PHENOMENOLOGICAL *p-p* AND *n-p* PHASE PARAMETERS*

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(Presented by G. BREIT)

This is a continuation of the nucleon-nucleon phase shift analysis by the Yale group [1, 2] in the 0—350 Mev range combining information at many energies making use of additional data. The procedures used differ from the older mainly regarding the treatments of experimental uncertainties and of one-pion exchange (OPE), combined employment of gradient and least squares adjustments and more systematic smoothing of correction functions. The quantity

$$\chi^2 = N_{eff} D = \sum_{i,j} w_{ij}^j (A_\delta^j \eta_{ij}^j - y_{ij}^j)^2 + \sum_j \varepsilon_j [A^j (A_\delta, \delta) - 1]^2 \quad (1)$$

was minimized [3]. Here j specifies a set of measurements at one energy, $i_j \equiv i$ a particular measurement, $w_i^j = 1/(\Delta y_i^j)^2$ the statistical weight of the measured value y_i^j having a relative uncertainty Δy_i^j, η_i^j the theoretical expression for y_i^j in terms of phase-parameters collectively denoted by δ, $\varepsilon_j = 1/(\Delta A^j)^2$ the statistical weight of the measurement of the normalizing A^j as $A^j = 1$. The A_δ^j is a parameter playing formally the role of a δ. The y_i^j is considered as corresponding to the «theoretical» quantity $A_\delta^j \eta_i^j = \hat{\eta}_i^j$ while $A^j (A_\delta, \delta)$ has the form $A^j (\delta_\delta, \delta) = A_\delta$. The form of (1) is such that the error matrix method can be directly applied. Eliminating the A^j from the equation the elements of the equivalent matrix are obtained as $m_{ss'}^{(n-f)} = m_{ss'}^{(n-f)} - \sum_r m_{sr} [1/m_{rr}^{(f)}] m_{rs'}$; $r = 1, 2, \ldots, f$; $s, s' > f$; f = number of different $j = n_j$:

$$\hat{\eta}_1 = A^1(\ ), \ldots, \hat{\eta}_f = A^f(\ ); \quad \hat{\eta}_{f+1} = A_\delta^1 \eta_1 (\delta), \ldots, \hat{\eta}_{f+N} = A_\delta^f \eta_N (\delta);$$

$$m_{A_\delta^k, A_\delta^l} = \left[\varepsilon_k + \sum_i w_i^k (\eta_i^k)^2\right] \delta_{kl}; \quad \Delta a_{s'} = \sum_s [(\hat{m}^{(n-f)})^{-1}]_{s's} \{b_s - \sum_r m_{sr} [m_{rr}^{(f)}]^{-1} b_r\}; \quad b_p = \sum_i w_i (\partial \eta_i^0/\partial a_p) \times (y_i - \eta_i^0); \quad m_p, \ A_\delta^j = \sum_i w_i^f A_\delta^j (\partial \eta^j/\partial \delta_p) \eta_i^j;$$

$$m_{p,q} = \sum_{i,j} w_i^j (A^j)^2 (\partial \eta_i^j/\partial \delta_p)(\partial \eta_i^j/\partial \delta_q).$$

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The matrix $[m^{(n-f)}]^{-1}$ was used in place of m^{-1} in the first reference of [1] after replace-

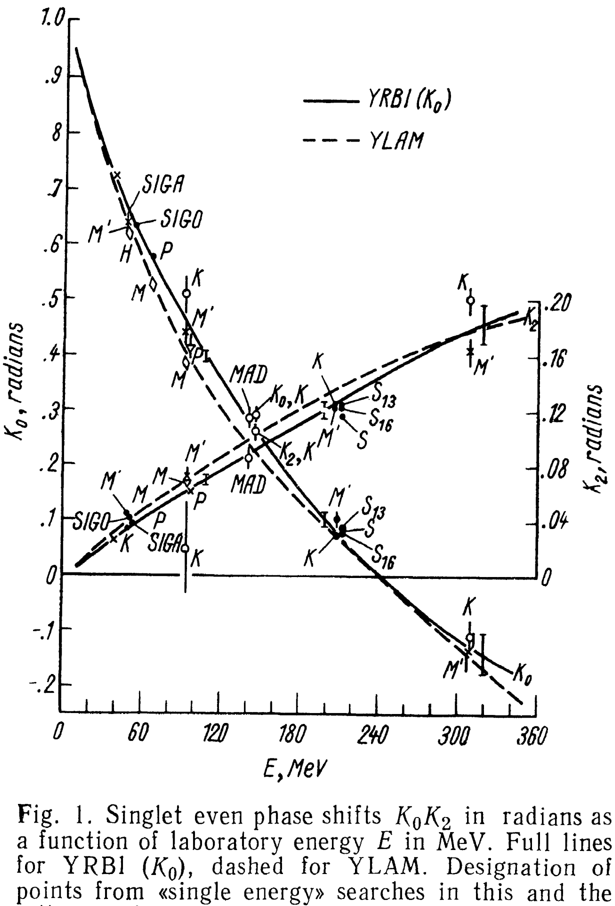

Fig. 1. Singlet even phase shifts $K_0 K_2$ in radians as a function of laboratory energy E in MeV. Full lines for YRBI (K_0), dashed for YLAM. Designation of points from «single energy» searches in this and the following figures; M from [4], K from [5], MAD from [6]; S from [7], S_{13} and S_{16} from [8], P from [9], H from [10], SIGA from [11] with searched phases supplemented by those from Amati, Leader and Vitals's dispersion treatment, SIGA with unsearched OPE phases [11], M' from [12].

ment of $\partial/\partial\delta_p$ by $\Sigma(\partial\delta_p/\partial a_{pq})\partial/\partial\delta_p$ in order to calculate $\langle\Delta a_{pr}\,\Delta a_{qs}\rangle$ in the notation of that reference. If the «measured» $\sigma(\theta)$ are obtained from relative values normalized to an available measured σ_{tot} the listed $\sigma(\theta)$ are used with $\varepsilon_j = 0$ and the σ_{tot} are treated as separate independent data. OPE is used for any (L, j) pair at energies below $E_m(L, J)$ only when no definite improvement results from releasing the phase-parameter for $E < E_m\,(L, J)$. Preliminary searches were made by the gradient method; the final by the least squares equations employing Eq. (1) in the linear approximation to the variation of the $\eta_i^j A^j(\;)$. In previous work graphical smoothing was done between the employment of sets of correction function [1]. In newer work this was supplemented by a least squares adjustment of a smooth analytically specified function covering a wide energy range in order to subordinate the influence of subjective judgement.

New data were aided in two main stages. In the first 722 data were used for $p-p$ and 758 for $n-p$ searches as compared with 541 for YLAM and 293 for YLAN3M. In the second stage 780 data at 97 energies were used for $p-p$, 762 data at 85 energies for $n-p$ searches. For $p-p$ the change in OPE treatment as well as elimination of most data lumping and abridgements in compounding experimental uncertainties gave phase starting with YLAM. Replacement of K_0 from YLAM by the t from YRBI, inclusion of newer data and of the newer error treatment gave then after researching, including that of K_0 the phase set YRBI (K_0); similar employment of K_0 from YLAM gave phase set YLAM (K_0). The [3] F_2 start was from the Yale potential. The shape of the K_0 versus E plot for YRBI (K_0) and that of a few other phase-parameters is smoother than for YLAM and YLAM (K_0). The $T=1$ parameters of YRBI (K_0) were used in searches for $T=0$ phases. Starting with YLAN3M and treating OPE as for $p-p$, the phase set YLAN4M was obtained. Some of the $T=1$ phases are shown in Fig. 1, 2, 3. Dashed lines are used for YLAM and full ones for YRBI (K_0). The legend to Fig. 1 explains designation for points from single energy determinations of other workers [4—12]. Coupling parameters are not shown. Single energy determinations agree sometimes better with one than the other of the two «all energies» fits. Fig. 4, 5 show some of the $T=0$ phases for YLAN3M and YLAN4M, with dashed and full lines of the former and latter respectively. Uncertainties of YRBI (K_0), YLAN4M obtained by the parallel shift prodedure [1] are indicated by error bars and do not allow for relative variations of phases within rather wide energy regions. The error limits may therefore be too

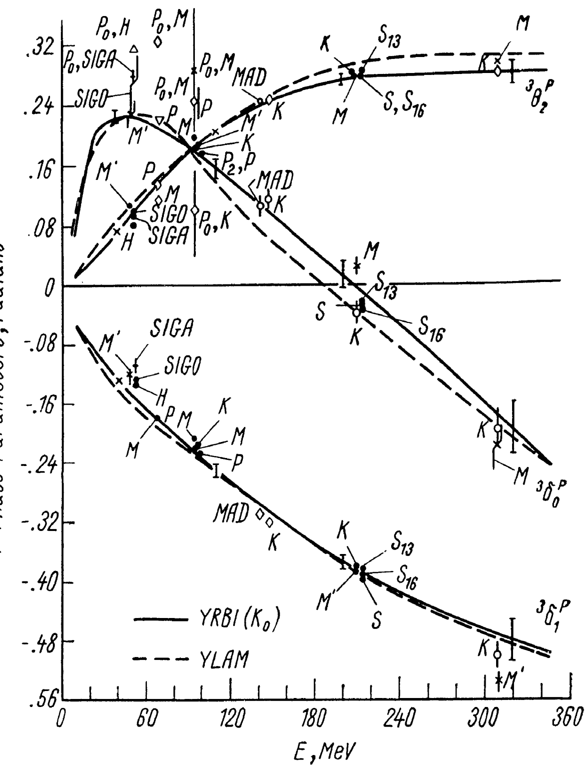

Fig. 2. Phase parameters $^3\delta_0$, $^3\delta_1^p$, $^3\delta_2^p$. Points designated as in Fig. 1.

narrow. Furthermore they do not allow for other grooves in the hypersurface of D versus the phase parameters. Joining to data below 9.7 MeV for $p-p$ and 13.7 MeV for $n-p$ can be improved and the possibility of different fits with D values not significantly larger than those for YRBI (K_0) and YLAN4M exists.

Illustrations of differences in quality of reproduction of experimental values by means of the new as compared with the old fits are shown in Fig. 6. Much of the new $p-p$ data consists of $P(\theta)$ measurements. The influence of the 213 MeV p-p and 350 MeV n-p data is clear from comparisons with $\sigma\,(\theta)$ in Fig. 6. Outstanding discrepancies of YRBI (K_0) with some of the measured values of triple scattering parameters exist especially for A and partly for D.

Magnetic moment effects with allowance for wave distortion [13] have been included

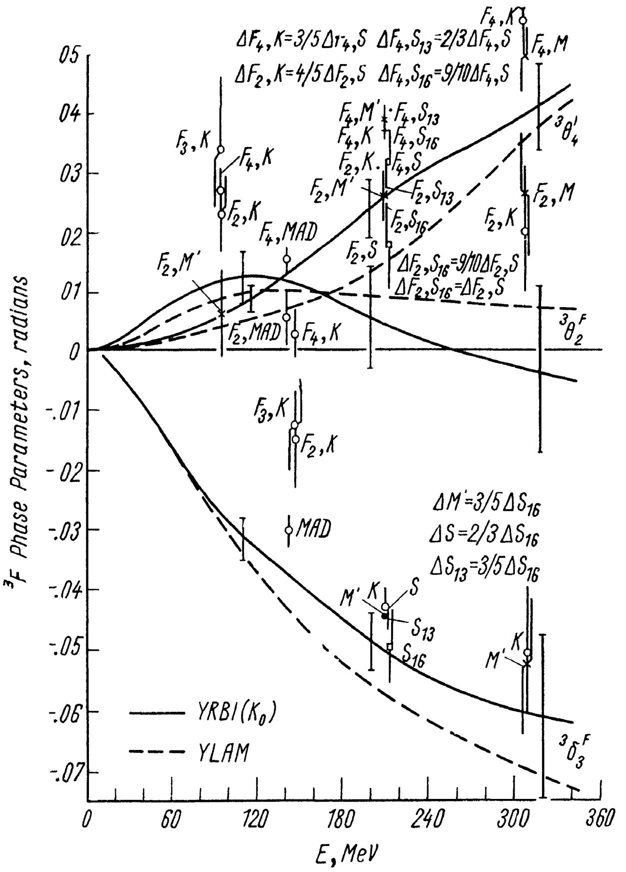

Fig. 3. Phase parameters $^3\theta_2^F$, $^3\delta_3^F$, $^2\theta_4^F$. Points as in Fig. 1.

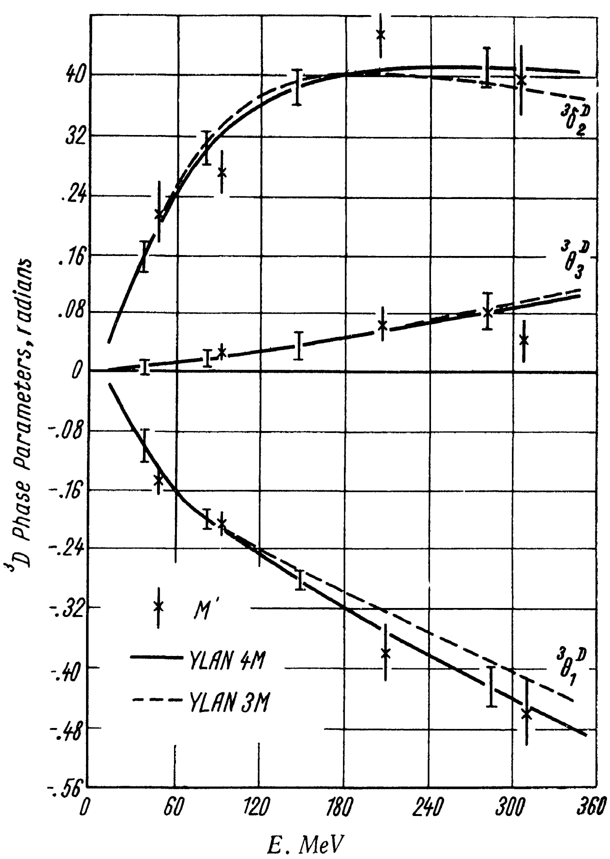

Fig. 4. Phase parameters $^3\theta_1^D$, $^3\delta_2^D$, $^3\theta_3^D$: M' from [12].

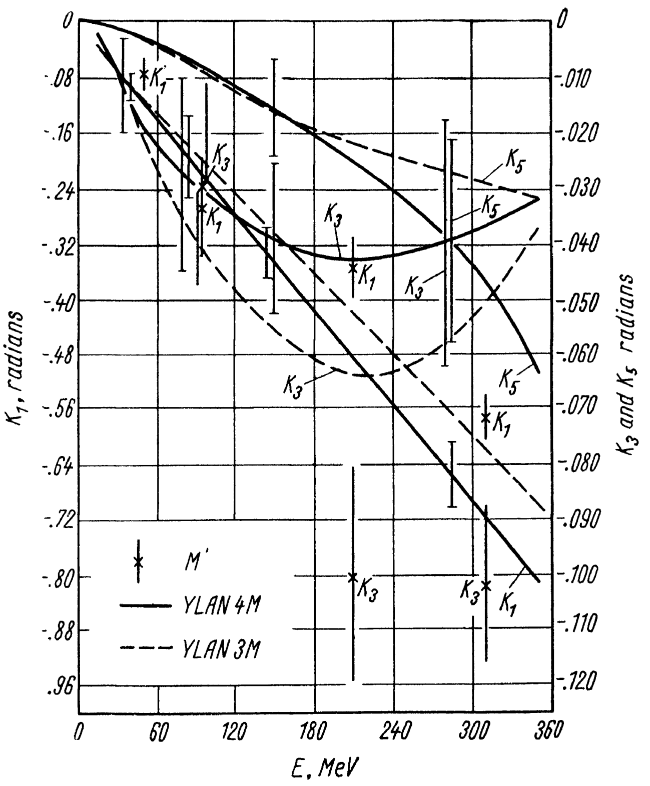

Fig. 5. Phase shifts K_1, K_3, K_5; M' from [12].

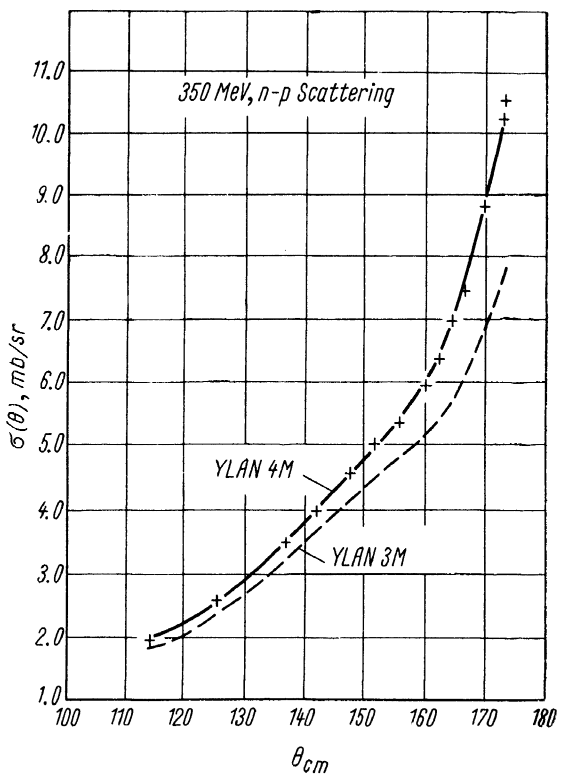

Fig. 6. Comparison of YLAN $3M$ and YLAN $4M$ with experimental differential cross section at $E_{\mathrm{lab}} = 350$ MeV.

throughout leaving OPE as the only long range effect. Preliminary values of the pion nucleon coupling constant which remain to be further improved especially regarding the presetting effect [14] correspond to $(g_0^2)_{p-p} = 14.57 \pm 0.42$ and $(g_0^2)_{n-p} = 13.87 \pm 0.24$, the fractional spin-spin modification of OPE is 0.046 ± 0.062 for *p-p* and 0.004 ± 0.037 for *n-p*; the central addition 0.18 ± 0.15 and -0.16 ± 0.51 respectively. No definite indication of violation of long range charge independence or of the theoretical form of the OPE is apparent in new tests of short range charge independence either.

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DISCUSSION

Yu. M. Kazarinov

Dr. Breit, what is the energy dependence of the parameter ρ_1 in your new analysis? Here there seems to be strong disagreement between your result and the data of previous analyses performed for fixed energies.

G. Breit

I understand that Dr. Kazarinow obtains a negative value for this parameter and that this result agrees with that of Perring witn the limits of error. I do not have a slide along but can make a sketch. The values obtained from the many energy analysis are not too certain *, but since more data are used the effect of experimental errors is expected to be less serious than for single energy analysis. The sign of ϱ_1 obtained in the many energy analysis is the same as that obtained in models of the interaction (potential models).

Wilson

I wish to remark that Professor Breit's comparison of charge independence applies only to high angular momentum states given by the periphery of the nucleon.

I do not see how this is inconsistent with the difference between the PP effective range and the NP singlet effective range that one obtains on a naive interpretation of the data. The latter includes core terms and could be caused by higher mass mesons.

G. Breit

I did not mean to say that tests of long range charge independence show that there can be no disagreement in the effective ranges of *n-p* and *p-p* in the $^1S_0$ state although they have a bearing on the question. I had primarily in mind the tests of short range charge independence performed by our group. In these the $T = 1$ phases determined from *p-p* are compared with those from *n-p* data employing the $^1S_0$ as well as other phases. The energies involved are higher than those that matter for the effective range.

It might also be mentioned especially in connection with the discussions at the Paris Conference that the long range tail of the one pion exchange potential introduces higher terms in the effective range expansion. As the wave function maxima go through the potential tail a wavyness is introduced in the phase shift — energy dependence. Such a phenomenon can be seen in an old Yale group paper of about 6-8-years ago. When the higher terms are introduced in the analysis the conclusion regarding the *n-p* effective range being the smaller one becomes more doubtful.

* Note in proof. At the time of publication of YLAN3M and of other fits there were other premising fits with a different sign of ϱ_1 either in the whole or in part of the 13-350 MeV energy region. In particular YLAN3 from which YLAN3M arose was negative at the lower energies and positive above about 200 MeV.