ln 2$
α	$\sqrt{2/3}/M_{Pl}$	$\ln 2/\phi_0^2$	$\ln 2/\phi_0^2$	$\ln 2/\phi_0^2$
n_s	0.966	0.966	0.966	0.966
r	0.042	0.076	0.018	0.098
N_*	52	64	62	50
H_*/M_{Pl}	1.11	1.08	1.28	1.00
energy scales	$\phi_{\text{in}}\simeq3.18M_{Pl}$	$\phi_{\text{in}}\simeq1.91M_{Pl}$	$\phi_{\text{in}}\simeq3.89M_{Pl}$	$\phi_{\text{in}}\simeq2.10M_{Pl}$
	$\phi_{\text{end}}\simeq1.03M_{Pl}$	$\phi_{\text{end}}\simeq4.88M_{Pl}$	$\phi_{\text{end}}\simeq1.90M_{Pl}$	$\phi_{\text{end}}\simeq4.89M_{Pl}$

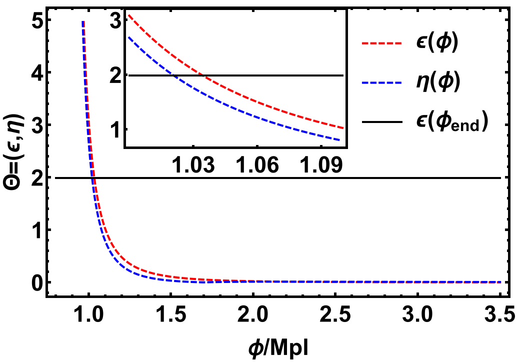

Figure 2. Slow roll parameters for the potential $V_S(\phi)$, equation (31), with $\phi_0=\sqrt{3/2}M_{Pl}\ln 2$, $\chi=1.66$, $\alpha=\sqrt{2/3}/M_{Pl}$ and $V_0=-\chi\phi_0^4/4$ in a minimally coupled scenario.

We want to emphasize that in large-field inflationary models, the inflaton field typically evolves over a super-Planckian range during the slow roll regime (see e.g. [2]). When dealing with single-field scenarios, we are aware that large super-Planckian field excursions may be in tension with quantum gravity theories (see also [83]). However, we assume modest excursions $\Delta\phi\sim O(1)\times M_{\text{pl}}$, that are typically less constrained, [84], and in agreement with recent observations related to the CMB radiation [85].

Then imposing that the e-folding number, equation (26), satisfies the constraint $N=70$, we find the initial condition for the field ϕ.

So, from equation (18), we can compute the initial condition on the time variation of the field, say

$$\dot{\phi}_{in}\simeq-\frac{\mathcal{V}'(\phi_{in})}{3H_{in}}. \tag{34}$$

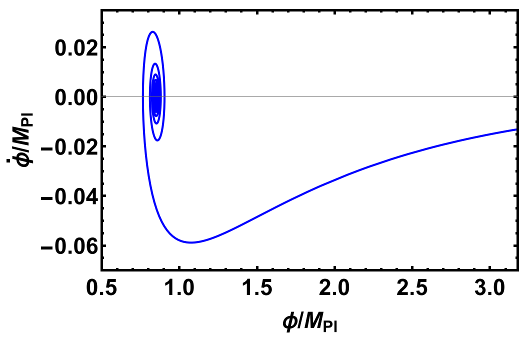

Figure 3. Inflaton configuration space for the potential $V_S(\phi)$, equation (31), with $\phi_0 = \sqrt{3/2}M_{Pl}\ln 2$, $\chi = 1.66$, $\alpha = \sqrt{2/3}/M_{Pl}$ and $V_0 = -\chi\phi_0^4/4$ in a minimally coupled scenario.

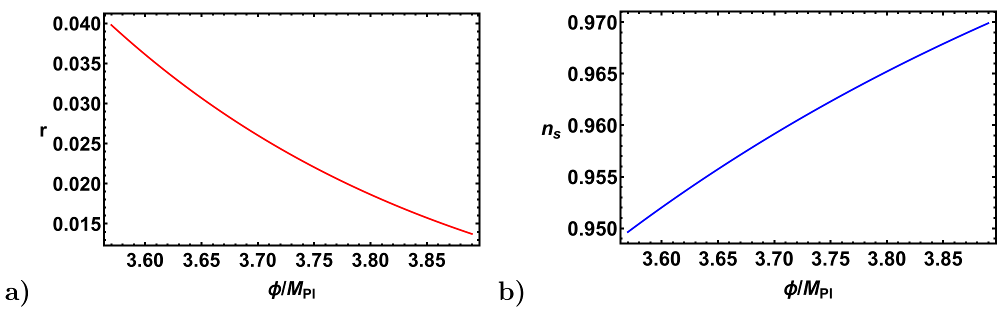

Figure 4. In (a) the tensor-to-scalar ratio, r defined in equation (27*a*), and in (b) the spectral index, n_s defined in equation (27*b*), of the potential $V_W(\phi)$ introduced in equation (32), in a small field regime and in a minimally coupled scenario.

As we discussed previously, we expect that, for small and large field potentials, the inflaton ϕ evolves from smaller to larger values and viceversa aiming to reach the minimum of the potential. The corresponding computed initial and final conditions are summarized in table 2.

Finally, solving the equation of motion of the inflationary Universe, equation (14), we obtain the configuration space evolution as shown in figure 3. The field naturally goes towards the minimum of the potential, and it presents a chaotic evolution. The chaotic evolution is characterized by an attractor around the field value where the potential reaches its minimum, causing the inflaton to oscillate around that point. This phenomenon is commonly referred to as the *graceful exit*.

The last step is to verify the viability of such models comparing the value of the tensor-to-scalar ratio and of the spectral index with respect to the observational constraints. Thus, in order to account the horizon crossing, we compute these parameters for all the potentials from the beginning of inflation, namely $N \simeq 65$, to $N \simeq 45$.

Our computed plots, in figure 4, clearly demonstrate that inflationary models satisfy the observational data, as they provide a good fit with the measured values of the tensor-to-scalar ratio r and the spectral index n_s. In particular, by fixing the values of n_s, we determine the e-folding number N_* corresponding to the horizon crossing of the pivot scale, along with their respective values of r. Despite the encouraging results yielded by the tensor-to-scalar ratio r and the spectral index n_s, it is important to remark that our pivot scale Hubble parameter exceeds the constraints provided by the Planck mission [23]. However, this discrepancy depends on the choice of the free parameters involved in our computation, and mainly with the choice of vacuum energy offset. In our results, the offsets of the investigated potentials were arbitrarily fixed adopting the aforementioned cancellation mechanism, with precise values of the constants. Accordingly, in future works, it would be quite interesting to rescale the employed vacuum energy magnitude, in order to obtain a more precise alignment between the cancellation mechanism and the experimental inflationary constraints.

All these results are summarized in table 2.

4. Non-minimally coupled generalised K-essence inflation

At primordial times, as stated in section 1, there is no reason to exclude *a priori* a non-minimal coupling between the inflaton and scalar curvature, R. Further, introducing a non-minimal coupling and analyzing the resulting dynamics to determine whether these non-minimally coupled potentials can give rise to a suitable inflationary phase, particularly in the presence of strong gravitational fields. Recent studies have demonstrated particle creation when considering potentials arising from the interaction between the inflaton field and the Ricci scalar [86, 87]. Analogously, the interacting terms have also been utilized to investigate the expected deviations in entanglement and particle production [76, 77]. Quite relevantly, such effective potentials have been employed to describe geometric quasi-particles [78], unifying the phenomenon of gravitational particle creation for both dark matter and baryons [88].

An interacting Yukawa-like term in equation (2), representing the simplest interacting Lagrangian, can therefore be written as

$$\mathcal{L}_{\rm int} = \frac{1}{2}\xi R\phi^2, \tag{35}$$

giving rise to a new effective potential, $V_{\rm eff}(\phi) = V(\phi) + \frac{1}{2}\xi R\phi^2$, where the coupling constant, ξ, is considered as a free parameter and the $1/2$ factor is arbitrary.

First, we focus on the range $-4 < \log_{10}\xi < 4$, namely a plausible bound provided by the Planck mission [23], whereas later we further extend the range of ξ, invoking $|\xi| \ll 1$, by virtue of the analogy with particle physics, where a very small ξ is essential to guarantee that gravity does not significantly change at local scales.

Indeed, from equation (35), the effective Newtonian gravitational constant, $G_{\rm eff}$, couples to ϕ by

$$G_{\rm eff} = \frac{G}{1-\xi\chi\phi^2}. \tag{36}$$

The effective constant might be consistent with our present constraints on G. However, we remark that the condition $G_{\rm eff} \simeq G$ is restored after the transition, while during the transition is not strictly necessary as due to the metastable phase itself, see e.g. [30, 31]. Regarding the sign of ξ, when $\xi > 0$, we ask for $G_{\rm eff} > 0$, yielding

$$\phi^2 < \frac{1}{\chi\xi}, \tag{37}$$

leading to a multiplicative degeneracy between χ and ξ.

It is important to note that the specific implications of positive or negative non-minimal coupling can have significant consequences for the large scale dynamics of our resulting inflationary dynamics and depend on the structure of the involved inflationary potentials.

The coupling term imposed by the interacting Lagrangian, equation (35), written as a Yukawa-like contribution gives rise to conceptual complications. Firstly, we would like to emphasize that the dynamics can be studied through classical field equations. Specifically, for non-minimally coupled single inflaton fields, quantum corrections can be safely neglected, as discussed in e.g. [89] and the references therein. However, the main concern is to single out the 'right frame', either Jordan or Einstein one, to work in [90–92].

Phrasing it differently, the corresponding dynamics can be studied in the above quoted two different frames [93, 94] having that

- in the Jordan frame, the action is left unaltered. The interacting term, $\mathcal{L}_{int}$, provides the type of interaction;
- in the Einstein frame, a conformal transformation is applied on the metric, getting rid of the interaction.

In principle, transitioning from one frame to another could potentially carry implications for the associated physics, even though this aspect remains not entirely comprehended at present, see e.g. [95].

Below, let us focus separately on each of the two frames, adopting our generalised K-essence fluid.

4.1. The Jordan frame

Now, we focus on the Jordan frame. As introduced at the beginning, in this frame, we need to recover new Einstein's field equations through the variation of the total action

$$\begin{aligned}\frac{1-\chi\xi\phi^2}{\chi}G_{\mu\nu} = {}&(2-2\xi)\,\partial_\mu\phi\partial_\nu\phi + 2\xi\phi\,(g_{\mu\nu}\Box - \partial_\mu\partial_\nu)\,\phi \\ &- g_{\mu\nu}\left[\left(\frac{1}{2}-2\xi\right)\partial_\alpha\phi\partial^\alpha\phi + \mathcal{V}(\phi)\right].\end{aligned} \tag{38}$$

The first Friedmann equation, obtained taking into account the non-minimal coupled interacting term, equation (35), leads to

$$H^2 = \frac{\chi}{3\,(1-\chi\xi\phi^2)}\left(\dot{\phi}^2\mathcal{L}_{,X} + \mathcal{V}(\phi)\right), \tag{39}$$

having

$$\ddot{\phi} + \frac{3}{2}H\dot{\phi} + \frac{\mathcal{V}'(\phi) + \xi R\phi}{2\mathcal{L}_{,X}} \simeq 0, \tag{40}$$

where we approximate $R' = \frac{\dot{R}}{\dot{\phi}} \simeq 0$, since we delve into slow roll. Indeed, since we assume inflation occurring in a quasi-de Sitter phase, the scalar curvature, R,

$$R = \frac{\chi}{(1-\chi\xi\phi^2)}\left[(6\xi-2)\,\dot{\phi}^2 + 4\mathcal{V}(\phi) + 6\xi\phi\ddot{\phi}\right], \tag{41}$$

can be considered roughly constant. Manifestly, the third term of equation (41) can be regarded as a second-order term compared to the rest. Consequently, as the field ϕ increases, the curvature R also increases. This situation can present a significant challenge when aiming to smoothly transition out of inflation and reach the minimum of the effective potential.

On the one hand, in fact, in the case of small field inflation with a positive coupling strength, the interacting Lagrangian has an opposite sign than the potential one. Conversely, in the case of negative ξ both contributions decrease as the field decreases. On the other hand, for large field inflation the situation is perfectly symmetrical to small fields.

Hence, to address this concern, we study both the negative and positive coupling for each potential. Further, we discuss whether, and for which choice of parameters, the effective potentials in our hands lead to a well-defined inflationary stages, while ensuring the consistency of the model with observational data.

4.2. The Einstein frame

In the Einstein frame, in order to get rid of the non-minimally coupled term we perform the following conformal transformation [96, 97]

$$g_{\mu\nu}^{(E)} = g_{\mu\nu}\Omega^2(\phi)\ ,\ \Omega^2 = 1 - \xi\chi\phi^2. \tag{42}$$

Now the action becomes

$$S_E = \int d^4x\sqrt{-g^{(E)}}\left[\frac{R^{(E)}}{2\chi}\frac{1-\xi\chi\phi^2}{\Omega^2} - \mathcal{L}^{(E)}\right], \tag{43}$$

with the positions

$$\mathcal{L}^{(E)} = \frac{\mathcal{L}}{\Omega^4}, \tag{44}$$

$$X_E = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi\left(\frac{\Omega^2 + 6\frac{1-\xi\chi\phi^2}{\chi}\left(\frac{\partial\Omega}{\partial\phi}\right)^2}{\Omega^4}\right). \tag{45}$$

Recasting the above action into the canonical minimally coupled scheme, we introduce a new field, h, and a transformed potential, $\mathcal{V}_E(\phi)$, defined as

$$h(\phi) = \sqrt{\frac{6}{\chi}}\tanh^{-1}\left(\frac{\sqrt{6\chi}\xi\phi}{\sqrt{1+\phi^2\chi\xi(-1+6\xi)}}\right) - \sqrt{\frac{-1+6\xi}{\chi\xi}}\sinh^{-1}\left(\sqrt{\chi\xi(-1+6\xi)}\phi\right), \tag{46}$$

$$\mathcal{V}_E(h) = \frac{\mathcal{V}(\phi(h))}{(1-\xi\chi\phi^2(h))^2}, \tag{47}$$

where the subscript 'E' recalls that the quantities are calculated in the Einstein frame.

Passing from ϕ to h is not analytic, in general. We can therefore consider some limiting cases. For example, one can consider the usual conformal coupling [32], i.e. $\xi = 1/6$ providing

$$\phi = \frac{1}{\sqrt{\chi\xi}}\tanh\left(\sqrt{\chi\xi}h\right), \tag{48}$$

that represents an upper limit for ξ, in view of the constraints that we imposed on ξ. Indeed, exceeding $\xi \simeq 1/6$ with larger values yields slow roll parameters that seem not to agree with the most recent bounds. In other words, we can undertake $\xi \simeq 1/6$ as an upper cut-off scale.

Thus, dismissing strong interactions, again the condition $|\xi| \gg 1$ holds, ending up with the following two main cases:

$$\phi \simeq \frac{1}{\sqrt{\chi\xi}} \sin\left(\sqrt{\chi\xi} h\right), \qquad 0 < \xi \ll 1, \tag{49a}$$

$$\phi \simeq -\frac{1}{\sqrt{\chi\xi\left(6\xi - 1\right)}} \sinh\left(\frac{\sqrt{\chi\xi} h}{\sqrt{6\xi - 1}}\right), \qquad -1 \lesssim \xi < 0. \tag{49b}$$

We can now study the inflationary dynamics, in both positive and negative weakly-interacting cases, with the same tools introduced for the minimally coupled scenario, dealing with h and the transformed potentials, $\mathcal{V}_E(h)$.

4.3. Non-minimally coupled inflationary dynamics

At this point we study whether the non-minimal coupling, $\frac{1}{2}\xi R\phi^2$, can lead to a consistent inflationary scenario or not, in the limit of weak interactions, $|\xi| \ll 1$ and we focus on both the Einstein and the Jordan frame.

We start performing the transformation between the field ϕ and h, equations (49*a*) and (49*b*), to write the potentials in the Einstein frame, :

$$\mathcal{V}_E\left(h\right) = \frac{\mathcal{V}\left(\phi\left(h\right)\right)}{\left(1 - \xi\chi\phi^2\left(h\right)\right)^2}, \tag{50}$$

$$\phi^{(+)} \simeq \frac{1}{\sqrt{\chi\xi}} \sin\left(\sqrt{\chi\xi} h\right), \tag{51}$$

$$\phi^{(-)} \simeq -\frac{1}{\sqrt{\chi\xi\left(6\xi - 1\right)}} \sinh\left(\frac{\sqrt{\chi\xi} h}{\sqrt{6\xi - 1}}\right). \tag{52}$$

In order to satisfy the aforementioned conditions for the potentials we fix the free parameters as reported in table 3. Within the potentials transformed in the Einstein frame, the main difference in behaviour arises for $\mathcal{V}_\Omega^E$ and $\mathcal{V}_W^E$. As evident from the figure 5, the coupling makes the potential walls infinite for both the models. Thus, the two potentials manifest a similar behavior, i.e. showing a quite analogous dynamics. Following the previous prescription for the minimally coupled case, we compute the slow roll parameters and the initial and final conditions.

In the Jordan frame, because of the scalar curvature, the slow roll parameters are no longer defined in the usual way, via equations (20*a*) and (20*b*). Thus, taking the previous initial conditions on the field h, from equations (49*a*) and (49*b*), we have

$$\dot{\phi} \simeq \cos\left(\sqrt{\chi\xi} h\right) \dot{h}, \qquad \xi > 0, \tag{53}$$

$$\dot{\phi} \simeq \left(1 - 6\xi\right) \cosh\left(-\frac{\sqrt{\chi\xi} h}{\sqrt{6\xi - 1}}\right) \dot{h}, \qquad \xi < 0. \tag{54}$$

Finally we numerically solve the equations of motion, equations (14)–(40), respectively within the Einstein and the Jordan frame. The evolution of the inflaton field in the configuration

Table 3. Resuming table of the non-minimally coupled inflationary features addressed in this work. We present the choice of the parameters $\xi, \alpha, \phi_0, \chi, V_0$ for the inflationary potentials introduced in section 3.2, the initial and final conditions of the non-minimally coupled inflationary stage. The label $\pm$ indicates where the coupling strength is positive or negative. The choice of the parameters ξ, ϕ_0, and consequently of χ, is made to provide a good agreement with the slow roll conditions first and with observational constraints. Specifically, V_0 and α are fixed as explained in section 3.2, whereas the initial and final conditions are recovered as developed in section 4.3. Finally, to confirm the agreement of the considered models with observational constraints, we show the values of r, n_s, N_*, H_* evaluated at the pivot scale.

$\xi \neq 0$	V_S	V_W^{small}	V_W^{large}	$V_\Omega^{(+)}$	$V_\Omega^{(-)}$
V_0	$-\chi\phi_0^4/4$	0	$-\chi\phi_0^4/4$	0	0
χ	26.56	0.02	0.10	$0.03 \cdot 10^{-1}$	$0.01 \cdot 10^{-1}$
ϕ_0/M_{Pl}	$\sqrt{\frac{3}{2}\frac{\ln 2}{2}}$	$3\sqrt{\frac{3}{2}\ln 2}$	$2\sqrt{\frac{3}{2}\ln 2}$	$5\sqrt{\frac{3}{2}\ln 2}$	$6\sqrt{\frac{3}{2}\ln 2}$
α	$\sqrt{\frac{2}{3}}\frac{1}{M_{Pl}}$	$\frac{\ln 2}{\phi_0^2}$	$\frac{\ln 2}{\phi_0^2}$	$\frac{\ln 2}{\phi_0^2}$	$\frac{\ln 2}{\phi_0^2}$
n_s	0.966	0.966	0.966	0.966	0.966
r	0.055	0.038	0.065	0.062	0.076
N_*	48	62	45	58	60
H_*/M_{Pl}	2.76	1.25	1.53	1.13	1.06
energy scales	$h_{\text{in}} \simeq 2.83 M_{Pl}$	$h_{\text{in}} \simeq 0.87 M_{Pl}$	$h_{\text{in}} \simeq 4.25 M_{Pl}$	$h_{\text{in}} \simeq 1.63 M_{Pl}$	$h_{\text{in}} \simeq 2.02 M_{Pl}$
	$h_{\text{end}} \simeq 0.61 M_{Pl}$	$h_{\text{end}} \simeq 3.02 M_{Pl}$	$h_{\text{end}} \simeq 1.91 M_{Pl}$	$h_{\text{end}} \simeq 4.23 M_{Pl}$	$h_{end} \simeq 4.84 M_{Pl}$
coupling strength	0.0002	0.0050	0.0005	0.0005	-0.0001

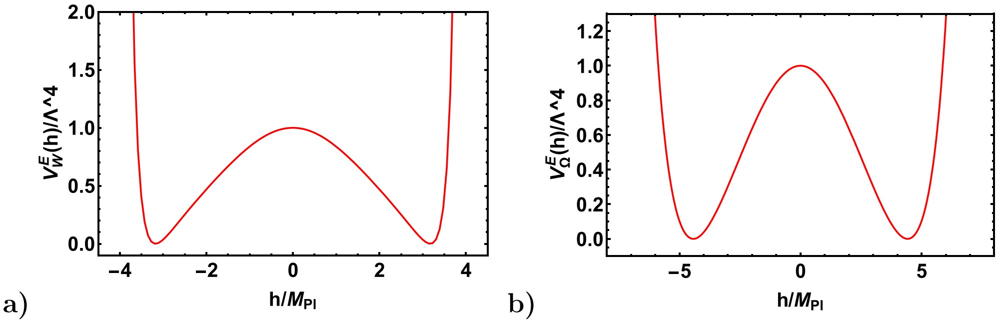

Figure 5. Inflationary potentials introduced in a non-minimally coupled scenario: (a) $V_W(\phi)$, equation (32), in a small field regime; (b) $V_\Omega(\phi)$, equation (33), with a positive; The choice of parameters is shown in table 3. The plots refer to as the potentials V without considering the kinetic term, namely $\mathcal{V} = V - K_0$.

space is totally analogous to the minimal coupled case in figure 3, namely it is characterized by a slow roll phase, followed by a well-defined attractor behavior around the minimum of the potential for all the potentials addressed, but V_Ω that, despite an inflationary stage in both frames, does not present a chaotic graceful exit in the Jordan frame. Details are reported in figure 6.

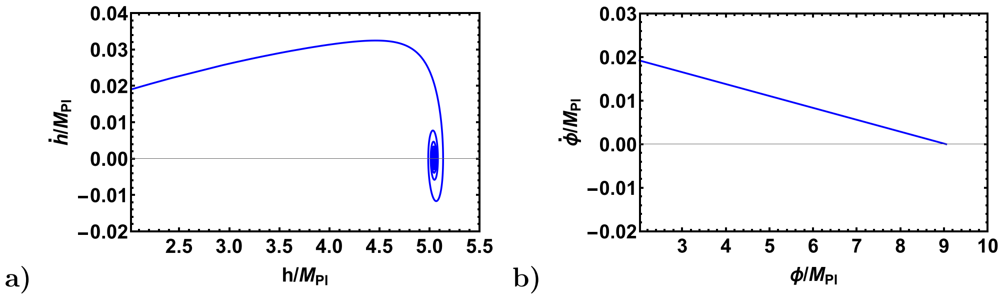

Figure 6. Inflation configuration space of the potential $V_\Omega(\phi)$, equation (33), with a negative coupling strength in a non-minimally coupled scenario: (a) in the Einstein frame; (b) in the Jordan frame. The choice of parameters is shown in table 3.

The non-minimally coupled generalised K-essence model proves to be successful in generating a suitable inflationary stage. Some interesting features are that the symmetry breaking potentials exhibit similar dynamics due to the coupling between the inflaton field and curvature and that the energy scales where inflation occurs are much higher with respect the others potentials.

Finally we focus on the comparison between the forecasted tensor-to-scalar ratio and spectral index and their observational counterparts. The values, presented in table 3, are generally consistent across the majority of the addressed potentials. Indeed, in some cases, we obtain values that conform to only certain data set bounds and not to all of them. Nevertheless, it is important to note that this prescription for the models introduces a fine-tuning issue. The values of r and n_s are strongly dependent on the parameters ϕ_0 and ξ. In particular, achieving a good fit to the observational data requires careful tuning of these parameters. This implies that a specific combination of values for ϕ_0 and ξ is needed to obtain the desired inflationary predictions. Such fine-tuning may be seen as a limitation for the models, as it suggests a sensitivity to the specific choices of these parameters.

Further, the analyses conducted in both the Einstein frame and Jordan frame consistently confirm the equivalence between the two frames. Phrasing it differently, the dynamics of the inflationary stage and the resulting predictions for observables remain consistent and unaffected by the choice of frame.

5. Outlooks and perspectives

In this work, we investigated a generalized version of K-essence models that exhibits a vanishing speed of perturbations, implying that its properties resemble those of a matter-like fluid. This characteristic holds significant implications, particularly within the realm of small perturbation theory.

In this respect, our main target was to show how inflation can be characterized by this exotic K-essence fluid.

Interestingly, from this generalised K-essence puzzle, we put forth that potentials addressing the *classical* cosmological constant problem can be formulated under precise conditions.

For the sake of clearness, we faced the issue of cancelling the classical contribution inside the effective cosmological constant definition, usually provided by the value of the potential at its minimum. Thus, remarkably we stressed that our approach does not fully-clarify the reasons to have a very small cosmological constant today, albeit promising results toward this can be found in [45, 50].

In the aforementioned works, a method to cancel out cosmological constant surplus is clearly described. This is achieved through a phase transition induced by a symmetry-breaking mechanism, which we employed as the background in our inflationary scenario.

We remarked that, during the transition, the generalised K-essence fluid can exhibit a phase of strong acceleration, reinterpreted in terms of inflation. Hence, we discussed the kinds of potentials associated with this phase, constructing them from very general assumptions. Particularly, we modeled three possible potentials, split into two main classes.

On the one hand, the first has been categorized as Starobinsky-like paradigms, where we moved from a general double exponential potential, yielding a large field potential, V_S, closely resembling, in its functional form, the pure Starobinsky one.

The second class of potentials is rooted in symmetry breaking frameworks, constructed through careful selections of the free parameters within the double exponential hypothesis. In this context, we introduced the V_W and V_Ω models. The latter exhibits infinite potential walls and is characterized by its small field nature, whereas the former, featuring finite potential walls, displays both small and large field behaviors.

Accordingly, we conducted an investigation into the dynamics of inflation in both of these scenarios for small and large fields.

Further, we delved into minimal and non-minimal couplings. Then, to accomplish this, we incorporated a Yukawa-like interacting term, giving rise to effective coupled potentials.

To this end, in the Einstein frame, we computed the action, deriving the analytical form of the transformed field h. Afterwards, we provided approximated expressions for specific limiting case, with a focus on the weak interaction limit, i.e. $|\xi| \ll 1$. Conversely, in the Jordan frame, we explicitly found the modified equation of motion, and assumed a quasi-de Sitter phase characterized by $\dot{R} \simeq 0$.

In this regard, all the here-investigated potentials showed a well-defined inflationary epoch in both the minimally and non-minimally coupled cases. However, limits on the coupling constants have also been found, suggesting $|\xi| \sim 10^{-3} \div 10^{-4}$. In addition, the non-minimally coupled case with a negative coupling strength appeared disfavored, showing that only V_Ω manages to provide an inflationary stage, albeit with limitations on the fine-tuning of free coefficients.

As a byproduct of our findings, the large field potentials have been found to better adapt to the inflationary evolution in the generalised K-essence scenario than small fields, that instead appear quite disfavored. For large field, we showed that the involved potentials might exhibit an offset fixed by $V_0 = -\frac{\chi\phi_0^4}{4}$, healing *de facto* the *classical* cosmological constant problem.

In all these cases, we remarked a strategy to unify old with chaotic inflation under the same standards. Indeed, after the slow roll stage, a graceful exit from inflation takes place naturally, for both the coupled and uncoupled frameworks. In this respect, we also showed that the inflationary dynamics is independent of the choice of frame, i.e. Jordan or Einstein one,confirming previous findings [98]. This prerogative may suggest the equivalence between the Jordan and Einstein frames in terms of particle production, as required for cancelling the cosmological constant, Λ_{vac}, during the transition.

Concluding, among all the models we investigated, symmetry breaking potentials have emerged as the least compatible with observational constraints.

On the contrary, the most well-aligned model is our Starobinsky-like one. Our study also enriches the understanding toward obtaining the Starobinsky-like picture without passing from extensions of Einstein's gravity and/or phenomenological extensions, see e.g. [99–102]. Notice that our scenario slightly departs from the pure Starobinsky model, since the free constants appear slightly different.

More importantly, the way we inferred the model itself is *completely different* from a second-order generalization of Einstein's gravity, i.e. from the original technique developed to obtain it. In this respect, we notice that our potential can adapt to couple with the Ricci curvature, while the original Starobinsky potential cannot be coupled since it is derived from geometrical perspective via a conformal transformation [103]. More broadly, we introduced a class of potentials analogous to $\alpha-$attractor models, but starting with totally different physical hypothesis. In such a way, our results imply that the generalised K-essence scenario seems to prefer *large fields*, originating from the generic double exponential ansatz, in equation (28) with $c = d = 1$.

Next, the analyses of the tensor-to-scalar ratio and the spectral index reveal that the choice of coupling has a significant impact on the compatibility with the cosmic microwave background anisotropies. Although the potentials yield consistent values for the tensor-to-scalar ratio and spectral index, we observe a strong dependence on the parameter choice, except for the Starobinsky-like potential.

Overall, these findings support the viability of the generalised K-essence framework for inflationary scenarios and highlight the role of non-minimal coupling in providing consistent inflationary dynamics with desirable properties.

In future works, we intend to better examine the energy offset of our models to improve the agreement with Planck's observations. Indeed, in this work, the energy constraints of the potentials were conventionally fixed, within the cancellation mechanism picture, regardless direct observations made by the Planck satellite. Since the free parameters of our models directly affect the magnitude of the Hubble parameter, H_*, rescaling the free constants, and particularly the vacuum energy offset, would offer the chance to better align with experimental bounds.

Further, we aim to explore additional potentials, particularly focusing on the nature of the generalised K-essence fluid. A possible hint that would explain the role played by the fourth-order symmetry breaking potential would be to match ϕ with the Higgs field, unifying *de facto* our model with the Higgs inflation [7, 97].

Moreover, we underlined throughout the text that, in order to delete the vacuum energy cosmological constant, the potential might transform into particles, likely different from baryons, that are produced during the inflationary stage. This prerogative is extremely different from the standard model in which particles, under the form of baryons, emerge in the reheating phase [71]. Hence, one aspect that we aim to investigate will be to better characterize particle production during generalised K-essence inflation.

Data availability statement

No new data were created or analysed in this study.

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