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# 1/2 spin-isospin fermions close to the unitary limit

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**Abstract.** The equal mass three-fermion system having 1/2 spin-isospin symmetry is studied around the unitary limit. The two body system has two different scattering lengths,  $a_0$  and  $a_1$ , corresponding to the spin singlet state  $S = 0$  and the spin triplet state  $S = 1$ , respectively. The unitary limit is defined when the two quantities  $a_0, a_1 \rightarrow \infty$ . The three-nucleon system is located very close to this limit, the singlet and triplet  $n - p$  scattering lengths are large with respect to the range of the nuclear interaction. The ratio of the two is about  $a_0/a_1 \approx -4.31$ . This value defines a plane in which  $a_0$  and  $a_1$  can be varied. Using a nucleon-nucleon spin dependent potential with variable strength it is possible to study the behavior of the three-nucleon binding energy along that plane. This analysis can be considered an extension of the Efimov plot for three bosons to the case of three 1/2-spin-isospin fermions.

## 1. Introduction

Weakly bound states have always attracted some attention because the particles have a large probability to be outside the interaction range and the system has universal characteristics. The limiting case is the unitary limit in which the two-body scattering length diverges,  $1/a = 0$ . In this case the two-body system is resonant and can be described reasonably well in the zero-range approximation. In a series of papers [1, 2] V. Efimov has shown that a system of three identical bosons interacting through a two-body short-range potential has a geometrical series of energy levels at the unitary limit accumulating at zero energy. The ratio between the energies of two consecutive states is constant and, remarkably it results independent of the particular form of the interaction. This behavior has been denoted Efimov effect and its universal characteristic has given to this effect a particular relevance. In fact in the last two decades an enormous amount of work has been dedicated to study this effect in different fields as molecular, atomic, nuclear and particle physics.

The spectrum of the three-boson system close to the unitary limit, in the zero-range limit, is described by the Efimov radial law

$$\begin{aligned} E_3^n / (\hbar^2 / ma^2) &= \tan^2 \xi \\ \kappa_* a &= e^{(n-n^*)\pi/s_0} \frac{e^{-\Delta(\xi)/2s_0}}{\cos \xi}, \end{aligned} \quad (1)$$

where  $\Delta(\xi)$  is an universal function. A parametrization of this function in the range  $-\pi < \xi < -\pi/4$  can be found in Ref. [3]. The quantity  $s_0 \approx 1.00624$  is a universal number and  $\kappa_*$

defines the energy  $\hbar^2\kappa_*^2/m$  of the level  $n = n^*$  at the unitary limit. Knowing the value of  $\kappa_*$  the complete spectrum is determined as a function of  $a$ . The above equations have been derived in the zero-range limit in which the two-body energy is  $E_2 = \hbar^2/ma^2$ . The three-body sector shows a discrete scale invariance (DSI), the two-body low energy observables can be written in terms of  $a$ , making this quantity a control parameter (see for instance Ref. [3] and references therein). When the two-body potential has a finite range, the Efimov radial law can be modified as it was discussed in Refs. [4, 5]. In the following the case in which the particles are 1/2-spin-isospin fermions [6] is discussed.

## 2. Effective description of 1/2-spin-isospin fermions close to the unitary limit

The zero-range energy spectrum given by Eq.(1) is unbounded from below as has been shown by Thomas [7]. Finite-range potentials cure this pathology limiting the energy spectrum of the three-body system from below being the lowest state the ground state of the system. For values of  $a$  much bigger than the range of the interaction the three-body energy spectrum has many excited state with this number diverging as  $a \rightarrow \infty$ . Eq.(1) is still valid for the highest levels, however the ground and first excited levels show significant finite-range effects. In order to take into account these corrections the Efimov radial law is modified as follow [4, 5],

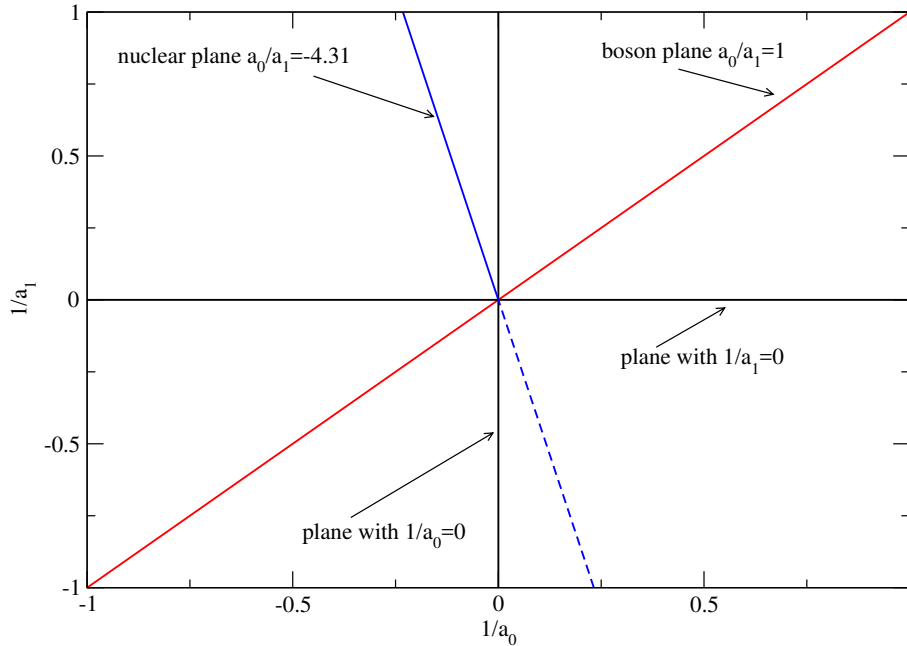
$$\begin{aligned} E_3^n/E_2 &= \tan^2 \xi \\ \kappa_n^3 a_B + \Gamma_n^3 &= \frac{e^{-\Delta(\xi)/2s_0}}{\cos \xi}. \end{aligned} \quad (2)$$

Two modifications have been introduced, the first one takes into account the fact that in the case of finite range interactions the two-body scattering length  $a$  and the energy length,  $a_B$ , defined by  $E_2 = \hbar^2/ma_B^2$ , with  $E_2$  the two-body binding energy if  $a > 0$ , or the two-body virtual-state energy in the opposite case,  $a < 0$ , are slightly different. The second modification consists in the introduction of a finite-range parameter, the shift  $\Gamma_n^3$ , depending on the energy level. Recently it has been shown that the value of the shift is almost the same for very different potentials as the atomic helium-helium LM2M2 interaction of Aziz [8] and a two-parameter gaussian potential in which the strength and range have been fixed to reproduce the values of  $E_2$  and  $a$  given by that interaction. Other examples are a combination of yukawians, as the MT-III interaction [9], describing the  $s$ -wave triplet  $n-p$  state or a nonlocal gaussian potential. Interestingly the value of the shift for the ground state in the three cases results to be almost the same,  $\Gamma_0^3 \approx 0.8$  [10, 11].

Following those findings we would like to discuss the case of fermions having 1/2 isospin symmetry in which the interaction is different in the singlet and triple state as is the case of the two-nucleon system. The experimental data for the zero energy  $p-n$  system are  $a_0 = -23.740 \pm 0.020$  fm and  $a_1 = 5.424 \pm 0.003$  fm and the respective effective range values are  $r_{eff}^0 = 2.77 \pm 0.05$  fm and  $r_{eff}^1 = 1.753 \pm 0.008$  fm (as reported in Ref. [12]). To describe these data it is possible to use a gaussian potential

$$V(r) = V_S e^{-r^2/r_S^2}, \quad (3)$$

with different strength in the singlet ( $S = 0$ ) and triplet ( $S = 1$ ) states. To simplify the description, the same range (in the following we consider  $r_0 = r_1 = 1.65$  fm) is considered. The values  $V_0 = -37.90$  MeV and  $V_1 = -60.575$  MeV approximate well the experimental data and, for  $S = 1$ , the deuteron binding energy is well described too. Varying the strengths  $V_S$  it is possible to explore the three-nucleon binding energy as the values of  $a_S$  move toward the unitary limit. This study is done fixing the ratio  $a_0/a_1 = -4.31$  to its experimental value, accordingly both strengths are related. It should be noticed that there are other possibilities to make this study. For example, the ratio  $a_0/a_1 = 1$  corresponds to explore the three-boson system. In Fig.1 selected planes (the three-body energy defines the third axis not shown in the figure) are



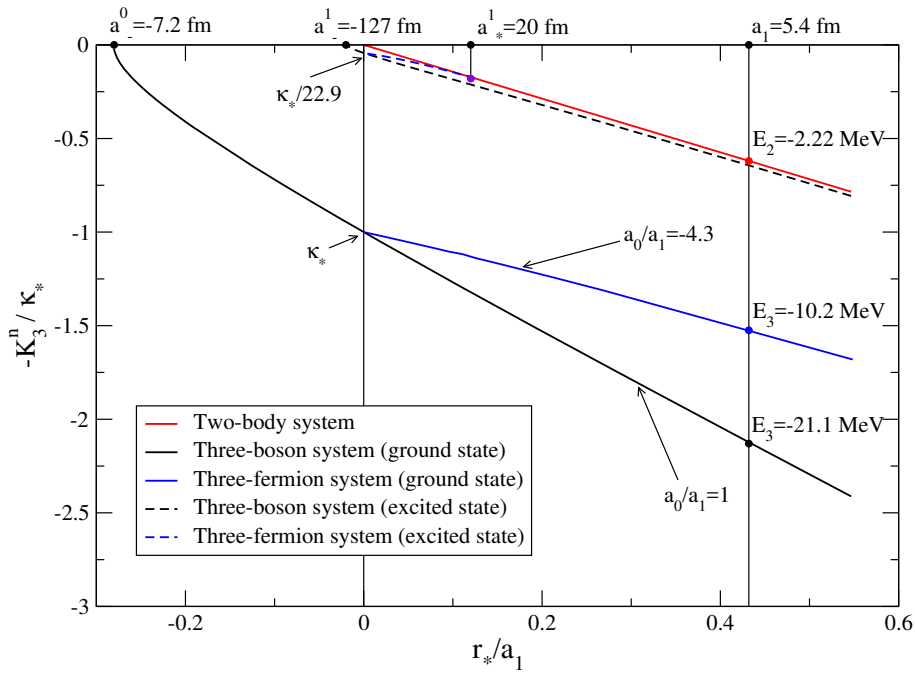
**Figure 1.** Selected planes to study the three-body spectrum as function of the inverse of the  $a_0$  and  $a_1$  scattering lengths.

shown determined by the values of  $a_0$  and  $a_1$ . As mentioned, when the two scattering lengths are equal,  $a_0/a_1 = 1$ , the corresponding plane is the boson plane for three equal particles very well studied (see for instance Ref. [4]). In the figure this is indicated by the (red) solid line. The (blue) solid line corresponds to a plane in which the nuclear physical point is included, this plane is called the nuclear plane. The continuation to the fourth quadrant given by the (blue) dashed line does not give new information since in this case the singlet and triplet potentials are exchanged and the system has the same binding energy. The two axes correspond to planes in which the singlet or triplet scattering lengths are on the unitary limit. The energy values  $E_3^n$  could be different in the different planes, however when the two scattering lengths are at the unitary limit the energy value is unique and it is that of the boson spectrum.

### 3. Three-body spectrum of 1/2-spin-isospin fermions close to the unitary limit

The spin-dependent potential defined in Eq.(3) can be used to explore the spectrum of the three-nucleon system close to the unitary limit. With the strengths and range defined in the previous section the low energy two-body data are reasonable well described. Extending the analysis to the three-body case a binding energy of  $E_3 = -10.2$  MeV is obtained to be compared to the experimental value of the triton  $E(^3\text{H}) = -8.48$  MeV. The observed over binding is common characteristic when attractive gaussian potentials are used. In order to have a quantitative agreement with the experimental value a three-body force has to be included (see for instance Ref. [13]). Here we are interested to study the variation of the three-body binding energy as a function of  $a_1$  with the ratio  $a_0/a_1$  fixed to the nuclear value, thus we do not include such a three-body force.

The results of the present study are given in terms of the binding momentum, defined from the relation  $(\hbar^2/m)(K_3^n)^2 = E_3^n$ , with  $n = 0, 1$  for the ground and first excited state respectively. In Fig.2 the binding momentum,  $K_3^n$ , is shown as a function of  $a_1$  (in units of  $r_*$ , the value of the effective range at the unitary limit) stressing that we have fixed the ratio  $a_0/a_1$  around  $-4.31$ . The (blue) solid line describes the ground state whereas the (blue) dot-dashed line describes the



**Figure 2.** The binding momentum  $K_3^n$  for the ground state ( $n = 0$ ) and first excited state ( $n = 1$ ) as a function of the inverse of the triplet scattering length  $a_1$ .

first excited state. For comparison the binding momentum values in the case of three bosons are given too by the (black) solid and dashed lines. The (red) solid line is the two-body binding momentum. The binding momentum  $K_3^n$  is shown in units of  $\kappa_*$ , the binding momentum of the ground state at the unitary limit and both, the fermion and boson values, are the same and correspond to a binding energy of  $(\hbar^2/m)\kappa_*^2 = 3.6$  MeV. The excited state in this limit has a binding momentum equal to  $K_*/22.9$  showing a slight finite-range correction (the zero-range theory predicts the value  $\kappa_*/22.7$ ). In the figure the (negative) values of the triplet scattering lengths,  $a_0^0$  and  $a_0^1$ , at which the three-boson ground and excited states disappear into the three-nucleon continuum are shown. Their ratio of about 17.6 is well below the prediction of 22.7 given by the zero-range theory. In the case of the three-fermion system it is interesting to notice that the first excited state disappears into the  $n-d$  continuum at the value  $a_*^1 \approx 20$  fm, well before matching the nuclear physical point corresponding to  $a_1 = 5.4$  fm). Conversely, in the case of the three-boson system, the excited state remains bound and always below  $E_2$ . This fact has been studied before in the case of a system of three helium atoms. The present analysis shows that in the case of three nucleons the difference in the single and triple potential strengths is such that the excited state disappears into the  $n-d$  continuum before reaching the physical point. Evidence of the presence of this state embedded in the  $n-d$  continuum has been given in the literature [14, 15] and the curvature of the  $n-d$  effective range function close to the threshold is one of this [16, 17]. Finally, at the physical value, the two- and three-body energies are explicitly displayed. As it is well known, in order to reproduce the triton binding energy a three-body force has to be included.

The spectrum of the three-fermion system can be described by Eq.(2) with a value of the shift depending on the ratio  $a_0/a_1$ . For the ground state level the numerical results can be described with  $\Gamma_3^0 \approx -0.2$ , a negative value. If we compare this value to the value of about 0.8 obtained in the case of three bosons we can conclude that there is a particular value of the ratio at which the shift  $\Gamma_3^0 \approx 0$ . In this case Eq.(2) will be equivalent to the zero-range equation. However this

would valid for the ground state level, the shift depends on the level and, as we have shown, the ratio with the excited state at the unitary limit is 22.9 (not 22.7) for a gaussian potential independently of the shift value.

#### 4. Conclusions and Outlook

The present study analyzes the particularities of the Efimov plot in the case of 1/2-spin-isospin fermions. In order to make this study a particular way of constructing the Efimov plot has been selected. The ratio of the singlet and triplet scattering lengths has been kept fixed, accordingly the binding momentum  $K_3^n$  and  $a_1$  define a plane similar to the boson case, in particular the values at the unitary limit coincide. The present work has to be consider a first step in the study of the Efimov plot for three nucleons. A deeper analysis including some results in the four-nucleon case and considering a three-body force are given in Ref. [13]. Extensions of the present study consist in the investigation of universal characteristic of the virtual state of the triton and the breakup reaction close to the deuteron threshold [18, 19].

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