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RESEARCH

Recent progress in classical string cosmology

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Abstract
I will summarise recent work on constructing regular bouncing string cosmologies in the framework of Hohm and Zwiebach's classification of all order α' corrections compatible with $O(d, d)$ symmetry.

1 Defining Classical String Cosmology (CSC)

Let me start by recalling that perturbative string theory enjoys two distinct expansions:

- 1. A loop expansion with an obvious analog in Quantum Field Theory (QFT). The novelty here is just that string coupling g_s (unlike the gauge coupling of QFT or Newton's constant in General Relativity) is itself a (scalar) field ϕ, called the dilaton. I'm using the convention in which $g_s^2 = e^{\phi}$.
- 2. The so-called α'-expansion

It's worth recalling that, thanks to expansion 2., expansion 1. appears to be free of ultraviolet (UV) divergences.

By definition, Classical String Cosmology (CSC) is the cosmology that comes out of string theory at lowest (tree-level) order in expansion 1. but supposedly exact (see below) in the expansion 2.. In other words it refers to a weak-coupling limit without assuming slowly varying fields (or small curvature of spacetime). We will thus consider the tree-level effective action of the ubiquitous massleess fields of string theory, the (string-frame) metric $g_{\mu\nu}$, the dilaton ϕ and the Kalb-Ramond antisymmetric field $B_{\mu\nu}$, imagining to have integrated out the heavy degrees of freedom (that implies that the derivative expansion of the effective action can exhibit divergences/singularities when one enters the strong curvature regime).

We will mostly consider here the case of a homogeneous (spatially flat but in general anisotropic) set of fields $g_{\mu\nu}$, ϕ, $B_{\mu\nu}$ in $D = d + 1$ spacetime dimensions, in which case the system exhibits d abelian isometries corresponding to spatial translations in d-dimensions. In the rest of this paper I will report on recent progress made on studying this class of string cosmologies.

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2 Symmetries of CSC and their perturbative realization

At the beginning of the nineties it was understood that these d-abelian isometries lead to an $O(d, d; R)$ (simply $O(d, d)$ hereafter) symmetry of the CSC equations [1], [2], [3]. This non-compact, continuous symmetry has been argued to hold non-perturbatively in the α' expansion [4].

More recently it was rediscovered in double field theory [5] where it naturally follows from the doubling of the spacial coordinates (while the original formulation uses the doubling typical of a Hamiltonian approach).

When acting on a solution, $O(d, d)$ generates, in general, inequivalent solutions. To illustrate this point, consider a discrete $O(d, d)$ subgroup, known as scale-factor-duality (SFD) [6], [7], which survives even in the absence of a $B_{\mu\nu}$ background (but crucially in the presence of ϕ). SFD is a symmetry under the discrete group Z_2^d:

$$Z_2^d \equiv a_i \to \tilde{a}_i \equiv a_i^{\eta_i} \; ; \; \bar{\phi} \equiv \phi - \sum_i \log a_i \to \tilde{\phi} - \sum_i \log \tilde{a}_i \; ;$$
$$\eta_i = \pm 1 \text{ (independently for each i)} . \tag{1}$$

Here $a_i(t)$ represent the scale-factors of a Bianchi I-type cosmology and $\bar{\phi}$ defines the so-called shifted dilaton which is invariant under SFD transformations. Note that this is a large group of symmetry transformations which is not shared by General Relativity because of the non-existence of the dilaton field in GR (Newton's constant in GR is just a number).

On the other hand, GR and string theory share another discrete cosmological symmetry, that of time reversal $t \to -t$. As we shall see, it turns out to be very useful to add such a transformation to the SFD (or $O(d, d)$) symmetry group. In the same way as time-reversal connects two physically different cosmologies, so does SFD. These are just transformations that allow us to go from one solution of the field equations to another (generally inequivalent) solution.

Solutions of the field equations that follow from the effective action of string theory correspond to a possible background-field configuration compatible with the two-dimensional conformal invariance which is necessary for a consistent quantum string propagation. These solutions are often referred to as "string vacua". Unfortunately, although many of them can be constructed perturbatively in α', only a handful are known to be exact solutions (by constructing explicitely the corresponding CFT).

Coming back to regarding the solutions as (cosmological) vacua of string theory we can ask the standard question we are accustomed to in QFT: In a context in which the symmetry is *not explicitly* broken is it *spontaneously* broken? This question is easily answered by the solutions of the lowest-order effective action. Taking just for simplicity of illustration, the case of an isotropic cosmology, in which the symmetry group becomes $Z_2(t - rev) \otimes Z_2(SFD)$, one finds the four distinct solutions shown in Fig. 1. They are clearly connected by the symmetry group and they are all four physically inequivalent. They also all singular, either in the past (big-bang-like) or in the future (big-crunch-like). Such solutions clearly break spontaneously $Z_2 \otimes Z_2$ to nothing. This is unlike the case of the exact trivial solution in which (for a critical

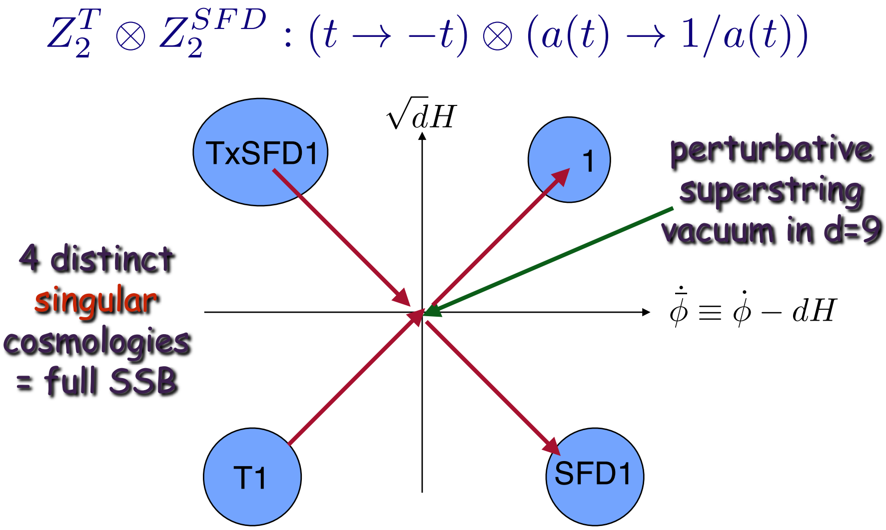

Fig. 1 Parametric plots (in the $\dot{\bar{\phi}} - H$ plane) of four distinct perturbative isotropic solutions. Starting with anyone of them, say the one labelled 1, we can go the other three by applying time-reversal (T1), SFD (SFD1), or both (TxSFD1)

value of D) spacetime is flat and the dilaton is constant (the origin in Fig. 1), in which case there is no spontaneous breaking. As we shall argue below, when all-order α' corrections are taken into account, the breaking pattern may well end up being in between these two extreme (no breaking vs. total breaking) cases.

3 The pre-Big bang scenario: a quick reminder

The pre-Big Bang (PBB) cosmological scenario [8] [9] is deeply rooted in the symmetries of CSC described in the previous Section and on the assumption of "Asymptotic Past Triviality" [10] which amounts to the following postulate:

The Universe started at vanishingly small curvature and coupling and thus its early evolution can be reliably described using the well-known tree-level, small-curvature string effective action.

It is easily checked that generic solutions evolve, at least initially, towards growing curvature and coupling. As long as both are still sufficiently small one gets a very specific inflationary cosmology that has been dubbed "dilaton driven inflation" (DDI). The name comes from the fact that the growth of the dilaton entails the growth of the effective Newton constant which in turn, through an analog of the Friedman equation, accelerates the expansion rate.

However, if one were to trust the tree-level, small-curvature string effective action all the way, this would lead to a curvature (and/or coupling) singularity in a finite proper time. But, obviously, before reaching the singularity higher-curvature and/or higher-loop corrections will come into play. An act of faith was made at the time assuming that:

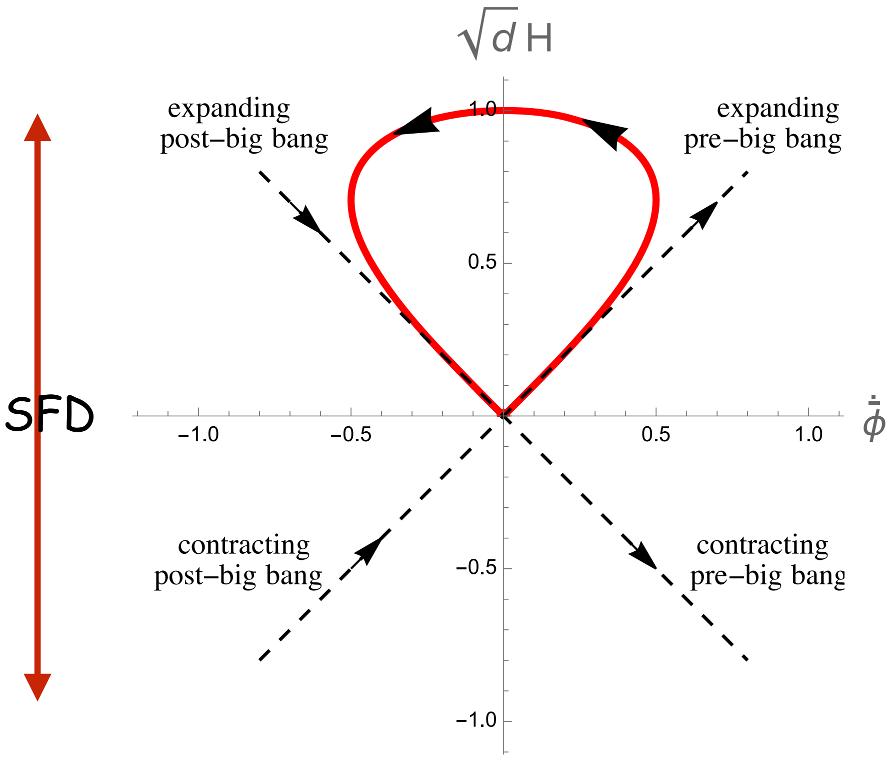

Fig. 2 Parametric plot (in the $\dot{\bar{\phi}} - H$ plane) illustrating the basic PBB assumption about how, through higher order corrections, two singular solutions of Fig. 1 may get smoothly connected through a curvature bounce (a growing H followed by a decreasing H)

A combination of high-curvature/large-coupling corrections makes it possible to avoid the singularity and to connect the early-time (pre-bang) solution to the the $SFD \otimes T$-related one describing (after adding important duality-breaking corrections) the post-bang era lasting till today.

The idea is shown pictorially in Fig. 2, which should be contrasted with the four lowest-order singular branches of Fig. 1. Note that the regular solution is actually invariant under a Z_2 sugroup of the original $Z_2 \otimes Z_2$ symmetry.

The interesting point is that scenarios like this one can have observable consequences to the point that they can be falsified. Large-scale perturbations were computed long ago and gave (n_S, n_T are the usual spectral indices with $n_S \sim 1$ and $n_T \sim 0$ corresponding to nearly scale-invariant spectra):

- A blue spectrum of tensor perturbations ($n_T \sim 3$)
- A blue spectrum of adiabatic scalar curvature perturbations ($n_S \sim 4$)
- A rather unique mechanism to produce a potentially interesting spectrum of electromagnetic perturbations [11], [12] that could play the role of seeds for generating the observed intergalactic cosmic magnetic fields.

Of course, such a blue-tilted spectrum of curvature perturbations is incompatible with CMB data. In order to save the model phenomenologically, one has to appeal to the stringy axion. Its spectrum of isocurvature perturbations can be converted to curvature perturbations via the "curvaton" mechanism and this can lead to a realistic scenario [13] if the primordial axion perturbations are nearly scale invariant (which is perfectly possible although not automatic) On the other hand tensor spectrum stays blue so that a generic prediction of the model is that the ratio r of tensor to scalar perturbation will remain very small and dominated by a well-known [14] second order contribution from scalar perturbations.

4 Recent progress on PBB's "act of faith"

Recently, some interesting progress has been made on the most delicate and important point of the PBB scenario: the avoidance of the singularity and the smooth joining of the inflationary branch to the decelerating post-bounce expansion. This progress is very much based on the few-years-old work by Hohm, Zwiebach and their collaborators. I will first recall the basic claim by Hohm and Zwiebach concerning an all-order reformulation of $O(d,d)$-covariant string cosmology. I will then introduce a more recent Hamiltonian-like reformulation of their work allowing to give a sufficient criterion for having regular solutions.

4.1 All-order α' corrections a la Hohm and Zwiebach (HZ)

A few years ago, progress on understanding the $O(d,d)$ properties of α'-corrections (initiated long ago by K. Meissner [15]) culminated in a claim by Hohm and Zwiebach (HZ) [16] about the general structure of the $O(d,d)$-invariant cosmological effective action that can be obtained by extensive use of integration by parts and field-redefinitions. For the purpose of this talk we will need their result for the particular case of a Bianchi I-type cosmology in the absence of a $B_{\mu\nu}$ background. It reads:

$$S = -\frac{1}{2}\int dt N e^{-\overline{\phi}}\left[N^{-2}\dot{\overline{\phi}}^2 + F\left(N^{-1}\dot{\hat{b}}_i\right) + 2V(\phi)\right] \; ; \; F = -N^{-2}\sum_i \dot{\hat{b}}_i^2 + \dots \tag{2}$$

where the dot denotes time derivatives (we are using units in which $\lambda_s = \sqrt{\frac{\alpha'}{2}} = 1$). Here $N^{-1}\dot{\hat{b}}_i = H_i$ is the ith Hubble parameter, and the function $F(H_i)$ in the isotropic case reduces to the function introduced in [16]: it can be written as an infinite (and not necessarily convergent) series of even powers of the Hubble parameter, and includes, in principle, the all-order α' corrections predicted by a given string model [1]. For later use we have added to the action a potential term $V(\phi)$ that can possibly come from loop and/or non-perturbative corrections and breaks the $O(d,d)$ symmetry. However, for now we shall set $V = 0$.

[1] These are only known, unfortunately, at a relatively low order.

By varying the action (2) with respect to N, $\hat{b}_i$ and ϕ, and defining $f_i = \partial F/\partial H_i$, we obtain the following Euler-Lagrange equations in the cosmic-time gauge $N = 1$:

$$\dot{\overline{\phi}}^2 = F - \sum_i f_i H_i, \qquad \dot{f_i} = f_i \dot{\overline{\phi}}, \qquad 2\ddot{\overline{\phi}} = -\sum_i f_i H_i. \tag{3}$$

To zeroth order in α' one has $F = -\sum H_i^2$, and recovers the well known (see e.g. [8], [9]) tree-level low-curvature string cosmology equations. Two properties of (2) are worth emphasizing: i) It only contains first-order derivatives, ii) the shifted dilaton occurs in its lowest-order form. Both properties turn out to be important for the developments described hereafter.

In the HZ formulation the basic quantity determining the kind of solutions (in particular regular vs. singular behaviour) is the F-function, known only to a few orders in α'. In general, a smooth, analytic F gives a singular solution. However, a paper by Wang, Wu, Yang, Ying [17], [18] showed an example of regular solution. It was constructed by a trial and error procedure, looked highly fine-tuned, and did not specify the underlying F-function. By analysing it in more detail, Gasperini and I found an explanation [19] for why their solution exists. It suggested the existence of a much larger class of regular, bouncing solutions. That required a reformulation of the HZ approach.

4.2 Hamiltonian/Routhiam reformulation of HZ and regular bouncing solutions

As argued in [19], one can conveniently adopt a "partial Hamiltonian" approach to the action (2) (also known as *Routhian* approach in a classical mechanics context, see e.g. [20]), and perform a Legendre transformation on just a subset of the original coordinates $N, \overline{\phi}, \hat{b}_i$, in our case on just the latter d Lagrangian coordinates $\hat{b}_i$. Denoting by π_i the momentum conjugate to $\hat{b}_i$ one defines:

$$\pi_i = \frac{\partial L}{\partial \dot{\hat{b}}_i} = -\frac{1}{2} e^{-\overline{\phi}} \frac{\partial F}{\partial H_i} = -\frac{1}{2} e^{-\overline{\phi}} f_i \equiv e^{-\overline{\phi}} z_i, \tag{4}$$

where we have introduced, for later use, the more useful rescaled momenta $z_i = -f_i/2$. The associated Legendre transformation defines the so-called Routhian $\mathcal{R}(N, \overline{\phi}, \pi_i)$

$$\mathcal{R}(N, \overline{\phi}, \pi_i) = N e^{-\overline{\phi}} \left[\frac{1}{2} N^{-2} \dot{\overline{\phi}}^2 + h(z_i) + V\left(\overline{\phi} + \sum_i \hat{b}_i \right) \right], \quad \frac{\partial \mathcal{R}}{\partial \pi_i} = \dot{\hat{b}}_i$$
$$h(z_i) \equiv \frac{1}{2} \left(F - \sum_i \dot{\hat{b}}_i \frac{\partial F}{\partial \dot{\hat{b}}_i} \right) = \frac{1}{2} \sum z_i^2 + \mathcal{O}(\alpha') + \dots, \qquad \frac{\partial h}{\partial z_i} = H_i, \tag{5}$$

where the second equation on the first line is to be used to express $\dot{\hat{b}}_i$ in terms of the π_i and we have introduced [19] a reduced "Hamiltonian" $h(z_i)$. Note that the last equation for $H_i = N^{-1}\dot{\hat{b}}_i$ basically inverts the functions $f_i = f_i(H_j)$, giving $H_i = H_i(z_j) = H_i(-\frac{1}{2}f_j)$.

Hence, in this new context, a given model is specified by the choice of $h(z_i)$ and of $V(\phi)$, and equations (3) can be rewritten in Routhian language as a combination of the Euler-Lagrange equations for N and $\overline{\phi}$ and the Hamilton equations for $\hat{b}_i$, π_i, namely $\partial\mathcal{R}/\partial\pi_i = \dot{\hat{b}}_i$, $\partial\mathcal{R}/\partial\hat{b}_i = -\dot{\pi}_i$. Using (4), and after setting at the end $N = 1$, these equations can be finally rewritten in terms of z_i as:

$$\dot{\overline{\phi}}^2 = 2\,h(z_i) + 2V, \qquad \dot{z}_i = z_i\,\dot{\overline{\phi}} - \frac{\partial V}{\partial \phi}, \qquad \ddot{\overline{\phi}} = \sum_i z_i\,\frac{\partial h}{\partial z_i} + \frac{\partial V}{\partial \phi}\,, \tag{6}$$

where, as usual, the first equation (the Hamiltonian constraint) together with the second set imply the last equation. These are the equivalent of equations (3) in the Routhian formalism.

4.2.1 Regular bounces for $V = 0$

For $V = 0$ the action (2) is SFD-invariant. In Ref. [19] it was shown that the condition on $h(z)$ for a regular bounce basically amounts to the requirement that $h(z)$, after growing from its vanishing value at $z = 0$, bends down and has a second zero at some finite value z_0. In that case, generically:

$$\dot{\overline{\phi}} \to \pm\sqrt{(z - z_0)} \text{ for } z \to z_0. \tag{7}$$

According to the APT hypothesis initially $\dot{\overline{\phi}} > 0$ in the asymptotic past and one has to pick the plus sign in (7) as one approaches $z = z_0$. Then, in order to have a regular bounce, $\dot{\overline{\phi}}$ has to change to the negative-sign branch. In fact the evolution after the bounce is nothing but the $SFD \otimes T$ transformed of the one before the bounce.

Inventing functions $h(z)$ satisfying the above constraint is obviously quite trivial, one of the simplest example being given by:

$$h(z) = \frac{1}{2}z^2\left(1 - \frac{1}{4}z^2\right). \tag{8}$$

The example (8) however is only apparently simple. If one goes back from this simple Hamiltonian to the corresponding Lagrangian (i.e. to the corresponding $F(H)$) one finds the latter has a rather complicated analytic structure with branch points occurring at value of H of $\mathcal{O}(1)$ in string units something we believe to be physically acceptable as a consequence of having integrated out the massive string modes. The parametric plot of the solution corresponding to the choice (8) is shown in Fig. 3 and precisely agrees with the regular solution discovered in [17].

A few comments about the solution shown in Fig. 3 are in order:

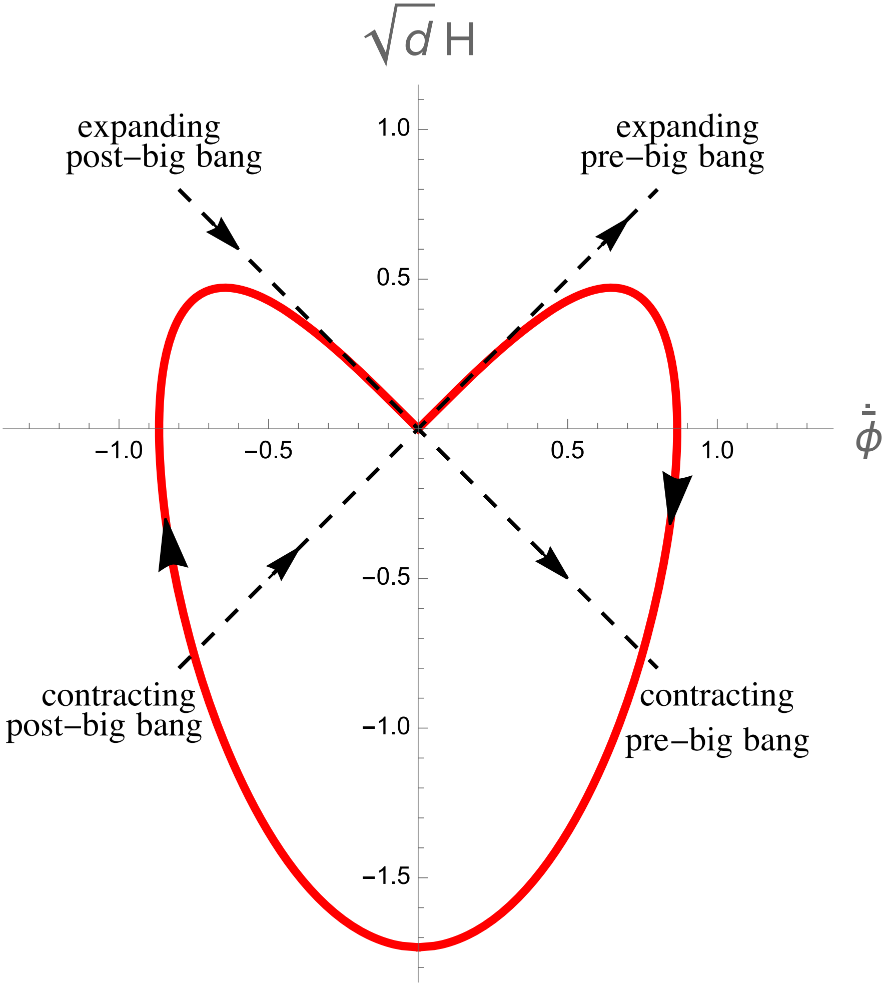

Fig. 3 Parametric plot (in the $\dot{\bar{\phi}} - H$ plane) of the regular solution corresponding to the choice (8) for $h(z)$. Although its early- and late-time behaviors resemble the ones of Fig. 2, the regular bounce now occurs through a (short) period of contraction in the string frame

- 1. Contrary to the original expectation of an “anti-clockwise" bounce (meaning an ever expanding Universe in the string frame) the bounce occurs “clockwise", i.e. it involves a string-frame contracting phase.
- 2. Yet, the solution is invariant under the Z_2 subgroup $T \times SFD$ of the original $Z_2 \otimes Z_2$. Only *two* distinct “vacua? are generated by the full $Z_2 \otimes Z_2$: part of the original spontaneously broken symmetry is now restored.
- 3. Qualitative properties of the solution in both the string (Fig. 4) and Einstein frames (Fig. 5) can be easily studied.

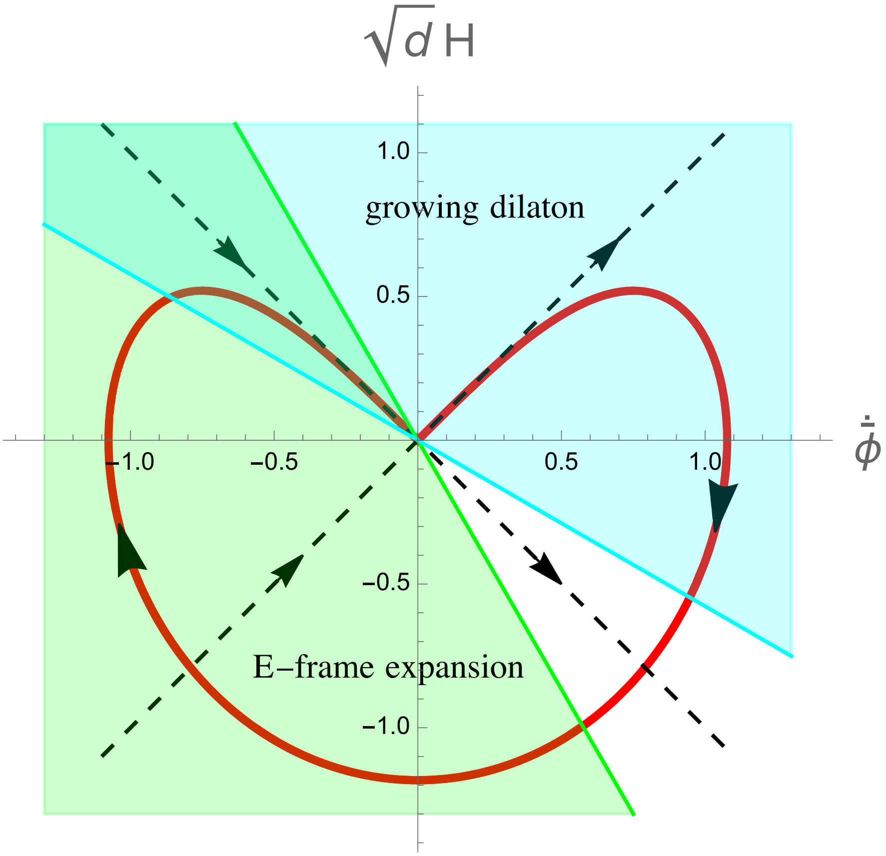

Fig. 4 A regular bouncing cosmology similar to that of Fig. 3 in which we also show the regions in the parametric plot where the coupling (dilaton) is growing or decreasing as well as those that correspond to an expansion or to a contraction in the Einstein frame

4.2.2 Adding a non-perturbative potential

In the old days (nineties) it was found to be difficult, if not impossible, to achieve dilaton stabilization through a non-perturbative potential. There was some kind of no-go (or hard to go?) theorem (see e.g. [21]) about such a "graceful exit" from the prebounce phase. But, at the time, one was trying to stabilize the dilaton before having gone through the bounce. After a regular bounce is achieved through α' corrections the game becomes much easier: it is sufficient to add a (non-perturbative) potential.

We have considered [22] several kinds of non-perturbative potentials as shown in Fig. 6. Typically, we have assumed a local minimum of the potential in the strong coupling regime $g_s^2 = \mathcal{O}(1)$. Besides we have considered different possible strong bare coupling ($\phi \to +\infty$) limits Fig. 7.

Depending on the choice of the dilaton potential and on initial conditions various late-time scenarios can emerge:

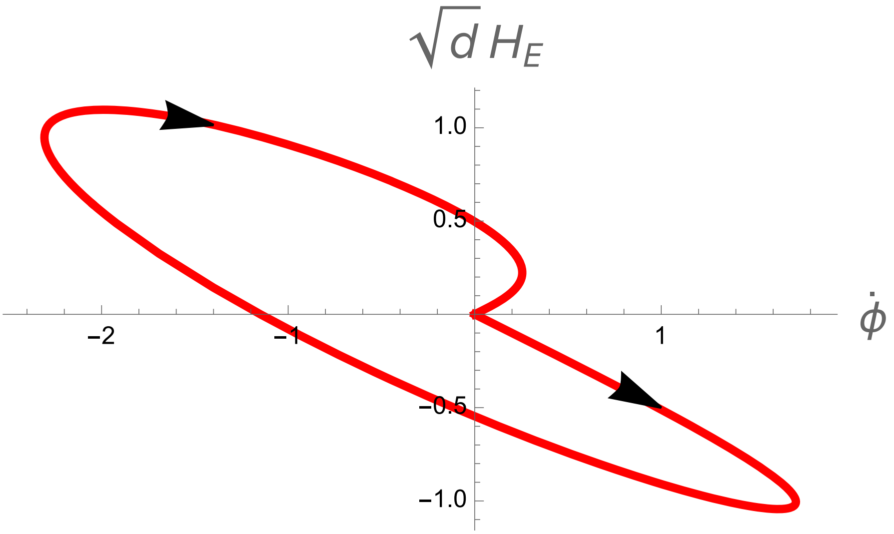

Fig. 5 The bouncing cosmology of Fig. 4 seen as a bounce of the Einstein frame scale factor

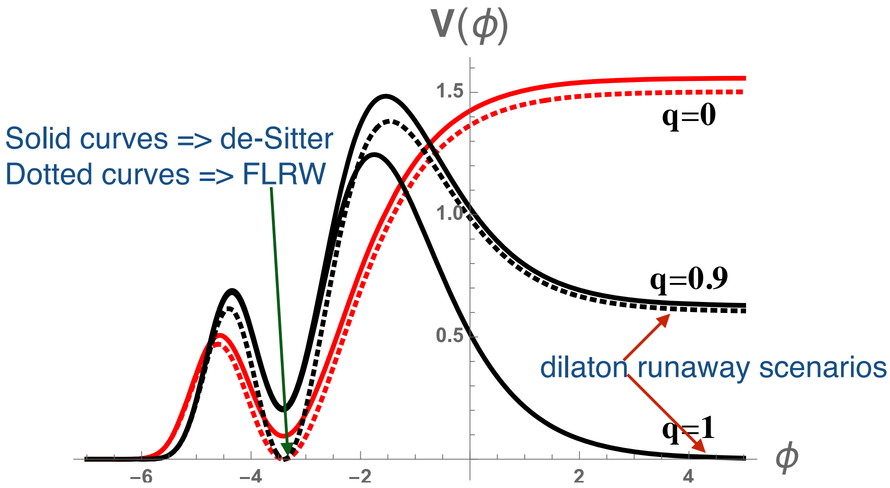

Fig. 6 A sample of non-perturbative potentials considered in [22]

- 1. Attraction to an absolute minimum of the potential with $V = 0$. In this case one has to play the standard PBB game with inflation occurring before the bounce.
- 2. Attraction to a (metastable) minimum of the potential with $V > 0$. Here one can use the framework as a model of initial conditions for conventional inflation. An example of such a solution is shown in Fig. 8.
- 3. The dilaton climbs over the potential and ends up at positive infinity. This can lead to the so-called dilaton runaway scenario [23], [24].
- 4. The dilaton bounces back to negative infinity, a rather uninteresting scenario.

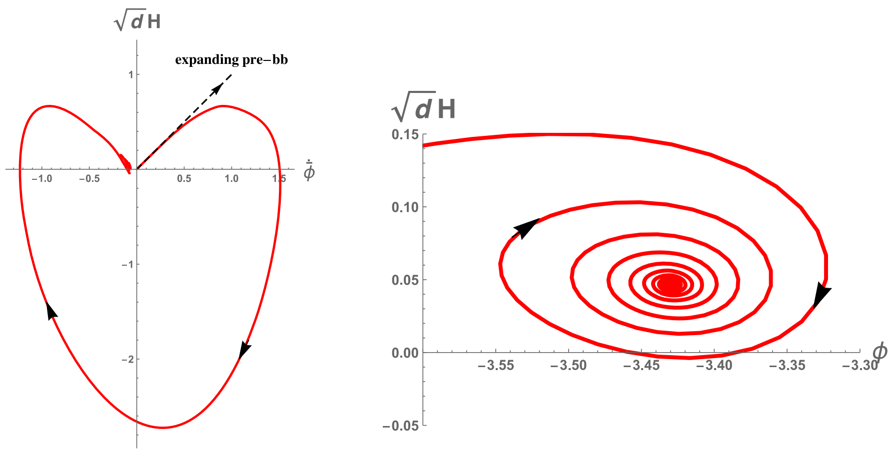

Fig. 7 Parametric plots of a regular solution having a de-Sitter attractor at late times. The right panel is a blow-up of the late time evolution

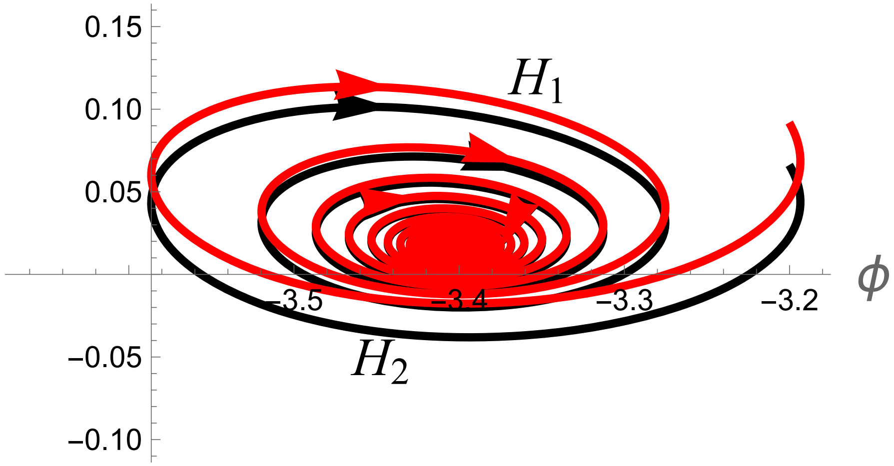

Fig. 8 Parametric plot showing an isotropic late time attractor for the case of two initially different scale factors

For each potential one can determine numerically the basin (of initial conditions) leading to each one of these scenarios.

So far we have only discussed isotropic cosmologies but the discussion of anisotropic Bianchi type I cosmologies can be easily carried out. In doing so we have found a new interesting isotropization mechanism. An old criticism of the PBB scenario was that it was hard to remove an initial non-vanishing shear (see e.g. [25]). Very interestingly, the higher derivative (stringy) corrections also help in this respect! A simple example with two different initial scale factors converging to an isotropic cosmology at late times is shown in Fig. 9. So far so good, what's next?

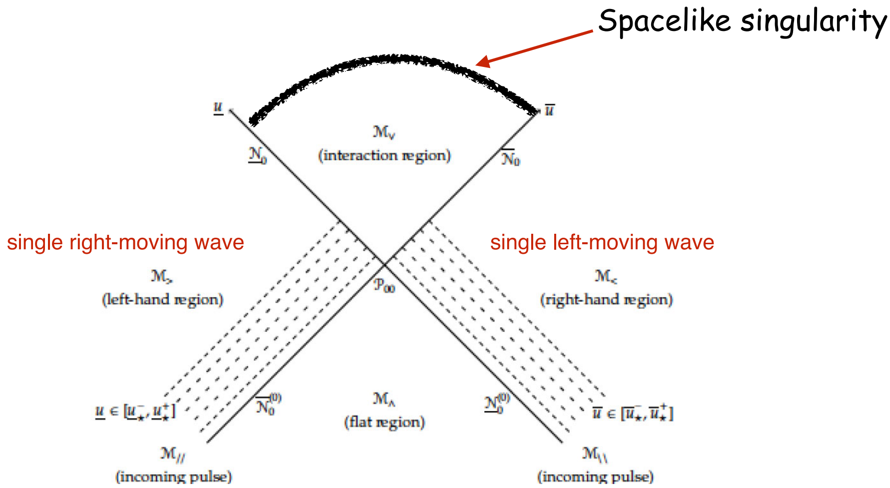

Fig. 9 Schematic diagram of the collision of two infinite-front plane waves (with suppressed coordinates of the planes). In three of the four regions we have an exact string background. In the fourth region –and in the absence of string corrections– a singularity develops (figure is from [34])

Obviously, one would like to compute perturbations on top of these homogeneous solutions. Two obvious questions come to mind:

Q1: Can one do this within an (approximate) $O(d, d)$ framework?
Q2: Can we make the HZ/GV framework descend from a covariant string effective action when the solution is restricted to one with d Abelian isometries?

We are trying to address Q2 in the general case but we are still puzzled, so I prefer not to report on this topic today. Rather, in the last part of my talk I'll jump to a different, yet related, topic which is still in progress.

5 Plane-wave collisions with α' corrections

Instead of a $(d+1)$cosmological background with d abelian isometries consider a model in $D = d + 2$ spacetime dimensions and d abelian isometries[2]: the collision of two (gravitational + dilatonic) plane waves, studied (and solved!) in the GR literature since the seventies [27], [28], [29]).

The solutions all exhibit naked singularities at late times. In the presence of a dilaton these can be Kasner-like without BKL oscillations (with space-dependent Kasner exponents).

This model has been used as an example of how generic, arbitrarily weak initial data may evolve into a cosmology of the PBB type [30], [31]. Questions:

Q1. Can the introduction of corrections a la HZ/GV remove the singularity and lead to a regular bounce?

[2] Its $O(d, d)$ covariant formulation was given in [26].

Q2. Since the would-be singularity is velocity-dominated (small spatial gradients) does one recover an $O(d+1, d+1)$ symmetry near the bounce?

We now have derived constraints and constraint-compatible e.o.m. in the presence of quite generic higher-derivative corrections and are ready to solve numerically the PDEs (ODEs in the non-interacting regions). No results for today!

6 Conclusion

To summarize:

- Thanks to the work of Hohm, Zwiebach and collaborators there has been renewed interest on Classical String Cosmology in the last few years.
- The HZ approach to all-order α' corrections compatible with the expected $O(d, d)$ symmetry allows to address the question of whether or not some low-curvature cosmological solutions can be uplifted to regular cosmological string backgrounds.
- A Hamiltonian/Routhian reformulation of HZ helps finding criteria for this to happen. Such criteria involve no fine-tuning and can be seen as quite generic.
- Adding a non-perturbative potential the problems of dilaton stabilization and shear-suppression appear to be easily solved leading to possible late-time attractors of the FLRW or de-Sitter type.

…and for the future

- In order to compute perturbations one would need to derive the HZ framework from a covariant setup. This seems to impose further constraints on the possible forms of the corrections: a question clearly deserving further study.
- It would be interesting to apply the whole approach to less symmetric situations, such as the collision of null plane waves (exact string backgrounds before the collision) and ask under which conditions the curvature singularity (unavoidable in the GR limit) is cured by classical string corrections.
- Even farther into the future: the collision of finite-fronted waves and the fate of the black-hole they lead to if a Closed Trapped Surface is formed at collision time [32], [33].

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Data Availability No datasets were generated or analysed during the current study.

Declarations

Competing interests The authors declare no competing interests.

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