The Origin of Fields (Point-like Particles)

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We are suggesting that our world is in fact the 4-dimensional Minkowski space-time with the $SU_c(3)\times SU_L(2)\times U(1)\times SU_f(3)$ gauge symmetry built-in from the outset. The complex scalar fields would not be there (in view of the born-repulsive nature) but we have three exceptions: the complex scalar fields $\Phi(1,2)$ (the Standard-Model Higgs), $\Phi(3,1)$ (the purely family Higgs), and $\Phi(3,2)$ (the mixed family Higgs), here with the first family label and the second $SU_L(2)$ label. They help the gauge fields to form the "background" for everything, generating all the masses if necessary. The quark world is acceptable because of the $SU_c(3)\times SU_L(2)\times U(1)$ (i.e., 123) symmetry while the lepton world is acceptable in view of the $SU_L(2)\times U(1)\times SU_f(3)$ (another 123) symmetry.

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1. The Complex Scalar Fields, to Begin with

In the standard 20th-century textbooks on quantum field theory, the introduction of the complex scalar fields usually ends up with Klein-Gordon equation; it is very rare to discuss the domain of the renormalizable theories. In the 4-dimensional Minkowski space-time, the $\lambda(\phi^\dagger(x)\phi(x))^2$ interaction turns out to be this exception, and just the case in the 4-dimensional space-time, not in other dimensions.

What is more is that this λ is dimensionless; it should be a pure number in this 4-dimensional Minkowski space-time, because of its dimensionless-ness.

Because of its repulsive nature, the complex scalar field cannot be seen in nature - explaining why the complex scalar fields cannot be observed by us.

Thus, we may start thinking of our world - the $SU_c(3)\times SU_L(2)\times U(1) SU_f(3)$ force-fields 4-dimensional Minkowski space-time. All objects in this world have some specific labels under the force-fields gauge group - that defines our world [1].

Then, in our world, we have the force fields born with it. In other words, we have the various gauge fields that would need the longitudinal components, because each gauge field are born with only two components (degrees of freedom). Thus, the complex scalar fields are called for, via spontaneous symmetry breaking (SSB) through the Higgs mechanisms.

It is much more "powerful" by going through the Higgs mechanism with more Higgs fields, $\Phi(1,2)$ (the standard Higgs), $\Phi(3,2)$ (the mixed family Higgs), and $\Phi(3,1)$ (the purely family Higgs) in the origin of mass [2] and in this paper. Because they are "related" to each other, they can interact attractively to lower the energy, to overcome the curse of the single complex scalar field.

As an important note, we try to attach the specific meaning to the "point-like" Dirac field (particle) or "point-like" complex scalar field, when there is no size description in the equation, i.e., we could not find the size parameter such as in the simple Dirac equation. We say "point-like" rather than "point" because of the quantum principle, which we believe in "in the physics sense". In the $SU_c(3)\times SU_L(2)\times U(1)\times SU_f(3)$ 4-dimensional Minkowski space-time (or our world), this is what we talk about.

We may ask, at the level of the building blocks of matter, why we do not see complex scalar fields? These fields have the max-

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imal overlap (in symmetries) with the background - the 4-dimensional Minkowski space-time. We try to "show" that, in the 4-dimensional Minkowski space-time, the dimensionless self-interaction $\lambda(\phi^\dagger\phi)^2$ for the complex scalar field ϕ plays the role of all these - why we don't see these fields and furthermore they appear only in the context of some Higgs mechanisms.

As in the origin of mass [2], there would be no mass terms for the complex scalar fields if the temperature is high enough. But the dimensionless self-interaction $\lambda(\phi^\dagger\phi)^2$ wouldn't go away at these high temperature - thus playing the key roles of everything else. But this is a repulsive interaction; so, it needs the "related" complex scalar field(s) to make the story complete.

In fact, as we start talking about the "origin" of some object or the "existence" of something, we have to realize, and stick to, the philosophical meaning of this word. Something, if it exists, should have some effect on its environment, which has some impact on us. Thus, the vacuum itself does not exist unless the change of the vacuum can be observed. A particular field, or a particle, exists only if it has some impact on its environment, or on us.

Or, more precisely, a particle does not exist if it does not interact with some existing particle, such as the electron or the photon. This is the meaning of "existence". A complex scalar field does not exist if it has no interaction with something in existence. Thus, it is essential to have some "related" field that interacts with something observable, directly of indirectly. The self-interaction $\lambda(\phi^\dagger\phi)^2$ would be useless but the interaction $\lambda'(\phi_a^\dagger\phi_b)\cdot(\phi_b^\dagger\phi_b)$, with some observable ϕ_b, would do - so, the "related" field is important in this context.

The basic philosophy of this paper is as follows. The leptons, such as the electron, are ruled as to be "exist". Therefore, the Standard-Model (SM) Higgs $\Phi(1,2)$ exists since it coupled with the leptons; so does the mixed family Higgs triplet/doublet $\Phi(3,2)$. The pure family Higgs triplet $\Phi(3,1)$ exist since it couples with $\Phi(3,2)$. The criterion applies for the quarks, also.

One basic guideline is the energy; the (positive) energy cannot be created from the vacuum or from nothing.

We will try to "show" the following: In the 4-dimensional Minkowski space-time with the $SU_c(3)\times SU_L(2)\times U(1)\times SU_f(3)$ force-fields (gauge-groups) structure (say, our Space), the three Higgs $\Phi(1,2)$, $\Phi(3,2)$, and $\Phi(3,1)$ (complex scalar fields) exist, while the other complex scalar fields should not exist. These Higgs and the gauge bosons (mediating the various forces) provide the "background" for everything else. The quark world, having the (123) structure, can be accommodated in this background. Likewise, the lepton world, of another (123) structure, can be accommodated as well by this background.

To be precise, "our world" is the 4-dimensional Minkowski space-time with the $SU_c(3)\times SU_L(2)\times U(1)\times SU_f(3)$ force-fields (gauge-fields) group structure [1].

Among many things, we are questioning why the electrons are there. Why are muons there? This is the so-called "the origin problem" or "the generation problem". Originally, we have no way to answer these questions, thus stopped asking the questions. In fact, the questions are still there. In fact, the similarity of the muon to the electron might mean that they are members of the multiplet in some group, as from lessons in group theory. With the origin of mass [2], all these questions are answered. As the next step, we should be thinking whether there would be a better definition of the Standard Model [1] and thus we could try to answer the origin of the particles, the point-like particles.

Basically, we live in the 4-dimensional Minkowski space-time with some force-field "structure". We would eventually try to answer why we have these quarks and why we have these leptons, and how they are interact in the way it is. In this 4-dimensional Minkowski space-time, we first recognize that the dimensionless $\lambda(\phi^\dagger\phi)^2$ interaction for a complex scalar field ϕ is rather unique. It offers a natural explanation [2] of the origin of mass. In this 4-dimensional Minkowski space-time, everything is dimensionless, $\lambda(\phi^\dagger\phi)^2$ helps to generate the various masses; a couple of them make the entire world. These scalar fields are born to be repulsive; the term $\lambda(\phi^\dagger\phi)^2$ (with positive λ) means the lump of positive energy. If two complex scalar fields $\phi_{a,b}$ are "related", such as $\Phi(1,2)$ and $\Phi(3,2)$, then an attractive interaction $-2\lambda(\phi_a^\dagger\phi_b)\cdot(\phi_b^\dagger\phi_a)$ becomes possible. Thus, a few "related" complex scalar fields could exist but, owing to the repulsive nature of the $\lambda(\phi^\dagger\phi)^2$, the complex scalar field, without any "relative", cannot exist.

Thus, in our world, all the complex scalar fields are born with the $\lambda(\phi^\dagger\phi)^2$ interaction with a certain positive number λ. At the higher temperature where no particles have masses, this would be the only kind of the interactions unless the particles are related (with the same non-trivial index under some group in our world). When the temperature is sufficiently low, everything enters the phase with masses.

So, we may pose a tricky question as follows: "Is it necessary that the complex scalar field ϕ in the 4-dimensional Minkowski space-time must come with a repulsive $\lambda(\phi^\dagger\phi)^2$ interaction with a fixed λ?" Or, in fact we could ask the question alternatively: "In the 4-dimensional Minkowski space-time, the complex scalar field $\phi(x)$, if observable, should come with the repulsive interaction $\lambda(\phi^\dagger\phi)^2$ with $\lambda = \frac{1}{8}$. Is it true?" Maybe some number close to $\frac{1}{8}$ is what we find [2].

Imagine that we have the complex scalar field with only the kinetic-energy term in our world - but it also has nothing to initiate the interactions with the others; maybe they are there but no interaction means that others or we cannot see them. Besides, our world is stable, not meta-stable, or the vacuum should not decay away; so, the complex scalar field with only the kinetic term had "decayed" long time ago.

It is indeed that the $\lambda(\phi^\dagger\phi)^2$ interaction is rather unique in the 4-dimensional Minkowski space-time. It precedes the complex scalar field *without* the λ interaction, because of no interaction with its surroundings, beside its "meta-stability".

Of course, another possible answer lies in the spontaneous symmetry breakings (SSB) occurring for generalized Higgs mechanisms - they need the repulsive $\lambda(\phi^\dagger\phi)^2$ interaction to stabilize the whole system. All the masses come from SSB's [2] and the complex scalar fields exist for that, and maybe only for that.

2. Point-like Dirac Fields

The Dirac equation comes from the linearization of the Einstein equation - it carries the one-half spin and the emerging two-component part describes its antiparticle. The mystery seems to lie in the spin, rather than in its antiparticle.

Dirac's invention of Dirac equation for the electron is remarkable for several aspects: There is no size parameter in the equation, thus describing a point-like particle. Second, it describes the electron, an observable particle in existence. It turns out that we could use the electron to "define" the existence of the entire world. It means that the entire world corresponds to a set of rigorous mathematics.

As for our guiding principle of energy, the origin of the Dirac equation doesn't disturb the energy, but creates the extra (new) space for something to exist. So, Dirac's invention is in fact the discovery of this extra space.

In our world, i.e., the 4-dimensional Minkowski space-time with the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ force-fields structure, the allowed point-like Dirac particle must be the member of multiplet under the overall group. Here we label "the quark world" or "the lepton world" since each world represents a multiplet under the overall group, i.e., $SU_c(3)\times SU_L(2)\times U(1)\times SU_f(3)$. Here "point-like" refers to some physical "point" that satisfies "relativity" and "quantum principle".

In other words, we should not say that "the electron" exists, but rather we should say that the lepton world exists.

We specify "our Space" as "the

$SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ 4-dimensional Minkowski space-time" by telling the overall force-fields group explicitly. Any object which we are talking about must have the overall group assignment, in addition to the transformation property under the Lorentz group. We call such object as "the basic unit" [3], because altogether they are the basic construction units of "the Standard Model". Note that the so-called "building blocks of matter" are *not* the basic units, but the left-handed triplet-doublet lepton multiplet $((\tau_L, \nu_{\tau,L}, (\mu_L, \nu_{\mu,L})), (e_L, \nu_{e,L}))(columns)$ is one basic unit in our Standard Model [3].

In all our discussions or related "derivations", the "size" of the particle never enters - that is what the "point-like" stands for. It is not a "point" in the mathematical sense; it is "point-like" in the quantum-mechanics sense; and the geometrical meaning and the physical meaning could be quite different; e.g., do we impose "uncertainty relations" in defining a "point"?

But should we have these quarks/leptons? For a general 4-dimensional Minkowski space-time, it would be awfully difficult to get the answer, if there is one. However, if we start with the 4-dimensional Minkowski space-time with some structure, it would be easier to get a meaningful answer. That is why we offer a precise definition of the Standard Model [1] - it is the Standard Model in the 4-dimensional Minkowski space-time with the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ gauge-group (force-field) structure.

Our Space is the 4-dimensional Minkowski space-time and the structure, the group structure, is the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$. It means that every object has the designated group property and it transforms in a certain way under the 4-dimensional Lorentz group. As the nature is described by the Standard Model, every object should have the designated group and Lorentz group transformation properties.

Thus, we live in the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ Minkowski space-time. It supports the quark world - it has the (123) group. It also can support the lepton world - as it has another (123) group. (123) makes it free of QED Landau ghosts and makes the whole thing asymptotically free. We realize that it is very important to have (123) - to have everything consistent among themselves.

3. The Lepton World or the Quark World

As mentioned earlier, in the "$SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ Minkowski space-time", the standard (123) supports the quark world while another (123) supports the lepton world - here the standard (123) means "$SU_c(3) \times SU_L(2) \times U(1)$" and another (123) means "$SU_L(2) \times U(1) \times SU_f(3)$". On the surface, we need (123) to take care of QED Landau ghost and (123) makes it asymptotically free, thus behaving very nicely at relatively low energies. We should look for the deeper reasons why this is so - and the mathematical reason cited above is not the physical reason which we are looking for.

In fact, it is important to realize the natural separation of another (123) from the standard (123). Presence of Landau ghost in the $SU_L(2) \times U(1)$ alone, in our opinion, signals the need of $SU_f(3)$ for the protection. So, although we haven't seen the family gauge bosons so far, we anticipate such protection somewhere if going up in energy.

Maybe we should use the language, to be precise [1], that the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ Minkowski space-time supports the quark world and it also supports the lepton world.

Thus, we regard [3] $((\nu_\tau,\, \tau)_L,\, (\nu_\mu,\, \mu)_L,\, (\nu_e,\, e)_L)$ $(columns)$ $(\equiv \Psi(3,2))$ as the $SU_f(3)$ triplet and $SU_L(2)$ doublet. It is essential to complete the (extended) Standard Model [4] by working out the Higgs dynamics in detail [2]. It is also essential to realize the role of neutrino oscillations - it is the change of a neutrino in one generation (flavor) into that in another generation; or, we need to have the coupling

$ih\bar{\Psi}_L(3,2) \times \Psi_R(3,1) \cdot \Phi(3,2)$, exactly the coupling introduced by Hwang and Yan [3]. Then, it is clear [4] that the mixed family Higgs $\Phi(3,2)$ must be there. The remaining purely family Higgs $\Phi(3,1)$ helps to complete the picture, so that the eight gauge bosons are massive in the $SU_f(3)$ family gauge theory [5].

The Kinetic Terms:

We work with the Lie group $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ as the gauge group. Each basic unit is made up from quarks (of six flavors, of three colors, and of the two helicities) and leptons (of three generations and of the two helicities); each basic unit has one kinetic term and has the definite transformation property under the overall group, $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$. The story for each basic unit is fixed if the so-called "gauge-invariant derivative", i.e. D_μ in the kinetic-energy term $-\bar{\Psi}\gamma_\mu D_\mu \Psi$, is given for a given basic unit [6].

We have, for the up-type right-handed quarks u_R, c_R, and t_R,

$$D_\mu = \partial_\mu - ig_c \frac{\lambda^a}{2} G_\mu^a - i\frac{2}{3} g' B_\mu, \qquad (1)$$

and, for the rotated down-type right-handed quarks d'_R, s'_R, and b'_R,

$$D_\mu = \partial_\mu - ig_c \frac{\lambda^a}{2} G_\mu^a - i(-\frac{1}{3}) g' B_\mu. \qquad (2)$$

On the other hand, we have, for the $SU_L(2)$ quark doublets,

$$D_\mu = \partial_\mu - ig_c \frac{\lambda^a}{2} G_\mu^a - ig\frac{\vec{\tau}}{2} \cdot \vec{A}_\mu - i\frac{1}{6} g' B_\mu. \qquad (3)$$

For the lepton world, we introduce the family triplet, $(\nu_\tau^R, \nu_\mu^R, , \nu_e^R)$ (column), under $SU_f(3)$. Since the minimal Standard Model does not see the right-handed neutrinos, it would be a natural way to make an extension of the minimal Standard Model. Or, we have, for $(\nu_\tau^R, \nu_\mu^R, \nu_e^R)$,

$$D_\mu = \partial_\mu - i\kappa \frac{\bar{\lambda}^a}{2} F_\mu^a. \qquad (4)$$

and, for the left-handed $SU_f(3)$-triplet and $SU_L(2)$-doublet $((\nu_\tau^L, \tau^L), (\nu_\mu^L, \mu^L), (\nu_e^L, e^L))$ (all columns),

$$D_\mu = \partial_\mu - i\kappa \frac{\bar{\lambda}^a}{2} F_\mu^a - ig\frac{\vec{\tau}}{2} \cdot \vec{A}_\mu + i\frac{1}{2} g' B_\mu. \qquad (5)$$

The right-handed charged leptons form another triplet $\Psi_R^C(3,1)$ under $SU_f(3)$.

The Off-diagonal Mass Terms via Higgs:

In the quark world, we have only the SM Higgs $\Phi(1,2)$ so that all the quarks get the masses in the old-fashion way. On the other hand, in the lepton world, we have both the SM Higgs $\Phi(1,2)$ and the mixed family Higgs $\Phi(3,2)$, but the old-fashion way via the SM Higgs applies only to the charged leptons.

The neutrino mass term assumes the *unique* form:

$$i\frac{h}{2} \bar{\Psi}_L(3,2) \times \Psi_R(3,1) \cdot \Phi(3,2) + h.c., \qquad (6)$$

Here the Higgs field $\Phi(3,2)$ is the mixed family Higgs, because it carries some nontrivial $SU_L(2)$ charge. In fact, the charged part of $\Phi(3,2)$ does not experience the spontaneous symmetry breaking (SSB), as worked out explicitly in [2].

We wish to note, again, that, for charged leptons, the Standard-Model choice is $\Psi^\dagger(\bar{3},2)\Psi_R^C(3,1)\Phi(1,2) + c.c.$, which gives three leptons an equal mass. But, in view of that if (ϕ_1, ϕ_2) is an $SU(2)$ doublet then $(\phi_2^\dagger, -\phi_1^\dagger)$ is another doublet, we could form $\tilde{\Phi}^\dagger(3,2)$ from the doublet-triplet $\Phi(3,2)$.

$$i\frac{h^C}{2} \bar{\Psi}_L(3,2) \times \Psi_R^C(3,1) \cdot \tilde{\Phi}^\dagger(3,2) + h.c., \qquad (7)$$

which gives rise to the imaginary off-diagonal (hermitian) elements in the 3×3 mass matrix, so removing the equal masses of the charged leptons.

It is useful to talk about the lepton world and the quark world, *separately.* They exist at the different ranges. We suspect that they should be protected by different $SU(3)$ - being asymptotically free for acceptance by our world, or our Space. If the lepton world were not asymptotically free, many things would be up and down - the mathematics would run weird.

So, the quarks and the leptons "exist" since they have participated in all these interactions, or carry the charges of the force fields (i.e., the $SU_c(3)\times SU_L(2)\times U(1)\times SU_f(3)$ gauge fields). The word "existence" becomes easier to understand, as we talk about something beyond the various Higgs - the so-called "background" in our world.

4. The Three Higgs Fields, to Close Up Everything

The "$SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ Minkowski space-time" is a highly structured group, for obvious reasons and nontrivial gauge-invariance reasons. The complex scalar fields exist, not only to make up the longitudinal components of the gauge fields (in the U-gauge) but also make something equivalent (or, invisible). Thus, we try to combine all the groups together [1] in order to synthesize together to obtain something new. It is clear that combining the force-fields gauge groups with the 4-dimensional Minkowski space-time is a natural first step.

We believe that one of the most urgent questions in the next stage of particle physics is to study "the origin of mass" - a question that we have recently gained some understanding [2]. In that [2], we may set all the mass terms of the various Higgs to identically zero, except one spontaneous-symmetry-breaking (SSB) igniting term. All the mass terms are the results of this SSB, when switched on. Therefore, the "mass" is the result of this SSB - a generalized Higgs mechanism. Accordingly, when the temperature is higher than a certain critical temperature, the notion of "mass" does not exist.

In this mass-generation game, the set of the "various" Higgs includes the Standard-Model (SM) Higgs $\Phi(1,2)$, the mixed family Higgs $\Phi(3,2)$, and the pure family Higgs $\Phi(3,1)$, where the first label refers to the group $SU_f(3)$ while the second the group $SU_L(2)$ (singlets in other groups). The ignition could be on the pure family Higgs $\Phi(3,1)$ [2], but *not* on the SM Higgs $\Phi(1,2)$.

To begin with, all the Higgs mass terms are zero, except the SSB ignition term $\mu_2^2\Phi^\dagger(3,1)\Phi(3,1)$. Why the SM Higgs $\Phi(1,2)$ fails at this "ignition" task is another question which we might ask. The elusive Higgs $\Phi(3,1)$ does work as the "ignition" channel.

These related Higgs, being the scalar fields, act as the systems of energies, self-interacting (dimensionless) via $\lambda(\phi^\dagger\phi)^2$ and interacting equivalently with other Higgs. See the illustration in [1]. We conclude that these related three Higgs interact attractively with a universal λ. When the temperature is low enough, it becomes the "mass" phase, or the phase in which all the particles (SM or family Higgs, gauge bosons except the photon, quarks, and leptons) have masses.

In the 4-dimensional Minkowski space-time, two complex scalar fields ϕ_a and ϕ_b are said to be "related" such as that both are triplets of $SU(3)$, then the interaction $\eta(\phi_a^\dagger\phi_b)\cdot(\phi_b^\dagger\phi_a)$ exists, and maybe exists maximally (i.e., not violating the positive definiteness of the overall energy). This may be the salient property of the 4-dimensional Minkowski space-time.

In the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ Minkowski space-time, the three complex scalar fields $\Phi(1,2)$, $\Phi(3,2)$, and $\Phi(3,1)$ act like *one* complex scalar field - i.e., one "ignition" at some place, SSB happening everywhere; for example, see the origin of mass [2]. They have to be "related" in order to accomplish everything.

Remembering what we did in "The Origin of Mass" [2], we realize that, before the spontaneous symmetry breaking (SSB), the Standard Model does not contain any parameter that is pertaining to "mass", but, after the SSB, all particles in the Standard Model acquire the mass terms as it should - a way to explain "the origin of mass". In this way, we sort of tie "the origin of mass" to the effects of the SSB, or the generalized Higgs mechanism.

Thus, we have to have the various Higgs at our disposal, but not too many in view of "minimum Higgs hypothesis" or the repulsive nature of these scalar fields. In the model

[4], we have the Standard-Model Higgs $\Phi(1,2)$, the purely family Higgs $\Phi(3,1)$, and the mixed family Higgs $\Phi(3,2)$, with the first label for $SU_f(3)$ and the second for $SU_L(2)$. We need another triplet $\Phi(3,1)$ since all eight family gauge bosons are massive [5].

In another arXiv paper [1], we try to introduce the joint-group space, "$SU_c(3)\times SU_L(2)\times U(1)\times SU_f(3)$ Minkowski space-time", in the effort of trying to find out what would be the constraints on the complex scalar fields. First of all, we have to recognize the special importance of the dimensionless interaction $\lambda(\phi^\dagger\phi)^2$, the only pure number λ for the 4-dimensional low-spin fields. We find $\lambda=\frac{1}{8}$, without knowing the underlying reason. Secondly, those unrelated complex fields could be described by $\lambda(\phi_a^\dagger\phi_a+\phi_b^\dagger\phi_b)^2$ (with $a\neq b$), through a repulsive interaction. Thus, we can write an "attractive" interaction, $(\phi_a^\dagger\cdot\phi_b)\cdot(\phi_b^\dagger\cdot\phi_a)$, for only those related complex fields. We use this to understand the origin of mass [2].

Let us write down the terms for potentials among the three Higgs fields, subject to (1) that they are renormalizable, and (2) that symmetries are only broken spontaneously (the Higgs or induced Higgs mechanism). We write [4, 6]

$$\begin{aligned}V_{Higgs} = \; & \mu_2^2\Phi^\dagger(3,1)\Phi(3,1)+\lambda(\Phi^\dagger(1,2)\Phi(1,2)+cos\theta_P\Phi^\dagger(3,2)\Phi(3,2))^2\\ & +\lambda(-4cos\theta_P)(\Phi^\dagger(\bar{3},2)\Phi(1,2))(\Phi^\dagger(1,2)\Phi(3,2))\\ & +\lambda(\Phi^\dagger(3,1)\Phi(3,1)+sin\theta_P\Phi^\dagger(3,2)\Phi(3,2))^2+\lambda(-4sin\theta_P)(\Phi^\dagger(\bar{3},2)\Phi(3,1))(\Phi^\dagger(3,1)\Phi(3,2))\\ & +\lambda_2'\Phi^\dagger(\bar{3},1)\Phi(3,1)\Phi^\dagger(1,2)\Phi(1,2)+(terms\ in\ i\delta's\ and\ in\ decay).\end{aligned} \tag{9}$$

These are two prefect squares minus the other extremes, to guarantee the positive definiteness, when the minus μ_2^2 was left out. (θ_P may be referred to as "Pauchy's angle".)

From the expressions of u_iu_i and v^2, we obtain

$$v^2(3cos^2\theta_P-1)=sin\theta_Pcos\theta_Pw^2. \tag{10}$$

In the U-gauge, we choose to have

$$\begin{aligned}\Phi(1,2) &= (0,\frac{1}{\sqrt{2}}(v+\eta)),\\ \Phi^0(3,2) &= \frac{1}{\sqrt{2}}(u_1+\eta_1',u_2+\eta_2',u_3+\eta_3'),\\ \Phi(3,1) &= \frac{1}{\sqrt{2}}(w+\eta',0,0),\end{aligned} \tag{8}$$

all in columns. The five components of the complex triplet $\Phi(3,1)$ get absorbed by the $SU_f(3)$ family gauge bosons and the neutral part of $\Phi(3,2)$ has three real parts left - together making all eight family gauge bosons massive.

For the sake of simplicity, we will neglect the mixing (and the mixing inside $\eta'_{1,2,3}$) in this paper. To work out on "the origin of mass", we would drop out all "mass" terms to begin with.

In treating the problem with the renormalization group (RG) equations, we realize that, even though to begin withwe set all the mass terms to zero, they would climb back so easily in the case of the complex scalar fields - as judged by the RG flow diagrams. This is why have to analyze different problems from a general lagrangian as in [4].

Let us illustrate some typical results of [2]. We begin with [2]

And the SSB-driven η' yields

$$w^2(1-2sin^2\theta_P)=-\frac{\mu_2^2}{\lambda}+(sin2\theta_P-tan\theta_P)v^2. \tag{11}$$

These two equations show that it is necessary to have the driving term, since $\mu_2^2=0$ implies that everything is zero. Also, $\theta=45^\circ$ is the (lower) limit.

The mass squared of the SM Higgs η is $2\lambda cos\theta_Pu_iu_i$ (noting the factor of two), as known to be $(125\ GeV)^2$. The famous v^2 is the number divided by 2λ, or $(125\ GeV)^2/(2\lambda)$. Using PDG's for e, $sin^2\theta_W$, and the W-mass [9], we find $v^2 = 255\ GeV$. So, we set $\lambda = \frac{1}{8}$, a simple model indeed.

The mass squared of η' is $-2(\mu_2^2 - sin\theta_P u_1^2 + sin\theta_P(u_2^2 + u_3^2))$. The other condensates are $u_1^2 = cos\theta_P v^2 + sin\theta_P w^2$ and $u_{2,3}^2 = cos\theta_P v^2 - sin\theta_P w^2$ while the mass squared of η'_1 is $u_1^2\lambda$, those of $\eta'_{2,3}$ be $u_{2,3}^2\lambda$. The mixings among η'_i themselves are neglected in the paper.

There is no SSB for the charged Higgs $\Phi^+(3,2)$. The mass squared of ϕ_1 is $\lambda(cos\theta_P v^2 - sin\theta_P w^2) + \frac{\lambda}{2}u_i u_i$ while $\phi_{2,3}$ be $\lambda(cos\theta_P v^2 + sin\theta_P w^2) + \frac{\lambda}{2}u_i u_i$.

A further look of these equations tells that $3cos^2\theta_P - 1 > 0$ and $2sin^2\theta_P - 1 > 0$. A narrow range of θ_P is allowed (greater than 45° while less than 57.4°, which is determined by the group structure). For illustration, let us choose $cos\theta_0 = 0.6$ and work out the numbers as follows: (Note that $\lambda = \frac{1}{8}$ is used.)

$$
\begin{aligned}
& 6w^2 = v^2, \quad -\mu_2^2/\lambda = 0.32v^2; \\
\eta: \quad & m^2(\eta) = (125\,GeV)^2, \quad v^2 = (250\,GeV)^2; \\
\eta': \quad & m^2(\eta') = (51.03\,GeV)^2, \quad w^2 = v^2/6; \\
\eta'_1: \quad & m^2(\eta'_1) = (107\,GeV)^2, \quad u_1^2 = 0.7333v^2; \\
\eta'_{2,3}: \quad & m^2(\eta'_{2,3}) = (85.4\,GeV)^2, \quad u_{2,3} = 0.4667v^2; \\
\phi_1: \ & mass = 100.8\,GeV; \qquad \phi_{2,3}: mass = 110.6\,GeV.
\end{aligned} \tag{12}
$$

All numbers appear to be reasonable. Since the new objects need to be accessed in the lepton world, it would be a challenge for our experimental colleagues.

As for the range of validity, $\frac{1}{3} \leq cos^2\theta_P \leq \frac{1}{2}$. The first limit refers to $w^2 = 0$ while the second for $\mu_2^2 = 0$.

We may fix up the various couplings, using our common senses. The cross-dot products would be similar to κ, the basic coupling of the family gauge bosons. The electroweak coupling g is 0.6300 while the strong QCD coupling $g_s = 3.545$ (order of unity); my first guess for κ would be about 0.1 (which is rather small). The masses of the family gauge bosons would be estimated by using $\frac{1}{2}\kappa \cdot w$, so slightly less than $10\,GeV$. (In the numerical example with $cos\theta_P = 0.6$, we have $6w^2 = v^2$ or $w = 102\ GeV$. This gives $m = 5\ GeV$ as the estimate.) So, the range of the family forces, existing in the lepton world, would be 0.04 $fermi$.

In [2], the term that ignites the SSB is chosen to be with η', the purely family Higgs. This in turn ignites EW SSB and others. It explains the origin of all the masses, in terms of the spontaneous symmetry breaking (SSB). SSB in $\Phi(3,2)$ is driven by $\Phi(3,1)$, while SSB in $\Phi(1,2)$ from the driven SSB by $\Phi(3,2)$, as well. The different, but related, scalar fields can accomplish so much, to our surprise. And these Higgs are exactly those which the gauge fields (i.e., the force-fields) are calling for.

If we look at the Standard Model more closely, the Nature ignites the purely family Higgs channel and then the electroweak Higgs channel SSB passively (having the major prediction $v = 2m_{Standard \quad Model}$). What follows is then that the linear terms of $\eta'_{1,2,3}$ have to survive (because of the same as the mass terms), and so on. It is very interesting, indeed.

5. The Scenario Re-Cast

In the 4-dimensional Minkowski space-time, the complex scalar field $\phi(x)$ is described, generally and renormalizably, by

$$\mathcal{L} = -(\partial_\mu \phi)^\dagger (\partial_\mu \phi) - M^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2. \quad (13)$$

If $\lambda < 0$, the system collapses (i.e. unbounded from below). If $\lambda > 0$, it is repulsive so that the system cannot build up by itself. The interesting question is that λ is dimensionless - a pure number that characterizes the 4-dimensional Minkowski space-time (maybe $\lambda = \frac{1}{8}$, but we need the proof).

The force fields are described by the gauge fields in the group $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$. They need complex scalar fields to complete the picture via generalized Higgs mechanisms. The longitudinal components are missing in the purely gauge-fields description such that complex scalar fields are needed for the Higgs mechanisms. Thus, in the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ 4-dimensional Minkowski space-time (i.e., our Space), the world is already set up in terms of the force fields - the gauge fields and the complex scalar fields via the special Higgs mechanisms, say, the "background".

It yields, and only yields, three Higgs fields $\Phi(1,2)$, $\Phi(3,2)$, and $\Phi(3,1)$. The "related" Higgs fields, such as $\Phi(1,2)$ and $\Phi(3,2)$ with $SU_L(2)$ doublets, can overcome the "self-repulsive" nature and become useful and be able to live in this world.

Let's turn our attention to the lepton world: We know that they couple to the $SU_L(2) \times U(1)$ gauge sector (which makes them visible). To interpret the ordering via three generations, we proposed the force-fields nature of the $SU_f(3)$ gauge sector. Neutrino oscillations provide a direct proof that generations can switch among themselves. By introducing the $SU_f(3)$ gauge sector to the original $SU_L(2) \times U(1)$, the lepton world is free from the Landau ghost and is asymptotically free. So, the lepton world is accepted.

How about the quark world? It is already a perfect world since it couples to the $SU_c(3) \times SU_L(2) \times U(1)$ (i.e. the standard (123)) gauge sector. It exhibits the "size" effect, i.e., that the quark world exists only within a given volume; or, it exhibits the temperature effect, i.e., that it undergoes the phase transition (into something else). The quark world is so much different from the lepton world - that the two $SU(3)$ is so much different.

This completes the story of how the building blocks of matter build up the entire matter world.

6. Is it a Complete Theory?

"Is it a complete theory?" Even though it is a very difficult question to ask, we have to ask and try to answer, eventually.

Does the sum of all ultraviolet divergences of given order (and of the same characteristics) give rise to some finite number or zero? If we look at a specific diagram, such as the self-energy diagram in Ch. 10 of [6], ultraviolet divergence is certainly there - the issue that troubled all famous theoretical physicists for the entire 20th Century. Maybe in the 21st Century, there might be some breakthroughs that would decide whether the quantum field theory would be here to stay.

Fortunately, the electron self-energy diagram which we just mentioned is not alone since the highly coupled theory such as the Standard Model has many other diagrams which have the same characteristics. In a complete theory, we sum all of them (of the same characteristics) and ultraviolet divergences would cancel among themselves. It is clear that the $U(1)$ gauge part, i.e., QED, is *not* a complete theory. The Standard Model offers us a candidate of the complete theory.

We have been rather persistent in addressing the question whether it is a complete theory - in the origin of mass [2], we ask this question because we are not so sure if this solution for the origin of mass is true or not (despite all the beautiful numbers); in a precise definition of the Standard Model [1], tests on the complete theory were discussed; and, early on, the fine-tuning problem for introducing super-symmetry

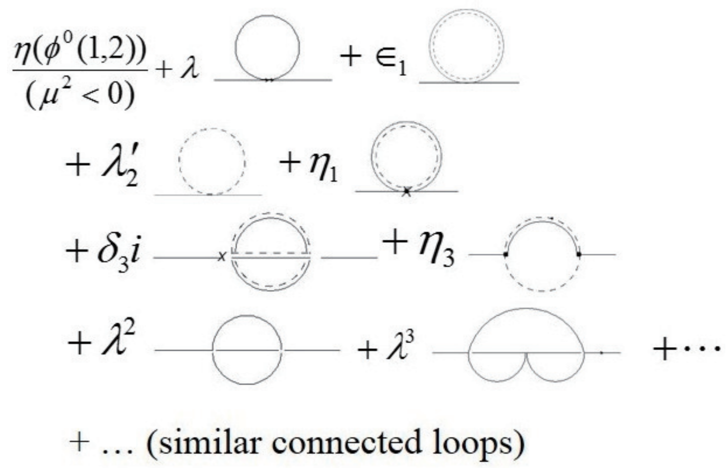

Fig. 1. The within-Higgs diagrams for the Standard-Model Higgs $\Phi(1,2)$.

particles [7] was raised.

We may try to initiate a study of this question for the Standard Model in the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ Minkowski space-time. First, the "building blocks of matter" seems to be complete in this joint-group space. Next, all the fermions are "point-like" Dirac particles in this 4-dimensional Minkowski space-time. In fact, the Standard Model may provide a *consist and complete* mathematical framework (system), which we may investigate further in the mathematical sense.

In what follows, we use the dimensional regularization [6] and in the U-gauge. Here the causal requirement is not reinforced. Every diagram in the U-gauge can be given an answer, though the pole at the 4-dimension may be over-simplified. Since the procedure is so far the only procedure that gives the "answers" for ultraviolet divergences, the exercises may eventually prove to make some sense, for organizations and other purposes.

For the diagrams for the wave-function renormalizations for the Standard-Model Higgs, the mixed family Higgs, and purely family-triplet Higgs, respectively, by Fig. 1, Fig. 2, and Fig. 3, plus those with the (Dirac) fermion loop diagrams and those with the loops involving gauge bosons, we quote the results on quadratic divergences [2]. The couplings in these figures and those in some equations of this paper were given in, e.g., [1].

In short, in Fig. 1, we show the wave-function renormalization of the Standard-Model Higgs $\Phi(1,2)$, among the Higgs, in the U-gauge. The lowest-order loop diagrams, from the above interaction lagrangian, are shown from 1(b) [in λ] to 1(g) [in η_3], where the first five are of quadratic divergence while the last one of logarithmic divergence. The higher-order connected loop diagrams, many of them and of quadratic divergence multiplied by logarithmic divergences, are also troublesome.

The one-loop diagrams involving the quark (or charged lepton), when simplified, are sums of quadratic and logarithmic divergences.

Using dimensional regularization (i.e. the appendix of Ch. 10, the Wu-Hwang book, Ref. [6]), we obtain the one-loop and quadratic-divergence results as follows. In the dimensional regularization, the factor $\Gamma(1-\frac{n}{2})$ stands for where the quadratic divergence appears. Maybe the fractional dimensions, which are represented as finite numbers, could get some meaning, but we have to remember that, as a drawback, we bypass the $-i\epsilon$ in the propagators.

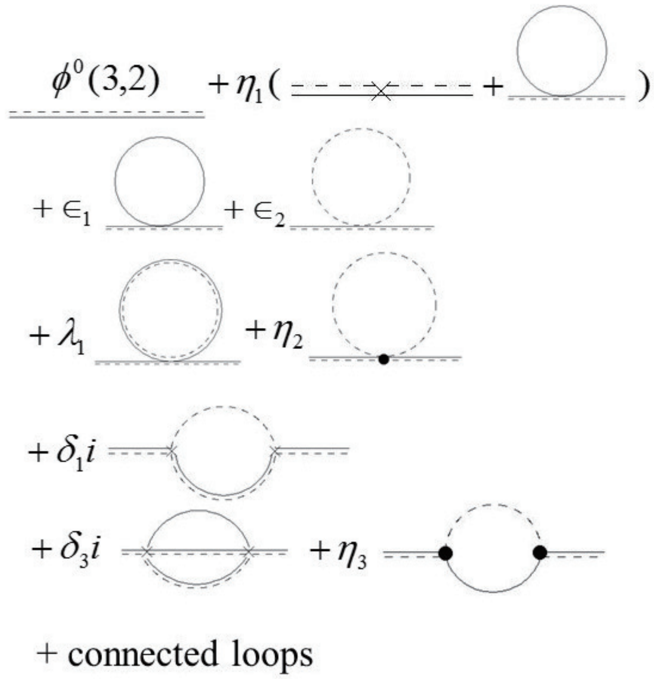

Fig. 2. The diagrams for the mixed family Higgs $\Phi(3,2)$.

In follows, we concentrate only on those ultraviolet divergences of quadratic order:

$$\begin{gathered}
-4\cdot\frac{n}{2}\cdot(S_q+S_{c.l.})\Gamma(1-\frac{n}{2})\\
+\{3\lambda m^2(\eta)+\frac{\epsilon_1}{2}\sum_i m^2(\eta_i')+\epsilon_1\sum_i m^2(\phi_i)\\
+\frac{\lambda_2'}{2}m^2(\eta')+\frac{\eta_1}{2}\sum_i m^2(\eta_i')+others\}\Gamma(1-\frac{n}{2})\\
\equiv 0;\\
S_q=\sum_{quarks}3\cdot G_i^2\cdot(m_i^2-\frac{1}{6}m^2(\eta)),\\
S_{c.l.}=\sum_{c.l.}G_i^2\cdot(m_i^2-\frac{1}{6}m^2(\eta)).
\end{gathered}\tag{14}$$

Or, we have

$$\begin{gathered}
-4\cdot\frac{n}{2}\cdot(S_q+S_{c.l.})\Gamma(1-\frac{n}{2})\\
+\{\lambda(3m^2(\eta)-cos\theta_P\sum_i m^2(\eta_i')+2cos\theta_P\sum_i m^2(\phi_i))+\frac{\lambda_2'}{2}m^2(\eta')+others\}\Gamma(1-\frac{n}{2})\\
\equiv 0.
\end{gathered}\tag{15}$$

Here we "reinforce" "$\equiv 0$" for the sake of talking about a "complete" theory. The "others" are from the loops of the various gauge bosons - fairly messy as far as this equation is concerned.

First of all, we focus our attention only on the quadratic divergences, since these are "the highest divergences" in the first loops - relatively easy to "collect" and the most important divergences altogether; if we cannot do anything about them, then the game is over.

The last equations tell us that it is a game for masses and mass-related couplings. The Higgs mass also enter the terms for the quarks and for the charged leptons. In fact, there is one aver-

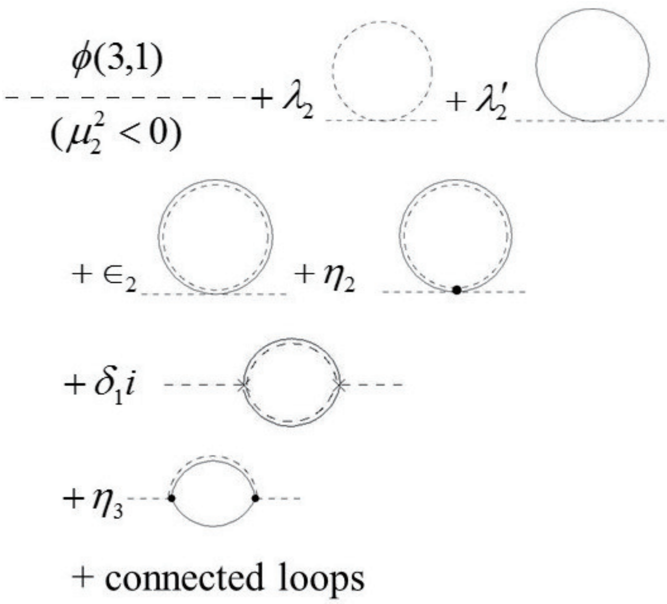

Fig. 3. The diagrams for the purely family Higgs $\Phi(3,1)$.

age mass for the charged leptons and the curl-dot contributions enter as higher order loops (differing from quadratic divergences).

Here the coefficients of $\Gamma(1-\frac{n}{2})$ are the coefficients of quadratic divergences (originally) while those of $\Gamma(2-\frac{n}{2})$ are the coefficients of logarithmic divergences - for the latter, divergence is less severe and the contributions could be everywhere.

From the R_ξ gauge (the Appendix of Ch. 13 [6]), the limit of $\xi \to 0$, which gives rise to the U-gauge, also means the increase in power k of two. The sudden switch-on of $\Gamma(1-\frac{n}{2})$, at $\xi \to 0$ means that the categorization over ultraviolet divergences should be dealt with in a careful way.

On Figs. 2, we have for η'_1, again for quadratic divergences,

$$\begin{aligned}
&-4\cdot\tfrac{n}{2}\cdot T_{lepton}\Gamma(1-\tfrac{n}{2})\\
&+\{3\lambda_1 m^2(\eta'_1)+\tfrac{\epsilon_1+\eta_1}{2}m^2(\eta)\\
&+\tfrac{\epsilon_2+\eta_2}{2}m^2(\eta')+others\}\Gamma(1-\tfrac{n}{2})\equiv 0;\\
&T_{lepton}=\sum H_i^2\cdot(m_i^2-\tfrac{1}{6}m^2(\eta'_1)).
\end{aligned} \quad (16)$$

Here T_{lepton} is defined in accordance with the curl-dot product in neutrinos or in charged leptons. Since only leptons are involved under the another (123) theory, Fig. 2 is slightly simpler than Fig. 2; but both figures are fairly complicated when we try to take into account the "others", i.e., those from the various gauge bosons.

Let's try to do Fig. 3 in some details. The lagrangian for η' is given by

$$\begin{aligned}
&-4\lambda sin\theta[\tfrac{1}{4}(w+\eta')^2(u_1+\eta'_1)^2+\tfrac{1}{2}(w+\eta')^2\phi_1^\dagger\phi_1]\\
&+\lambda\{\tfrac{1}{4}(w+\eta')^4+2sin\theta[\tfrac{1}{4}(w+\eta')^2(u_i+\eta'_i)(u_i+\eta'_i)+\tfrac{1}{2}(w+\eta')^2\phi_i^\dagger\phi_i]\\
&+sin^2\theta(\tfrac{1}{2}(u_i+\eta'_i)(u_i+\eta'_i)\phi_i^\dagger\phi_i)\}\\
&+\mu_2^2\tfrac{1}{2}(w+\eta')^2.
\end{aligned} \quad (17)$$

The figures 3(c) and (d) now split into four loops - in η_1', in $\eta_{2,3}'$, in ϕ_1, and in $\phi_{2,3}$; those in η_1 and in ϕ_1 are responsible for the negative signs. Or, we have, for the quadratic divergences,

$$\{3m^2(\eta') + \frac{1}{2}sin\theta[(\sum_i m^2(\eta_i') - 2m^2(\eta_1')) + 2(\sum_i m^2(\phi_i) - 2m^2(\phi_1))] + others\}\Gamma(1-\frac{n}{2}) \equiv 0, \quad (18)$$

where the common λ's are deleted. Again, the "others" indicates that there are contributions from the family gauge bosons, etc.[1]

We note that the result for Fig. 3, which does not involve the quarks nor the leptons, might be implemented relatively easily. The result for Fig. 2, which involves the leptons, could be implemented in the next step. Finally, we could consider the constraint from Fig. 1. Unfortunately, the terms in "others" (i.e., in the various gauge boson) are fairly complicated and the transmutations on the poles occur in the U-gauge are involved. Altogether, it should be workable if the theory is complete; if it is not complete, the identities should be broken somewhere.

We are sorry that we have been buried ourselves so deep in the complexities of the loop diagrams. The dimensional regularization, if it could be adopted, coupling with the U-gauge (with, only with, all "physical" present) may give some useful results - provided that the transmutation mentioned above could be dealt with.

If we deal with a complete theory, such as the Standard Model of ours, we could require that all these quadratic-divergent parts cancel out completely. In the above, the coefficients of $\Gamma(1-\frac{n}{2})$ are summed up to zero - one-loop order and in fact eventually to all orders to be more precise. In our complete theory, we have quarks, leptons, Higgs, family Higgs, and the various kinds of gauge bosons, subject to $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ gauge symmetry. If we would be counting the diagrams, such as Fig. 1, there would be too many. Thus, eventually, we hope to "prove" the cancelation theorems via symmetry: but this has to be the next thing after some of the couplings get fixed or relations get worked out.

7. The General Remarks

So, this world is very special. It is based on the 4-dimensional Minkowski space-time with the gauge group $SU_c(3)\times SU_L(2)\times U(1)\times SU_f(3)$ built in from the outset. We realize that the complex scalar field would be self-repulsive if alone and that two "related" complex scalar fields could interact attractively (and so exist) and they become the very-much-wanted longitudinal components of the gauge fields. The quark world would be accepted because of the (123) symmetry. Further, the lepton world could be accepted in view of another (123) symmetry.

What would go wrong if there would be no $SU_f(3)$? The lepton sector would suffer from lack of asymptotical freedom if we could forget Landau ghosts at the kinematics much further away. Basically, the theory would blow up in the case of no asymptotical freedom. In other words, we should try to keep the $SU_f(3)$ by all means.

To keep $SU_f(3)$ for the lepton world amounts to a suggestion of another very short range of the family forces (e.g., slightly less than 10 GeV or around 0.04 Fermi for our example). When the separation of two leptons is much than this very short range, it is dominated by the family forces and the system becomes almost free (according to "asymptotic freedom") - so, it is easy to accept the lepton world by our world.

The (123) symmetry, or another (123) symmetry, yields predictions so precise, in compar-

[1] *In Eqs. (15), (16) and (18), the cancellations should occur for the causality ones (with - iϵ ones). We don't know how to fix up this requirement with the dimensional regularization.*

ison with the experimental data, such that it seems to be useless to ask for another unification of forces. There are reasons to reject this - we do not understand why this world contains more baryons than anti-baryons, we do not know the origin of CP violations (via complex numbers in the "real" world), etc.

There are a couple of less urgent questions: First, there are mixing mass terms such as $\eta\eta_i$, of which the exact meaning needs to be further clarified. Remember that there are linear terms in $\eta_{1,2,3}$ while there are SSB's in η' and η. Second, the neutrino mass term is purely a off-diagonal 3×3 matrix but the perturbation theory via Feynman rules is diagonal in this matrix space. Some issues might be diagonalized away; let's wait and see. Barring from our ignorance on these questions, the following remarks may be modified slightly.

On the experimental side, one way to verify the Standard Model is the experimental search for the family Higgs η'_1, or $\eta'_{2,3}$, or charged family Higgs ϕ_1^+ and $\phi_{2,3}^+$, or pure family Higgs η', in a $200\,GeV$ e^-e^+ collider, since these family particles can only be accessed in the lepton channels. Maybe it was a little early to shut down the LEP-II operations at CERN. Note in the notations that, to emphasize the role of "family", we deliberately put the τ channel as the 1st channel.

The active search for the cross-generation Higgs η'_1 is through "the family collider [8]", of a $\mu^\mp e^\pm$ collider, since two generations of leptons must be simultaneously involved in the search of family Higgs η'_1. The technology may be not quite ready in developing the "unstable" $\mu^\pm$ beams; thus, the option of the e^-e^+ collider in this study should be there. Here the indirect search for the "charged" Higgs $\phi_{1,2}$ (from $\Phi(3,2)$) may be seriously considered.

On the theoretical side, we note that the implication of the family gauge theory is in fact a multi-GeV or sub-sub-fermi gauge theory - the leptons are shielded from this $SU_f(3)$ theory against the QED Landau's ghost. An active search of this force clearly should be encouraged. On the other hand, the $g-2$ anomaly certainly deserves another serious look in this context.

Now, in the Standard Model, the masses of quarks are diagonal, or the singlets in the $SU_f(3)$ space, those of the three charged leptons are $m_0 + a\lambda_2 + b\lambda_5 + c\lambda_7$ (before diagonalization) and the masses of neutrinos are purely off-diagonal, i.e. $a'\lambda_2 + b'\lambda_5 + c'\lambda_7$. This result is very interesting and very intriguing. How to develop a formalism with the off-diagonal masses should be the important task of all the theoretical physicists, especially if the diagonalization might be insufficient to deal with the problem.

This result follows from the above curl-dot product, or, the $\epsilon^{abc}\bar{\Psi}_{L,a}\Psi_{R.b}\Phi_c$ product, i.e. the $SU_f(3)$ operation, in writing the coupling(s) to the right-handed lepton triplets. In fact, we have $a'/a* = b'/b* = c'/c*$ for the coupling strengths. QCD is also $SU(3)$ and baryons are constructed of three triplets of quarks - our studies of $SU(3)$ could go deeper yet.

In addition, neutrinos oscillate among themselves, giving rise to a lepton-flavor-violating interaction (LFV) [10]. There are other oscillation stories, such as the oscillation in the $K^0 - \bar{K}^0$ system, but there is a fundamental "intrinsic" difference here - the $K^0 - \bar{K}^0$ system is composite while neutrinos are "point-like" Dirac particles. We have standard Feynman diagrams for the kaon oscillations but similar diagrams do not exist for point-like neutrino oscillations - our proposal solves the problem in a unique way.

Thinking it through, it is true that neutrino masses and neutrino oscillations may be regarded as one of the most important experimental facts over the last thirty years [9]. Treating neutrinos as "point-like" Dirac particles, the problem of neutrinos oscillations between different generations indeed presents us something fundamental and deep.

On the other hand, certain LFV processes such as $\mu \rightarrow e + \gamma$ [9], $\mu + A \rightarrow A^* + e$, $e^+ + e^- \rightarrow \mu^+ + e^-$, etc., are closely related to the most cited picture of neutrino oscillations [9]. In early publications [10], it was pointed out that the cross-generation or off-diagonal neutrino-Higgs interaction may serve as the detailed mechanism of neutrino oscillations, with some vacuum expectation value of the family Higgs, $\Phi(3,1)$ and $\Phi^0(3,2)$. So, even though we haven't seen, directly, the family gauge bosons and family Higgs particles, we already see the manifestations of their vacuum expectation values.

8. The Philosophical End

Basically, we assume that our Space is the 4-dimensional Minkowski space-time with the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ gauge-group structure built in from the outset; in this Space, the various Higgs "exist" such that all gauge bosons, except the photon, are either confined or massive - this provides the background to support the quark world as well as to support the lepton world. This is the origin of "point-like particles", or of "fields".

There are three philosophical questions in our minds: First, why do we have so special the complex scalar fields in the 4-dimensional Minkowski space-time? Can we prove $\lambda = \frac{1}{8}$? Second, why do we need the ignition channel, which turns out to be in the purely family channel ($\mu_2^2 < 0$)? Third, the dimensional regularization, even though it can even deal with the U-gauge, may not give whole story regarding the leading ultraviolet divergences. In the beginning of this 21th century, we might have this ghost story to resurface again. But this should be, as our knowledge accumulates in the process.

The story of the three Higgs is in fact rather friendly. Three complex scalar fields, so similar to one another, write the whole story together. They come in to stabilize the gauge fields to make them the various force fields - the so-called "background" of our world.

The complex scalar fields, unless "related" in our world, do not exist because of the self-repulsive-ness. The quark world is accepted because of the (123) symmetry. The lepton world is also accepted in view of another (123) symmetry. So far, we only know of two acceptable worlds that are built up from point-like Dirac particles. We "believe" that the (123) symmetry, or something similar, is required as the ticket of entrance into this world.

Thus, in our Space (our world) every complex scalar field except the three different Higgs $\Phi(1,2)$, $\Phi(3,2)$, and $\Phi(3,1)$ is "inert", i.e., it does not interact with particles in the observable list. The three Higgs are those defining the "minimum Higgs hypothesis" [11], subject to renormalizability.

As another hypothesis in [11], it was proposed earlier (five years ago) that we could work with another working rule - "Dirac similarity principle", based on eighty years of experience. It is quite strange that all quarks and all leptons are "point-like" Dirac particles, although this is true only to the extent that we know. There is some magic for these "point-like" Dirac particles. Our Space with the background made of the various Higgs and gauge-fields could accept the quark world of (123) and also accept the lepton world of another (123). The (123) means that it is well behaved under $SU(3) \times SU_L(2) \times U(1)$ for $Q \to \infty$.

It may be easy to understand "Dirac similarity principle" from ABC of group theory, but it is easy to forget for the high-braw theoretical physicists. The members of the same multiplet must be of the same category of objects - so, everything of spin $\frac{1}{2}$ can be connected to the electron, thus that it is described by Dirac equation, not something else. The observable point-like Dirac particles form a tight-kitted set of group.

Now, we understand [1] that our Space (our world) is the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ Minkowski space-time, that can only accommodate the scalar fields ϕ with a natural-born $\lambda(\phi^\dagger\phi)^2$ "repulsive" self-interaction only in the exceptional cases (when they could become the longitudinal components of the gauge field). This is in accord with "minimum Higgs hypothesis".

We may add that, under two working hypotheses or our Standard Model (in the 4-dimensional Minkowski space-time with the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ gauge-group structure built in from the outset), we should be able to close the Universe; that is, all the dark-matter particles and all the ordinary-matter particles are accounted for. Our Standard Model provides a description of the entire matter world - i.e., the 25% dark-matter world and the 5% ordinary-matter world.

References

[1] W-Y. Pauchy Hwang, arXiv:1409.6077v1 [hep-ph], 22 Sep 2014.

[2] W-Y. Pauchy Hwang, The Universe, Vol. **2**, No. 2, 47 (2014).

[3] W-Y. Pauchy Hwang and Tung-Mow Yan, The Universe, Vol. **1**, No. 1, 5 (2013); arXiv:1212.4944v1 [hep-ph], 20 Dec 2012.

[4] W-Y. Pauchy Hwang, arXiv:1304.4705 [hep-ph], 17 April 2013.

[5] W-Y. Pauchy Hwang, Nucl. Phys. **A844**, 40c (2010); W-Y. Pauchy Hwang, International J. Mod. Phys. **A24**, 3366 (2009); the idea first appeared in hep-ph, arXiv: 0808.2091; talk presented at 2008 CosPA Symposium (Pohang, Korea, October 2008), Intern. J. Mod. Phys. Conf. Series **1**, 5 (2011); plenary talk at the 3rd International Meeting on Frontiers of Physics, 12-16 January 2009, Kuala Lumpur, Malaysia, published in American Institute of Physics 978-0-7354-0687-2/09, pp. 25-30 (2009).

[6] Ta-You Wu and W-Y. Pauchy Hwang, "Relatistic Quantum Mechanics and Quantum Fields" (World Scientific 1991); Francis Halzen and Alan D. Martin, "Quarks and Leptons" (John Wiley and Sons, Inc. 1984); E.D. Commins and P.H. Bucksbaum, "Weak Interactions of Leptons and Quarks" (Cambridge University Press 1983). We use the first book for the notations and the metric(s).

[7] W-Y. Pauchy Hwang, The Universe, Vol. **2**, No. 1, 41 (2014).

[8] W-Y. Pauchy Hwang, arXiv:1409.6296v1 [hep-ph], 22 Sep 2014.

[9] Particle Data Group, J. Phys. G: Nucl. Part. Phys. **37**, 1 (2010); and its biennual publications.

[10] W-Y. Pauchy Hwang, Hyperfine Interactions **215**, 105 (2013).

[11] W-Y. P. Hwang, arXiv:11070156v1 [hep-ph], 1 Jul 2011), Plenary talk given at the 10th International Conference on Low Energy Antiproton Physics (Vancouver, Canada, April 27 - May 1, 2011). "Dirac similarity principle" and "minimum Higgs hypothesis" first appeared in public in the talk as well as in writing.