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Autoclustering in baryon spectra

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Abstract. A nearest-neighbor analysis of baryon mass spectra reveals a striking autoclustering of resonances to swarms of increasing sizes. Each cluster contains K binomials of opposite parities whose spins range from $1/2$ to $K-1/2$ and a mono-parity state of the highest spin $K+1/2$ in the swarm. The clusters with $K=1$, 3, and 5 are observed in both the nucleon and the Δ excitations (up to the two nucleon states F_{17}, $H_{1,11}$ with respective masses around 1700 MeV and 2200 MeV, and the three Δ states P_{31}, P_{33}, and D_{33} with masses around 2500 MeV). Clusters with K even and non-zero are unoccupied so far. We trace back above regularity pattern to internal nucleon and Δ structures dominated by a quark–di-quark configuration and its respective rotational-vibrational excitations. Clusters of the above type are appealing because upon boosting they transform (up to form factors) as a Lorentz tensor of rank- K with Dirac components, i.e. as $\psi_{\mu_1\ldots\mu_K}$, and thus allow for a covariant description of resonances in flight.

1 Order in excited light-quark baryons

The structure of the nucleon spectrum is far from being settled despite its long history. This situation relates to the fact that the first facility that measured nucleon levels, the Los Alamos Meson Physics Facility (LAMPF) failed to find all the states that were possible as excitations of three quarks. Later on, the Thomas Jefferson National Accelerator Facility (TJNAF) was designed to search (among others) for those "missing resonances". At present, all data have been collected and are awaiting evaluation [1].

In a series of papers [2] I performed a near st neighbour analysis of data on mass distribution of nucleon resonances reported in Ref. [3] and drew attention to the not overlookable (by the unbiased eye) increase of state densities in a few narrow mass bands and its exact replica in the $\Delta(1232)$ spectrum (see Fig. 1).

The first group of nearly degenerate resonances consists of two equal spin-$\frac{1}{2}$ of opposite parities (one parity binomial) and a mono-parity spin– $\frac{3}{2}^-$ state. The second group starts with three parity binomials with spins ranging from $\frac{1}{2}^\pm$ to $\frac{5}{2}^\pm$, and terminates with a mono-parity spin-$\frac{7}{2}^+$ resonance. Finally, the third group begins with five parity binomials with spins ranging from $\frac{1}{2}^\pm$ to $\frac{9}{2}^\pm$, and terminates by a mono-parity spin $\frac{11}{2}^+$ resonance (see Ref. [6] for the complete N and $\Delta(1232)$ spectra). A comparison between the N and $\Delta(1232)$ spectra shows that they are identical up to two unoccupied resonances on the nucleon side (these are the counterparts of the F_{37} and $H_{3,11}$ states of the Δ excitations) and up to three unoccupied states on the Δ side (these are the counterparts of the nucleon P_{11}, P_{13}, and D_{13} states from the third group). The Δ(1600) resonance which is most probably and independent hybrid state, is the only state that at present seems to drop out of our systematics.

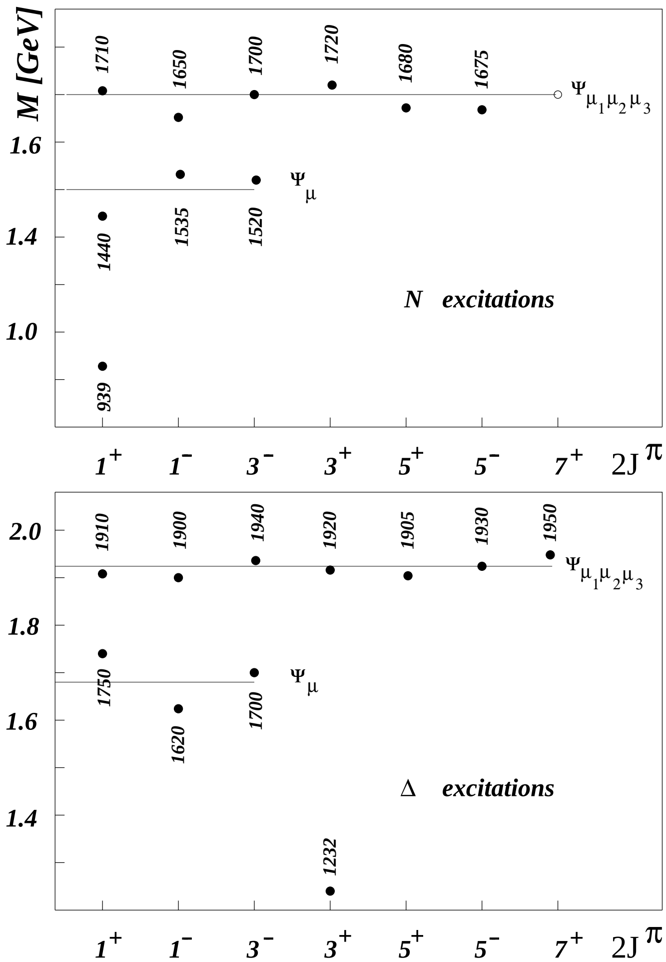

Fig. 1. Summary of the data on the nucleon and the Δ resonances. The breaking of the mass degeneracy for each of the clusters at about 5% may in fact be an artifact of the data analysis, as has been suggested by Höhler [4]. The filled circles represent known resonances, while the sole empty circle corresponds to a prediction. Figure taken from [5].

The existence of identical nucleon- and Δ crops of resonances raises the question as to what extent are we facing here a new type of symmetry which was not anticipated by any model or theory before. The next section devotes itself to answering this question.

2 Spectroscopy of autoclustering

2.1 Relevance of the quark–di-quark configuration

To the extent QCD prescribes baryons to be constituted of three quarks in a color singlet state, one may feel encouraged to exploit for the description of baryonic systems algebraic models developed for the purposes of triatomic molecules, a path pursued by Refs. [7].

In the dynamical limit $U(7) \longrightarrow U(3)\times U(4)$ of the three quark system, two of the quarks act as an independent entity, a di-quark (Dq), while the third quark (q) acts as a spectator. The di-quark approximation [8] turned out to be rather convenient in particular in describing various properties of the ground state baryons [9], [10].

The necessity for having a quark–di-quark configuration within the nucleon is independently supported by arguments related to spin in QCD. In Refs. [11], and [12] the notion of spin in QCD was re-visited in connection with the proton spin puzzle. As it is well known, the spins of the valence quarks are by themselves not sufficient to explain the spin-$\frac{1}{2}$ of the nucleon. Rather, one needs to account for the orbital angular momentum of the quarks (here denoted by L_{QCD}) and the angular momentum carried by the gluons (so called field angular momentum, G_{QCD}):

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_{QCD} + G_{QCD}$$
$$= \int d^3x\Big[\frac{1}{2}\bar{\psi}\boldsymbol{\gamma}\gamma_5\psi \;+\; \psi^{\dagger}(\boldsymbol{x}\times(-\mathbf{D}))\psi + \boldsymbol{x}\times(\mathbf{E}^a \times \mathbf{B}^a)\Big]\,.$$

In so doing one encounters the problem that neither L_{QCD}, nor G_{QCD} satisfy the spin $su(2)$ algebra. If at least $(L_{QCD} + G_{QCD})$ is to do so,

$$\Big[\left(L^i_{QCD} + G^i_{QCD}\right), \left(L^j_{QCD} + G^j_{QCD}\right)\Big] = i\epsilon^{ijk}\left(L^k_{QCD} + G^k_{QCD}\right)\,, \tag{1}$$

then $\mathbf{E}^{i;a}$ has to be *restricted* to a chromo-electric charge, while $\mathbf{B}^{i;a}$ has to be a chromo-magnetic dipole according to,

$$E^{i;a} = \frac{gx'^{\,i}}{r'^{\,3}}T^a\,,\quad B^{ia} = \Big(\frac{3x^ix^lm^l}{r^5} - \frac{m^i}{r^3}\Big)T^a\,, \tag{2}$$

where $x'^{\,i} = x^i - R^i$. The above color fields are the perturbative one-gluon approximation typical for a di-quark-quark structure. The di-quark and the quark are in turn the sources of the color Coulomb field, and the color magnetic dipole field. In terms of color and flavor degrees of freedom, the nucleon wave function indeed has the required quark–di-quark form $|p_\uparrow\rangle = \frac{\epsilon_{ijk}}{\sqrt{18}}\,[u^+_{i\downarrow}d^+_{j\uparrow} - u^+_{i\uparrow}d^+_{j\downarrow}]\,u^+_{k\uparrow}\,|0\rangle$.
A similar situation appears when looking for covariant QCD solutions in form of a membrane with the three open ends being associated with the valence quarks. When such a membrane stretches to a string, so that a linear action (so called gonihedric string) can be used, one again encounters that very K-cluster degeneracies in the excitations spectra of the baryons, this time as a part of an infinite

tower of states. The result was reported by Savvidy in Ref. [13]. Thus the covariant spin-description provides an independent argument in favor of a dominant quark-di-quark configuration in the structure of the nucleon, while search for covariant resonant QCD solutions leads once again to infinite K-cluster towers.
Within the context of the quark–di-quark (q-Dq) model, the ideas of the rovibron model, known from the spectroscopy of diatomic molecules [14] acquires importance as a tool for the description of the rotational-vibrational (rovibron) excitations of the q–Dq system.

2.2 The quark rovibron

In the rovibron model (RVM) the relative q–Dq motion is described by means of four types of boson creation operators s^+, p_1^+, p_0^+, and p_{-1}^+. The operators s^+ and p_m^+ in turn transform as rank-0, and rank-1 spherical tensors, i.e. the magnetic quantum number m takes in turn the values $m = 1, 0$, and -1. In order to construct boson-annihilation operators that also transform as spherical tensors, one introduces the four operators $\widetilde{s} = s$, and $\widetilde{p}_m = (-1)^m \, p_{-m}$. Constructing rank-$k$ tensor product of any rank-k_1 and rank-k_2 tensors, say, $A^{k_1}_{m_1}$ and $A^{k_2}_{m_2}$, is standard and given by

$$[A^{k_1} \otimes A^{k_2}]^k_m = \sum_{m_1, m_2} (k_1 m_1 k_2 m_2 | k m) \, A^{k_1}_{m_1} A^{k_2}_{m_2} \, . \tag{3}$$

Here, $(k_1 m_1 k_2 m_2|km)$ are the standard $O(3)$ Clebsch-Gordan coefficients.

Now, the lowest states of the two-body system are identified with N boson states and are characterized by the ket-vectors $|n_s \, n_p \, l \, m\rangle$ (or, a linear combination of them) within a properly defined Fock space. The constant $N = n_s + n_p$ stands for the total number of s- and p bosons and plays the róle of a parameter of the theory. In molecular physics, the parameter N is usually associated with the number of molecular bound states. The group symmetry of the rovibron model is well known to be $U(4)$. The fifteen generators of the associated $su(4)$ algebra are determined as the following set of bilinears

$$\begin{aligned} A_{00} &= s^+\widetilde{s} \, , \qquad A_{0m} = s^+\widetilde{p}_m \, , \\ A_{m0} &= p^+_m\widetilde{s} \, , \qquad A_{mm'} = p^+_m\widetilde{p}_{m'} \, . \end{aligned} \tag{4}$$

The $u(4)$ algebra is then recovered by the following commutation relations

$$[A_{\alpha\beta}, A_{\gamma\delta}]_- = \delta_{\beta\gamma} A_{\alpha\delta} - \delta_{\alpha\delta} A_{\gamma\beta} \, . \tag{5}$$

The operators associated with physical observables can then be expressed as combinations of the $u(4)$ generators. To be specific, the three-dimensional angular momentum takes the form

$$L_m = \sqrt{2} \, [p^+ \otimes \widetilde{p}]^1_m \, . \tag{6}$$

Further operators are (D_m)– and (D'_m) defined as

$$D_m = [p^+ \otimes \widetilde{s} + s^+ \otimes \widetilde{p}]^1_m \, , \tag{7}$$

$$D'_m = i[p^+ \otimes \widetilde{s} - s^+ \otimes \widetilde{p}]^1_m \, , \tag{8}$$

respectively. Here, $\mathbf{D}$ plays the róle of the electric dipole operator.

Finally, a quadrupole operator Q_m can be constructed as

$$Q_m = [p^+ \otimes \widetilde{p}]^2_m\,, \quad \text{with} \quad m = -2, ..., +2\,. \tag{9}$$

The $u(4)$ algebra has the two algebras $su(3)$, and $so(4)$, as respective sub-algebras. The $so(4)$ sub-algebra of interest here, is constituted by the three components of the angular momentum operator L_m, on the one side, and the three components of the operator D'_m, on the other side. The chain of reducing $U(4)$ down to $O(3)$

$$U(4) \supset O(4) \supset O(3)\,, \tag{10}$$

corresponds to an exactly soluble RVM limit. The Hamiltonian of the RVM in this case is constructed as a properly chosen function of the Casimir operators of the algebras of the subgroups entering the chain. For example, in case one approaches $O(3)$ via $O(4)$, the Hamiltonian of a dynamical $SO(4)$ symmetry can be cast into the form [15]:

$$H_{RVM} = H_0 - f_1\,\left(4\mathcal{C}_2\,(so(4)) + 1\right)^{-1} + f_2\,\mathcal{C}_2(so(4))\,. \tag{11}$$

The Casimir operator $\mathcal{C}_2\,(so(4))$ is defined accordingly as

$$\mathcal{C}_2\,(so(4)) = \frac{1}{4}\left(\mathbf{L}^{\,2} + \mathbf{D}'^{\,2}\right) \tag{12}$$

and has an eigenvalue of $\frac{K}{2}\left(\frac{K}{2}+1\right)$. Here, the parameter set has been chosen as

$$H_0 = M_{N/\Delta} + f_1\,, \quad f_1 = 600\text{ MeV}\,, \quad f_2^N = 70\text{ MeV}\,, \quad f_2^\Delta = 40\text{ MeV}\,. \tag{13}$$

Thus, the $SO(4)$ dynamical symmetry limit of the RVM picture of baryon structure motivates existence of quasi-degenerate resonances gathering to crops in both the nucleon- and Δ baryon spectra. The Hamiltonian that will fit masses of the reported cluster states is exactly the one in Eq. (11).

In order to demonstrate how the RVM applies to baryon spectroscopy, let us consider the case of q-Dq states associated with $N = 5$ and for the case of a $SO(4)$ dynamical symmetry. It is of common knowledge that the totally symmetric irreps of the $u(4)$ algebra with the Young scheme $[N]$ contain the $SO(4)$ irreps $\left(\frac{K}{2}, \frac{K}{2}\right)$ (here K plays the role of the four-dimensional angular momentum) with

$$K = N, N-2, ..., 1 \quad \text{or} \quad 0\,. \tag{14}$$

Each one of the K- irreps contains $SO(3)$ multiplets with three dimensional angular momentum

$$l = K, K-1, K-2, ..., 1, 0\,. \tag{15}$$

In applying the branching rules in Eqs. (14), (15) to the case $N = 5$, one encounters the series of levels

$$\begin{aligned} K &= 1: \quad l = 0, 1;\\ K &= 3: \quad l = 0, 1, 2, 3;\\ K &= 5: \quad l = 0, 1, 2, 3, 4, 5\,. \end{aligned} \tag{16}$$

The parity carried by these levels is $\eta(-1)^l$ where η is the parity of the relevant vacuum. In coupling now the angular momentum in Eq. (16) to the spin-$\frac{1}{2}$ of the three quarks in the nucleon, the following sequence of states is obtained:

$$\begin{aligned}
K = 1: &\quad \eta J^\pi = \frac{1}{2}^+, \frac{1}{2}^-, \frac{3}{2}^-;\\
K = 3: &\quad \eta J^\pi = \frac{1}{2}^+, \frac{1}{2}^-, \frac{3}{2}^-, \frac{3}{2}^+, \frac{5}{2}^+, \frac{5}{2}^-, \frac{7}{2}^-;\\
K = 5: &\quad \eta J^\pi = \frac{1}{2}^+, \frac{1}{2}^-, \frac{3}{2}^-, \frac{3}{2}^+, \frac{5}{2}^+, \frac{5}{2}^-, \frac{7}{2}^-, \frac{7}{2}^+, \frac{9}{2}^-, \frac{11}{2}^-.
\end{aligned} \tag{17}$$

Therefore, rovibron states of half-integer spin transform according to $\left(\frac{K}{2}, \frac{K}{2}\right) \otimes \left[\left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)\right]$ representations of $SO(4)$. The isospin structure is accounted for pragmatically through attaching to the K–clusters an isospin spinor χ^I with I taking the values $I = \frac{1}{2}$ and $I = \frac{3}{2}$ for the nucleon, and the Δ states, respectively. As illustrated by Fig. 1, the above quantum numbers cover both the nucleon and the Δ excitations.

The states in Eq. (17) are degenerate and the dynamical symmetry is $O(4)$. The above considerations apply to the rest frame. In order to describe clusters in flight one needs to subject the O(4) degenerate resonance states to a Lorentz boost.

The most efficient way to achieve this task is not to boost the spin by spin but rather the K multiplet as a whole, which takes one (up to form factors) to the K Lorentz tensors with Dirac spinor components, $\psi_{\mu_1 \ldots \mu_K}$.

2.3 Observed and unoccupied clusters within the rovibron model

The comparison of the states in Eq. (17) with the reported ones in Fig. 1 shows that the predicted sets are in agreement with the characteristics of the non-strange baryon excitations with masses below ~ 2500 MeV, provided, the parity η of the vacuum changes from scalar ($\eta = 1$) for the $K = 1$, to pseudoscalar ($\eta = -1$) for the $K = 3, 5$ clusters. A pseudoscalar "vacuum" can be modeled in terms of an excited composite di-quark carrying an internal angular momentum $L = 1^-$ and maximal spin $S = 1$. In one of the possibilities the total spin of such a system can be $|L - S| = 0^-$. To explain the properties of the ground state, one has to consider separately even N values, such as, say, $N' = 4$. In that case another branch of excitations, with $K = 4$, 2, and 0 will emerge. The $K = 0$ value characterizes the ground state, $K = 2$ corresponds to $(1,1) \otimes [\left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)]$, while $K = 4$ corresponds to $(2,2) \otimes [\left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)]$. These are the multiplets that we will associate with the "missing" resonances predicted by the rovibron model. In this manner, reported and "missing" resonances fall apart and populate distinct $U(4)$- and $SO(4)$ representations. In making observed and "missing" resonances distinguishable, reasons for their absence or, presence in the spectra are easier to be searched for. In accordance with Ref. [16] we here will treat the $N = 4$ states to be all of natural parities and identify them with the nucleon ($K = 0$), the natural parity $K = 2$, and the natural parity $K = 4$–clusters. We shall refer to the latter as 'missing" rovibron clusters. In Table I we list the masses of the K–clusters concluded from Eqs. (11), and (13).

Table 1. Predicted mass distribution of observed (obs), and missing (miss) rovibron clusters (in MeV) according to Eqs. (9,11). The sign of η in Eq. (15) determines natural- (η = +1), or, unnatural (η = −1) parity states. The experimental mass averages of the resonances from a given K–cluster have been labeled by "exp".

K	sign η	N^{obs}	N^{exp}	Δ^{obs}	Δ^{exp}	N^{miss}	Δ^{miss}
0	+	939	939	1232	1232		
1	+	1441	1498	1712	1690		
2	+					1612	1846
3	−	1764	1689	1944	1922		
4	+					1935	2048
5	−	2135	2102	2165	2276		

In Ref. [15] we presented the four dimensional Racah algebra that allows to calculate transition probabilities for electromagnetic de-excitations of the rovibron levels. The interested reader is invited to consult the quoted article for details. Here I restrict myself to reporting the following two results: (i) All resonances from a K- mode have same widths. (ii) As compared to the natural parity $K = 1$ states, the electromagnetic de-excitations of the unnatural parity $K = 3$ and $K = 5$ rovibron states appear strongly suppressed. To illustrate our predictions I compiled in Table 2 below data on experimentally observed total widths of resonances belonging to $K = 3$, and $K = 5$. The suppression of the electromagnetic

Table 2. Reported widths of resonance clusters

K	Resonance	width [in GeV]
3	$N\left(\frac{1}{2}^-;1650\right)$	0.15
3	$N\left(\frac{1}{2}^+;1710\right)$	0.10
3	$N\left(\frac{3}{2}^+;1720\right)$	0.15
3	$N\left(\frac{3}{2}^-;1700\right)$	0.15
3	$N\left(\frac{5}{2}^-;1675\right)$	0.15
3	$N\left(\frac{5}{2};^+ 1680\right)$	0.13
5	$N\left(\frac{3}{2}^+;1900\right)$	0.50
5	$N\left(\frac{5}{2}^+;2000\right)$	0.49

de–excitation modes of unnatural parity states to the nucleon (of natural parity) is shown in Table 3. It is due to the vanishing overlap between the scalar di-quark in the latter case, and the pseudo-scalar one, in the former. Non-vanishing widths can signal small admixtures from natural parity states of same spins belonging to even K number states from the "missing" resonances. For example, the significant

Table 3. Reported helicity amplitudes of resonances.

K	parity of the spin-0 di-quark	Resonance	$A^p_{\frac{1}{2}}$	$A^2_{\frac{3}{2}}$ $[10^{-3}\mathrm{GeV}^{-\frac{1}{2}}]$
3	−	$N\left(\frac{1}{2}^+;1710\right)$	9 ± 22	
3	−	$N\left(\frac{3}{2}^+;1720\right)$	18 ± 30	-19 ± 20
3	−	$N\left(\frac{3}{2}^-;1700\right)$	-18 ± 30	-2 ± 24
3	−	$N\left(\frac{5}{2}^-;1675\right)$	19 ± 8	15 ± 9
3	−	$N\left(\frac{5}{2};^{+}1680\right)$	-15 ± 6	133 ± 12
1	+	$N\left(\frac{3}{2}^-;1520\right)$	-24 ± 9	166 ± 5

value of $A^p_{\frac{3}{2}}$ for $N\left(\frac{5}{2}^+;1680\right)$ from $K = 3$ may appear as an effect of mixing with the $N\left(\frac{5}{2}^+;1612\right)$ state from the natural parity "missing" cluster with $K = 2$. This gives one the idea to use helicity amplitudes to extract "missing" states.

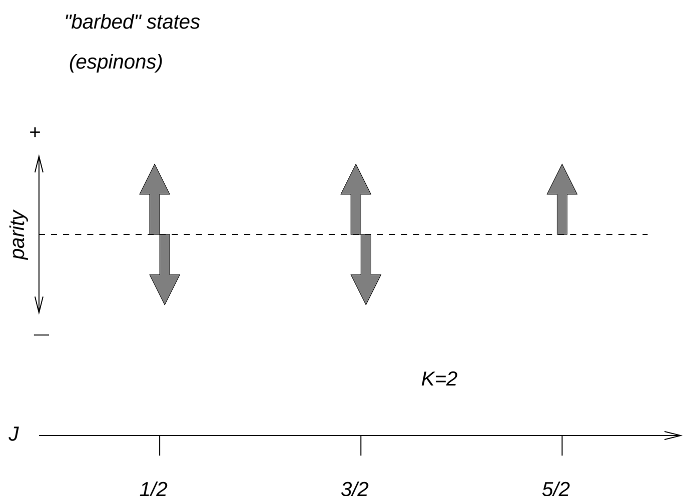

Fig. 2. K-excitation mode of a quark-diquark string: *barbed* states (*espinons*).

The above considerations show that a K-mode of an excited quark-di-quark string (be the diquark scalar, or, pseudoscalar) represents an independent entity (particle?) in its own rights which deserves its own name. To me the different spin facets of the K–cluster pointing into different "parity directions" as displayed in Fig. 2 look like barbs. That's why I suggest to refer to the K-clusters as *barbed* states to emphasize the aspect of alternating parity. Barbs could also be associated with thorns (Spanish, espino), and *espinons* could be another sound name for K-clusters.

3 Conclusions

Beyond pointing onto the phenomenon of an evident autoclustering in the spectra of the light quark baryons, it was argued that the swarms of resonances can be (i) explained as a consequence of rotational-vibrational modes of an excited quark-di-quark configuration, be the di-quark scalar, or, pseudoscalar, when at rest, and (ii) described covariantly in terms of $\psi_{\mu_1\ldots\mu_K}$, when in flight.

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