THE ONE-PION EXCHANGE MODEL AND ITS RELATION TO REGGE POLE EXCHANGE

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1. INTRODUCTION

Soon after the 30 GeV machine came into operation the one-pion exchange model [1] was proposed. It was the first dynamical model which provided some means of explaining phenomena observed at these high energies and it has even been extended up to cosmic ray energies. In this talk the main features of the model will be reviewed and some of its limitations and relationships with Regge poles will be discussed.

2. MAIN FEATURES OF THE MODEL AND ITS LIMITATIONS

The importance of one-pion exchange can be seen by considering peripheral collisions. In a semi-classical picture, the peripheral interaction is a glancing collision where the incident particle only interacts with the outer region of the target and receives a small momentum transfer. In this situation the long-range forces are the most important and in strong interactions the one-pion exchange provides the longest range force. Furthermore, the peripheral collisions seem to give a significant fraction of the total cross-section. This is because strong interaction cross-sections are of order

$$\sigma_{tot.} \sim \pi R^2$$

where R is the pion Compton wavelength. Cross-sections of such large magnitude are sensitive to the outer parts of the target and therefore to one-pion exchange. Another attractive feature is that one-pion exchange is relatively easy to analyse theoretically.

There are, however, a number of limitations to the application of this model:

1) A rather trivial limitation when electromagnetic interactions are also present is that the Coulomb force is of longer range. This gives a pole at zero momentum transfer (Fig. 1) and so it is necessary to avoid the resulting peak in the forward direction. At high energies the Coulomb peak is confined to a very small angles and presents no difficulty in practice.

2) A more important limitation is due to multi-particle exchange contributions, which are also present. For small momentum transfer the single

* Text based on notes by A. P. Contagouris and G. C. Oades.

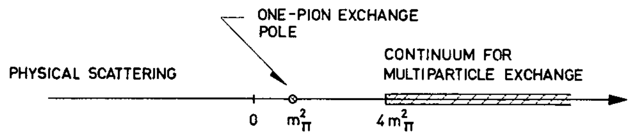

Fig.1

Position of singularities in the complex t-plane

pion pole may dominate the scattering, but for larger momentum transfer the two- and more-pion exchange contributions are at a comparable distanc (Fig. 1) and there is no longer any reason to neglect them.

3) Even for small momentum transfer the multi-particle contribution could be more important if the discontinuity across the cut is large enough. In terms of potentials, the one-pion pole gives a term of the form

$$g e^{-m_\pi r}/r \tag{1}$$

while the multi-particle continuum gives:

$$\int_{4m_\pi^2}^{\infty} d\mu^2 \; g(\mu^2)\, \frac{e^{-\mu r}}{r}\,; \tag{2}$$

if $g(\mu^2)$ is large enough, this term can be more important than (1). An example of this situation is provided by the diffraction peak in elastic scattering. To study the diffraction peak it is necessary to consider the absorptive part of the amplitude in the forward direction; the optical theorem gives

$$\mathrm{Im}\, f^{el}\,(t = o) = \sum_n f_n^* \rho_n f_n \tag{3}$$

where f_n is the amplitude for transition between the initial (or final) state and an intermediate state n and ρ_n is the phase space factor associated with this state n. In terms of diagrams Eq. (3) can be represented as shown in Fig. 2.

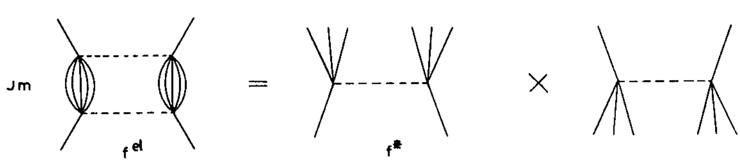

Fig.2

It is seen that at least two pions must be exchanged and therefore diffraction is associated with a shorter range term. There is a large contribution because all terms add coherently.

Consider as an example the process $p + p \to p + n + \pi^+$, which has been studied by SELLERI and FERRARI [2, 3, 4]. A typical one-pion exchange diagram is shown in Fig. 3.

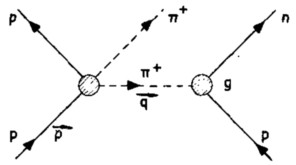

Fig.3

A one-pion exchange contribution to $p + p \to p + n + \pi^+$

This diagram gives a contribution to the cross-section of the form:

$$d\sigma/dt \;\alpha\; [g^2\, t/(t - m_\pi^2)^2]\; \sigma(n^+p \to \pi^+p). \qquad (4)$$

The factor t results from the negative parity of the pion. Other one-pion exchange diagrams give similar terms and also interference contributions. Selleri and Ferrari compared the final expression with experimental data between 1 and 3 GeV; they found good agreement provided they introduced a one-parameter cut-off for large momentum transfers. This is certainly a reasonable approach since the model is not expected to be valid at large momentum transfers, as pointed out above.

The last example was a particularly favourable case, since the pion-nucleon coupling constant and the low-energy cross-section were known. In general, less information is available. In this case, certain tests can be applied to the experimental data to see if the model is working. First, there is a specific prediction for the momentum transfer variation which should be satisfied at least for small momentum transfers. A second type of test has been proposed by YANG and TREIMAN [5]. In the left-hand side of Fig.3 consider the frame in which the incoming proton is at rest ($\vec{p} = 0$). A spin zero pion with momentum $\vec{q}$ is then scattered on a stationary proton; therefore the final state should have azimuthal symmetry about q. If the data show an isotropic distribution for rotation of the final 3-momentum about $\vec{q}$, they are consistent with the exchange of a single pion.

In a case when all proposed tests are satisfied so that the validity of the model is likely, it is possible to use the data in many ways. For example, in the case of $p + p \to p + n + \pi^+$, the data could be used to obtain the coupling constant g if the pion-nucleon cross-section is known. Alternatively, if the coupling constant is known, then the data could be used to obtain $\sigma(\pi^+p \to \pi^+p)$. It should be noted, however, that some extrapolation is always required. Either the experimental results must be continued to the pion pole $t = m^2$ so that physical values for $\sigma(\pi^+p \to \pi^+p)$ can be used; or else the physical values for $\sigma(\pi^+p \to \pi^+p)$ must be continued to the region $t \lesssim 0$ where the experiment $p + p \to p + n + \pi^+$ is conducted. Also, note that the tests made on the data refer to necessary but not sufficient conditions for the validity of the model.

As has been already stressed, with increasing momentum transfer corrections become necessary even in favourable cases. FERRARI and SELLERI [3] continue the physical π - N cross-section by use of Chew-Low theory; this is justified since they are still in the region of the 3-3 resonance even for larger t. They also introduce [4] a pion form factor $F_\pi(t)$ and express the cross-section in the form:

$$d\sigma/dt \propto g^2 F_\pi^2(t)\,[t/(t-m_\pi^2)^2]\,\sigma(\pi^+ p \to \pi^+ p). \qquad (5)$$

Then they obtain $F_\pi(t)$ by fitting the experimental data.

Of course, states with other quantum numbers, such as ρ and ω, can also be exchanged. As a first approximation one might hope to describe the scattering as an exchange of only a few such objects, each being treated with a form factor as was pion exchange. Formula (5) represents the optimistic limit in which only one object, the pion, is exchanged.

Suppose now that this procedure is applied at higher energies ($\gtrsim 2$ GeV). For simplicity, consider a case where only a pion and a ρ are exchanged. The amplitude can then be represented by

$$f = P_0(\cos\theta_t)\,G_\pi(t) + P_1(\cos\theta_t)\,G_\rho(t) \qquad (6)$$

where in terms of Mandelstam variables

$$\cos\theta_t = -1 - s/2q_t^2\ . \qquad (7)$$

The first term of (6) is due to the exchange of a pion (spin zero) and the second to that of a ρ - meson (spin one). Eq. (7) shows, then, that for large s the asymptotic contributions to the amplitude are correspondingly $\sim s^0$ and $\sim s^1$. To test these predictions we can use the available data on proton-proton scattering in the region 1-4 GeV (Brookhaven) and above 12 GeV (CERN) [6]. These data are derived from experiments in which fast outgoing protons of varying energy E' are detected at fixed angle for a given incoming proton energy E. A typical cross-section is shown in Fig. 4. Apart from the

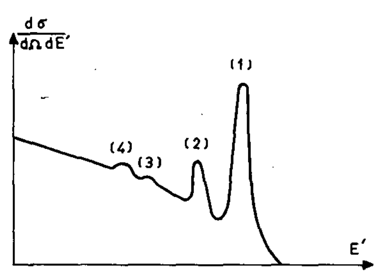

Fig. 4

High energy proton-proton differential cross-section

elastic peak (1), a number of other peaks are superimposed on the inelastic continuum. The positions of these peaks seem to be quite stable and are consistent with the interpretation that (2) corresponds to production of the $P_{3/2}$, T = 3/2 isobar, (3) to the $D_{3/2}$, T = 1/2 isobar and (4) to the $F_{3/2}$, T = 1/2 isobar. Now, as the energy rises for fixed (high) momentum transfer, the most striking feature is that the peak (2) disappears quickly, while (3) and (4) decrease only slowly. The observed decrease at $|t| \gg 1\ \mathrm{GeV}^2$ is found to be clearly more rapid than the predictions of a one-pion exchange model (asymptotic behaviour $\sim s^0$) [6]. Exchange of spin one (asympt. behaviour $\sim s^1$) or any finite number of higher spin particles will only make matters worse.

3. RELATIONSHIPS OF THE MODEL WITH REGGE POLES

An alternative way to handle the scattering is provided by the Regge pole hypothesis, according to which Regge poles of spin $J_i(t)$ are exchanged. This has the following features at high energies:

(1) The maximum spin exchanged at each t is finite, since the Regge pole is a kind of bound state resulting from forces of finite range in the crossed channel. Exchange of the other, lower spin, terms can be ignored at sufficiently high energy, leaving a simple expression for the amplitude growing as $s^{J_{max}(t)}$.

(2) The spin of each Regge pole varies with t, decreasing with increasing momentum transfer in the physical region of the process.

From the $s^{J(t)}$ growth of the amplitude it follows that exchange of a Regge pole with spin J (t) contributes to the differential cross-section an asymptotic term of the form

$$\frac{d\sigma}{dt} = F(t) \left(\frac{S}{2m^2}\right)^{2J(t)-2} . \tag{8}$$

Now, the proton-proton CERN experiments above 12 GeV establish the following upper limit in the cross-section for production of the 33 isobar:

$$(d\sigma/d\Omega_{\,lab})_{33} \lesssim 0.05\,(d\sigma/d\Omega_{\,lab})_{elastic.}$$

If this is combined with Eq.(8) and the p-p scattering data at lower energies ($\lesssim$ 4 GeV), the upper limit $J_{max}(t)$ for the spin of the exchanged pole can be determined [6] (Fig. 5)

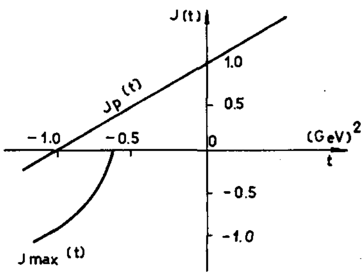

Fig. 5

The maximum exchanged spin consistent with 3-3 production and the Pomeranchuk Regge trajectory

It can be seen now that the decrease of J(t) for increasing physical $|t|$ and the possibility of having $J(t) < 0$ at sufficiently high momentum transfer may easily provide results compatible with $J_{max}(t)$ of Fig. 4. This is clearly not possible if the amplitude A behaves asymptotically like $\sim s^0$ (exchange of a particle of constant J(t) = 0), let alone if $A \sim s^n$ ($n \geqslant 1$).

The production of the second and third pion-nucleon isobar can be studies in the same spirit. In this case the results are compatible with a spin varying as $J_P(t)$ of Fig. 5. Notice that one pion exchange (J=0) does not dominate even at small $|t|$.

The difference in the spin of the exchanged Regge object for production of the $P_{3/2}$ isobar and for production of the $D_{3/2}$ or $F_{5/2}$ isobar can be easily understood from Fig. 6. In the first case the isospin of the exchanged parti-

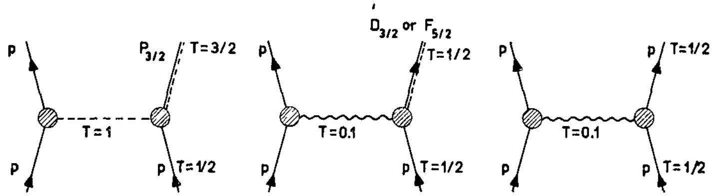

Fig. 6

Diagrams for the exchange of the leading Regge term in p-p T = 3/2 and T = 1/2 isobar production and elastic scattering

cle has to be T = 1; in the case of $D_{3/2}$ or $F_{5/2}$ production, as in the case of elastic scattering, T = 0 is also present and therefore the "Pomeranchuk" (or "vacuum") trajectory can be exchanged.

At this point a possible application of the $D_{3/2}$ or $F_{5/2}$ production to future very high energy accelerators may be mentioned. These accelerators will provide fast protons but their usefulness will be extended if collisions of the protons with a target produce a good secondary beam of fast pions. Cosmic ray data indicate the existence of such events, but a more quantitative estimate is desirable. Now, the functions J(t) and f(t) of formula (8) for $D_{3/2}$ and $F_{5/2}$ isobar production can be determined at present machine energies; then the formula can be extrapolated to very high energies. The calculation carried out in [6] along these lines indicates that a significant flux of pions with energy up to 2/3 of the original proton energy should be expected.

4. CONCLUSIONS

To summarize the situation in elastic or nearly-elastic events at high energy: We have presented some reasons why large momentum transfer events are better understood in terms of Regge pole exchange than in terms of exchange of a few particles like the π and ρ. As stated earlier, there was no strong reason for the one-pion to work in this region anyway. At low momentum transfer, if there is a Regge trajectory with higher J(t) than the pion trajectory, then this will dominate the scattering at sufficiently high energy. If there is no higher trajectory, or at intermediate energies where the factor $S^{J(t)}$ is not yet dominant, exchange of the pion Regge trajectory can control the scattering. In this case there is a correspondence with the one-pion exchange model at low momentum transfer where $J_\pi(t) \approx 0$ and, in fact, this correspondence is exact in the limit $t \rightarrow m^2_\pi$. From the work of Selleri and Ferrari, production of the 3-3 resonance in p - p scattering at 1 to 3 GeV is an example of this latter case.

Up to now, only a small class of inelastic processes has been discussed. I shall close with a few comments about our fragmentary understanding of more highly inelastic events. Many particle production, including energies above 30 BeV, has also been considered in terms of one-pion exchange. In place of the $p\pi^+$ final state in Fig. 2, for example, one allows all available final states for $p\pi^+$ scattering to emerge from the vertex on the left side. The resulting sum over amplitudes and phase space has an effect equivalent to a single amplitude growing like s^1 even though a spin zero particle was exchanged. In this way one recovers the possibility mentioned earlier, that one-pion exchange may give an appreciable part of the total cross-section.

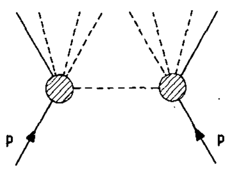

Fig. 7

One-pion exchange with many particle production

When one attempts to study the total cross-section, however, a problem arises. The factor s which has just been introduced has an effect similar to that associated with a spin-one exchange, which is known in general to introduce divergences. In fact, when the contributions of one-pion exchange terms are integrated over to obtain the total cross-section, it diverges logarithmically and some cut-off must be introduced. In the elastic case a precise formula is provided by the Regge pole hypothesis; as t falls below zero, J(t) decreases and the divergences are avoided in a natural way. In the inelastic case the mechanism of the cut-off is not understood but the data indicate the presence of strong damping at high momentum transfers so a cut-off is certainly present.

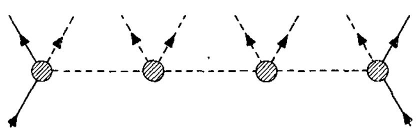

Fig. 8

Multi-particle production resulting from one-pion exchange

One pion exchange with many particle production relates the amplitude for high energy pp scattering, for example, to a product of amplitudes for high energy πp scattering (Fig. 7). A further step has been taken by AMATI et al. [7], GOEBE [8], and by F. and G. SALZMAN [9], who break down the amplitude into the product of several one-pion exchange terms (Fig. 8).

This step has the great advantage of reducing a high energy amplitude to a product of low energy amplitudes. It will be discussed in more detail in Fubini's lectures; a related approach which puts less emphasis on pion exchange is presented in my lecture on highly inelastic processes.

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