Phenomenology of dark tensor currents

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Abstract. In this work we suggest a dark mediator model which extends the usual U(1) Lagrangian by including new degrees of freedom coupling to Standard Model fermions. Such dark particles can contribute to the neutral pion decay. For interaction constant and the dark particle mass consistent with the observed anomalies in the nuclear decays by the ATOMKI group we show that the dark tensor particles naturally incorporate lepton universality violation which introduces sizable effect on the muon anomalous magnetic moment.

1. Introduction

The Standard Model of elementary particles and interactions has proven to be extremely successful theory and still there exist some unexplained phenomena like dark matter, dark energy, baryon asymmetry and others. In this work we will focus on some peculiarities within elementary particle physics and more specifically on the beryllium decay and the muon anomalous magnetic moment. Both of these effects are believed to have their explanation by adding dark sector particles, initially suggested by B. Holdom [2]. Ideally, we should seek for the least number of dark sector particle candidates which can solve both problems simultaneously. We know that the particle which might be the reason for the beryllium decay should be a with mass around 17 MeV and the perfect candidate is a vector particle, like the dark photon.

Some of the low-lying mesons in the Standard Model can be described by vector fields, interacting through tensor currents with quark doublets. We incorporate this idea in a phenomenological Lagrangian and explore the consequences of having tensor currents. We further take the same fields with their interaction constants and investigate the influence on the correction of quantum electrodynamics vertex. For such interaction, lepton universality violation is manifest and the correction for the muon magnetic moment is orders of magnitude larger than for the electron. In this sense, the constructed model can compensate for the discrepancy between theory and experiment for the muon magnetic moment and in the same time preserve the astonishingly good agreement between theory and experiment for the electron.

2. The model

The full Lagrangian that we explore is given by

$$\begin{aligned} \mathcal{L} = ig\overline{Q}\gamma^5\tau^3 Q\pi^0 - e\overline{Q}C\gamma_\mu A^\mu Q \\ -e_1\overline{\Psi}\gamma_\mu A_1^\mu\Psi - e_2\overline{\Psi}\gamma_\mu\gamma^5 A_2^\mu\Psi + ie_3\overline{\Psi}\gamma^5 A_3\Psi - \\ -ie_4\overline{\Psi}\frac{q^\mu}{|q|}\sigma_{\mu\nu}A_4^\nu\Psi + e_5\overline{\Psi}\frac{q^\mu}{|q|}\sigma_{\mu\nu}\gamma^5 A_5^\mu\Psi - m\overline{\Psi}\Psi, \end{aligned} \quad (1)$$

where we introduce the neutral pion field coupled to the first generation quark doublet Q through the constant g and the Pauli matrix τ^3. In addition we have a photon field A^μ, interacting with quarks through the matrix $C = \frac{1}{6}\tau^0 + \frac{1}{2}\tau^3$, a dark photon A_1^μ, dark pseudo-vector particle A_2^μ, dark pseudo-scalar A_3, dark tensor A_4^μ and dark pseudo-tensor A_5^μ interacting with any Standard model fermion Ψ. The constants of interactions e_i are dimensionless.

Lorentz invariance of the theory admit the existence of chiral particle partners. But the straightforward introduction of such terms leads to$(\overline{\Psi}\sigma^{\mu\nu}\Psi)^2 + (\overline{\Psi}\sigma^{\mu\nu}\gamma^5\Psi)^2 = 0$. Therefore, the incoming momentum of the particle q should be included in the vertex definition which can potentially stem from a low-energy approximation of string field theories.

3. Dark meson production in neutral pion decay

An accessible process resulting in photon production where new vector particles can possibly be observed is the neutral pion decay. The corresponding loop diagrams for the pion decay into a photon and a dark photon, A_1, are

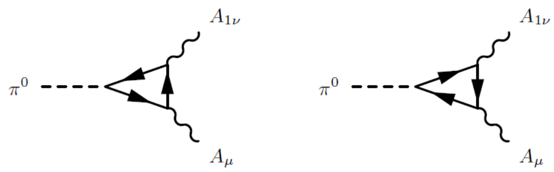

Figure 1. Diagrams for the π^0 decay into a photon and a A_1.

The decay width of the process is then

$$\Gamma_{\pi^0\to\gamma A_1} = \frac{1}{16\pi m_\pi^3}(m_\pi^2 - m_1^2)|\mathcal{M}|^2 = \\ = \frac{1}{128\pi^5 m_\pi^3}(m_\pi^2 - m_1^2)^3\epsilon^2 e^4 \frac{m^4}{f_\pi^2}\left(\frac{1}{2m^2} + \frac{m_1^2}{24m^4}\right)^2, \tag{2}$$

where we have used the result from the Goldberger-Treiman relation for the pion interaction constant $g = \frac{m}{f_\pi}$ and $e_1 = \epsilon e$. Here ϵ is a rescaling constant, m and m_π are the quark constituent mass and the pion mass respectively, and f_π is the pion decay constant.

In the same way we can consider other potential contribution to the pion decay with a photon in the final state. Since we require the new particle to have the potential to explain the anomalous beryllium decay observed by the ATOMKI collaboration, we will consider only particles which can decay into a pair of lepton and anti-lepton. This rules out all terms suggested in (1), except for the tensor and pseudo-tensor mesons.

The production of the tensor mesons in a pion decay can be calculated from the diagrams:

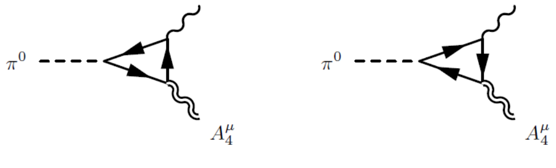

Figure 2. Diagrams for the π^0 decay into a photon and a tensor meson.

The decay width for $M < m$ and $m_\pi < m$ is then

$$\Gamma_{\pi^0\to\gamma A_{4,5}} = \frac{4(m_\pi^2 - M^2)}{\pi m_\pi^3 M^2}\left(\frac{m}{f_\pi}e'e\right)^2 \Bigg[\frac{\sqrt{M^2(4m^2-M^2)}(m_\pi^2-M^2)\arctan\left(\frac{\sqrt{M^2(4m^2-M^2)}}{2m^2-M^2}\right)}{M^2} - \sqrt{m_\pi^2(4m^2-m_\pi^2)}\arctan\left[\frac{\sqrt{m_\pi^2(4m^2-m_\pi^2)}}{2m^2-m_\pi^2}\right] + M^2\left(2+\frac{1}{16\pi^2}\left(\mathrm{Log}\left[1+\frac{\Lambda^2}{m^2}\right]-\frac{\Lambda^2}{\Lambda^2+m^2}\right)\right)\Bigg]^2. \tag{3}$$

where with M we denote the mass of either of the tensor mesons ($M = m_{4,5}$) and e' is the corresponding interaction constant ($e' = e_{4,5}$). Due to the ultraviolet divergence, a cut-off is introduced, Λ, which we choose to be the constituent quark mass m.

The energy distribution of the final state electron-positron pair can be used as a criteria to differentiate among the different mediators.

4. Leptonic processes with dark mesons

4.1. Dark meson decay into lepton-antilepton pair

The decays of the dark mesons into an elecron-positron pair are governed by the tree diagrams, shown in fig. 3.

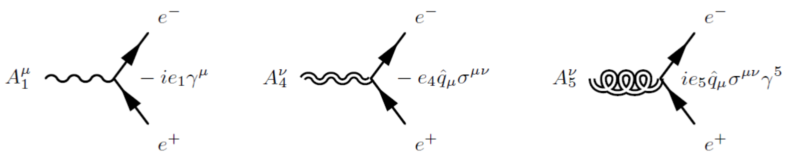

Figure 3. Basic Feynman rules for $A_{1,4,5}$ fields. A notation $\hat{q}^\mu = \frac{q^\mu}{|q|}$ was introduced.

The decay width of the dark photon into an electron-positron pair is

$$\Gamma_{1e^+e^-} = \frac{1}{3}\alpha\epsilon^2 m_1\left(1+\frac{2m_e^2}{m_1^2}\right)\sqrt{1-\frac{4m_e^2}{m_1^2}}, \tag{4}$$

where $\alpha = \frac{e^2}{4\pi}$ is the fine-structure constant.

For the decay of the tensor mesons we obtain that the tensor interaction vertex leads to non trivial dependence between the decay rate and the lepton mass [7]. This can naturally lead to lepton universality violation effects.

4.2. Contribution to anomalous magnetic moment

The current discrepancy between theory and experiment for the electron and the muon respectively is [3, 4]

$$\Delta a_e = \frac{g_e - 2}{2} = (4.8 \pm 3)\times 10^{-13} \text{ and } \Delta a_\mu = \frac{g_\mu - 2}{2} = (251 \pm 59)\times 10^{-11} \tag{5}$$

Now we can calculate the correction to the lepton anomalous magnetic moment due to the dark mesons.

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4.3. QED vertex correction due to tensor currents
Having all rules established allows us to compute the QED vertex corrections due to A_4^μ and A_5^μ.

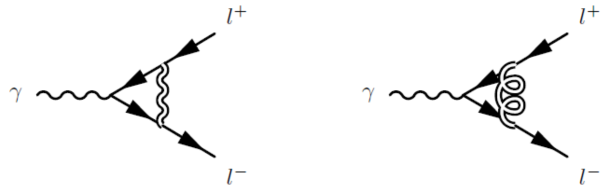

Figure 4. Feynman diagrams for the electromagnetic correction due to A_4^μ and A_5^μ dark bosons.

Here $l^\pm$ stands for a lepton/antilepton. The QED vertex can be expanded into form factors as

$$\Gamma^\mu = F_1(k^2)\gamma^\mu + F_2(k^2)\frac{i\sigma^{\mu\nu}}{2m}k_\nu, \tag{6}$$

where k is the photon momentum. Quantum corrections are calculated from loop diagrams and the anomalous magnetic moment is evaluated at $k = 0$. The two loop diagrams in (4) lead to

$$\begin{aligned}\delta_4\Gamma^\mu = \frac{e^2\epsilon_4^2}{16\pi^2}\Bigg(&-\frac{9m^2+2M^2}{m^2} + \frac{\left(8m^4-3m^2M^2-M^4\right)\ln\left(\frac{m^2}{M^2}\right)}{m^4} - \\ &-\frac{2\sqrt{M^2\left(M^2-4m^2\right)}\left(16m^4-m^2M^2-M^4\right)\ln\left(\frac{\sqrt{M^2-4m^2}+M}{2m}\right)}{m^4\left(4m^2-M^2\right)}\Bigg)\end{aligned} \tag{7}$$

$$\begin{aligned}\delta_5\Gamma^\mu = \frac{e^2\epsilon_5^2}{16\pi^2}\Bigg(&-\frac{3m^2-2M^2}{m^2} - \frac{M^2\left(3m^2-M^2\right)\ln\left(\frac{m^2}{M^2}\right)}{m^4} - \\ &-\frac{2\left(m^2-M^2\right)\sqrt{M^2\left(M^2-4m^2\right)}\ln\left(\frac{\sqrt{(M^2-4m^2)}+M}{2m}\right)}{m^4}\Bigg),\end{aligned} \tag{8}$$

where we take $m_4 = M$ or $m_5 = M$ and m is the lepton mass in the final state. The dependence of Δa is plotted in the figure 5 both for tensor (left) and pseudotensor (right) interaction.

These plots show that for the mass of the tensor meson in the range $\mathcal{O}(10)$ MeV, the discrepancy in the prediction for the anomalous magnetic moment of the muon can be compensated without influencing the prediction for the electron.

5. Conclusion
We considered a phenomenological model describing the interaction between Standard model fermions and bosons from the dark sector, motivated by the ambition to explain simultaneously the anomalous beryllium decay together with the discrepancy between theory and experiment for the anomalous magnetic moment of the muon. We show that both phenomena can be accounted for by adding two vector fields interacting through tensor currents, inspired by the already proved importance of tensor currents in observable matter. The production of the new mesons can occur in the decay of the neutral pion, which can be probed by looking for peaks in the dilepton invariant mass distribution in π^0-Dalitz decay. The correction to the QED vertex due to the tensor interacting particles for a certain strength of interaction is sizable for the muon and negligible for the electron, providing a natural explanation for the measured experimental value of a_μ [5, 6].

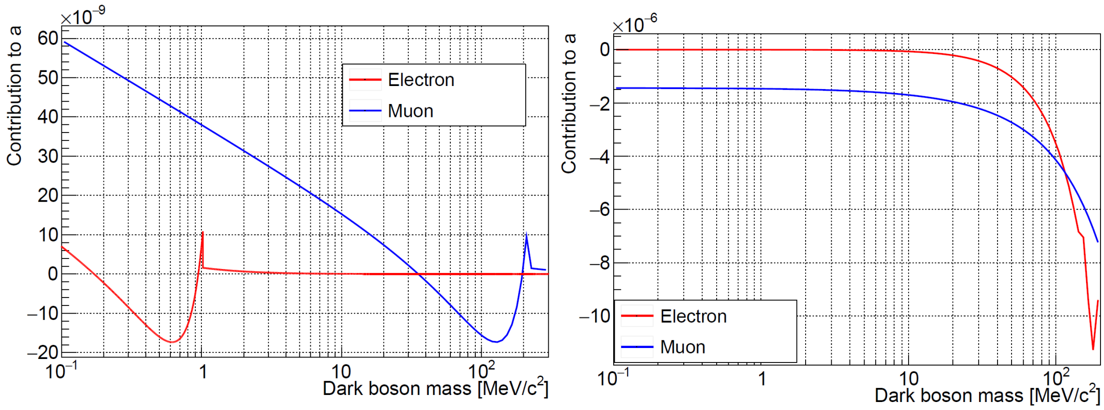

Figure 5. Contribution of the dark boson to $a = (g-2)/2$ as a function of the mass of the tensor boson. The left graph is for tensor and the right for pseudotensor interaction. We choose $\epsilon_{4/5} = 10^{-3}$.

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