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Updated SM calculations of σ_W/σ_Z and the W boson width

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Abstract

The central value and theoretical uncertainties on the cross section ratio σ_W/σ_Z are evaluated using the NNLO calculations and the latest MSTW PDFs.

The partial width, total width and branching ratios of the W boson in the Standard Model, in the light of the latest electroweak calculations, are also updated.

Contents

1 Introduction

In this note we consider the theoretical uncertainty on the ratio of leptonic rates for the inclusive production of W and Z bosons, namely,

$$\mathrm{R} \;=\; \frac{\sigma \times \mathrm{B}\left(p\overline{p} \rightarrow \mathrm{W}^{\pm} \rightarrow \ell^{\pm}\nu\right)}{\sigma \times \mathrm{B}\left(p\overline{p} \rightarrow \mathrm{Z}^{\circ} \rightarrow \ell^{+}\ell^{-}\right)}\,.$$

from which the W leptonic branching ratio and an indirect determination of the total W width can be extracted.

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Published data on R come from Run II measurements from the CDF collaboration [1], together with those from both the CDF [2] and DØ [3] collaborations from Run I. A note is in preparation of a Tevatron combination of these published values.

From the definition of R,

$$\begin{aligned} \mathrm{R} &= \frac{\sigma \cdot \mathrm{B}(\mathrm{W}^{\pm} \to \ell^{\pm}\nu)}{\sigma \cdot \mathrm{B}(\mathrm{Z}^{\circ} \to \ell^{+}\ell^{-})} \\ &= \frac{\sigma_{\mathrm{W}}}{\sigma_{\mathrm{Z}}} \cdot \frac{\Gamma(\mathrm{Z})}{\Gamma(\mathrm{Z}^{\circ} \to \ell^{+}\ell^{-})} \cdot \frac{\Gamma\left(\mathrm{W}^{\pm} \to \ell^{\pm}\nu\right)}{\Gamma(\mathrm{W})}, \end{aligned}$$

we can extract the branching ratio of $\mathrm{W}^{\pm} \to \ell^{\pm}\nu$, $\Gamma\left(\mathrm{W}^{\pm} \to \ell^{\pm}\nu\right)/\Gamma(\mathrm{W})$, by using a Standard Model calculation for $\sigma_{\mathrm{W}}/\sigma_{\mathrm{Z}}$ and the LEP measurement of the $\mathrm{Z}^{\circ} \to \mathrm{e}^{+}\mathrm{e}^{-}$ branching ratio, namely $B(\mathrm{Z}^{\circ} \to \mathrm{e}^{+}\mathrm{e}^{-}) = (3.3658 \pm 0.0023)\%$, assuming lepton universality [4].

In a previous Tevatron combination [5] of preliminary Run II results on R, together with those from Run I, the results of the calculation in Ref. [6] were used to assign a theoretical uncertainty on the ratio $\sigma_{\mathrm{W}}/\sigma_{\mathrm{Z}}$. In that study a program based on the QCD NNLO expression developed by Van Neerven, *et al.* [7, 8] was used and gave the ratio of cross sections as $\sigma_{\mathrm{W}}/\sigma_{\mathrm{Z}} = 3.361 \pm 0.024$. However, the calculation was tree-level as far as electroweak vertices are concerned. Consequently, there was an uncertainty in the definition of $\sin^2\theta_W$, which was accounted for by an additional uncertainty of ± 0.048. The value for the cross section ratio used was $\sigma_{\mathrm{W}}/\sigma_{\mathrm{Z}} = 3.361 \pm 0.054$. Subsequent updates of Ref. [6] in Ref. [9] gave, $\sigma_{\mathrm{W}}/\sigma_{\mathrm{Z}} = 3.370$ with an uncertainty for the electroweak component alone of ± 0.014. The CTEQ6.1 and MRST2001E PDF sets were used in these studies.

2 Details of the calculation

Recently updated sets of PDFs from the MSTW Collaboration (formerly MRST) have been made available [10]. These new NNLO PDFs are interfaced to the NNLO program [11], used to calculate $\sigma_{\mathrm{W}}, \sigma_{\mathrm{Z}}$ and $\sigma_{\mathrm{W}}/\sigma_{\mathrm{Z}}$, which is again based on the results of Van Neerven, *et al.* [7, 8]. The results presented here use this program, modified as discussed below.

The couplings of the Z boson to fermion-pairs have been changed from the Born-level formulation to using the effective couplings derived from fits to LEP and SLD Z boson data; namely using $\rho = 1.0050 \pm 0.0010$ and $\sin^2\theta_{eff} = 0.23153 \pm 0.00016$ [4].

The Cabibbo-Kobayashi-Maskawa [12, 13](CKM) matrix elements used are also modified from the default values used in the program. The values used are the unconstrained measured values from [4]. These values, rather than the unitarity constrained values are used because the value of R can be used to give constaints on unitarity of the CKM matrix and also to extract V_{cs}, which is poorly known from direct measurement. The CKM values and uncertainties used are given in Table 1.

Table 1: Values and uncertainties of the CKM matrix elements used and the resulting uncertainty on X. The values are those not constrained by Unitarity.

CKM element	value	uncertainty	ΔX
V_{ud}	0.97377	0.00027	0.0019
V_{us}	0.2257	0.0021	0.0016
V_{ub}	0.0043	0.0003	0.0000
V_{cd}	0.230	0.011	0.0039
V_{cs}	0.957	0.095	0.0050
V_{cb}	0.0416	0.0006	0.0000

The central value obtained is $X = \sigma_W/\sigma_Z = 3.363$. The uncertainties on this which have been investigated are from a) PDF variations, b) uncertainties in the Z boson electroweak parameters, and c) uncertainties in the W boson CKM elements.

For the PDF unceratinties the eigenvector method was used. The values of $X = \sigma_W/\sigma_Z$ were computed for the 15 pairs of eigenvectors. This gives 15 pairs of values ΔX_{up} and ΔX_{down}, corresponding to the "up" and "down" components of each pair. The positive uncertainty on X was taken to be ΔX_{up} if $\Delta X_{up} > 0$ and $\Delta X_{down} < 0$. The positive uncertainty on X was taken to be ΔX_{down} if $\Delta X_{down} > 0$ and $\Delta X_{up} < 0$ (and *vice versa* for the negative uncertainty). In the case where both the "up" and "down" variations are positive (negative) the value $\sqrt{(\Delta X_{up}^2 + \Delta X_{down}^2)/2}$ was taken to be the positive (negative) uncertainty and the other component was set to zero. The positive and negative components are then separately added in quadrature, giving $\Delta X_+ = 0.013$ and $\Delta X_- = 0.010$. We take the uncertainty on X from the PDFs to be $\pm$ 0.013.

Note that the uncertainties in $\alpha_s(M_Z)$ and $\alpha_s(M_W)$ are not explicitly taken into account in the eigenvector method. However, these are expected to largely cancel in the ratio considered here [10]. The value of the electromagnetic coupling constant at the M_Z scale, $\alpha(M_Z)$, is not directly used in these computations. Instead the values of G_F and the vector boson masses are used, thus absorbing some of the higher-order electroweak effects. The widths of the W and Z bosons are also not used directly in the cross section ratio calculation. This is, it is a zero-width approximation. Again finite width effects are expected to largely cancel in the ratio, but this has not explicitly been verified.

The uncertainty in the Z boson electroweak parameters $\rho = 1.0050 \pm 0.0010$ and $\sin^2\theta_{eff} = 0.23153 \pm 0.00016$ were obtained by changing the values of each parameter in turn by $\pm 1\sigma$ and adding the uncertainties in quadrature. The result is $\Delta X = \pm$ 0.003.

For the CKM uncertainties each of the CKM elements in Table 1 was moved by $\pm 1\sigma$ and adding the uncertainties in quadrature. The result is $\Delta X = \pm$ 0.050. The largest uncertainty comes from V_{cs}, which is poorly known from direct measurement. This makes the theory estimate of X significantly larger than previous estimates.

Combining these uncertainties in quadrature gives

$$X = \sigma_{\mathrm{W}}/\sigma_{\mathrm{Z}} = 3.363 \pm 0.052.$$

3 Branching ratios and widths of W boson in SM

The W-boson decays weakly into either a quark-antiquark pair or a lepton and its corresponding neutrino. The partial leptonic decay width is given by [14]

$$\Gamma(\mathrm{W} \to \mathrm{e}\overline{\nu}_{\mathrm{e}}) = \frac{G_{\mathrm{F}} M_{\mathrm{W}}^3}{6\pi\sqrt{2}}(1 + \delta_\ell^{SM}) = 226.6 \pm 0.2\, MeV. \quad (1)$$

The values G_{F} = (1.16637 ± 0.00001) x 10^{-5} GeV^{-2} [2] and M_{W} = 80.398 ± 0.025 GeV are used in the calculation. The uncertainty is dominated by that in M_{W}. Note that by using the values of G_{F} and M_{W} to determine the SM value of $\Gamma(\mathrm{W} \to \mathrm{e}\overline{\nu}_{\mathrm{e}})$, the electroweak corrections δ_ℓ^{SM} are small (δ_ℓ^{SM} = -0.34%), because the bulk of the corrections are absorbed in G_{F} and M_{W}.

The partial width to $q\bar{q}$ final states, for massless quarks, is given by

$$\Gamma(\mathrm{W} \to q_i\overline{q}_j) = f_{EW} f_{QCD} \Gamma(\mathrm{W} \to \mathrm{e}\overline{\nu}_{\mathrm{e}}) \mid V_{ij} \mid^2 . \quad (2)$$

where $\mathrm{f}_{EW} = (1 + \delta_q^{SM})$ and δ_q^{SM} is the electroweak correction, with δ_q^{SM} = -0.40% [14], and $\mathrm{f}_{QCD} = 3(1 + \alpha_{\mathrm{s}}(\mathrm{M_W})/\pi + 1.409(\alpha_{\mathrm{s}}(\mathrm{M_W})/\pi)^2 + ...)$ is a QCD colour correction factor and V_{ij} is the CKM matrix element for i=u,d and j=d,s,b.

The total width Γ_{W} in the SM is given approximately by

$$\Gamma_{\mathrm{W}} = (3 + 2f_{QCD})\Gamma(\mathrm{W} \to \mathrm{e}\overline{\nu}_{\mathrm{e}}) = 2.0932 \pm 0.0022\, GeV, \quad (3)$$

where the uncertainty from $\alpha_{\mathrm{s}}(\mathrm{M_W})$ =0.1196 ± 0.0021 is 1.0 MeV, and that from M_{W} is 2.0 MeV. The form in this equation is approximate and neglects the differences in the electroweak radiative corrections for leptons and quarks. This small effect is however included in the numerical value given.

From the above values the W leptonic branching ratio is computed to be

$$B(\mathrm{W} \to \ell\overline{\nu}_\ell) = (10.83 \pm 0.01)\%.\, GeV. \quad (4)$$

The CKM matrix elements entering into W decay are given in Table 1. The main $q\bar{q}$ decay modes are $u\bar{d}$ and $c\bar{s}$. The $q\bar{q}$ branching ratio thus gives mainly constraints on the matrix elements V_{ud} and V_{cs}. Since the former is well known from other measurements, the $q\bar{q}$ mode can be used to give V_{cs}. Also the W leptonic branching ratio can be used to test the CKM unitarity constraint.

[2] Including the new result from the MuLan Collaboration [15] gives (G_{F} = (1.166371 ± 0.000006) x 10^{-5} GeV^{-2}. The FAST Collaboration [16] also have a new result, namely G_{F} = (1.166353 ± 0.000009) x 10^{-5} GeV^{-2}. Using the updated world average value from [15] gives neglible changes to the results reported here.

4 Summary

The Standard Model value of X, the ratio of the total W boson to Z boson cross sections, has been estimated using the latest MSTW PDFs. An improved electroweak formalism for the Z boson has been used and, for the W boson production the latest direct CKM measurements have been used. The result is

$$X = \sigma_{\rm W}/\sigma_{\rm Z} = 3.363 \pm 0.052.$$

Various properties of the W boson in the Standard Model have also been updated using revised electroweak corrections [14]. The partial leptonic decay width is

$$\Gamma({\rm W} \rightarrow {\rm e}\overline{\nu}_{\rm e}) = 226.6 \pm 0.2\, MeV. \tag{5}$$

The total width $\Gamma_{\rm W}$ is

$$\Gamma_{\rm W} = 2.0932 \pm 0.0022\, GeV, \tag{6}$$

and the W leptonic branching ratio is computed to be

$$B({\rm W} \rightarrow \ell\overline{\nu}_{\ell}) = (10.83 \pm 0.01)\%.\, GeV. \tag{7}$$

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A small correction should be applied to the numbers given in the PRL such that the combined value is R $= 10.837 \pm 0.200$, with statistical and systematic uncertainties of 0.146 and 0.136 respectively.

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