METHODS AND MODELS FOR THE STUDY OF DECOHERENCE

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Abstract

We shall review methods used in the description of decoherence on particle probes in experiments due to surrounding media. This will include conventional media as well as a model for space-time foam arising from non-critical string theory.

1 The role of decoherence

Until recently in elementary particle physics the environment was not considered. Scatterings were calculated in a vacuum background and S-matrix elements were calculated within the paradigm of the standard gauge theory model. The latter is a successful theory overall. However, recently systems which oscillate coherently have been investigated with increasing precision, e.g. neutrino and neutral meson flavour oscillations. Clearly neutrinos produced in the sun, on going through it, encounter an obvious scattering environment. In laboratory experiments however there does not seem to be the need for such considerations; of course there are uncertainties in determining time and position which lead to features akin to decoherence [1]. However, triggered again by increased precision, the effect of fluctuations in the space-time metric due to space-time defects such as microscopic black holes, and D branes in string theory are being estimated. Given the smallness of the gravitational coupling compared to the other interactions in the past the search for such effects was regarded as optimisitc. Progress in experimental techniques is making such effects more testable [2].

In this Handbook it was considered to be desirable to split the discussion of decoherence between two chapters.This one will render a brief account of the methods of decoherence that are used in the analysis of experiments given in the companion chapter [3]. We shall demonstrate why there is a large universality class in the space of theories describing decoherence with most analyses using models from this class. However we should stress that the universality is for descriptions where the system-environment interaction is in some sense conventional. Indeed when we introduce descriptions emanating from string theory we can and do produce descriptions which can give qualitatively different effects [4]. Such non-conventional descriptions are to be expected since it is natural for quantum space-time to be somewhat different from the paradigm of Brownian phenomena in condensed matter. Moreover the manifestation of gravitational decoherence in a theory, which is diffeomorphic covariant at the classical level, is not just restricted to fluctuation and dissipation. It is pivotal in the breakdown of discrete symmetries such as CPT and more obviously T. This is an exciting role for decoherence because it gives rise to qualitatively new phenomena [5] which is being tested now and in the next generation of laboratory experiments.

This paper will be divided into three sections:

- decoherence in a general setting with a discussion of how coherence is lost and the implication for discrete symmetries
- generic treatment of system-reservoir interactions and the Lindblad formalism from Markovian approximations
- non-critical string theory and D-particle foam and the phenomenolgy of stochastic metrics

2 General Features of Decoherence

The fact that an environment $\mathcal{E}$ interacts with a system $\mathcal{S}$ and is affected by it is obvious whether they interact classically or quantum mechanically. However classically the measurement of $\mathcal{E}$ can only locally affect $\mathcal{S}$. This is in sharp contrast to the quantum mechanical situation where non-local effects can take place. The associated distinguishing property is that of entanglement. For the compound system $\mathcal{ES}$ Schmidt bases allow us to write the state $|\Psi\rangle$ as

$$|\Psi\rangle = \sum_n \sqrt{p_n}\, |\phi_n\rangle\, |\Phi_n\rangle$$

where the Hilbert space $H_{\mathcal{S}}$ of states $|\phi_n\rangle$ are associated with $\mathcal{S}$ and the Hilbert space $H_{\mathcal{E}}$ of states $|\Phi_n\rangle$are associated with $\mathcal{E}$. In the Schmidt basis the states for different n in the different spaces have to be mutually orthogonal i.e.

$$\langle\phi_n|\,\phi_m\rangle = \langle\Phi_n|\,\Phi_m\rangle = \delta_{nm}$$

and the non-negative coefficients p_n satisfy $\sum_n p_n^2 = 1$.

The corresponding density matrix is

$$\rho = \rho_{class.} + \sum_{n\neq m} \sqrt{p_n p_m}\,|\phi_n\rangle\langle\phi_m| \otimes |\Phi_n\rangle\langle\Phi_m|$$

where $\rho_{class.} = \sum_n p_n\,|\phi_n\rangle\langle\phi_n| \otimes |\Phi_n\rangle\langle\Phi_n|$. The term $\rho - \rho_{class.}$ is known as the entanglement. Clearly entanglement is a measure of the departure of the compound system from a product state of states of $\mathcal{S}$ and $\mathcal{E}$. A classic example of a pure entangled state is the EPR state (Einstein-Podolsky-Rosen) written conventionally in terms of spin $\frac{1}{2}$ systems

$$\frac{|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle}{\sqrt{2}}$$

which is clearly not factorisable. Now let us see how the interaction between $\mathcal{S}$ and $\mathcal{E}$ leads to decoherence by considering a simple interaction

$$\lambda H_{\mathcal{E}\mathcal{S}} = \sum_n |\phi_n\rangle\langle\phi_n| \otimes \widehat{A}_n$$

where $\widehat{A}_n$ are operators on the $H_{\mathcal{E}}$. For an initial pure unentangled state i.e. a product state

$$|\Psi\rangle = \sum_n c_n\,|\phi_n\rangle|\Theta_0\rangle$$

(where $|\Theta_0\rangle$ can be expressed in terms of the $|\Phi_n\rangle$s) under time evolution

$$|\phi_n\rangle|\Theta_0\rangle \xrightarrow{t} |\phi_n\rangle \exp\left(-i\widehat{A}_n t\right)|\Theta_0\rangle \equiv |\phi_n\rangle|\Theta_n(t)\rangle$$

The density matrix traced over the environment $\rho_{\mathcal{S}}(t)$ gives

$$\rho_{\mathcal{S}}(t) = \sum_{n,m} c_m^* c_n \langle\Theta_m(t)\,|\Theta_n(t)\rangle\,|\phi_m\rangle\langle\phi_n|$$

If the circumstances are such that $\langle\Theta_m(t)|\Theta_n(t)\rangle \longrightarrow \delta_{mn}$ as $t \longrightarrow \infty$, then asymptotically

$$\rho_S(t) \longrightarrow \sum_n |c_n|^2 |\phi_n\rangle\langle\phi_n|\,.$$

All coherences embodied by off-diagonal matrix elements have vanished, i.e. there is complete decoherence [6].

We will now consider an *associated* aspect of the interaction of the system with the environment, the lack of an invertible scattering matrix. Consider schematically three spaces $\mathfrak{H}_1, \mathfrak{H}_2$ and $\mathfrak{H}_3$ where $\mathfrak{H}_1$ is the space of states of the initial states, $\mathfrak{H}_2$ is the state space for inaccessible environmental degrees of freedom (e.g. states inside a black hole horizon) and $\mathfrak{H}_3$ is the space of final states. Within a scattering matrix formalism consider an in-state $\sum_A x_A |X_A\rangle_1 |0\rangle_2 |0\rangle_3$ (where the subscripts 1, 2 and 3 are related to the spaces $\mathfrak{H}_1, \mathfrak{H}_2$ and $\mathfrak{H}_3$) this is scattered to $\sum_A \mathbb{S}_A^{bc} x_A |0\rangle_1 \left|\overline{Y}^b\right\rangle_2 \left|\overline{Z}^c\right\rangle_3$ where the bar above the state labels indicates the CPT transform [7]. (On introducing the operator $\theta = CPT$ we have explicitly $\left|\overline{Y}^b\right\rangle = \theta|Y_b\rangle$ etc.) Now on tracing over the inaccessible degrees of freedom (in $\mathfrak{H}_2$) we obtain

$$|X_A\rangle\langle X_A| \longrightarrow \sum_{c,c'} \not{S}_{A\,A}^{\,c\,c'} \left|\overline{Z}^c\right\rangle\left\langle\overline{Z}^{c'}\right|$$

with the effective scattering matrix $\not{S}$ given by

$$\not{S}_{A\,A}^{\,c\,c'} = \sum_{b,b'} \mathbb{S}_A^{bc}\mathbb{S}_A^{*\,b'c'}.$$

This does not factorise, which it would have to, for $/S$ to be of the form $UU^\dagger$. Consequently evolution is non-unitary. This is generic to environmental decoherence. Of course with space-time defects the inaccessible degrees of freedom can be behind causal horizons.

For local relativistic interacting quantum field theories there is the CPT theorem. Such theories show unitary evolution. A violation of CPT for Wightman functions (i.e. unordered correlation functions for fields) implies violation of Lorentz invariance [8]. However CPT invariance of course is not sufficient for Lorentz invariance. For physical systems, which in the absence of gravity show CPT invariance, the incorporation of a gravitational environment can lead to non-unitary evolution as we have argued. In fact we shall sketch arguments from non-critical string theory which produce such non-unitary evolution. There is then a powerful argument due to Wald which argues that an operator θ incorporating strong CPT invariance does not exist. The argument proceeds via reduction ad absurdum. For strong CPT invariance to hold we should have in states and out states connected by $\$$ and θ and their operations commute in the following sense. For an in state ρ_{in} there is an out state ρ_{out} such that

$$\rho_{out} = \$\rho_{in}.$$

Also there is another out state $\rho'_{out} = \theta\rho_{in}$ associated with ρ_{in}. If the CPT transforms of states have the same $\$$ evolution as the untransformed states then there is strong CPT invariance. In such situations

$$\theta\ \$\theta\ \$\rho_{in} = \rho_{in}$$

and so

$$\theta\ \$\theta\ \$ = I,$$

i.e. $\$$ has an inverse. In most circumstances interaction with an environment produces dissipation and so the inverse of $\$$ would not exist. Hence the assumption of strong CPT is incompatible with non-unitary evolution [9].

3 Particles propagating in a medium and master equations

Particles reaching us from outside a laboratory always travel through some physical medium which can often be described by a conventional medium. For the moment we will be general and call the medium $\mathcal{E}$ and the particle $\mathcal{S}$. We are ignoring particle-particle interactions and so the approximation of a single body point of view is appropriate. This bipartite separation can be subtle since different degrees of freedom of the same particle can be distributed between $\mathcal{E}$ and $\mathcal{S}$. Initially (at $t = t_0$) the state ρ of the compound system is assumed to have a factorised form

$$\rho\left(t_0\right) = \rho_{\mathcal{S}} \otimes \rho_{\mathcal{E}} \tag{1}$$

with $\rho_{\mathcal{S}}$ being a normalised density operator on the Hilbert space $\mathfrak{H}_{\mathcal{S}}$ of states of $\mathcal{S}$ and analogously for $\rho_{\mathcal{E}}$. This condition may be not hold in the very early universe and for an ever present meidum such as space-time foam; $\mathcal{E}$ and $\mathcal{S}$ would then always be entangled. Certainly for laboratory experiments the condition 1 is acceptable [10] and the analysis is simplified. Write the total hamiltonian H as

$$H = H_{\mathcal{S}} + H_{\mathcal{E}} + \widehat{H}_{\mathcal{S}\mathcal{E}}$$

where $H_{\mathcal{S}\mathcal{E}}$ represents the interaction coupling the system and environment.The Heisenberg equation is

$$\frac{\partial \rho}{\partial t} = -i\left[H_{\mathcal{S}} + H_{\mathcal{E}} + H_{\mathcal{S}\mathcal{E}}, \rho\right] \equiv L\rho \tag{2}$$

and we will also find it useful to let $-i\left[H_{\mathcal{S}}, \rho\right] \equiv L_{\mathcal{S}}\rho$, $-i\left[H_{\mathcal{E}}, \rho\right] \equiv L_{\mathcal{E}}\rho$ and $-i\left[H_{\mathcal{S}\mathcal{E}}, \rho\right] \equiv L_{\mathcal{S}\mathcal{E}}\rho$. ρ evolves unitarily. For measuring with operators acting on $\mathfrak{H}_{\mathcal{S}}$ it is sufficient to consider

$$\rho_{\mathcal{S}} = Tr_B \rho \tag{3}$$

but given a $\rho_{\mathcal{S}}$ there is in general no unique ρ associated with it. Hence the evoultion of $\rho_{\mathcal{S}}$ is not well defined. However by choosing a reference environment state $\rho_{\mathcal{E}}$ satisfying

$$L_{\mathcal{E}}\rho_{\mathcal{E}} = 0 \tag{4}$$

we can associate with a $\rho_{\mathcal{S}}$ a unique state $\rho_{\mathcal{S}} \otimes \rho_{\mathcal{E}}$ of $S\mathcal{E}$. In this way a well defined evolution can be envisaged.

We will obtain a master equation for $\rho_{\mathcal{S}}$ by using the method of projectors [11]. Let us define

$$P\rho = \left(Tr_{\mathcal{E}}\rho\right) \otimes \rho_{\mathcal{E}}.$$

Clearly

$$P^2\rho = \left[Tr_{\mathcal{E}}\rho_{\mathcal{E}}\right]\left(Tr_{\mathcal{E}}\rho\right) \otimes \rho_{\mathcal{E}} = \left(Tr_{\mathcal{E}}\rho\right) \otimes \rho_{\mathcal{E}} = P\rho$$

and so P is a projector. Also we define $Q = 1 - P$. Acting on 2 with P gives

$$P\frac{\partial \rho}{\partial t} = PL\rho = PLP\rho + PLQ\rho. \tag{5}$$

Similarly

$$Q\frac{\partial \rho}{\partial t} = QL\rho = QLP\rho + QLQ\rho. \tag{6}$$

These give two coupled equations for $P\rho$ and $Q\rho$. 6 can be solved for $Q\rho$ on noting that

$$\left(\frac{\partial}{\partial t} - QL\right) Q\rho = QLP\rho$$

and then on formally integrating

$$\int_0^t \frac{\partial}{\partial t'}\left(e^{-QLt'} Q\rho\left(t'\right)\right) dt' = \int_0^t e^{-QLt'} QLP\rho\left(t'\right) dt'$$

i.e.

$$e^{-QLt} Q\rho(t) = Q\rho(0) + \int_0^t e^{-QLt'} QLP\rho\left(t'\right) dt'. \tag{7}$$

This expression for $Q\rho$ is substituted in 5 to give

$$Tr_{\mathcal{E}}\left[P\frac{\partial\rho}{\partial t}\right] = \frac{\partial\rho_{\mathcal{S}}}{\partial t} = Tr_{\mathcal{E}}\left[PLP\rho\right] + Tr_{\mathcal{E}}\left[PL\left(e^{-QLt} Q\rho(0) + \int_0^t e^{-QL\left(t-t'\right)} QL\right.\right.$$

and can be simplified further on noting that

$$PL_{\mathcal{E}}Q\rho = PL_{\mathcal{E}}\left(\rho - P\rho\right) = PL_{\mathcal{E}}\rho = 0 \Longrightarrow PL_{\mathcal{E}} = 0 \tag{8}$$

owing to the cyclic properties of traces. Also

$$PL_{\mathcal{S}}Q\rho = PL_{\mathcal{S}}\left(\rho - \rho_{\mathcal{S}} \otimes \rho_{\mathcal{E}}\right) = L_{\mathcal{S}}P\rho - \left(L_{\mathcal{S}}\rho_{\mathcal{S}}\right) \otimes \rho_{\mathcal{E}} = 0 \Longrightarrow PL_{\mathcal{S}} = PL_{\mathcal{S}}P. \tag{9}$$

Hence

$$\begin{aligned} Tr_{\mathcal{E}}\left(PL_{\mathcal{S}}P\rho\right) &= Tr_{\mathcal{E}}\left(PL_{\mathcal{S}}\rho\right) \\ &= Tr_{\mathcal{E}}\left(Tr_{\mathcal{E}}\left(L_{\mathcal{S}}\rho\right) \otimes \rho_{\mathcal{E}}\right) = Tr_{\mathcal{E}}\left(L_{\mathcal{S}}\rho\right) = L_{\mathcal{S}}\rho_{\mathcal{S}}. \end{aligned}$$

Also we assume that $H_{\mathcal{SE}} = V_{\mathcal{S}} \otimes V_{\mathcal{E}}$ (which is standard for local quantum field theory) and so

$$\begin{aligned} Tr_{\mathcal{E}}\left(PL_{\mathcal{SE}}P\rho\right) &= Tr_{\mathcal{E}}\left(PL_{\mathcal{SE}}\rho_{\mathcal{S}} \otimes \rho_{\mathcal{E}}\right) \\ &= Tr_{\mathcal{E}}\left[P\left(V_{\mathcal{S}}\rho_{\mathcal{S}}\right) \otimes \left(V_{\mathcal{E}}\rho_{\mathcal{E}}\right) - P\left(\rho_{\mathcal{S}}V_{\mathcal{S}}\right) \otimes \left(\rho_{\mathcal{E}}V_{\mathcal{E}}\right)\right] \\ &= Tr_{\mathcal{E}}\left[V_{\mathcal{S}}\rho_{\mathcal{S}} \otimes \rho_{\mathcal{E}}Tr_{\mathcal{E}}\left(V_{\mathcal{E}}\rho_{\mathcal{E}}\right) - \rho_{\mathcal{S}}V_{\mathcal{S}} \otimes \rho_{\mathcal{E}}Tr_{\mathcal{E}}\left(\rho_{\mathcal{E}}V_{\mathcal{E}}\right)\right] \\ &= \left[V_{\mathcal{S}}, \rho_{\mathcal{S}}\right] \otimes \rho_{\mathcal{E}}\left(Tr_{\mathcal{E}}\left(V_{\mathcal{E}}\rho_{\mathcal{E}}\right)\right) \\ &= Tr_{\mathcal{E}}\left(L_{\mathcal{SE}}\rho_{\mathcal{E}}\right)\rho_{\mathcal{S}}. \end{aligned}$$

The analysis would go through also when $H_{\mathcal{SE}}$ is a sum of factorised terms. Similarly on using 8 and 9

$$Tr_{\mathcal{E}}\left(PLe^{QLt}Q\rho\left(0\right)\right)=Tr_{\mathcal{E}}\left(L_{\mathcal{SE}}e^{QLt}Q\rho\left(0\right)\right)$$

and

$$Tr_{\mathcal{E}}\left(PLe^{QLt'}QLP\rho\left(t-t'\right)\right)=Tr_{\mathcal{E}}\left(L_{\mathcal{SE}}e^{QLt'}QL\rho_{S}\left(t-t'\right)\otimes\rho_{\mathcal{E}}\right).$$

In summary the master equation reduces to

$$\frac{\partial}{\partial t}\rho_{S}\left(t\right)=L_{\mathcal{S}}^{eff}\left[\rho_{S}\left(t\right)\right]+\int_{0}^{t}\mathcal{K}\left(t'\right)\left[\rho_{S}\left(t-t'\right)\right]+\mathcal{J}\left(t\right)\tag{10}$$

with

$$\begin{aligned}L_{\mathcal{S}}^{eff} &\equiv L_{S}+Tr_{\mathcal{E}}\left(L_{\mathcal{SE}}\rho_{\mathcal{E}}\right),\\ \mathcal{K}\left(t\right)\left[\rho_{S}\right] &= Tr_{\mathcal{E}}\left(L_{\mathcal{SE}}e^{QLt}QL\left[\rho_{S}\otimes\rho_{\mathcal{E}}\right]\right),\\ \mathcal{J}\left(t\right) &= Tr_{\mathcal{E}}\left(L_{\mathcal{SE}}e^{QLt}Q\rho\left(0\right)\right).\end{aligned}$$

In general it is an integro-differential equation with a memory kernel $\mathcal{K}$. Since the evolution of ρ is unitary, the positivity of ρ is maintained. The partial trace $\rho_S\left(t\right)$ of the positive operator ρ preserves the positivity. (10) is exact and so guarantees a positive $\rho_S\left(t\right)$. It is only when approximations (truncations) are made that positivity may be lost. The Markov approximation occurs if there is a timescale $\tau_{\mathcal{E}}$ associated with $\mathcal{K}\left(t\right)$ which is much shorter than $\tau_{\mathcal{S}}$ the natural time scale of the system $\mathcal{S}$ i.e. $\frac{\tau_{\mathcal{S}}}{\tau_{\mathcal{E}}}\longrightarrow\infty$. This Markov approximation has to be done carefully for otherwise positivity can be lost [12]. Mathematically there is another singular solution of this limit, $\tau_{\mathcal{S}}\longrightarrow\infty$ with $\tau_{\mathcal{E}}$ finite [13] which leads to the phenomenonology of dynamical semi-groups and the Lindblad formalism [14].

Definition 1 *Time evolutions Λ_t with $t\geq0$ form a dynamical semi-group if a) $\Lambda_{t_1}\circ\Lambda_{t_2}=\Lambda_{t_1+t_2}$, b) $Tr\left[\Lambda_t\rho\right]=Tr\left[\rho\right]$ for all t and ρ and c) are positive i.e map positive operators into positive operators.*

There are other technical conditions such as strong continuity which we will not dwell on. As far as applications are concerned the most important characterisation of dynamical semi-groups is that they arise from the singular limit mentioned above and are governed by the following theorem due to Lindblad:

Theorem 2 *If $P(\mathfrak{H})$ denotes the states on a Hilbert space $\mathfrak{H}$, and L is a bounded linear operator which is the generator of a dynamical semi-group (i.e. $\Lambda_t = e^{Lt}$), then*

$$L[\rho] = -i[H,\rho] + \frac{1}{2}\sum_j \left(\left[V_j\rho, V_j^\dagger\right] + \left[V_j, \rho V_j^\dagger\right]\right)$$

where $H\left(= H^\dagger\right)$, V_j and $\sum_j V_j^\dagger V_j$ are bounded linear operators on $\mathfrak{H}$.

This is the Lindblad form which has been used extensively in high energy physics phenomenology. $L[\rho]$, in the absence of the terms involving the Vs, is the Liouville operator. H is the hamiltonian which generally could be in the presence of a background stochastic classical metric [15)] (as we will discuss later). Such effects may generally arise from back-reaction of matter within a quantum theory of gravity [16)] which decoheres the gravitational state to give a stochastic ensemble description. In phenomenological analyses a theorem due to Gorini, Kossakaowski and Sudarshan [17)] on the structure of L, the generator of a quantum dynamical semi-group [14, 17)] is of importance. This states that for a non-negative matrix c_{kl} (i.e. a matrix with non-negative eigenvalues) such a generator is given by

$$\frac{d\rho}{dt} = \mathcal{L}[\rho] = -i[H,\rho] + \frac{1}{2}\sum_{k,l} c_{kl}\left([F_k\rho, F_l^\dagger] + [F_k, \rho F_l^\dagger]\right),$$

where $H = H^\dagger$ is a hermitian Hamiltonian, $\{F_k, k = 0, ..., n^2-1\}$ is a basis in $M_n(\mathbf{C})$ such that $F_0 = \frac{1}{\sqrt{n}} I_n$, $\mathrm{Tr}(F_k) = 0\ \forall k \neq 0$ and $\mathrm{Tr}(F_i^\dagger F_j) = \delta_{ij}$ [17)]. In applications we can take $F_i = \frac{\Lambda_i}{2}$ (where, for example, Λ_i are the Gell-Mann matrices) and satisfy the Lie algebra $[F_i, F_j] = i\sum_k f_{ijk}F_k, (i = 1, ...8)$, f_{ijk} being the standard structure constants, antisymmetric in all indices. It can always be arranged that the sum over k and l run over $1, \ldots, 8$. Without a microscopic model, in the three generation case, the precise physical significance of the matrix c_{kl} cannot be understood. Moreover a general parametrisation of c_{kl} is too complicated to have any predictive power.

It is precise in formulation but gives no inkling of its $\mathcal{SE}$ compound system progenitor. Therein lies its weakness [18] but nonetheless it has been useful in providing 'test' theories and estimating orders of magnitudes for the strength of effects. If the strength of effects are in accord with a theoretical picture then it has been customary to conclude that the source of the decoherence is compatible with the theoretical picture. Recently it has been argued that this may be too simplistic and it is necessary to delve into the background $\mathcal{SE}$ to be able to argue in favour of a picture.

4 Master Equations from (Non-critical) String Theory

When neutrinos from the Sun are produced (e.g. from the nuclear $p-p$ cycle) and pass through it, the nature of $\mathcal{E}$ and $L_{\mathcal{SE}}$ can be understood from the gauge theories of the weak interactions [19]. Consequently the programme outlined in the previous paragraph with a perturbative evaluation of $\mathcal{K}(t)$ is feasible in principle. However in recent years there has been a debate on whether microscopic black holes can induce quantum decoherence at a microscopic level. The presence of quantum-fluctuating microscopic horizons, of radius of the order of Planck length (10^{-35} m), may give space-time a "foamy" structure, causing decoherence of matter propagating in it. In particular, it has been suggested [20] that such Planck-scale black holes and other topological fluctuations in the space-time background cause a breakdown of the conventional S-matrix description of asymptotic scattering in local quantum field theory.Hence when we consider space-time foam we are on less firm ground for applying the Lindblad formalism. Clearly gleaning an understanding of the nature of space-time itself raises a huge number of foundational issues. String theory is one attempt to address such questions but is still far from the goal of clarifying strong gravity. There are some who even believe that gravity is an emergent feature and consequently that an attempt to understand the quantum aspects of gravity may be fundamentally futile. It is not appropriate to enter this debate here. As far as experiments are concerned, both now and in the near future, it is reasonable to ask what the current theories have to say concerning quantum effects where a nearly flat metric gravity is clearly reasonable.

The issue of quantum-gravity-induced decoherence is controversial and worthy of further phenomenological exploitation. We shall restrict ourselves to a specific framework for analyzing decoherent propagation of low-energy matter in foamy space-time backgrounds in the context of string theory [21, 22], the so-called Liouville-string [23] decoherence [24]. One motivation for using string theory is that it appears to be the best controlled theory of quantum gravity available to date. At this juncture we should also mention that there are other interesting approaches to quantum space-time foam, which also lead to experimental predictions, e.g. the "thermal bath" approach advocated in [25], according to which the foamy gravitational environment may behave as a thermal bath; this induces decoherence and diffusion in the propagating matter, as well as quantum damping in the evolution of low-energy observables, features which are, at least in principle, testable experimentally. As we shall see presently, similar behaviour is exhibited by the specific models of foam that we study here; the D-particle foam model of [26, 27] may characterize modern versions of string theory [22], and are based on point-like membrane defects in space-time (D-particles). Such considerations have more recently again come to the fore because of current neutrino data including LSND data [28]. There is experimental evidence, that the neutrino has mass which leads to neutrino oscillations. However LSND results appear consistent with the dominance of anti-neutrino oscillations $\overline{\nu}_e \rightleftarrows \overline{\nu}_\mu$ over neutrino oscillations. In particular, provided LSND results turn out to be correct, which at present is quite unclear, there is evidence for CPT violation. It has been suggested recently [5] that Planck scale quantum decoherence may be a relevant contribution to the CPT violation seen in the experiments of LSND. Other examples of flavour oscillating systems with quite different mass scales are furnished by $B\overline{B}$ and $K\overline{K}$ systems [29]. The former because of the large masses involved provides a particularly sensitive system for investigating the Planck scale fluctuations embodied by space-time foam. In all these cases, experiments, such as CPLEAR [30], provide very low bounds on CPT violation which are not inconsistent with estimates from dimensional analysis for the magnitudes of effects from space-time foam. These systems have been analyzed within a dynamical semigroup approach to quantum Markov processes. Once the framework has been accepted then a master equation for finite-dimensional systems ensued which was characterized by a small set of parameters. This approach is somewhat phenomenological and is primarily used to fit data [31, 32, 33]. Consequently it is important to obtain a better understanding of the nature of decoherence from a more fundamental viewpoint.

Given the very limited understanding of gravity at the quantum level, the analysis of modifications of the quantum Liouville equation implied by non-critical strings can only be approximate and should be regarded as circumstantial evidence in favour of the dissipative master equation. In the context of two-dimensional toy black holes [34] and in the presence of singular space-time fluctuations there are believed to be inherently unobservable delocalised modes which fail to decouple from light (the observed) states. The effective theory of the light states which are measured by local scattering experiments can be described by a non-critical Liouville string. This results in an irreversible temporal evolution in target space with decoherence and associated entropy production.

The following master equation for the evolution of stringy low-energy matter in a non-conformal σ-model can be derived [24]

$$\partial_t \rho = i\left[\rho, H\right] + :\beta^i \mathcal{G}_{ij}\left[g^j, \rho\right]: \tag{11}$$

where t denotes time (Liouville zero mode), the H is the effective low-energy matter Hamiltonian, g^i are the quantum background target space fields, β^i are the corresponding renormalization group β functions for scaling under Liouville dressings and $\mathcal{G}_{ij}$ is the Zamolodchikov metric [35, 36] in the moduli space of the string. The double colon symbol in (11) represents the operator ordering $: AB := [A, B]$. The index i labels the different background fields as well as space-time. Hence the summation over i, j in (11) corresponds to a discrete summation as well as a covariant integration $\int d^{D+1}y\,\sqrt{-g}$ where y denotes a set of $(D+1)$-dimensional target space-time co-ordinates and D is the space-time dimensionality of the original non-critical string.

The discovery of new solitonic structures in superstring theory [22] has dramatically changed the understanding of target space structure. These new non-perturbative objects are known as D-branes and their inclusion leads to a scattering picture of space-time fluctuations. Heuristically, when low energy matter given by a closed (or open) string propagating in a $(D+1)$-dimensional space-time collides with a very massive D-particle embedded in this space-time, the D-particle recoils as a result. Since there are no rigid bodies in general relativity the recoil fluctuations of the brane and their effectively stochastic back-reaction on space-time cannot be neglected. On the brane there are closed and open strings propagating. Each time these strings cross with a D-particle, there is a possibility of being attached to it, as indicated in Fig. 1. The entangled state causes a back reaction onto the space-time, which can be calculated perturbatively using logarithmic conformal field theory formalism [37].

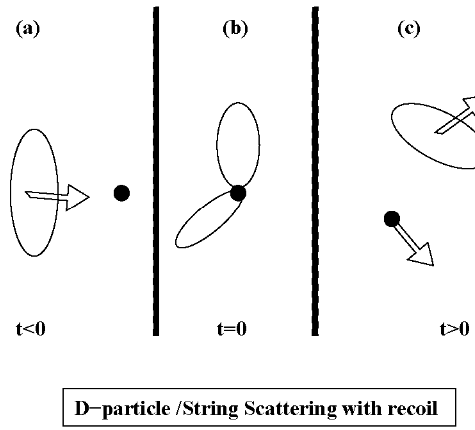

Figure 1: *Schematic picture of the scattering of a string matter state on a D-particle, including recoil of the latter. The sudden impulse at* $t = 0$, *implies a back reaction onto the space time, which is described by a logarithmic conformal field theory. The method allows for the perturbative calculation of the induced space-time distortion due to the entangled state in (b).*

Now for large Minkowski time t, the non-trivial changes from the flat metric produced from D-particle collisions are

$$g_{0i} \simeq \overline{u}_i \equiv \frac{u_i}{\varepsilon} \propto \frac{\Delta p_i}{M_P} \tag{12}$$

where u_i is the velocity and Δp_i is the momentum transfer during a collision, ε^{-2} is identified with t and M_P is the Planck mass (actually, to be more precise $M_P = M_s/g_s$, where $g_s < 1$ is the (weak) string coupling, and M_s is a string mass scale); so g_{0i} is constant in space-time but depends on the energy content of the low energy particle and the Ricci tensor $R_{MN} = 0$ where M and N are target space-time indices. Since we are interested in fluctuations of the metric the indices i will correspond to the pair M, N. However, recent astrophysical observations from different experiments all seem to indicate that 73% of the energy of the Universe is in the form of dark energy. Best fit models give the positive cosmological constant Einstein-Friedman Universe as a good candidate to explain these observations. For such de Sitter backgrounds $R_{MN} \propto \Omega g_{MN}$ with $\Omega > 0$ a cosmological constant. Also in a perturbative derivative expansion (in powers of α' where $\alpha' = l_s^2$ is the Regge slope of the string and l_s is the fundamental string length) in leading order

$$\beta_{\mu\nu} = \alpha' R_{\mu\nu} = \alpha' \Omega g_{\mu\nu} \tag{13}$$

and

$$\mathcal{G}_{ij} = \delta_{ij}. \tag{14}$$

This leads to

$$\partial_t \rho = i\,[\rho, H] + \alpha'\Omega : g_{MN}\left[g^{MN}, \rho\right] : \tag{15}$$

For a weak-graviton expansion about flat space-time, $g_{MN} = \eta_{MN} + h_{MN}$, and

$$h_{0i} \propto \frac{\Delta p_i}{M_P}. \tag{16}$$

If an antisymmetric ordering prescription is used, then the master equation for low energy string matter assumes the form

$$\partial_t \dot{\rho}_{Matter} = i\,[\rho_{Matter}, H] - \Omega\left[h_{0j}, \left[h^{0j}, \rho_{Matter}\right]\right] \tag{17}$$

(when α' is absorbed into Ω). In view of the previous discussion this can be rewritten as

$$\partial_t \dot{\rho}_{Matter} = i\,[\rho_{Matter}, H] - \Omega\left[\overline{u}_j, \left[\overline{u}^j, \rho_{Matter}\right]\right] \ . \tag{18}$$

thereby giving the *master equation for Liouville decoherence* in the model of a D-particle foam with a cosmological constant.

The above D-particle inspired approach deals with possible non-perturbativ quantum effects of gravitational degrees of freedom. The analysis is distinct from the phenomenology of dynamical semigroups which does not embody specific properties of gravity. Indeed the phenomenology is sufficiently generic that other mechanisms of decoherence such as the MSW effect can be incorporated within the same framework. Consequently an analysis which is less generic and is related to the specific decoherence implied by non-critical strings is necessary.It is sufficient to study a massive non-relativistic particle propagating in one dimension to establish qualitative features of D-particle decoherence. The environment will be taken to consist of both gravitational and non-gravitational degrees of freedom; hence we will consider a generalisation of quantum Brownian motion for a particle which has additional interactions with D-particles. This will allow us to compare qualitatively the decoherence due to different environments.The non-gravitational degrees of freedom in the environment (in a thermal state) are conventionally modelled by a collection of harmonic oscillators with masses m_n, frequency ω_n and co-ordinate operator $\widehat{q}_n$ coupled to the particle co-ordinate $\widehat{x}$ by an interaction of the form $\sum_n g_n \widehat{x}\widehat{q}_n$. The master equation which is derived can have time dependent coefficients due to the competing timescales, e.g. relaxation rate due to coupling to the thermal bath, the ratio of the time scale of the harmonic oscillator to the thermal time scale etc. However an ab initio calculation of the time-dependence is difficult to do in a rigorous manner. It is customary to characterise the non-gravitational environment by means of its spectral density $I(\omega)\left(=\sum_n \delta(\omega-\omega_n)\frac{g_n^2}{2m_n\omega_n}\right)$. The existence of the different time scales leads in general to non-trivial time dependences in the coefficients in the master equation which are difficult to calculate in a rigorous manner [38]. The dissipative term in (18) involves the momentum transfer operator due to recoil of the particle from collisions with D-particles (12). This transfer process will be modelled by a classical Gaussian random variable r which multiplies the momentum operator $\widehat{p}$ for the particle:

$$\overline{u_i} \quad \rightarrow \quad \frac{r}{M_P}\widehat{p} \tag{19}$$

Moreover the mean and variance of r are given by

$$\langle r\rangle = 0 \;, \qquad \text{and} \qquad \left\langle r^2\right\rangle = \sigma^2 \;. \tag{20}$$

On amalgamating the effects of the thermal and D-particle environments, we have for the reduced master equation [39] for the matter (particle) density matrix ρ (on dropping the Matter index)

$$i\frac{\partial}{\partial t}\rho = \frac{1}{2m}\left[\widehat{p}^{2},\rho\right] - i\Lambda\left[\widehat{x},\left[\widehat{x},\rho\right]\right] + \frac{\gamma}{2}\left[\widehat{x},\left\{\widehat{p},\rho\right\}\right] - i\Omega r^{2}\left[\widehat{p},\left[\widehat{p},\rho\right]\right] \tag{21}$$

where Λ, γ and Ω are real time-dependent coefficients. As discussed in [39] a possible model for $\Omega\left(t\right)$ is

$$\Omega\left(t\right) = \Omega_{0} + \frac{\widetilde{\gamma}}{a+t} + \frac{\widetilde{\Gamma}}{1+bt^{2}} \tag{22}$$

where ω_{0}, $\widetilde{\gamma}$, a, $\widetilde{\Gamma}$ and b are positive constants. The quantity $\widetilde{\gamma} < 1$ contains information on the density of D-particle defects on a four-dimensional world.The time dependence of γ and Λ can be calculated in the weak coupling limit for general n (i.e. ohmic, $n = 1$ and non-ohmic $n \neq 1$ environments) where

$$I\left(\omega\right) = \frac{2}{\pi}m\gamma_{0}\omega\left[\frac{\omega}{\varpi}\right]^{n-1}e^{-\omega^{2}/\varpi^{2}} \tag{23}$$

and ϖ is a cut-off frequency. The precise time dependence is governed by $\Lambda\left(t\right) - \int_{0}^{t} ds\, \nu\left(s\right)$ and $\gamma\left(t\right) - \int_{0}^{t} ds\, \nu\left(s\right) s$ where $\nu\left(s\right) - \int_{0}^{\infty} d\omega\, I\left(\omega\right) \coth\left(\beta\hbar\omega/2\right.$ For the ohmic case, in the limit $\hbar\varpi \ll k_{B}T$ followed by $\varpi \to \infty$, Λ and γ are given by $m\gamma_{0}k_{B}T$ and γ_{0} respectively after a rapid initial transient. For high temperatures Λ and γ have a powerlaw increase with t for the subohmic case whereas there is a rapid decrease in the supraohmic case.

5 CPT and Recoil

The above model of space-time foam refers to a specific string-inspired construction. However the form of the induced back reaction (12) onto the space-time has some generic features, and can be understood more generally in the context of effective theories of such models, which allows one to go beyond a specific non-critical (Liouville) model. Indeed, the D-particle defect can be viewed as an idealisation of some (virtual, quantum) black hole defect of the ground state of quantum gravity, viewed as a membrane wrapped around some small extra dimensions of the (stringy) space time, and thus appearing to a four-dimensional observer as an "effectively" point like defect. The back reaction on space-time due to the interaction of a pair of neutral mesons, such as those produced in a meson factory, with such defects can be studied generically as follows: consider the non-relativistic recoil motion of the heavy defect, whose coordinates in space-time,in the laboratory frame, are $y^i = y^i_0 + u^i t$, with u^i the (small) recoil velocity.One can then perform a (infinitesimal) general coordinate transformation $y^\mu \to x^\mu + \xi^\nu$ so as to go to the rest (or co-moving) frame of the defect after the scattering. From a passive point of view, for one of the mesons, this corresponds to an induced change in metric of space-time of the form (in the usual notation, where the parenthesis in indices denote symmetrisation) $\delta g_{\mu\nu} = \partial_{(\mu}\xi_{\nu)}$, which in the specific case of non-relativistic defect motion yields the off-diagonal metric elements (12). Such transformations cannot be performed simultaneously for both mesons, and moreover in a full theory of quantum gravity the recoil velocities fluctuate randomly, as we shall discuss later on. This means that the effects of the recoil of the space-time defect are observable. The mesons will feel such effects in the form of induced fluctuating metrics (12). It is crucial to note that the interaction of the matter particle (meson) with the foam defect may also result in a "flavour" change of the particle (e.g. the change of a neutral meson to its antiparticle). This feature can be understood in a D-particle Liouville model by noting that the scattering of the matter probe off the defect involves first a splitting of a closed string representing matter into two open ones, but with their ends attached to the D-particle, and then a joining of the string ends in order to re-emit a closed string matter state. The re-emitted (scattered) state may in general be characterised by phase, flavour and other quantum charges which may not be required to be conserved during black hole evaporation and disparate space-time-foam processes. In our application we shall restrict ourselves only to effects that lead to flavour changes. The modified form of the metric fluctuations (12) of each component of the metric tensor $g^{\alpha\beta}$ will not be simply given by the simple recoil distortion (12), but instead can be taken to have a 2×2 ("flavour") structure [4]:

$$
\begin{aligned}
g^{00} &= (-1 + r_4)\,\mathsf{1} \\
g^{01} &= g^{10} = r_0 \mathsf{1} + r_1\sigma_1 + r_2\sigma_2 + r_3\sigma_3 \\
g^{11} &= (1 + r_5)\,\mathsf{1}
\end{aligned}
\tag{24}
$$

where $\mathsf{1}$, is the identity and σ_i are the Pauli matrices. The above parametrisation has been taken for simplicity and we can also consider motion to be in the x- direction which is natural since the meson pairs move collinearly. A metric with this type of structure is compatible with the view that the D-particle defect is a "point-like" approximation for a compactified higher-dimensional brany black hole, whose no hair theorems permit non-conservation of flavour.(In the case of neutral mesons the concept of "flavour" refers to either particle/antiparticle species or the two mass eigenstates). The detailed application of this model to the ω effect for neutral mesons can be found in [4].

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