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Multiboson production at the LHC

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Abstract. In this contribution, I discuss the new topics of quantum entanglement and polarized cross sections in diboson production processes at the LHC. Motivations for triboson production studies are given.

1. Introduction

The CERN Large Hadron Collider (LHC) started operating since 2009 and has accumulated a lot of data, in particular in the production of massive diboson pairs, namely ZZ, WW, and WZ proccesses. The detailed studies of these processes allow us to probe deeply the Standard Model (SM) and in particular the electroweak (EW) symmetries and the spontaneous symmetry breaking mechanism. It also helps un in the search for new physics effects by providing new observables related to polarization of the gauge bosons. With 13 TeV data as well as the new data from run 3 and beyond, it is possible to study non-trivial observables such as the joint polarization of the massive gauge bosons. The new measurements from ATLAS and CMS collaborations in the WZ channel can be found in Refs. [1, 2], respectively.

In this contribution, the important topic of joint polarization of the massive gauge bosons in diboson production will be discussed. We take this opportunity to highlight some new entanglement results of the diboson systems produced at the LHC. Finally, our discussion is naturally extended to the case of triboson production.

2. Bell inequalities and entanglement in diboson

Entanglement in diboson production at the LHC is a new topic which has recently attracted a significant amount of attention (see e.g. [3, 4, 5, 6, 7] and references therein). In this talk, we discuss how entanglement can be quantified and measured at the LHC. The calculation method and results of this section are taken from Ref. [4].

Quantum entanglement is a new feature of quantum mechanics compared to classically local and deterministic formalisms [8]. This difference can be quantified using Bell inequality [9], which has been advanced and extended to different contexts (see e.g. [8]). Another observable called concurrence can be used to extablish the existence of entanglement [4].

For a mixed state of a bipartite system consisting of two subsystems A and B, which is the case of diboson production at the LHC, the concurrence $\mathcal{C}$ depends on the density matrix ρ: $\mathcal{C} = \mathcal{C}[\rho]$. It vanishes if and only if the state is separable, that is not entangled. For the case of two-qubit systems, an analytic solution for $\mathcal{C}$ is known. However, for systems of two qutrits (a massive gauge boson has three polarization states – hence called a qutrit), no analytic solution is known and we will use the following lower bound [10]

$$\left(\mathcal{C}[\rho]\right)^2 \geq \mathscr{C}_2[\rho], \tag{1}$$

with

$$\mathscr{C}_2[\rho] = 2\max\left(0,\ \mathrm{Tr}[\rho^2] - \mathrm{Tr}[(\rho_A)^2],\ \mathrm{Tr}[\rho^2] - \mathrm{Tr}[(\rho_B)^2]\right), \tag{2}$$

where $\rho_A = \mathrm{Tr}_B[\rho]$ and $\rho_B = \mathrm{Tr}_A[\rho]$ are the reduced density matrices. If $\mathscr{C}_2 > 0$ then we can conclude that the two bosons are entangled. The maximum value of the concurrence is $2/\sqrt{3}$ when the system is at a totally symmetric and maximally entangled pure state [4].

Alternatively, we can test the existence of quantum entanglement using the following so-called Bell observable

$$\mathcal{I}_3 = \mathrm{Tr}(\rho\mathcal{B}), \tag{3}$$

where the explicit form of the operator $\mathcal{B}$ is given in [4].

The Bell inequality (or rather the Collins-Gisin-Linden-Massar-Popescu inequality [11, 12], which is a generalized Bell inequality for systems made of two qutrits) then reads

$$\mathcal{I}_3 \leq 2, \tag{4}$$

which is obtained for the classical deterministic local models. For quantum mechanics the constraint is relaxed to $\mathcal{I}_3 \leq 4$.

Thus, if we can prove that $\mathcal{I}_3$ can be greater than 2 then the deterministic local formalisms are disproved and we have evidence to support quantum mechanics.

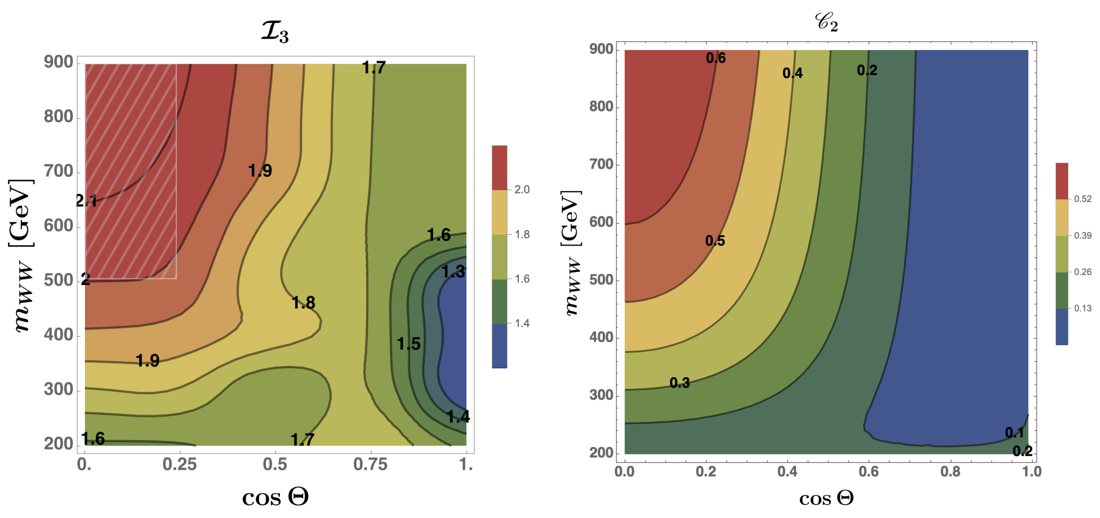

Figure 1. The Bell observable $\mathcal{I}_3$ (left) and concurrence lower bound $\mathscr{C}_2$ (right) as functions of two variables $\cos\Theta$ (scattering angle in the WW CM frame) and m_{WW} for the process $pp \to W^+W^- + X$ at $\sqrt{s} = 13$ TeV. Results taken from [4].

To calculate the above entanglement observables $\mathscr{C}_2$ and $\mathcal{I}_3$ we need to know the density matrix ρ for the process $pp \to V_1V_2 + X$, which is calculated from the density matrices of the parton-parton subprocesses.

For the partonic process $q_a\bar{q}_b \to V_1V_2$ the density matrix reads [4]

$$\rho_{ab}(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) = \sum_{\mu\mu'\nu\nu'} \frac{\mathcal{M}_{ab,\mu\nu}\mathcal{M}^{\dagger}_{ab,\mu'\nu'}}{|\overline{\mathcal{M}}_{ab}|^2} \mathcal{P}^{\mu\mu'}_{\lambda_1\lambda'_1}(p_1)\mathcal{P}^{\nu\nu'}_{\lambda_2\lambda'_2}(p_2) \tag{5}$$

where $\mathcal{P}^{\mu\nu}_{\lambda\lambda'} = \epsilon^{\mu}(p,\lambda)^{\star}\epsilon^{\nu}(p,\lambda')$, $\lambda_i = 1, 2, 3$ $(i = 1, 2)$ is the polarization index of the gauge bosons. In the squared amplitude $|\overline{\mathcal{M}}_{ab}|^2$, summation over the polarizations of the gauge bosons and the initial-state quarks is understood. The density matrix for the proton-proton process $pp \to V_1V_2 + X$ is than calculated as [4]

$$\rho = \sum_{\{q_a,q_b\}} w_{ab}\rho_{ab}, \tag{6}$$

where we have to sum over all possible initial-state parton pairs. The coefficients w_{ab} are given by

$$\begin{aligned} w_{ab} &= \frac{L_{ab} \otimes |\overline{\mathcal{M}}_{ab}|^2}{\sum_{\{q_a,q_b\}} L_{ab} \otimes |\overline{\mathcal{M}}_{ab}|^2}, \\ L_{ab} \otimes |\overline{\mathcal{M}}_{ab}|^2 &= \frac{4\tau}{\sqrt{s}} \int_{\tau}^{1/\tau} \frac{\mathrm{d}z}{z} q_a(\tau z, \mu_F)\bar{q}_b(\tau/z, \mu_F)|\overline{\mathcal{M}}_{ab}|^2, \end{aligned} \tag{7}$$

where $|\overline{\mathcal{M}}_{ab}|^2$ is a function of two variables $m_{VV} = \tau\sqrt{s}$ and the scattering angle Θ in the diboson center-of-mass (CM) frame, μ_F is the factorization scale.

From the above method one can proceed to calculate the density matrix ρ. Leading order (LO) results for the ZZ, W^+W^-, and WZ processes are given in Ref. [4]; where the WZ results are calculated in the limit $M_W = M_Z$.

To illustrate the results obtained with the above method, we display in Fig. 1 the results of the Bell observable $\mathcal{I}_3$ (left) and of the concurrence lower bound $\mathscr{C}_2$ (right), for the case of W^+W^- production at the LHC with $\sqrt{s} = 13$ TeV. The result of the Bell observable shows that $\mathcal{I}_3$ can be greater than 2, and this happens in the region of $0 < \cos(\Theta) < 0.4$ and $m_{WW} > 500$ GeV. The value of $\mathscr{C}_2$ is positive in the entire region of the phase space and is largest in the region where $\mathcal{I}_3$ is maximal. Similar results are obtained for the ZZ process, while $\mathcal{I}_3$ is always less than 2 for the WZ case [4]. The value of $\mathscr{C}_2$ is much smaller in the case of WZ than in the WW and ZZ cases. These results suggest that the amount of entanglement in the WZ process is smallest. Since the WW and ZZ cross sections in the region where $\mathcal{I}_3 > 2$ are suppressed, it has been found that the LHC data are not sufficient to establish the violation of the Bell inequality [4].

3. Doubly-polarized cross sections for diboson productions

The unpolarized cross section for the process $pp \to V_1V_2 \to 4$ leptons can be written as

$$\sigma_{\text{unpol}} = \sigma_{\text{LL}} + \sigma_{\text{LT}} + \sigma_{\text{TL}} + \sigma_{\text{TT}} + \sigma_{\text{interf}}, \tag{8}$$

where the index L stands for longitudinal polarization, T for transverse polarization, LT means that the gauge boson V_1 is longitudinal polarized while the V_2 is in the transverse mode, etc. The last term is called the interference contribution, originated from the interferences between the LL, LT, TL, and TT amplitudes.

So far, most measurement results at the LHC have been obtained for the unpolarized cross sections. With the accumulation of data, results for polarized cross sections have been recently

	NLO QCD+EW	ATLAS	Pull
$W^+_L Z_L$	0.056 ± 0.003	0.072 ± 0.016	-1.0
$W^+_L Z_T$	0.156 ± 0.013	0.119 ± 0.034	$+1.0$
$W^+_T Z_L$	0.151 ± 0.012	0.153 ± 0.033	-0.1
$W^+_T Z_T$	0.630 ± 0.041	0.660 ± 0.040	-0.5
$W^-_L Z_L$	0.059 ± 0.003	0.063 ± 0.016	-0.3
$W^-_L Z_T$	0.167 ± 0.014	0.11 ± 0.04	$+1.3$
$W^-_T Z_L$	0.155 ± 0.013	0.21 ± 0.04	-1.3
$W^-_T Z_T$	0.615 ± 0.041	0.62 ± 0.05	-0.1

Table 1. Comparison between the NLO QCD+EW predictions [13] and ATLAS measurements [1] for the polarization fractions $f_{ij} = \sigma_{ij}/\sigma_{\text{unpol}}$ $(i,j = \text{L},\text{T})$ of the $W^\pm Z$ processes. The pull is defined as (Theory − Experiment)$/\sigma$ where σ is the combined error calculated in quadrature.

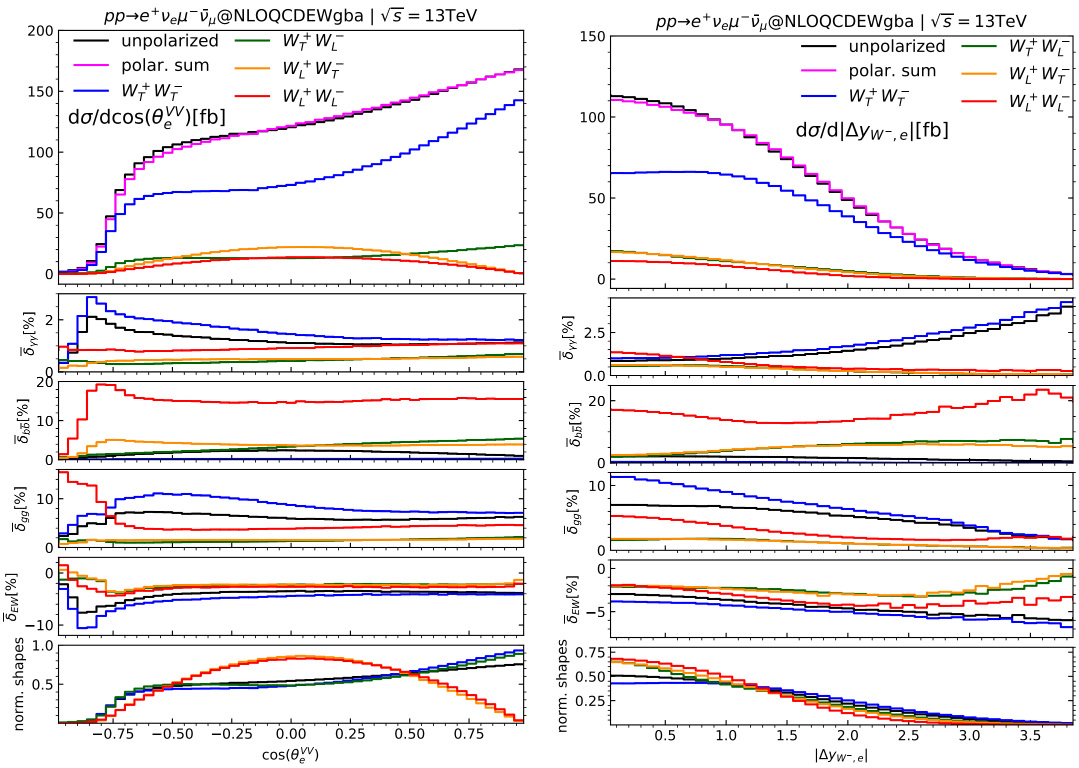

Figure 2. Distributions in $\cos\theta_e^{VV}$ (left) and $|y_{e^+} - y_{W^-}|$ (right) for the $pp \to W^+W^- \to e^+\nu_e\mu^-\bar{\nu}_\mu + X$ process. The big panel shows the absolute values of the cross sections including all contributions from the $q\bar{q}$, gg, $b\bar{b}$, $\gamma\gamma$ processes. The middle panels display the corrections with respect to the NLO QCD $q\bar{q}$ cross sections. The bottom panel shows the normalized shapes of the distributions plotted in the top panel. Results are taken from [14].

available for inclusive $W^\pm Z$ production at ATLAS [15, 1] and at CMS [2], same-sign $WWjj$ at CMS [16], and ZZ at ATLAS [17].

From the theoretical side, the doubly-polarized cross sections on the r.h.s. of Eq. (8) can be calculated using the double-pole approximation (DPA) [18, 19, 20]. This calculation has been performed at the next-to-leading order (NLO) QCD+EW level for WW [21, 14], ZZ [22], and WZ [23, 24, 13, 25]. For the case of WW, next-to-next-to-leading order (NNLO) QCD results are provided in [26]. Very recently, the NLO QCD amplitudes have been combined with parton-shower effects in the POWHEG-BOX framework for all inclusive diboson processes in [27].

These new results are exciting since they pave the way for precision comparisons between the SM predictions and measurements. An example of such a comparison is shown in Table 1, showing excellent agreement between the NLO QCD+EW predictions [13] and the ATLAS measurements [1] for the $W^{\pm}Z$ processes. Though this agreement is encouraging, we must pay attention to the large experimental uncertainties, which are from 6% to 36%. The theory errors are from 5% to 8% calculated from the scale uncertainties. Further works are therefore needed from both the experimental and theory sides to reduce the uncertainties to the level of a few percent.

With more data, differential cross sections will be measured for individual polarizations. In Fig. 2 we show the NLO QCD+EW $\cos\theta_e^{VV}$ (left) and $|y_{e^+} - y_{W^-}|$ (right) distributions recently obtained for the $pp \to W^+W^- \to e^+\nu_e\mu^-\bar{\nu}_\mu + X$ process [14]. These results show that different polarizations have interesting different shapes.

4. Triboson productions

The next exciting possibilities are multipartite entanglement and triple polarization studies in triboson production processes at the LHC. The list of processes inlcudes ZZZ, $ZZW^{\pm}$, ZW^+W^-, $W^+W^+W^-$, $W^-W^-W^+$. Existing calculations for the unpolarized cross sections of these processes can be found in the VBFNLO program [28] and in Refs. [29, 30, 31, 32] for the most recent theoretical results. New cross section measurements are presented in [33, 34, 35].

	$\sigma_{\rm LO}$ [fb]	$\sigma_{\rm NLO}$ [fb]	$\delta_{\rm EW}$ [%]	$\delta_{\rm QCD}$ [%]
TPA	0.093436(5)	0.14581(4)	-7.80(2)	57.82(3)
full	0.093436(5)	0.14572(4)	-8.26(2)	58.50(3)

Table 2. Results for the LO and the NLO cross sections and the NLO corrections ($\delta_{\rm EW}$ and $\delta_{\rm QCD}$) for the TPA and the full off-shell calculation for the process $pp \to e^-\bar{\nu}_e\mu^+\nu_\mu\tau^+\nu_\tau + X$ at $\sqrt{s} = 13$ TeV. The errors in the parentheses are of statistical nature from Monte Carlo integrations. Results are taken from [32].

Similar to the case of diboson, in order to calculate the triply-polarized cross sections we have to use the triple-pole approximation (TPA). TPA results for the case of WWW production with leptonic decays have been presented in [32]. The comparison between the TPA results and the full ones at LO and NLO is highlighted in Table 2. These results show that, with the chosen kinematic cuts, the TPA is an excellent approximation. This is very promising for the triple polarization studies.

Finally, we display in Fig. 3 selected results for the distributions in the transverse mass $M_{{\rm T},3\ell}$ of the three charged-lepton system (left) and in the difference of the pseudorapidities of the two positively charged leptons $\Delta\eta_{\mu^+\tau^+}$ (right) at LO and NLO also for the $W^-W^+W^+$ process, taken from [32]. These results show the important of NLO corrections and will be useful for cross-checking purpose, when future polarized cross section calculations are performed.

5. Conclusions

With an integrated luminosity of 300 fb^{-1} at the LHC Run 3 and of 3000 fb^{-1} at the High-Lumi LHC, there is a strong foundation to expect that high precision measurements will be achieved

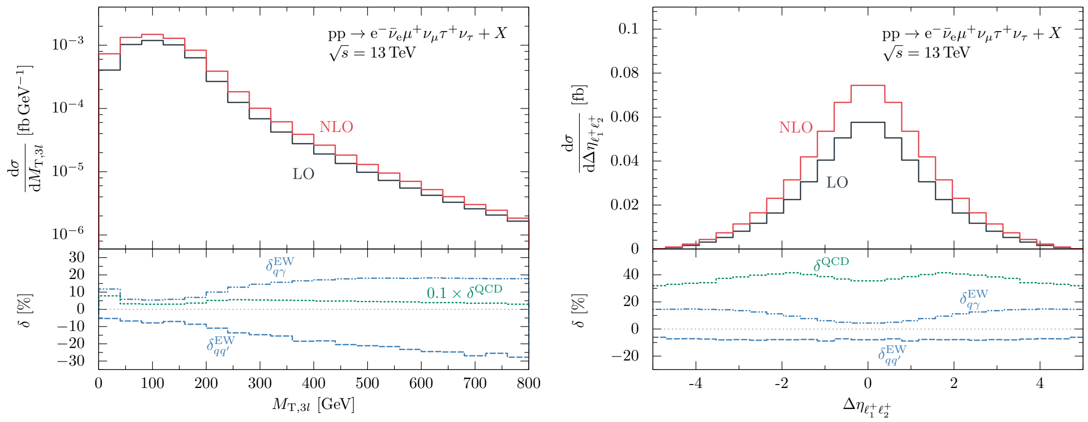

Figure 3. Distributions in the transverse mass $M_{\mathrm{T},3\ell}$ of the three charged-lepton system (left) and in the difference of the pseudorapidities of the two positively charged leptons $\Delta\eta_{\mu^+\tau^+}$ (right) for the process $pp \to e^-\bar{\nu}_e\mu^+\nu_\mu\tau^+\nu_\tau + X$ at $\sqrt{s} = 13$ TeV. Results are taken from [32].

for disboson and triboson productions. In this contribution, we have discussed how to quantify entanglement and test the Bell inequality in diboson processes. It was found that disproving the Bell inequality of $\mathcal{I}_3 \leq 2$ with the LHC data is very challenging due to small values of the cross sections. However, results at the future lepton-lepton colliders are more promising [4]. The topic of doubly-polarized cross sections in diboson production has also been touched, showing excellent agreement between the ATLAS measurements and the NLO QCD+EW predictions of the SM for the $W^\pm Z$ processes. Finally, we give motivations for extending the above diboson calculations to the triboson production. Some existing results for unpolarized cross sections, in particular the triple-pole approximation results, will be useful for polarization studies. We hope that the measurements of triply-polarized cross sections will come soon.

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