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The Symmetries of Nature

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Abstract. The study of the symmetries of nature has fascinated scientists for eons. The application of the formal mathematical description of symmetries during the last century has produced many breakthroughs in our understanding of the substructure of matter. In this talk, a number of these advances are discussed, and the important role that George Sudarshan played in their development is emphasized

1. Personal remarks and history

I first met George in a hotel room in New York at the then Annual APS meeting, in either 1957 or 1958. We were introduced by Werner B. Teutsch, whom most of you, except Steve, Susumu and George, don't know or recall. Werner and I had been old friends from Penn- the University of Pennsylvania. Werner had done his thesis on Positronium with Vernon Hughes at Penn in 1954 and then went to the Institute for Advanced Study. After that, he went to Yale and Tufts. While at Tufts he met George, who was visiting at Harvard. They started to work together. Their first paper was in the very first issue of Physical Review Letters, with a distinguished set of co-authors. S. Weinberg, R.E. Marshak, S. Okubo [1]. The topic was, "Divergenceless currents and K-meson decay." I, too, had a paper in that first issue of Physical Review Letters. It was entitled, "Configuration mixing in the C^{14} ground state," co-authored with Elizabeth Baranger [2].

Through Werner, my friendship with George grew over the course of the years, and in 1964, George and Jack Leitner offered me a Professorship at Syracuse University. After much soul-searching, I declined and stayed at the National Bureau of Standards (now called NIST). Shortly thereafter, George came to UT-Austin, where we are today. In a strange twist, I, too, later followed George in becoming a Texan, with a 4 year stint at the SSC, starting in 1990. I had a wonderful time at the SSC, until its demise.

2. Symmetry

What is it?

a. Leon Lederman, in an article for "My Einstein," a recent book by John Brockman [3], discussing Dirac's prediction of antimatter, said, "Dirac's urge to elegance and beauty had uncovered a revolution in physics: the existence of antimatter. For every particle - electrons, protons, neutrons, quarks - there must be an antiparticle. What Dirac's epiphany illustrates is the deep influence of the concept of symmetry on the physics of the twentieth century because symmetry thrives in music and mathematics, its influence in physics not only sparked a revolution in theoretical science, but also acted as a unifying connection to the humanities."

b. In a slightly different vein, my friend Peter Kaus said, recently, "Symmetry is the magic word that distinguishes theory from coincidence."

c. Punching in "Symmetry" on Google yields 36,400,000 entries.

d. Wikipedia [4] lists the entries below.

1 Mathematical model for symmetry
1.1 Non-isometric symmetry
2 Directional symmetry
3 Reflection symmetry
4 Rotational symmetry
5 Translational symmetry
6 Glide reflection symmetry
7 Rotoreflection symmetry
8 Screw axis symmetry
9 Symmetry combinations
10 Color
11 Similarity vs. sameness
12 More on symmetry in geometry
13 Symmetry in mathematics
14 Symmetry in logic
15 Generalization of symmetry
16 Symmetry in physics
17 Symmetry in biology
18 Symmetry in chemistry
19 Symmetry in the arts and crafts
19.1 Architecture
19.2 Pottery
19.3 Quilts
19.4 Carpets, rugs
19.5 Music
19.5.1 Form
19.5.2 Pitch structures
19.5.3 Equivalency
19.6 Other arts and crafts
19.7 Aesthetics
20 Symmetry in games and puzzles
21 Symmetry in literature
22 Symmetry in telecommunications
23 Moral symmetry

e. Brief Review [4], [5]

If G is a symmetry group of a theory describing a physical system - i.e., the fundamental equations of the theory are invariant under the transformations of G - the states of the system transform into each other according to some representation of the group G. The group transformations are mathematically represented by operations relating the states to each other. These operations are the operators acting on the state space that correspond to the physical observables. The observables representing the action of the symmetries of the theory in the state space, and therefore commuting with the Hamiltonian of the system, play the role of the conserved quantities. The eigenvalues of the invariants of the symmetry group provide the labels for classifying the irreducible representations of the group.

An important role played by symmetry is that of classification - for example, the classification of crystals using their varied symmetry properties, or the classification of elementary particles by means of the irreducible representations of some symmetry groups.

The requirement of invariance with respect to a transformation group imposes severe restrictions on the form that a theory may take, limiting the types of quantities that may appear in the theory as well as the form of its fundamental equations. An example is Einstein's use of general covariance when searching for his gravitational equations.

The group theoretical treatment of physical symmetries, with the resulting possibility of unifying different types of symmetries by means of a unification of the corresponding transformation groups, has provided the technical resources for symmetry to play a powerful role in theoretical unification. We assume that symmetry means invariance under any kind of transformation. So an object is symmetric with respect to a given mathematical operation, if when applied to the object, this operation doesn't change the object.

f. One of the simplest symmetries that we know, an SU(2), is that which describes the functioning of a traffic light. We need one operator to tell us whether the light is red or green, another to take us from red to green and a third to take us from green to red.

3. Internal Symmetries

a. In this session there are 3 talks, in addition to mine.

George Sudarshan, No-Go Theorems and the Exclusion Principle
M.Y. Han, Duke University, Raleigh, NC

E.C.G. Sudarshan and Symmetry in Classical Dyamics, Optics and Quantum Mechanics
N. Mukunda, Indian Institute of Science, Bangalore, India

Why Quantum Dynamics is Linear, How One Qubit Almost Completely Reveals the Dynamics of Two, and Other Things Learned Following George's Lead
Tom Jordan, University of Minnesota, Duluth, MN

Inasmuch as they will give detailed descriptions of many topics dear to George's heart, I'll mainly concentrate on internal symmetries, with some excursions into the possibility of combining them with Lorentz Invariance.

b. George's contributions to the study of symmetries are legion. They started almost at the beginning of his career. Consider the 1957 paper by Marshak, Okubo, and Sudarshan, "Consequences of charge Independence for the Magnetic Moments and Masses of Sigma Hyperons. [6]"

In this paper they obtain a sum rule

$$\mu_+ + \mu_- = 2\mu_0 \qquad (1)$$

for the magnetic moments of the strange baryons, invoking charge independent interactions of strongly coupled isospin multiplets. This paper was written before SU(3) entered the scene. On their first page, there is a list of the multiplets considered. These were independent at the time, only later being grouped in SU(3) multiplets.

c. SU(3) versus G2

During the period 1960-1961 a big question was whether the proper classification group for mesons and baryons was G2 or SU(3). Behrends was a proponent of G2 [7], whereas Gell-Mann [8] and Ne'eman [9] wanted SU(3). The deciding factor was the prediction for the number of pseudoscalar mesons. At the time, there existed three pions and four kaons, all pseudoscalars. G2 predicted that there should be seven pseudoscalars, whereas SU(3) predicted that there should be eight, an additional I = 0, Y = 0 meson. The issue was settled with the discovery of the η(548) meson. SU(3) was the correct choice.

d. Introduction to Internal Symmetries

In 1961, the game changed a bit. It looked like the pseudoscalar mesons could be accommodated in an octet of SU(3), but it wasn't totally clear whether the baryons should be in octets, or whether a competing model, the Sakata model [10], which had the physical proton, neutron and lambda particles as fundamental triplets, was the right way to go.

Fortunately, we (C. A. Levinson, H. J. Lipkin, S. Meshkov, A. Salam and R. Munir) [11], .were able to kill the Sakata model by looking at the prediction for proton anti-proton annihilation going into $K_L K_S$ compared to $K_L K_L$.

A good thing about the Sakata model was that groups like Ikeda, Ogawa, and Ohnuki [12], and Sawada and Yonezawa [13] produced tables where they combined BBB and $B\overline{B}$ from which I was able to abstract a complete set of SU(3) 8 x 8 Clebsch-Gordan coefficients.

This set of tables kept us in business for years.

e. SU(2) and SU(3) - broken

Once it was established that SU(3) was the way to go, there were some obvious problems. Just looking at the masses of mesons and baryons, in their respective multiplets, it was clear that there was a large symmetry breaking going on. This was explained by Gell-Mann [14] and Okubo [15], in 1962. They assumed that the symmetry breaker transformed like an I =0, Y=0 member of an octet. This neatly explained the observed splittings.

In that era we had something going for us – data - and lots of it. It was a fun time. I was working with Carl Levinson and Harry Lipkin, and then with Gaurang Yodh and George Snow.

We made copious use of Weyl reflections [16, 17] and applied them to decay widths and scattering amplitudes in hadronic processes. We invented the U-spin and V-spin subgroups of SU(3) [18] and observed that the photon is a U-spin scalar [19]. This was very useful in dealing with electromagnetic processes [19]. There were lots of interesting relationships and experimental tests among scattering amplitudes [20].

During this time, Sudarshan was firing on all cylinders, as well. By methods very different from ours - the common thread was the use of Weyl reflections - George and coworkers produced a host of relations, many of which he presented at the 1963 Athens, Ohio conference [21], where I gave our first public talk on U-spin and V-spin. Many of these relations were based on work done in collaboration with Allan Macfarlane and C. Dullemond [22], [23], [24] using the Shmushkevich method [25], [26].

The years 1963–1967 were years of prodigious physics output for George.

	Total	Symmetry
1963	7	3
1964	11	9
1965	18	12
1966	9	6
1967	11	6

f. Exciting times - SU(3), Quarks

In 1963 the Ξ^* (1520) was found at the mass predicted for a decuplet, where the spacing is linear, by the Gell-Mann Okubo mass formula. This led to the general belief that SU(3) was a good symmetry. In 1964 the Ω^- was found, again where it was predicted to be, and Gell-Mann [27] and Zweig [28] invented the quark model (Zweig called them aces). At that time there were only d, u, and s quarks.

g. SU(6) and Color

In 1964, quarks were given spin by Beg, Lee, Pais [29], Pais [30], Radicati and Gursey [31] and slightly later by Sakita and Wali [32].When quarks are given a spin, there is a spin statistics problem.

The SU(6) multiplets come from combining three quarks, 6x6x6 = 56 +70 + 70 + 20, where 56 = 10 x 4 + 8 x 2. SU(6) is broken into SU(3) flavor x SU(2) spin.

The problem is that 56 is a symmetric combination. Wally Greenberg, on leave from Maryland at IAS, invoked parastatistics, now called color, and explicitly wrote down the states in an SU(6) x O(3) model [33], though he didn't call it that. The symmetry problem for the 56 was solved by combining it with an antisymmetric color singlet giving a totally antisymmetric state.

Note that, also in 1964, George and Mahanthappa [34], in a paper entitled, "SU(6) x 0(3) Structure of Strongly Interacting Particles," also examined this problem, as did Richard Dalitz [35].

4. Combining Internal and space-time symmetries

It was clearly interesting to combine internal and space-time symmetries. This effort took place all over the world through 1964 and 1965. My memory - a bit hazy since it was 41 years ago, was going to the second Coral Gables Conference in January 1965 and hearing presentations by Salam, and several other groups claiming to have solved the problem. They didn't! George [36] discussed the problem but didn't claim to have solved it.

A bit later that year, I went to visit at Weizmann Institute and Harry Lipkin and I found that we could combine internal symmetries with a restricted version of the Lorentz transformation. We could do this for collinear processes such as decays, but not for scattering amplitudes.

We named the relevant SU(2) W-spin and called the combined symmetry $SU(6)_W$ [37,38]. W stood for Weizmann Institute. We did this for constituent quarks and learned that Dashen and Gell-Mann

[39] had done the same for current quarks. Barnes, Carruthers, and
Von Hippel [40] also did analogous work. The W spin operators are invariant under Lorentz transformations in the z direction. The W-spin classification for a particle with arbitrary momentum in the z direction is the same as the classification at rest.

The generators of $SU(2)_W$ are:

$$W_z = \sigma_z/2$$

$$W_x = \beta\sigma_x/2$$

$$W_y = \beta\sigma_y/2$$

beta is the intrinsic parity of spin 1/2 particles in the rest frame. The virtue of this symmetry is that it correctly describes decays that are forbidden in the standard SU(6) approach.

5. Gauge symmetries, supersymmetry and neutrino symmetries

a. Gauge Symmetries

The 1970s was the era of Unified Gauge Theories. The works of Georgi, Glashow, Quinn and Weinberg [41], [42] stressed the role of the gauge group SU(5) and its breakdown into SU(3) x SU(2) x U(1). In addition, Georgi [43] and Fritzsch and Minkowski [44] as well as Ramond, Reiss, and Harvey [45] emphasized the role of SO(10).

b. Supersymmetry

A success! We already have half of the spectrum that results from symmetry breaking.
Now we need the other half – the SUSY particles. We'll see what the LHC brings us.

c. Neutrino symmetries

Do the neutrinos follow the pattern of the quarks and charged leptons? Quark mass splittings have been described starting with a "Democratic" mass matrix, with symmetry
S3 x S3 [46], [47], [48], [49] and adding successive breaking terms in S2 x S2 and S1 x S1. [50], [51]. This gives one large mass, split from two much smaller masses.

For neutrinos, we don't know what the pattern is yet. We do know that one state is roughly an equal mixture of ν_μ and ν_τ, and might like to think that this is the heaviest neutrino. Clearly this situation is different from the almost pure, top, bottom, and τ masses. Some attempts are made using S3 and S2 symmetries [52], [53].

6. Outlook

The use of symmetry principles and the group theory that describes them has been a monumental success in Physics. Invoking a new principle of group theory inertia , there is no reason for this to stop. Undoubtedly, new symmetry principles will arise and simplify our views of the universe. George Sudarshan has spent a lifetime doing just this, and we can gladly say, Thank You, George.

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