SPACE-CHARGE EFFECTS ON FREQUENCY OF FREE OSCILLATIONS IN AN ISOCHRONOUS CYCLOTRON

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(Presented by Yu. I. Shvabe)

There exist several papers [1--5] which studied theoretically the influence of the spatial charge on the motion of particles within accelerators. It is quite important in the design of strong current accelerators to check the theoretical conclusions experimentally on existing devices. The present paper reports on the dependence of the axial oscillation frequency on the density of the space charge of the accelerated particles. Appropriate measurements were made on a cyclotron with a spiral magnetic field [6] during molecular hydrogen acceleration up to 12 MeV.

1. THE DENSITY OF CHARGED PARTICLES WITHIN A RELATIVISTIC CYCLOTRON AND ITS INFLUENCE ON THE FREQUENCY OF AXIAL OSCILLATIONS

Assuming a uniform distribution of charges of the accelerated ions within the beam along the vertical, Δz, and azimuth, $\Delta\varphi$, the relationship between the charge density, $\varkappa$, and the current, i, of ions incident on the target can be expressed by the relationship:

$$\varkappa = \frac{i}{\Delta z \Delta\varphi r \dot{r}}, \qquad (1)$$

where r is the radius and $\dot{r}$ is the radial velocity of the ions.

The changes in the charge density within a relativistic cyclotron as function of the radius and, consequently, of the energy E for the accelerated ions is determined by the expression:

$$\varkappa(r) = i\frac{E_0}{\Delta E}\left(\frac{E}{E_0}\right)^3 \frac{2\pi}{\Delta z \Delta\varphi r_{\infty}^2 \omega}. \qquad (2)$$

Here E_0 is the rest energy of the ion; $r_{\infty} = c/\omega$; ω is the angular velocity of ion rotation; ΔE is the energy increment of ions per turn.

The azimuthal extent of the beam, $\Delta\varphi r$, is significantly larger than the vertical dimensions of the beam, Δz, (with the exception of the central region) and the relative changes in the beam density along the radius over a length equal to the aperture of the dee does not exceed a few percent even at an energy of several hundred millions of electron volts. Consequently, the defocusing force, f_z, acting on an individual particle and due to the space charge of the beam can be represented with sufficient accuracy by the equation:

$$f_z(r) = 4\pi\varkappa(r)\, ze\left(\frac{E_0}{E}\right)^2. \tag{3}$$

The axial particle oscillations under the influence of the magnetic forces within a cyclotron with spiral magnetic field structure are described by a linear approximation (after neglecting the small terms) by the equation [6]:

$$z'' + \left\{-n + \frac{\varepsilon^2}{2}\left[1 + \left(\frac{R}{N\lambda\!\!\!^{-}}\right)^2\right] - \frac{\varepsilon R}{\lambda\!\!\!^{-}}\cos\left(\frac{R}{\lambda\!\!\!^{-}} - N\varphi\right)\right\} z = 0, \tag{4}$$

where ε is the degree of variation; $2\pi\lambda\!\!\!^{-}$ is the radial pitch of the magnetic field structure; N is the number of spirals; n =

$n = \frac{R}{H}\cdot\frac{dH(r)}{dr}\Big|_{r=R}$; $R = \frac{pc}{eH(R)}$; H is the magnetic field strength averaged over the azimuth; p is the momentum of the particle. From the solution of equation (4) it follows that in the first approximation the axial oscillation may be considered as a harmonic with the frequency Q_z and

$$Q_z = \sqrt{-n + \varepsilon^2\left[\left(\frac{R}{N\lambda\!\!\!^{-}}\right)^2 + \frac{1}{2}\right]}\ . \tag{5}$$

In case of relativistic cyclotrons the accuracy of equation (5) is 1--2 percent which is sufficient for several practical computations.

The defocusing field, f_z, reduces the frequency of axial oscillations by an amount ΔQ_z. Within the framework of the linear theory

$$\frac{\Delta Q_z}{Q_z} = 1 - \sqrt{1 - \frac{8\pi^2 ie}{\Delta E\,\Delta\varphi\,\Delta z\,\omega Q_z^2}}. \tag{6}$$

For instance, in the case of the isochronous cyclotron of the Laboratory of Nuclear Problems $Q_z = 0.1$, $\Delta E = 30$ keV, $\omega = 65.4\cdot 10^6\ \mathrm{sec}^{-1}$. If we assume that $\Delta\varphi = 0.5$ radian and $\Delta z = 1.6$ cm, then in the case of a 20 μA current of accelerated ions, one must expect according to equation (6) a frequency decrease of about 2.5 percent. Such a frequency shift can be clearly recorded, and, consequently, it should be possible to study the above-mentioned effect experimentally at the isochronous cyclotron of the Laboratory of Nuclear Problems.

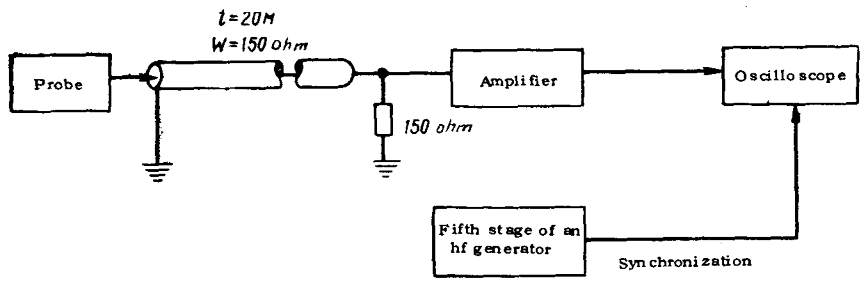

Figure 1. Block diagram of the azimuthal beam extension measurement within an isochronous cyclotron.

2. MEASUREMENT OF THE AZIMUTHAL EXTENSION OF THE BUNCH

Before proceeding to qualitative studies of the axial oscillation frequency changes one must first find the azimuthal extent $\Delta\varphi$ of the bunch of accelerated particles at various radii in the accelerator and also the vertical dimensions of the beam. The vertical dimensions were measured by means of calibrated vertical quartz targets and at various radii in the accelerator the result was within 1.5--2 cm.

The azimuthal extent of the bunch was measured using targets mounted on a movable probe. The shape of the current pulses collected by the target corresponds to the azimuthal charge distribution within the bunch at the radius of the target. The block diagram of the measuring device is shown in Figure 1. We maintained traveling waves within the cable connecting the target with the registering device by connecting a matching impedance directly at the input to the high gain measuring amplifiers. This matching impedance also served as the leakage resistance of the target. The distributed parameter of the amplifier had a 3 nsec time constant, and its amplification factor was about 10^3; the oscilloscope sensitivity was 1 V/cm with a time constant of 7 nsec and a sweep speed of 20 nsec/cm.

To reduce hf pickup the target was shielded by a grounded tungsten screen having a narrow slit through which the particles could reach the probe target. Using the method described, the extent of the bunch over radii ranging from 25.5--53 cm changed from 5 to 20 nsec (taking into account the pass-band of the amplifier and the oscilloscope) and, consequently, the azimuthal dimension of the bunch, $\Delta\varphi$, is within the 36--60° limits. Figure 2 shows the shape of the pulse on the oscilloscope screen.

3. FREQUENCY CHANGES OF THE FREE AXIAL OSCILLATIONS

We utilized the resonant method for the excitation of free oscillations by an external electric field formed between two electrodes symmetrically spaced with respect to the median plane of the orbit. The radial extension of the orbits, $\Delta R = 2$ cm, azimuthal $l = 15$ cm, while the distance between the electrodes was $d = 2$ cm. The increase in amplitude of the axial oscillations

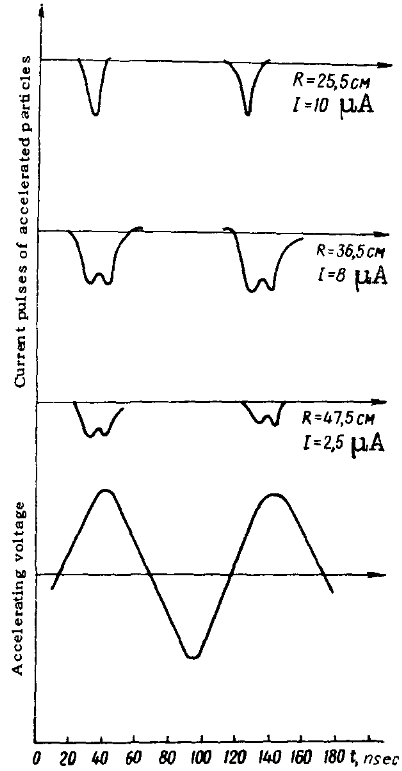

Figure 2. Shape and duration of the current pulse at the target within an isochronous cyclotron.

during the application of the external electric field can be found from the expression:

$$z = \frac{1}{2Q_z} \cdot \frac{eU}{\Delta E} \cdot \frac{l\Delta R}{d} \qquad (7)$$

(U is the amplitude of the electrode voltage).

With $Q_z = 0.15$, to achieve an amplitude increase of 1 cm the electrodes must be supplied by a voltage of 400--600 V. Such a voltage was generated by a push-pull generator having a frequency range of 0.5--2 Mc and a guaranteed frequency accuracy to within ±0.5 percent. To eliminate secondary electron emission the electrodes were subjected to a positive bias of 100 V.

The resonant frequency of the external electrode voltage, corresponding to the frequency of the axial oscillations, was adjusted for a strictly sustained accelerating condition of the accelerator by adjusting to the minimum current reaching the tantalum measuring target placed between the deflecting electrodes. The target had a power dissipation capacity of 200 W.

Figure 3 shows the current reaching the measuring target, as a function of the frequency of the electric excitation field, for various radii of the accelerator and small values of beam current (up to 1 μA). The change in the free oscillation frequency is negligibly small and the measured values for the resonant frequency correspond to the frequency of free axial oscillations, Q_z, determined only by the "stiffness" of the magnetic system. As can be seen in Figure 3, the accuracy of the determination of the axial oscillation frequency from the minimum current at the target is within ±(2.5--5) percent. On Figure 4 the experimental values of the axial oscillation frequency for various radii are compared to the calculated values computed from equation (5) where the values of n and ε were found during the measurements of the magnetic field of the cyclotron.

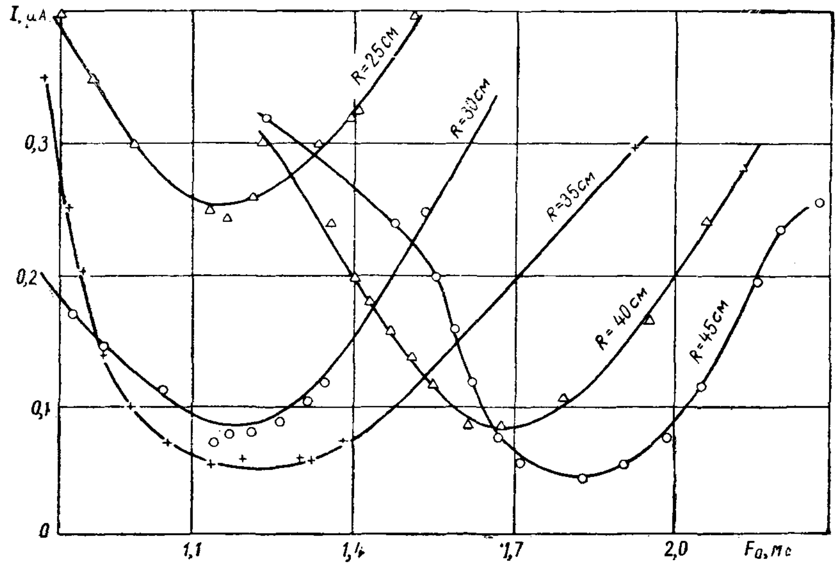

Figure 3. Target current as function of the frequency of electron excitation voltage.

The experimental investigation of the influence of the magnitude of the beam current on the axial oscillation frequency was carried out within the current range of 5 to 25 μA. The resonant frequency of the external excitation field was determined by the minimum value of the target current and a subsequent statistical processing of a large number of measurements at one point (according to the location of the target and the current of the accelerated ions). The root mean square error of all these measurements was reduced to 0.5--1 percent which is two to three times less than the effect studied. The preliminary series of measurements of the resonant frequency at various electrode voltages indicated that the possible influence of a decrease in beam current, within the region of deflecting electrodes, on the measured resonant frequency is less than the experimental error.

In Figure 5 are shown the results of the experimental determination of $\Delta Q_z/Q_z$ at various radii and a charge density corresponding to a beam current of 20 μA. In Figure 6, one finds the dependence of $\Delta Q_z/Q_z$ on the accelerator beam current for two accelerator radii. These graphs show that within the limits of experimental error, the dependence of the effect on the beam density and the measured values for $\Delta Q_z/Q_z$ agree with the results of the theoretical discussions.

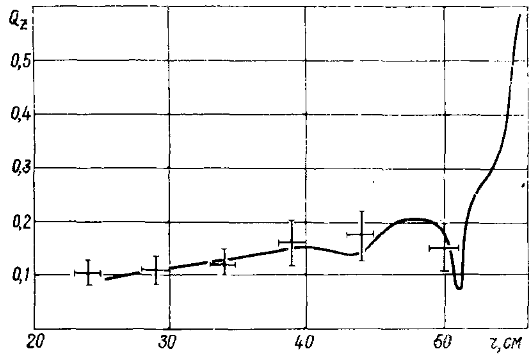

Figure 4. Calculated (solid line) and experimental values of the frequency of free axial oscillation within an isochronous cyclotron.

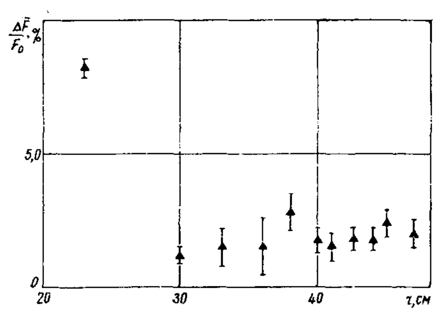

Figure 5. Decrease in axial oscillation frequency within an isochronous cyclotron for a beam current of 20 μA at various radii.

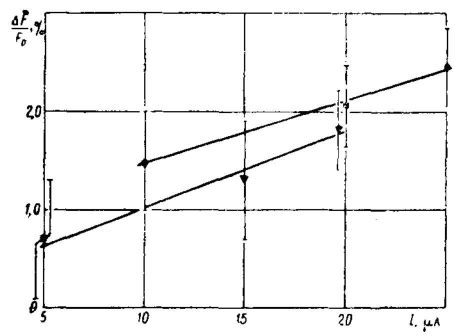

Figure 6. Decrease in axial oscillation frequency within an isochronous cyclotron as function of the beam current [●) R = 44 cm, ▼) R = 36.5 cm].

4. THE LIMITING INTENSITY OF THE INTERNAL BEAM WITHIN A RELATIVISTIC CYCLOTRON

It follows from equation (6) that the magnitude of the limiting current within a relativistic cyclotron is determined by the coulombic repulsion effect of the accelerated particles ($\Delta Q_z = Q_z$) and is equal to

$$i_{max} = \frac{Q_z^2 \Delta\varphi \Delta z \omega}{8\pi^2 e} \Delta E. \qquad (8)$$

On the other hand, according to equation (5), within a relativistic cyclotron at the start of the acceleration, $Q_z = 0$ and during the process of acceleration this quantity increases. Choosing an appropriate function for the increase in amplitude, $\varepsilon(r)$, along the radius one can, starting from a certain radius, maintain the frequency of these oscillations constant within a definite range, although it is desirable that the value of Q_z remains less than 0.5. The present-day methods for shimming of the magnetic field permit, with reasonable amplification limits, the maintenance of the given axial oscillation frequency to an accuracy of ±0.1. Consequently, the best acceptable range of variations for Q_z is in the 0.2--0.4 domain. In such a case, for a cyclotron having an energy of several hundred millions of electron volts and taking $\Delta\varphi = 0.5$ radian, $\Delta z = 2$ cm, $\Delta E = 200$ keV, $\omega = 0.75 \cdot 10^8$ sec^{-1}, we obtain a limiting current of 8.5 mA. It is quite true that the value obtained above is purely theoretical and even at lower charge densities there will occur losses of the beam at the dees.

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Our results show that the space charge effect does not prevent the achievement of beam intensities of the order of several milliamperes in a relativistic cyclotron. This effect causes only a spatial shift of the zone for resonant interaction of the oscillations.

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DISCUSSION

L. C. Teng

Could you say a few words about the derivation of your formula and the assumptions made?

Yu. I. Shvabe

The expressions describing the space charge influence were obtained starting from the linear equations of free oscillations, taking into account the electromagnetic field of the accelerated particles. We assumed that the particles within the bunch are uniformly distributed along the azimuth and that the vertical dimensions of the bunch are much smaller than its azimuthal extent.