The CMB bispectrum from secondary anisotropies: the Lensing-Integrated Sachs Wolfe-Rees Sciama contribution

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We investigated the CMB bispectrum signal arising from the cross correlation between lensing and the Integrated Sachs Wolfe/Rees Sciama effects. The ISW and RS effects arise because of time varying gravitational potential due, respectively, to the linear and non-linear growth of structure in the evolving universe. Both the lensing and the ISW-RS effects are related to the matter gravitational potential and thus are correlated phenomena, giving rise to a non-vanishing three points correlation function. The LRS signal is expected to be detected at high statistical significance from ongoing and future CMB experiments and, being a late time effect, it can be a powerful probe of the late time universe. Moreover, we showed that this bispectrum signal, if not accounted properly, can bias the estimation of the amplitude and variance of the local primordial non-Gaussianity. Finally we built CMB simulations with the LRS signal and we implemented and tested the optimal estimator for this specific bispectrum.

1 Introduction

One of the most relevant mechanism that can generate non-Gaussianity from secondary Cosmic Microwave Background (CMB) anisotropies is the coupling between weak lensing and the Integrated Sachs Wolfe (ISW) [1] Rees Sciama (RS) [2] effects. This is in fact the leading contribution to the CMB secondary bispectrum with a blackbody frequency dependence [3,4,5]. Weak lensing of the CMB is caused by gradients in the matter gravitational potential that distorts the CMB photon geodesics. The ISW and RS effects on the other hand arise because of time varying gravitational potential due, respectively, to the linear and non-linear growth of structure in the evolving universe. Both the lensing and the ISW-RS effect are then related to the matter gravitational potential and thus are correlated phenomena. This gives rise to a non-vanishing three points correlation function or, analogously, a non-vanishing bispectrum, its Fourier counterpart. Further, lensing and the RS effect are related to non-linear processes which are therefore highly non-Gaussian. The CMB bispectrum signal arising from the cross correlation between lensing and ISW/RS (from now on referred as LRS) is expected to have an high signal-to-noise from ongoing and future CMB experiments so that it will be detectable in the near future with an high statistical significance [4, 5, 6, 7, 8]. This will open the possibility to exploit the cosmological information related to the late time evolution encoded in the LRS signal. Moreover the LRS bispectrum can be a problem for the estimation of the primary local non-Gaussianity from future data since it can be a serious contaminant [7, 9].

Ongoing CMB experiments like e.g. Planck and future experiments like CORE will then require a detailed reconstruction of the Lensing-ISW RS bispectrum either to be able to separate out correctly the LRS contribution when estimating the local primary non-Gaussian parameter f_{NL} or to exploit the cosmological information encoded in the signal.

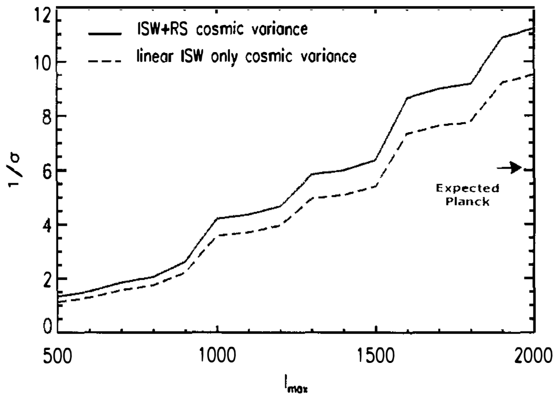

Figure 1: Expected statistical detection significance of the LRS bispectrum in the case of a cosmic variance limited full sky experiment as a function of the maximum multipole ℓ_{max}. The red arrow indicates a more realistic statistical detection significance expected for Planck at $\ell_{max} = 2000$.

2 The Lensing-ISW-RS bispectrum signal

The CMB Lensing-ISW-Rees Sciama bispectrum takes the form[3,4,7]:

$$B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} \equiv \langle a^{m_1}_{\ell_1} a^{m_2}_{\ell_2} a^{m_3}_{\ell_3} \rangle = \langle a^{m_1 P}_{\ell_1} a^{m_2 L}_{\ell_2} a^{m_3 RS}_{\ell_3} \rangle + 5 \text{ Permutations}. \quad (1)$$

This becomes: $B^{m_1 m_2 m_3 (L-RS)}_{\ell_1 \ell_2 \ell_3} = \mathcal{G}^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} b^{L-ISW/RS}_{\ell_1 \ell_2 \ell_3}$, where $\mathcal{G}^{m_1 m_2 m_3}_{\ell_1 \ell_2 l_3}$ is the Gaunt integral and the reduced bispectrum is given by

$$b^{L-ISW/RS}_{\ell_1 \ell_2 \ell_3} = \left[\frac{\ell_1(\ell_1+1) - \ell_2(\ell_2+1) + l_3(\ell_3+1)}{2} C^P_{\ell_1} q_{\ell_3} + (5\ perm.) \right], \quad (2)$$

Here C^P_ℓ is the primary angular CMB temperature power spectrum and q_ℓ are the coefficients which express the statistical expectation of the correlation between the lensing and the ISW-RS effect:

$$q_\ell \equiv \langle \phi^{*m}_{L\ell} a^{RSm}_\ell \rangle \simeq 2 \int_0^{z_{ls}} \frac{r(z_{ls}) - r(z)}{r(z_{ls}) r(z)^3} \left[\frac{\partial}{\partial z} P_\phi(k, z) \right]_{k=\frac{\ell}{r(z)}} dz. \quad (3)$$

Here P_ϕ is the gravitational potential power spectrum and the above equation accounts for both the linear ISW and the non-linear Rees-Sciama effect.

Fig.1 shows the statistical detection significance S/N expected for the LRS bispectrum signal as a function of the maximum multipole ℓ_{max}:

$$\text{S/N} = \sqrt{\sum_{\ell_{min} \le \ell_1 \le \ell_2 \le \ell_3}^{\ell_{max}} \frac{B^{L-RS}_{\ell_1 \ell_2 \ell_3} B^{L-RS}_{\ell_1 \ell_2 \ell_3}}{\Delta_{\ell_1 \ell_2 \ell_3} C_{\ell_1} C_{\ell_2} C_{\ell_3}}}. \quad (4)$$

3 The bias to the primary local Non-Gaussianity

The LRS bias to primary local f_{NL} is given by:

$$\hat{f}_{NL} = \frac{\hat{S}}{N}, \text{ where } \hat{S} = \sum_{2 \le \ell_1 \ell_2 \ell_3} \frac{B^{L-RS}_{\ell_1 \ell_2 \ell_3} B^P_{\ell_1 \ell_2 \ell_3}}{C_{\ell_1} C_{\ell_2} C_{\ell_3}} \text{ and } N = \sum_{2 \le \ell_1 \ell_2 \ell_3} \frac{(B^P_{\ell_1 \ell_2 \ell_3})^2}{C_{\ell_1} C_{\ell_2} C_{\ell_3}}. \quad (5)$$

We find that a bispectrum estimator optimized for constraining primordial non-Gaussianity of the local type would measure an effective $f_{NL} = 10$ for $\ell_{max} = 1000$ due to the presence of the primary-lensing-Rees-Sciama correlation. If not accounted for, this introduces a contamination in the constraints on primordial non-Gaussianity from the CMB bispectrum. For forthcoming data this bias will be larger than the $1 - \sigma$ error and thus non-negligible.

4 Optimal estimator and simulations

Estimator

Here we are interested in the L-RS case, for which the angular bispectrum, parametrized by the amplitude parameter f_{NL}^{LRS}, is:

$$\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle = \mathcal{G}_{\ell_1 \ell_2 l_3}^{m_1 m_2 m_3} f_{NL}^{LRS} b_{\ell_1 \ell_2 \ell_3}^{LRS}, \quad (6)$$

where $\mathcal{G}_{\ell_1 \ell_2 l_3}^{m_1 m_2 m_3}$ is the Gaunt integral, $b_{\ell_1 \ell_2 \ell_3}^{LRS}$ is the LRS reduced bispectrum as defined in equation (2) and $q_\ell \equiv \langle \phi_{\ell m}^{*L} a_{\ell m}^{RS} \rangle$ are the lensing-ISW/RS cross-correlation coefficients (which account for both the linear ISW and the non-linear Rees-Sciama effect) of equation (3).

It is then possible to build an optimal estimator for f_{NL}^{LRS} by maximizing the PDF with respect to this parameter. So, by solving $d \ln P / d f_{\rm NL}^{L-RS} = 0$, this is given by:

$$f_{NL}^{L-RS} = (F^{-1}) S_{LRS}, \quad (7)$$

where (F^{-1}) is the inverse of the L-RS Fisher matrix.

Assuming that the only NG contribution is coming from the L-RS term, S_{L-RS} is given by the data as:

$$S_{L-RS} \equiv \frac{1}{6} \sum_{all\ \ell m} \mathcal{G}_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} b_{\ell_1 \ell_2 \ell_3}^{L-RS} \left[(C^{-1}a)_{\ell_1 m_1} (C^{-1}a)_{\ell_2 m_2} (C^{-1}a)_{\ell_3 m_3} - 3 (C^{-1})_{\ell_1 m_1, \ell_2 m_2} (C^{-1}a)_{\ell_3 m_3} \right], \quad (8)$$

Simulations: the separable mode expansion method

Following[10] and[11], the non-Gaussian part of the CMB angular coefficients can be defined starting from a given CMB power spectrum C_ℓ and reduced bispectrum $b_{\ell_1 \ell_2 \ell_3}$. For the LRS bispectrum these take the form:

$$[a_{\ell m}^{NG}]_{LRS} = \int d^2\hat{\mathbf{n}} \sum_{\ell_2 m_2 \ell_3 m_3} b_{\ell \ell_2 \ell_3}^{LRS} Y_\ell^{m*}(\hat{\mathbf{n}}) \frac{a_{\ell_2 m_2}^G Y_{\ell_2}^{m_2}(\hat{\mathbf{n}})}{C_{\ell_2}} \frac{a_{\ell_3 m_3}^G Y_{\ell_3}^{m_3}(\hat{\mathbf{n}})}{C_{\ell_3}}, \quad (9)$$

where, again, $b_{\ell \ell_2 \ell_3}^{LRS}$ is the reduced bispectrum of eq.(2). The angular coefficients containing the wanted signal will then be: $a_{\ell m} = a_{\ell m}^G + [a_{\ell m}^{NG}]_{LRS}$, where $a_{\ell m}^G$ is the Gaussian part.

Results

I tested the estimator and the LRS coefficients built with the separable modes expansion method with 100 runs at full resolution ($N_{side} = 2048$) and up to $\ell_{max} = 1000$. According to the definition of f_{NL}^{LRS} the expected value is 1 with 1-σ error predicted from theory for $\ell_{max} = 1000$ of $\simeq 0.25$. The simulations give $f_{NL}^{LRS} = 1.11$ with averaged 1-σ error 0.36 as shown in Fig.2.

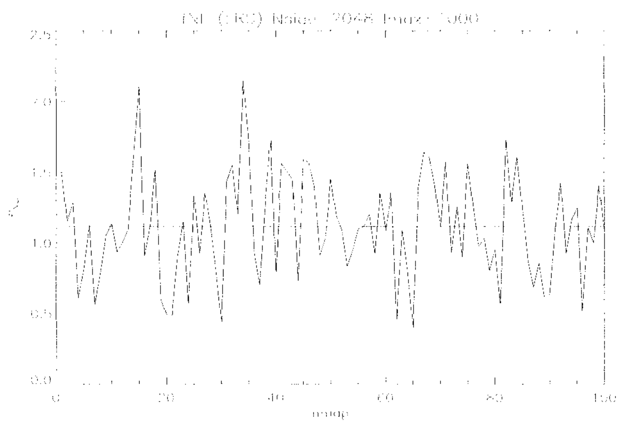

Figure 2: f_{NL}^{LRS} (eq. 7) obtained by testing the LRS estimator (cfr eq. 8) on 100 CMB maps (full sky and cosmic variance limited, N_{side} = 2048) containing the LRS signal simulated by using the separable mode expansion method (cfr eq. 9).

5 Conclusions

In this work we studied the LRS bispectrum signal. We showed that it can be a significant contaminant to the bispectrum signal from primordial non-Gaussianity of the local type. In particular both signals are frequency-independent and are maximized for nearly squeezed configurations, which in fact are the configurations that contribute the most to the S/N. If not included in the modeling, the primary-lensing-Rees-Sciama contribution yields an effective f_{NL} of 10 when using a bispectrum estimator optimized for local non-Gaussianity. Considering that expected 1-σ errors on f_{NL} are < 10 from forthcoming experiments, the contribution from this signal must be included in future constraints on f_{NL} from the Cosmic Microwave Background bispectrum. Within this picture it is extremely important to be able to model the LRS bispectrum either to be able to avoid contaminations either for exploiting it as a cosmological observable in view of future data. I presented the formalism and the numerical implementation for generating CMB non-gaussian maps which contain the LRS signal. I also implemented and tested numerically the Non-gaussian estimator optimized for the LRS bispectrum.

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