

Measurement quantization

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We present principles of Measurement Quantization (MQ) and approaches to measurement that support the discreteness of measure. Several claims are addressed. Notably, that measure is discrete with respect to the internal frame, non-discrete with respect to the system frame and that length is contracted due to the discreteness of measure. We address the relation of angular measure to momentum, the physical significance of count bounds and that the fundamental measures — more precise expressions for Planck's units — are an emergent property of the internal frame. Quantum experiments by Shwartz, et. al and CODATA provide physical support. We predict and derive values for elementary charge and the gravitational, Hubble, reduced Planck, electric, magnetic, Coulomb, and fine structure constants. We then correlate gravity with electromagnetism (unification). We present expressions for galactic rotation, dark matter, dark energy, and accelerating expansion. MQ advances over Loop Quantum Gravity with two frames, the difference which leads to the physical constants and the laws of nature. We correlate the quantum and cosmological, describing an inflation free quantum epoch, why it ceases and expansion. Therein are solutions to the horizon problem and homogenous, isotropic properties of the universe. Predictions include length contraction unrelated to special or general relativity (SR/GR), 13-digit measures of the gravitational constant, the Planck momentum, and universal mass accretion. A calculation of CMB age, quantity, present-day density and temperature provides additional support. Also offered, discrete solutions to the size and age of the universe, ground state orbital, SR, GR, and equivalence.

Keywords: Measurement Quantization; discrete measure; fundamental measures; Planck units; frames of reference.

Mathematics Subject Classification 2020: 85-08, 85A04, 85A40, 83F05, 52C99, 28-08, 28E99

1. Introduction

We demonstrate three features of our universe, that

- measure with respect to the Measurement Frame of the observer is discrete.
- measure with respect to the Target Frame of the universe is non-discrete.

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- a description of the difference between these frames allows us to resolve expressions and values for the physical constants and the laws of nature.

Physical support for these claims is accomplished using the Measurement Quantization (MQ) approach to classical expression. For those new to MQ, this approach is characterized by one feature, an expansion of the existing nomenclature. Instead of describing, for instance, the speed of light is such that $ct_f = l_f$, we expand this nomenclature using measurement pairs, each measure has some count of its corresponding reference measure. Thus, we would write $c = n_L l_f / n_T t_f$. Note that we have changed the order of terms purposefully. This paper presents physical support for the reference measures and with that, the physical significance of a nomenclature expansion.

This is accomplished in several ways but is most directly confirmed by a measure of length contraction, a feature of discrete measure that increases with decreasing distance. Not discussed with respect to other discrete interpretations of nature, MQ recognizes that a discrete measurement framework is an emergent feature of physically significant references in nature. And where the notion of length can be described by the Pythagorean theorem as a function of whole unit counts of the reference length, we recognize that there also exists a remainder, a fractional count of the reference (i.e. $(1^2 + 1^2)^{1/2} = 1.414 \dots$) that is lost with each increment of t_f . This appears physically as length contraction $2Q_L n_{Lr}$ (Appendix D — a function of the fractional count Q_L relative to the length count n_{Lr} — and can be verified with respect to the existing measurement data, as found with each CODATA publication of the physical constants [1–3]. This effect is not related to Special or General Relativity (SR/GR) [4].

Accompanying this paper, we add two supplements.^a

Both are Excel spreadsheets. The first spreadsheet (Supplement 1) contains calculations of all expressions in this document. The second spreadsheet (Supplement 2) contains calculations of all expressions in the body of this paper and supporting papers that address applications of MQ.

We would like to bring to the reader's attention several features that are not a part of the MQ approach. For one, the fundamental measures (Appendix W) are not axioms to the MQ approach. MQ does not consider approaches such as hidden or free variables. MQ does not entertain multiple dimensions above the three spatial and one time dimensions we can readily measure. MQ does not entertain disciplines such as Loop Quantum Gravity (LQG) [5, 6], String Theory [7], or Supersymmetry. MQ is not predicated on new axioms or new physics beyond what is implicit to the existing expressions of classical theory. That is to say, MQ is an approach to writing classical expressions achieved with a physically significant expansion of the existing nomenclature.

^ahttps://www.researchgate.net/publication/361148724_OnlineResources.

The purpose of this paper is to use the existing expressions of classical mechanics to demonstrate discrete measure, to make predictions not previously made (i.e. length contraction) and then show that those predictions are verifiable with respect to the existing measurement data.

2. Methods

The MQ approach to classical expression addresses the physical significance of measure, bounds to measure and those parameters which define the constants and laws of nature. MQ is characterized by one notable feature. We expand the existing nomenclature. With MQ, we consider the description of phenomena as a product of counts n_L , n_M , and n_T of fundamental measures l_f , m_f , t_f and θ_{si} . This last term represents a momentum as well as an angle. This will be discussed more as we proceed.

The fundamental measures are similar to Planck's Units [8], but account for a length contraction effect associated with discrete measure (Appendix D). And with these principles, we approach classical descriptions of nature that demonstrate 13 digits of correspondence. There are six papers which demonstrate applications of the MQ approach [9–14]. That said, the focus of this paper is to address the physical significance of measure with respect to three frames of reference — the Reference, Measurement and Target Frames (Appendix K) — specifically, the observations presented above.

3. Results

So, where do we begin? To answer this question, we pose a simple query. How does our existing nomenclature accommodate descriptions of discrete phenomena? Notably, it does not. Existing classical nomenclature has limited ability, for instance, to distinguish a geometry from a measurement. Classical nomenclature also cannot resolve relations such as the count of l_f per m_f at an event horizon. And for similar reasons, the existing nomenclature is challenged to distinguish properties of a system (i.e. the universe) from more complex relationships, such as observations of an expanding universe. Without an accommodating nomenclature, certain questions are difficult to assess. MQ addresses this shortcoming.

What has been accomplished thus far? There has been some progress with regards to these needs — mostly a function of work by Planck. Describing discrete phenomena requires at a minimum, two notions, fundamental measures and counts of those measures. Planck introduced his notion of fundamental measures in 1899, today known as Planck's units. Planck's units are not understood in the same way as that resolved with the MQ approach. For one, the Planck units are interpreted as the smallest measures.

With respect to Planck's units, we bring to the reader's attention an important modification to this interpretation. The notion of smallest measures refers only to

the Planck length and the Planck time. The Planck mass is, many orders in magnitude, larger than all particle phenomena. It is understood or at least expected that when a person speaks of Planck scale phenomena, they are referring to measures at or just above the Planck length and the Planck time.

We would be negligent if we gave all the credit to Planck. In 1881, George Stoney was the first to introduce the notion of fundamental units of measure [15]. He introduced this about two decades prior to Planck. The Stoney units differ from those of Planck by a scalar but are notably different in physical approach. Planck took a more general approach describing his measures as a function of G , c and $\hbar$.

With respect to both implementations, neither match the MQ approach. The existing notion of units is more like a factorization of all remaining nomenclatures, much like comparing SI units to natural units. Without establishing physical significance, unit systems are scaling exercises, for which there are several in use today. The units we propose are not just the smallest units. We show that they are physically significant, indivisible reference phenomena.

The second component feature which distinguishes the MQ nomenclature is a notion of counts. Planck was the first to introduce counting with his energy expression $E = nh\nu$. Describing the quantized nature of what he termed, a quantum of electromagnetic action, he introduced the count n into his equation. The term was intended to describe the discrete behavior of blackbody radiation. But as for classifying counts associated with length, mass, or time, this was never intended. And for this reason, the minimum components needed to describe phenomena in nature never matured.

This brings us to a formal definition of the MQ approach, a physically significant nomenclature that consists of two classes of terms. Given the notions of length, mass, and time, we describe phenomena as a function of counts n_L , n_M , and n_T of fundamental units of measure l_f , m_f , and t_f . Importantly, we do not presume these terms have physically significant values. Their significance, if any, is resolved from their application to the expressions for the speed of light, escape velocity and the uncertainty principle.

So, how do we demonstrate their significance? To date, the most common approach has focused on direct measurement. Unfortunately, experimental approaches thus far have fallen short of their calculated value by several orders in magnitude [16]. We have considered such experiments and recognize that the Planck units are distinct. That is, they do not share properties and as such a measure of the three units as a collection — that is the Planck momentum — is as physically significant as their measure independently. We can predict the value of such a measure using 2018 CODATA values for guidance.

$$\theta_{si} = \frac{l_f m_f}{2t_f} = 3.26239 \text{ kg m s}^{-1}. \quad (1)$$

Before carrying out such an experiment, we check the existing literature to see what has already been done. And notably, we find that this measure has been made, but only with respect to a radian measure. We would normally disregard such a

cross-dimensional coincidence if not for additional research whereby we demonstrate that the angle of polarization and the momentum of the beam are equal in value (Appendix S) at the lower measurement bound. Moreover, both measures can be made at the same time. We recognize the physics of this relation as a unique property of our universe.

First published by Schwartz and Harris in their 2011 paper entitled, *Polarization Entangled Photons at X-Ray Energies* [17], several measures are resolved; the value of θ_{si} being $3.26239 \text{ rad} \pm 2 \mu\text{rad}$ [18, 19]. Schwartz and Harris were neither investigating Planck units nor the relation of measure to other measures. The purpose of their experiments was to demonstrate that X-rays could be efficiently entangled when aligned in specific Bell states [20, 21]. They approached a description of the experiment with a distinct combination of Bell state principles implemented using classical expressions describing electromagnetic phenomena. The expression is largely incompatible with descriptions resolved using MQ, but nevertheless resolve the same numerical results. Notably, in 2022, the relation of angle and mass was mathematically correlated by Chen *et al.* in their paper entitled, *Conserved quantities in general relativity — the view from null infinity* [22], thus providing additional support for this relation.

We will present several descriptions but begin with a vector approach, which provides a close bridge between the classical notions of MQ, and the Bell approach implemented by Schwartz and Harris (Appendix A).

$$\begin{aligned} & (n_{sx} \cos(\theta), n_{sy} \sin(\theta)) + (n_{ix} \cos(\theta), n_{iy} \sin(\theta)) \\ & = (n_{px} \cos(2\pi - \theta), n_{py} \sin(2\pi - \theta)) + G'' \end{aligned} \quad (2)$$

Importantly, $\theta_s = \theta_i$ and $\theta_p = 2\pi - \theta$. This is to say that this particular measure reflects a beam of X-rays equal with respect to the angle of polarization for the signal and the idler such that the momentum of the beam is divided.

With three descriptions of the measure of θ_{si} (i.e. $2\theta_{si} l_f m_f / t_f$) — the Bell/vector approach, the momentum approach and the purely vector approach above — we now ask what has been learned. For instance, have we demonstrated that measure is discrete or countable? To this, we would say no. Indeed, we have achieved a measure with six digits of physical significance and yes, we have resolved supporting measurements that demonstrate a greater understanding of the fundamental measures. But we have not demonstrated that measure is discrete or countable. We have not in any way established that the fundamental measures are physically significant. And we see no advancement in our position even if the fundamental measures (i.e. Planck's units) could be measured with greater precision as a collection or independently. For these reasons, we conclude that unit measurements are not a useful approach in our quest to resolve a physically significant description of discrete phenomena.

Another solution, we propose, would be to use the MQ nomenclature to solve for a single solution set of the counts, n_L , n_M , and n_T . With that, we would demonstrate not only what the smallest measures were, but that there were no smaller measures. And we would be able to use that result to then demonstrate that measure

is discrete. The correlation between those expressions used to resolve the counts and the fundamental measures would also establish their physical significance.

We call to the reader's attention that a unique solution set is not necessarily straight-forward. Take for instance the expression for the speed of light, $c = n_L l_f / n_T t_f$ such that $n_L = n_T$. We could propose that $n_L = n_T = 1$, but just as easily we could propose any factor of that solution. If we then consider the expression for escape velocity at the event horizon $n_{Lr} > 2n_M$ (Appendix G), we extend our description to m_f , thus revealing that n_M is half of whatever value assigned to n_L and n_T . Thus, while we can resolve a solution set, we have not resolved a unique solution set. This is necessary in order to demonstrate discreteness.

With this in mind, we add one more expression, the MQ reduced expression for Heisenberg's uncertainty principle (Appendix B) [23] as it pertains to the conjugate pair representing the position and momentum of a particle.

$$c = n_L l_f / n_T t_f, \quad (3)$$

$$n_{Lr} > 2n_M, \quad (4)$$

$$2n_{Lr} n_M n_L = n_T. \quad (5)$$

We call to the reader's attention that the uncertainty principle is reduced to an expression consisting only of count terms. This is a notable advantage of the MQ approach. Keep in mind, the MQ nomenclature is in itself physically insignificant. But it does separate the geometry of a phenomenon n_L , n_M , and n_T , from the dimensional measures of that phenomenon, l_f , m_f , and t_f . With respect to the uncertainty principle, there are no measure terms. This is especially insightful in that we have always interpreted this expression as a description of uncertainty in measurement. It is difficult to justify such an interpretation with a description that has no component measures. And that is our point.

The uncertainty principle describes a geometry, specifically a lower count bound of fundamental measures. Granted, we have not demonstrated that the fundamental measures are physically significant. So, we will hold this observation until the end.

We would also like to clarify two similar MQ terms, n_{Lr} and n_L . We could use a single term instead of two but find MQ descriptions clearer when we distinguish a description regarding motion from a description regarding gravitational curvature. For this reason, we use the term n_{Lr} to describe a count of l_f between an observable and a center of mass. Conversely, we use the term n_L to describe the change in position of an observable. n_L can also be used to describe the relation of an observer to an observable.

These expressions can be combined in such a way as to resolve a unique solution set for the three counts, but there is yet a new problem. And that problem occurred before the calculation. Where? There exists circular reasoning with the reduction of these expressions to their component terms. We can demonstrate this by deriving the relation between Planck's expressions.

$$l_p = \left(\frac{\hbar G}{c^3} \right)^{1/2}, \quad (6)$$

$$t_p = \left(\frac{\hbar G}{c^5} \right)^{1/2}, \quad (7)$$

$$m_p = \left(\frac{\hbar c}{G} \right)^{1/2}. \quad (8)$$

We begin with any expression and then seek additional expressions to resolve the remaining two. Starting with l_p and including the expression for the speed of light $ct_p = l_p$, we resolve the expression for t_p . Notably, c is a component term to the definition of l_p and t_p , but neither expression is a contributor to the problem we wish to discuss. The problem occurs when we attempt to resolve the relation of m_p to l_p or t_p . This requires an expression including G or some description of gravitational curvature. One example would include the expression for escape velocity (Appendix G).

To apply the transform and then resolve a solution for m_p , the term m_p would have to be a component term of the expression for escape velocity. And that component term has to come from G . But how do we resolve the component terms for G ? That is the question which leads to circular thought. To gain some insight to the problem, we begin by using Planck's expressions for t_p and m_p to resolve their difference.

$$t_p^2 = \frac{\hbar G}{c^5} = \frac{\hbar c}{G} \frac{G^2}{c^6}, \quad (9)$$

$$m_p^2 = \frac{\hbar c}{G} \left(\frac{G^2 m_p^2}{c^6 t_p^2} \right). \quad (10)$$

Ignoring those terms immediately to the left and right of the second equality which are known to be equal by definition of the Planck mass, the remaining terms in the parenthesis are resolved. We recognize that they must equal the value of one.

$$\frac{G^2 m_p^2}{c^6 t_p^2} = 1, \quad (11)$$

$$G = c^3 \frac{t_p}{m_p}. \quad (12)$$

With this we resolve the component terms to G . In summary, we take the difference between t_p and m_p to resolve G , and then use G with t_p to resolve m_p . This is circular. If we take the new position that m_p is not resolved as a function of known physical correlations, we must also agree that we cannot know the component terms for G nor is there a means to resolve them.

The observation demonstrates that any known classical approach to G as a function of c and $\hbar$ cannot proceed. The result also calls into question the physical significance of m_p as a class of measure that should have equal properties to that of l_p and t_p . Supporting this interpretation, we readily recognize that m_p is not the smallest measure.

We also call to the reader's attention that the proposed solution for G also assumes that $n_M = 1/2$ and with that, all remaining count solutions are built. In short, the classical approach defines the values we are working to solve.

It follows that we must abandon Planck's expressions and start anew. We do so with a deeper investigation of the properties of distance using the Pythagorean theorem.

$$(n_{La}l_f)^2 + (n_{Lb}l_f)^2 = (n_{Lc}l_f)^2, \quad (13)$$

$$n_{La}^2 l_f^2 + n_{Lb}^2 l_f^2 = n_{Lc}^2 l_f^2, \quad (14)$$

$$n_{La}^2 + n_{Lb}^2 = n_{Lc}^2. \quad (15)$$

As before we call to the reader's attention that l_f has been canceled out of this expression. This is to say that the notion of distance is also not a physically significant component of this description. Rather, the notion of distance regards a geometry of counts relatively between physically significant measures of the reference l_f . Admittedly, the distinction can be subtle, but we find it immensely useful to know whether an expression reflects geometry, or an observed phenomenon is derived from physical measurement.

We proceed by considering the Pythagorean theorem as would be described using a right-angle triangle (Appendix C) while also considering the MQ definition of frames of reference (Appendix K). There are three sides and there are three frames.

Side a — we designate as our reference count, having a value of one. **Side a** describes the Reference Frame and as a reference, **side a** will always have a count of one for all considerations of **side b**, the Measurement Frame. Finally, **side c** represents a more precise calculation using those values assigned to **sides a** and **b**.

Observe that **side c** always equals **side b** plus some fractional count Q_L less than one-half. As such and to better conform with the existing naming standards, we replace the terms n_{Lb} and n_{Lc} with n_{Lr} and $n_{Lr} + Q_L$. And with this, we ask a question. If measure with respect to the internal Measurement Frame is discrete, then what becomes of Q_L ?

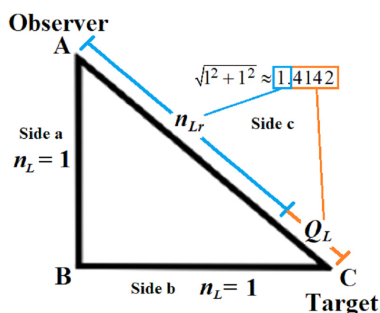


Fig. 1. Count of l_f between an observer and target. Adapted from [9]

Recall, **side c** represents the non-discrete Target Frame of the universe in which phenomena reside, but from the discrete Measurement Frame of Q_L is physically insignificant. As such, we recognize that Q_L is lost with each increment n_T in elapsed time t_f . And this loss we recognize as gravitational curvature Q_L/n_{Lr} .

To transform the expression into SI units, we multiply by l_f and then divide by t_f^2 , squared to reflect the accelerative nature of gravitational curvature. This gives us the expression

$$\frac{Q_L l_f}{n_{Lr} t_f^2}. \quad (16)$$

With this, we also recognize that we have defined curvature relative to the Target Frame. We will need to apply a transform — also known as a *metric differential* (Appendix T) — to resolve its Measurement Frame definition. This is accomplished by multiplying by the rate of expansion c at the leading edge of the system (i.e. the universe) as defined relative to that leading edge. We then divide by the radial rate of expansion θ_{si} (Appendix AZ). After reducing terms, we set the expression equal to Newton's expression for gravitational curvature, G/r^2 .

$$\frac{Q_L c^3}{r \theta_{si}} \approx \frac{G}{r^2}. \quad (17)$$

We have used the approximation symbol not because there is a precision issue with the MQ solution (those terms to the left of the equality), but because Newton's expression (right) does not account for a length contraction effect associated with discrete measure. We call this effect the *Informativity differential* $2Q_L n_{Lr}$ (Appendix D). This effect is a function of distance (i.e. the count n_{Lr} of l_f), largest at quantum distances and decreasing with increasing distance. For this reason, $\hbar$ is significantly affected when compared with G , for instance when working with expressions like Planck's units. The problem is resolved where all terms are resolved relative to the same frame (i.e. the same count n_{Lr}).

When using the MQ nomenclature, we are able to resolve this issue. Specifically, we do not italicize the term if a term is resolved at the upper count limit. For any other measure, we italicize the term. Thus, we can write the expression above as

$$\frac{Q_L c^3}{r \theta_{si}} = \frac{G}{r^2}, \quad (18)$$

without additional explanation. Because G is italicized, we recognize that its value is a function of the selected count n_{Lr} (i.e. $r = n_{Lr} l_f$) adjusted for the *Informativity differential* and that count is the same as that associated with the measure of the reduced Planck constant. For reference, that is about 84.6005496647(07) l_f (Appendix AB), also known as the *blackbody demarcation*.

While not meant as a diversion from our initial investigation, it is important to understand the effects of length contraction before presenting a final picture regarding the properties of measure. For this reason, we will further investigate published values for G and $\hbar$ over the last three CODATA publications [1–3].

We also call to the reader's attention that the *Informativity differential* is physically distinct from relativity. To be clear, the two phenomena have nothing in common. Relativity describes the distortion of measure — length, mass, and time — with respect to an inertial and/or gravitational mass. The *Informativity differential* describes the distortion of length as a consequence of the loss of Q_L with each increment in elapsed time t_f . One might ask, why does this effect not apply to the remaining dimensions. And to this we remind the reader that Q_L is resolved as a function of three-dimensional space. Using the Pythagorean theorem, we need only two dimensions such that the third is a by-product of the first two. In short, there is no distortion in the measures of time or mass as both are one-dimensional. This observation also places new constraints on multidimensional models as any prediction of dimensions in excess of one must also exhibit contraction with respect to the Measurement Frame. Experimental approaches to a verification of these claims are to follow shortly.

To begin, we recognize that any solution to a component term in a physically significant expression should resolve a result that has the same digits as any other expression consisting of different terms. This is to say that we should be able to use Planck's unit expressions for length, mass, and time to solve for the value of G and that all solutions should be the same. Unfortunately, this is not the case. Regardless of the publication year considered, results differ in the seventh digit (Table 1). As our concern is focused on this difference, we will look only at the 2018 publication, although any publication will show the same discrepancy.

The difference, as some might suspect, is because of the effects described by the *Informativity differential*. We can expose that problem by focusing our analysis on the first two solutions, lines one and two which present solutions to G using l_p and t_p . Notably, both solutions are dependent on c and $\hbar$, both which are defined with nine or more digits of physical significance. This leaves l_p and t_p , which are correlated by $ct_p = l_p$, again with nine digits of physical significance. These observations are in conflict with our initial observation of variance in the seventh digit. If all terms are defined with a precision two digits more than the observed variance, then what has gone wrong with our mathematical equality?

We could take a look at this problem in a slightly different way. Knowing that something is amiss and having only two measurements — c and $\hbar$ — we could modify

Table 1. Calculations of G using Planck's unit expressions.

CODATA ^a	2010 ($10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$)	2014 ($10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$)	2018 ($10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$)
$G = l_p^2 c^3 / \hbar$	6.673845(89)	6.674088(32)	6.674301(29)
$G = t_p^2 c^5 / \hbar$	6.673846(87)	6.674089(29)	6.674302(86)
$G = \hbar c / m_p^2$	6.67382(75)	6.674078(07)	6.674300(63)

^aPlanck's Unit expressions are resolved using those values provided in CODATA publications for 2010, 2014 and 2018 [1–3].

one or both so as to resolve a final solution. The challenge with this approach is that the solution appears valid only with respect to the associated n_{Lr} of that solution. This would not be known when using a classical approach, but nevertheless has challenged the CODATA collaboration for more than a decade as there appears to be no solution that is agreeable to all expressions.

We propose that we use the MQ approach to resolve both G and $\hbar$ using the same frame of reference. We can resolve G at a quantum distance that corresponds with the measure associated with blackbody radiation (Appendix AB) or we can resolve $\hbar$ at the upper count limit, the traditional approach to the measure of G . The latter inherits less questions and as noted we do not italicize $\hbar$ when a term represents a measure associated with the upper count limit (Appendix L).

$$\hbar = \lim_{n_{Lr} \rightarrow \infty} \frac{\theta_{si} l_f}{Q_L n_{Lr}} = 2\theta_{si} l_f = 1.05453 \cdot 10^{-34} \text{ Js.} \quad (19)$$

With this, we have part of the story. We would also need to know values for the fundamental measures. While we have not addressed this, they can be derived using the solution for discrete gravity (Appendix W) presented earlier. And with those results along with the solution for $\hbar$, we find that all solutions to G using Planck's unit expressions are the same, $6.6740779428(56) \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ to the extent of precision offered by the component measures.

A second consideration of the effects of length contraction regards experiments selected to represent a best measure of G and $\hbar$. Using our knowledge of the *Informativity differential*, we find that we can calculate all values assigned by the collaboration. That is to say, the selected experiments conform to a macroscopic or electromagnetic approach to measure and as such values can be resolved either as a function of n_{Lr} at the demarcation associated with electromagnetic phenomena or near the upper count limit (Table 2).

Table 2. Matching CODATA values for $\hbar$, G and the Planck units with MQ calculated values.

Year	$\hbar$ (10^{-34} Js)	l_p (10^{-35} m)	t_p (10^{-44} s)	m_p (10^{-8} kg)	G (10^{-11} $\text{m}^3 \text{ kg}^{-1} \text{ s}^{-2}$)
2010 ^a	1.054571726(47)	1.616199(97)	5.39106(32)	2.17651(13)	6.67384(80)
2014 ^a	1.054571800(13)	1.616229(38)	5.39116(13)	2.176470(51)	6.67408(31)
2018 ^a	1.054571817	1.616255(18)	5.391247(60)	2.176434(24)	6.67430(15)
$f(\hbar, G)^b$	1.0545349844(45)	1.6161716871(50)	5.3909684650(90)	2.1764705491(26)	6.6738448362(53)
$f(\hbar, G)^b$	1.0545349844(45)	1.6161999120(35)	5.3910626131(72)	2.1764325398(24)	6.6740779428(56)
$f(\hbar, G)^b$	1.0545718176(46)	1.6162281374(12)	5.3911567628(97)	2.1764705491(26)	6.6740779428(56)
$f(\hbar, G)^b$	1.0545718176(46)	1.6161999120(35)	5.3910626131(72)	2.1765085590(91)	6.6738448362(53)

^aCODATA publications for 2010, 2014 and 2018 [1–3].

^bUsing precise G and $\hbar$, we account for the *Informativity differential* at the *blackbody demarcation* $n_{Lr} = 84.6005496647(07)$ (i.e. italicized), the upper count limit (i.e. $\lim_{n_{Lr} \rightarrow \infty}$) (not italicized), or a mix of the two.

This table is laid out such that the first three lines describe G and $\hbar$ respective of each of the last three CODATA publications. The latter four lines are the result of calculations made using Planck's unit expressions, a function of G and $\hbar$ at the upper count limit (i.e. $G=c^3 t_f/m_f$, $\hbar = 2\theta_{sil_f}$) and G and $\hbar$ at the *blackbody demarcation* (Appendices Appendix AB, Appendix L and Appendix M). As well, we offer three examples, that are associated with blackbody radiation (Appendix AB), charge couplings (Appendix AE) and elementary charge (Appendix AH). In each case, the first seven digits of the demarcation are the same, 84.60055. The remaining difference is a function of the phenomenon being measured. Moreover, this is part of what allows the collaboration to vary $\hbar$ from year to year with minimal effect on the Planck unit calculations. Given that $\hbar$ is most often measured as a function of blackbody radiation, we present expressions for G and $\hbar$ respective of the *blackbody demarcation*.

$$\hbar = \lim_{n_{Lr}=84.6005496647(07)} \left[\frac{\theta_{sil_f}}{Q_L n_{Lr}} \right], \quad (20)$$

$$G = \lim_{n_{Lr}=84.6005496647(07)} \left[2Q_L n_{Lr} c^3 \frac{t_f}{m_f} \right]. \quad (21)$$

We then align solutions for both in the latter four lines, demonstrating a five- or six-digit correspondence with 9 of the 15 elections offered by the CODATA collaboration. Two additional results represent variations of $\hbar$ and the remaining four — in bold — are a function of G to be discussed shortly. Importantly, an understanding of the *Informativity differential* is all that is needed to resolve each value selected in the last three publication years.

With respect to the physical measures of G , G , $\hbar$ and $\hbar$, matches exist for all CODATA elections except the 2018 result for G which then skews the Planck unit calculations. Matches occur in columns and are identified with differently dashed lines. Notably, the bolded result for $G = 6.67430 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ is outside the permitted range associated with length contraction, a function of n_{Lr} associated with the phenomenon $(6.67384\text{--}6.67408) \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$. So how is this resolved?

There are several paths that can lead to such a result, most of which regard a misunderstanding of the *Informativity differential*. Consider an approach with respect to the ground state orbital of an atom (Appendix BL). To complete the calculation, we divide both sides by t_f and then replace those terms which define momentum $p = l_f m_f / t_f$.

$$\frac{l_f m_f}{t_f} = \frac{\alpha_p a_0 m_e}{t_f}. \quad (22)$$

We then solve for t_f .

$$t_f = \frac{\alpha_p a_0 m_e}{p}. \quad (23)$$

We use the defined value for the speed of light $ct_f = l_f$ to resolve a value for l_f . And with Planck's expression we resolve a value for m_f which finally allows us to calculate G . Granted, this is an exception to the prior argument that m_f is uncorrelated to l_f and t_f . We overlook this fact as the exception is not known in the existing literature and as such not a contributor to those expressions representing our current understanding of physical law.

$$m_f = \alpha \frac{a_0 m_e}{l_f} = 2.1765086 \cdot 10^{-8} \text{ kg}. \quad (24)$$

With this, we bring to the reader's attention that these calculations may have been carried out by other collaborators; the relevant inputs being those values used for l_f and t_f . Consider a momentum that resolves a value of l_f equal to the 2014 CODATA value $1.616229(38) \cdot 10^{-35} \text{ m}$. With l_f establishing the value for t_f , the researcher takes these as inputs and must decide from there what value will be used for the fine structure constant to resolve m_f . That is, without a thorough understanding of the prior calculations, the researcher might use Planck's expression for the fine structure constant to solve for $m_f = f(\alpha_p) = 2.1763929 \cdot 10^{-8} \text{ kg}$ or they might use the electromagnetic expression as recognized from the previously referenced CODATA publications to solve for $m_f = f(\alpha_c) = 2.176468910 \cdot 10^{-8} \text{ kg}$ [2].

$$G = f(\alpha_p) = c^3 \frac{t}{m} = 6.6743218 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}, \quad (25)$$

$$G = f(\alpha_c) = c^3 \frac{t}{m} = 6.6740887 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}. \quad (26)$$

The $\alpha_c^{-1} = 137.036$ approach produces the 2014 CODATA value for G while the $\alpha_p^{-1} = 137.041$ approach (Appendix AA) produces the 2018 CODATA value, the latter outside of the permitted range for G . And it is this latter approach which skews the three Planck unit calculations bolded in Table 2. Notably, the α_p result matches the CODATA to one part in the sixth digit, the total of significant digits offered (i.e. a 0.00001(15) difference). The magnitude of this difference also matches our calculation, as we have resolved this result at the upper limit rather than a count n_{Lr} of l_f appropriate to an ordinary laboratory measurement. In other words, there is no difference.

Finally, we bring to the reader's attention that the skew in G is symmetric. If we take the average of the 2010 and 2018 elected values for G , we find the result equal to the 2014 election.

$$\left(\frac{6.67384(80) + 6.67430(15)}{2} = 6.67407(48) \right) \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}. \quad (27)$$

We bring all these results to the reader's attention not because of the physical support for understanding the effects of the *Informativity differential*, but to demonstrate the simplicity of this effect and how accounting for it resolves the challenges put forth before the CODATA collaboration. The results are not as coincidental as they may seem at first sight. As noted earlier, experiments take one of

two forms, either a function of the demarcation count or the upper count limit, measurements and phenomena associated with different distances. For this reason, resolving CODATA selections is a straight-forward exercise of combining different distance measurements of G and $\hbar$ in combinations consistent with the reference experiments.

With this grounding, we can turn our attention back to the initial discussion, a unique solution set to counts of the fundamental measures. As noted early on, we observed a problem with circular reasoning when using Planck's unit expressions. With this we developed a new approach to gravitational curvature, a discrete description of gravity (Appendix C). Using discrete gravity, we can now resolve a new solution to the gravitational constant (Appendix F). The final expression is the same, but the solution is now resolved without use of Planck's unit expressions. As such, we avoid the circular reasoning previously mentioned. In the same way, we can also use the expression for discrete gravity to resolve a fundamental solution for escape velocity (Appendix H). As before, the result expression is the same, but the solution is independent of Planck's expressions.

With these, we can then use the expressions for the speed of light, escape velocity and the uncertainty principle to find a solution set for the counts (Appendix I). This is resolved to find that $n_L = n_T = 1$ and $n_M = 1/2$. This solution establishes that these are the smallest counts of the fundamental measures and that any other count must be a multiple of these counts. We can extend these requirements to the remainder of the measurement domain to show that measure is discrete (Appendix J). This is essential to complete a description of the fundamental measures as physically significant references with respect to Measurement Frame.

With this observation, we then recognize that the Target Frame of the universe has no external reference. For this reason, the Target Frame must be non-discrete and can be mathematically described as shown with a right-angle triangle in the derivation of discrete gravity. This is to say that phenomena are measured discretely with respect to the Measurement Frame but reside non-discretely with respect to the Target Frame. The difference between these frames is what allows us to resolve expressions and values for the physical constants [14], for instance, the value for G as we have just presented above.

Using the expression for discrete gravity we can then resolve the simplest relation between the fundamental measures $l_f m_f = 2\theta_{sit} f$. We call this the *fundamental expression* (Appendix V). And we can also use discrete gravity to solve for the values associated with each of the fundamental measures (Appendix W). A comparison of the fundamental measures with the Planck unit expressions reveal that they differ by the *Informativity differential*, as expected (Appendix X). Likewise, if we compare Newton's expression for gravitational curvature with that of MQ, again we find that they differ by the *Informativity differential* (Appendix Y). On all counts, MQ descriptions match with the experiment and provide a consistent description of nature.

We can also return to our initial discussion regarding θ_{si} and the units associated with this measure. As noted early on, with respect to the Shwartz and Harris Bell state model, they offer a description of θ_{si} in terms of a Bell state nomenclature. We offer the classical form of this measure as half of the Planck momentum (Appendix Q). We also offer a vector description reflecting those experiments carried out by Shwartz *et al.* (Appendix A). And we offer an electromagnetic version (Appendix P), that being a classical version translated with Planck's expression for length to describe θ_{si} as a function of the reduced Planck constant. With respect to all these descriptions, each is resolved as a function of the Measurement Frame. But what about the Target Frame?

A Target Frame description of θ_{si} cannot have a unit-based description because, as we have observed, the universe has no external reference. Without an external reference, there can be no physically significant reference to serve as a source definition for the three dimensions. It follows that descriptions resolved with respect to the Target Frame are dimensionless. One example of this is the *unity expression* (Appendix AA), an expression which describes the relation of θ_{si} to the count change n_L (i.e. expansion of the universe) at the first increment t_f of the expansionary epoch.

$$\left(\frac{1}{2\theta_{si}}\right)^{2/3} + \frac{n_L^2}{n_{Lc}^2} = 1. \quad (28)$$

A unity expression can be any Pythagorean expression equal to one, but we single out this expression in that it correlates the fundamental measures directly to the radial rate of *universal expansion* (Appendix AZ). The term n_L is a description of an increasing count of l_f along the radius of the universe. The term n_{Lc} describes the count of l_f associated with the motion of light in vacuum. Both are non-dimensional scalars, such that $2\theta_{si}$ must also be non-dimensional. This was expected, in that the expression is defined with respect to the Target Frame. Notably, the expression also takes the form of the Pythagorean theorem with a cube root added to the first term, one root for each of the three spatial dimensions. Recall, $2\theta_{si} = l_f m_f / t_f$. And with this we have presented several descriptions showing θ_{si} as having units of radians, momentum, or no units at all. But in each case, the value is the same (Appendix S, Appendix AD). And it is for this reason we use the term θ_{si} for all situations, not because a radian measure is appropriate, but to call attention to the unique characteristics associated with this term.

To conclude this portion of the presentation, we offer a derivation of the fine structure constant (Appendix AA). The derivation offers us the opportunity to tie together all the disciplines offered thus far into a single calculation with 12 digits of precision and having two classical expressions with which to compare those calculations. The solutions demonstrate application of the *metric differential* (Appendix T) needed to transform a Target Frame expression to the Measurement Frame and an accounting of the length contraction effects described by the

Informativity differential. The calculation follows an approach used repeatedly for the derivation of nearly all the physical constants in nature.

So as to provide some overview before beginning, we briefly describe the process. First, we resolve a local measurement, what is called a demarcation. Demarcations are solutions to the count n_{Lr} associated with the measure of a phenomenon. We can offer three examples, the *blackbody demarcation* (Appendix AB), the *charge coupling demarcation* (Appendix AE) and the *elementary charge demarcation* (Appendix AH). When comparing demarcations, we find that the first seven digits are the same, 84.60055. If we only desire seven or less digits of physical significance, we can use any phenomenon to resolve a demarcation.

With the demarcation we then use the *fundamental expression* to resolve the corresponding count n_θ of θ_{si} in the Target Frame for a demarcation count n_{Lr} of l_f with respect to the Measurement Frame. This count represents the count of θ_{si} associated with the fundamental form of the inverse fine structure constant α_f^{-1} . Notably, we consider this expression as our starting point. With this, we then apply the *metric differential* to resolve the Planck-like form α_p^{-1} . This name reflects that comparison with any Planck expression will yield a precise match, to the extent permitted by the physical significance of the least precise measure. And finally, we apply the *Informativity differential* to resolve the classical form of the expression α_c^{-1} , what is measured in the laboratory.

With this outline, we begin with the *fundamental expression* (Appendix V).

$$(n_L)l_f \frac{m_f}{t_f} = (2n_\theta)\theta_{si}. \quad (29)$$

This tells us that the corresponding count n_θ will be half of the demarcation count n_{Lr} measured locally. We then use the *blackbody demarcation* to resolve the count n_θ with respect to the Target Frame.

$$n_\theta = \text{RND}(n_{Lr}/2) = \text{RND}(84.6005496647(07)/2) = 42. \quad (30)$$

And this gives us the inverse fine structure constant in its fundamental form (Table 3). You can see this result listed in the first column of the table (Table 3).

Moving forward, we apply a transform to resolve the Measurement Frame solution. We call this transform the *metric differential* (Appendix T). It takes a different form depending on the units associated with the expression being transformed. For

Table 3. Fundamental, Planck-like and classical solutions to the inverse fine structure constant.

Measure	$\alpha_f^{-1} = 42\theta_{si}$	$\alpha_p^{-1} = m_e a_0 / m_f l_f$	$\alpha_c^{-1} = 4\pi\epsilon_0 \hbar c / e^2$
CODATA ^a	—	137. 04078552(97)	137.035999084(21)
MQ(α)	137.020392764(84)	137.040785529(68)	137.035999083(49)
S&H	137.02039	137.04078	137.03600
Difference	—	0.00000000(00)	0.000000000(72)

^a2018 CODATA publication [3].

instance, the *metric differential* associated with gravitational curvature is c/θ_{si} . Notably, this can be reduced to $2/m_f$ or half of m_f as in $n_M = 1/2$. The transform for elementary charge is complex requiring a great deal more calculation than what is already associated with the fine structure constant. And the *metric differential* for the fine structure constant, being a non-dimensional quality is the simplest of all. It is the product $42\theta_{si}$ minus the whole-unit portion of the same product. Thus, the Planck-like form of the fine structure constant is (Appendix AA)

$$Q_{n_\theta} = 42\theta_{si} - \text{RND}(42\theta_{si}), \quad (31)$$

$$\alpha_p^{-1} = \alpha_f^{-1} + Q_{n_\theta} = 42\theta_{si} + (42\theta_{si} - \text{RND}(42\theta_{si})), \quad (32)$$

$$\alpha_p^{-1} = 84\theta_{si} - \text{RND}(42\theta_{si}) = 137.041. \quad (33)$$

Lastly, we apply the *Informativity differential* $2Q_{Ln_{Lr}}$. This gives us the classical form α_c^{-1} of the expression (Appendix AA), what is measured in the laboratory.

$$\alpha_c^{-1} = 2Q_{Ln_{Lr}}(84\theta_{si} - \text{RND}(42\theta_{si})) = 137.036. \quad (34)$$

If we then compare these results to the CODATA [2], we find no difference. It should be noted that the comparison is based on an initial measure of θ_{si} by Schwartz and Harris, which is constrained to six digits of physical significance. We are, of course, motivated to find a solution having greater precision, given that most electromagnetic phenomena are known to 14 digits of physical significance. Using an n_{Lr} approximated by the *blackbody demarcation* (Appendix AB), we can achieve a measure of the fine structure constant and

$$2\alpha = \frac{1}{n_{Lr}\theta_{si}} \frac{\theta_{si}}{(\theta_{si}^2 + (n_{Lr}\theta_{si})^2)^{1/2} - n_{Lr}\theta_{si}} \frac{\theta_{si}}{(84\theta_{si} - \text{RND}(42\theta_{si}))}. \quad (35)$$

Is this physically circular? No, provided you do not use the same source measure. For instance, we use Planck's expression for the ground state orbital of an atom (Appendix BL) to resolve a calculated value for α and then use that to resolve θ_{si} . Planck's expression is a function of the measures for electron mass, ground state orbital distance and the speed of light, the latter which is defined. This is then compared with the 2018 CODATA value for α which is calculated based on source measures for elementary charge, the electric and reduced Planck constants, and the speed of light. Thus, we use one set of distinct source measures to predict the calculated value resolved using a different set of source measures, thus avoiding overlap of the source with the prediction.

A second potential conflict regards the source measure used to resolve n_{Lr} , the demarcation associated with a phenomenon. We avoid this by using the demarcation associated with blackbody radiation as an approximation to that for α . In that the n_{Lr} for α is resolved with the expression $n_{Lr}\theta_{si} = 276$, its use would return the value for θ_{si} thus effecting circular reasoning. For now, the demarcation approximation is sufficient, but over time new methods should be developed for the measure of demarcations therein improving this calculation.

For simplicity, we could just use the Schwartz and Harris results for our source data. Or we could use studies based on charge couplings. Regardless of the selected method, the conclusions are the same. We demonstrate an MQ calculation with no difference when compared to unrelated measurement phenomena.

A final consideration often used when using MQ to resolve descriptions of phenomena regards scaling operations. Specifically, the *fundamental expression* is scaled from the quantum to the cosmological thus correlating these two domains. But is this scaling physically significant? For one, scaling fixes the rate of expansion as a constant, such that the radial rate of *universal expansion* is θ_{si} . Scaling also implies that the universe is flat, a perfect balance such that its rate of expansion corresponds to its critical density. All of these points-of-view are intensely debated perspectives which come implicit to scaling. We begin by demonstrating the mathematical implementation of scaling.

$$l_f m_f = 2\theta_{si} t_f, \quad (36)$$

$$n_L l_f n_M m_f = 2\theta_{si} n_T t_f, \quad (37)$$

$$(n_T t_f n_L l_f) l_f m_f = 2\theta_{si} t_f (n_T t_f n_L l_f), \quad (38)$$

$$(n_L l_f) m_f \left(\frac{n_T t_f c}{n_L l_f} \right) = 2\theta_{si} (n_T t_f). \quad (39)$$

Such that the diameter of the universe is $D_U = n_{Lu} l_f$ and the age of the universe is $A_U = n_{Tu} t_f$, then

$$D_U m_f \left(\frac{A_U c}{D_U} \right) = 2\theta_{si} A_U. \quad (40)$$

Such that

$$\frac{2\theta_{si} A_U}{D_U} = \frac{2\theta_{si} t_f}{l_f} \frac{n_T}{n_L} = \frac{2\theta_{si}}{c} \frac{n_T}{n_L} = m_f \frac{n_T}{n_L}, \quad (41)$$

$$m_f \frac{n_{Tu}}{n_{Lu}} = m_f \frac{1}{m_f} = 1. \quad (42)$$

And finally

$$D_U = 2\theta_{si} A_U = 2 \cdot 3.26239 \cdot 13.794 = 90.003 \text{ gly}, \quad (43)$$

$$m_f = \frac{D_U}{A_U c} = \frac{90.003}{13.794 \cdot 299792458} = 2.1764 \cdot 10^{-8} \text{ kg}. \quad (44)$$

With MQ there are some notable paradoxes that are difficult to avoid — specifically, $c = n_L l_f / n_T t_f$ — such that l_f and t_f are physically significant references, scaling of n_L and/or n_T is applied such that $n_L = n_T$. The argument for a break down in scaling is an argument against an “observed” speed of light for all domain considerations. Not only is this problematic, but arguments for differences invite the obvious dilemma of where such domains should end, and new domains should begin. What are the physical circumstances of these domain boundaries? MQ does

Table 4. Tests extending the *fundamental expression* across the measurement domain.

Variances	Target frame variances from local measurement			
	Variance Test	Target	Measured	Difference
M_{CMB} $= f(n_L l_f)$	$A_{qe} \theta_{si}^4 / n_L l_f$	$1.50160 \cdot 10^{50} \text{ kg}$	$1.50149 \cdot 10^{50} \pm 2.910^{35} \text{ kg}^a$	$0.00011 \cdot 10^{50} \text{ kg}$
M_{vis} $= f(n_{Tu})$	$M_{vis} = \Omega_{vis} n_{Tu} M_{acr}$	$1.4794 \cdot 10^{53} \text{ kg}$	$(1.48 \pm 0.01) \cdot 10^{53} \text{ kg}^b$	$0.00 \cdot 10^{53} \text{ kg}$
T_{CMB} $= f(n_M m_f)$	$((n_M m_f)_{qe} c^3 / 4\sigma V_{qe})^{1/4}$	2.72553 K	$2.72548 \pm 0.00057 \text{ K}^a$	0.00005 K
Ω_{vis} $= f(\theta_{si})$	$\Omega_{vis} = 2 / \theta_{si} (\theta_{si}^2 + 2)$	4.8488348953%	$4.97 \pm 0.82\%^c$	0.12%

^aA consolidate study of CMB temperature measurements prior to 2009 [24].

^bAstrophysical constants [25].

^c2018 Planck Collaboration Results [26].

not suggest a change in the relation of measures with respect to differently considered scales. It follows, such that the observed speed of light is constant regardless of its scope of measurement, then the universe must be flat.

To approach this from a physical perspective, we need a phenomenon that occurs on a cosmological scale, can be measured in the local environment, and ideally formed in the earliest epoch. Measurements of the CMB meet all of these criteria and offer us the opportunity to compare the theoretical calculations of MQ across both epoch and measurement domains. Specifically, MQ predicts that the CMB is the result of universal mass accretion during the quantum epoch (Appendix BE). Given the fixed period of elapsed time associated with this epoch (Appendix BG) and the fixed rate of accretion, we can resolve the total associated energy of the CMB (Appendix BH). We then use the measured age of the universe to resolve the present-day density and temperature and compare that with local measurements made in the present epoch, thus cross comparing both epoch and measurement domains. Such that the MQ calculations of CMB formation during the quantum epoch match the local measurements during the expansionary epoch, we resolve the correlation in the “difference” column of Table 4.

As expected, there is no physically significant difference outside the noted precision associated with each measure. Moreover, any differences resolved moving forward, we predict, will remain within the precision error of the measurement. Why can we make this prediction? Because any deviation from a scaling of the *fundamental expression* opens the door to significant failures of well-tested phenomena, such as the speed of light in a vacuum.

4. Discussion

With this, we have completed a solution that ties together many concepts in a single result. Some may wonder why such a foreign series of calculations would produce correspondence. We would ask a more fundamental question. Where is classical mechanics less precise?

This completes a high-level review of the foundational components of MQ. Importantly, MQ is an approach, not a new form of physics, not new axioms, or

new ideas outside those principles consistent with classical theory. We expand our existing nomenclature merely to see if something new can be learned. And with that expansion new observations are made.

Appendix A. Angular Description of θ_{si}

Consider a beam of X-rays such that the signal and idler are equal.

$$\theta = \arccos\left(\frac{G_x}{n_{sx} + n_{ix} - n_{px}}\right), \quad (\text{A.1})$$

$$\theta = \arcsin\left(\frac{G_y}{n_{iy} + n_{iy} + n_{py}}\right), \quad (\text{A.2})$$

$$(\cos(\theta)(n_{sx} + n_{ix} - n_{px}), \sin(\theta)(n_{iy} + n_{iy} + n_{py})) = \vec{G}, \quad (\text{A.3})$$

$$(n_{sx} \cos(\theta) + n_{ix} \cos(\theta) - n_{px} \cos(\theta), n_{iy} \sin(\theta) + n_{iy} \sin(\theta) + n_{py} \sin(\theta)) = \vec{G}, \quad (\text{A.4})$$

$$(n_{sx} \cos(\theta), n_{sy} \sin(\theta)) + (n_{ix} \cos(\theta), n_{iy} \sin(\theta)) - (n_{px} \cos(\theta), -n_{py} \sin(\theta)) = \vec{G}, \quad (\text{A.5})$$

$$(n_{sx} \cos(\theta), n_{sy} \sin(\theta)) + (n_{ix} \cos(\theta), n_{iy} \sin(\theta)) - (n_{px} \cos(\theta), -n_{py} \sin(\theta)) = \vec{G}, \quad (\text{A.6})$$

$$(n_{sx} \cos(\theta), n_{sy} \sin(\theta)) + (n_{ix} \cos(\theta), n_{iy} \sin(\theta)) = (n_{px} \cos(2\pi - \theta), n_{py} \sin(2\pi - \theta)) + \vec{G}. \quad (\text{A.7})$$

Thus, $\theta_s = \theta_i$ and $\theta_p = 2\pi - \theta$ and half the Planck momentum are equal in value

$$\theta_{si} = \frac{n_L l_f n_M m_f}{2n_T t_f} = 3.26239 \text{ kg m s}^{-1}. \quad (\text{A.8})$$

One solution is

$$(\text{definition } c = n_L l_f / n_T t_f) \quad n_L = n_T = 1, \quad (\text{A.9})$$

$$n_m = 1/2. \quad (\text{A.10})$$

Table A.1 presents the Shwartz and Harris model describing their measurements.

Table A.1. Shwartz and Harris model of radian measures of the $\mathbf{k}$ vectors of the pump, signal and idler for maximally entangled Bell state at the degenerate Adapted from [17].

Bell State	θ_p	θ_s	θ_i
$(H_s, V_i\rangle + V_s, H_i\rangle)/\sqrt{2}$	0.1208 3.02079	-0.1208 3.26239	-0.1208 3.26239

Appendix D. Length Contraction (Informativity Differential)

$$a^2 + b^2 = c^2, \quad (\text{D.1})$$

$$1^2 + n_{Lb}^2 = n_{Lc}^2. \quad (\text{D.2})$$

Such that $n_{La} = 1$, $n_{Lb} = n_{Lc}$ by definition, we replace n_{Lb} with n_{Lr} and $n_{Lc} = n_{Lr} + Q_L$. Q_L is always the remainder of a more precise calculation $f(b)$.

$$1^2 + n_{Lr}^2 = (n_{Lr} + Q_L)^2, \quad (\text{D.3})$$

$$Q_L + n_{Lr} = \sqrt{1 + n_{Lr}^2}, \quad (\text{D.4})$$

$$Q_L = \sqrt{1 + n_{Lr}^2} - n_{Lr}, \quad (\text{D.5})$$

$$Q_L n_{Lr} = n_{Lr} (\sqrt{1 + n_{Lr}^2} - n_{Lr}). \quad (\text{D.6})$$

$2Q_L n_{Lr}$ is the *Informativity differential*. Alternatively,

$$Q_L + n_{Lr} = \sqrt{1 + n_{Lr}^2}, \quad (\text{D.7})$$

$$Q_L^2 + 2Q_L n_{Lr} + n_{Lr}^2 = 1 + n_{Lr}^2, \quad (\text{D.8})$$

$$Q_L^2 + 2Q_L n_{Lr} = 1, \quad (\text{D.9})$$

$$\frac{Q_L^2}{2} + Q_L n_{Lr} = \frac{1}{2}, \quad (\text{D.10})$$

$$Q_L(Q_L + 2n_{Lr}) = 1. \quad (\text{D.11})$$

Such that $Q_L^2/2$ approaches 0 as $n_{Lr} \rightarrow \infty$ then

$$\lim_{n_{Lr} \rightarrow \infty} Q_L n_{Lr} = \frac{1}{2}. \quad (\text{D.12})$$

Appendix E. Classical Solution to the Gravitational Constant

$$c = \frac{n_L l_f}{n_T t_f}, \quad (\text{E.1})$$

$$v_e = \left(\frac{2GM}{r} \right)^{1/2} = \left(\frac{2Gn_M m_f}{n_L l_f} \right)^{1/2}. \quad (\text{E.2})$$

Such that $n_L = n_T = 1$ (Eq. (A.9)) and $n_M = 1/2$ (Eq. (A.10)), or any factor of these values, then

$$\frac{n_L^2 l_f^2}{n_T^2 t_f^2} = \frac{2Gn_M m_f}{n_L l_f}, \quad (\text{E.3})$$

$$G = \frac{n_L^3 l_f^3}{m_f n_T^2 t_f^2} = \frac{l_f^3}{t_f^2} \frac{1}{m_f} = \left(\frac{l_f}{t_f} \right)^3 \left(\frac{t_f}{m_f} \right). \quad (\text{E.4})$$

Appendix F. Discrete Solution to the Gravitational Constant

Such that half the Planck momentum is defined as $p = l_f m_f / 2t_f$ and where this momentum p and angular measure θ_{si} are shown to be equal in value using the Schwartz and Harris experiments measure of θ for the signal and idler of polarization entangled photons at the degenerate frequency, we use the expression for discrete gravity (Appendix C) and reduce. This is accomplished where $Q_L n_{Lr} = 1/2$ at the upper count limit, Eq. (D.12).

$$G = \frac{Q_L r c^3}{\theta_{si}} = \frac{Q_L n_{Lr} l_f c^3}{\theta_{si}} = Q_L n_{Lr} l_f c^3 \frac{2t_f}{l_f m_f}, \quad (\text{F.1})$$

$$G = \left(\frac{l_f}{t_f} \right)^3 \left(\frac{t_f}{m_f} \right). \quad (\text{F.2})$$

Appendix G. Escape Velocity as a Function of Classical G

$$v_e = \left(\frac{2GM}{r} \right)^{1/2}, \quad (\text{G.1})$$

$$c^2 > \frac{2}{n_{Lr} l_f} \left(\frac{l_f^3}{t_f^3} \frac{t_f}{m_f} \right) n_M m_f, \quad (\text{G.2})$$

$$n_{Lr} > 2n_M. \quad (\text{G.3})$$

Appendix H. Escape Velocity as a Function of Discrete G

$$v_e = \left(\frac{2GM}{r} \right)^{1/2}, \quad (\text{H.1})$$

$$c > \left(\frac{2}{r} \frac{Q_L r c^3}{\theta_{si}} \frac{n_M \theta_{si}}{Q_L n_{Lr} c} \right)^{1/2} > \left(\frac{2n_M c^2}{n_{Lr}} \right)^{1/2}, \quad (\text{H.2})$$

$$n_{Lr} > 2n_M. \quad (\text{H.3})$$

Appendix I. Solving the Minimum Measure Counts

Such that $n_{Lr} = 2n_M$ at the bound, $n_L = n_T$ from $c = n_L l_f / n_T t_f$ and $2n_{Lr} n_M n_L = n_T$ (Eq. (B.7)), then

$$2n_{Lr} n_M n_L = n_T, \quad (\text{I.1})$$

$$2n_{Lr} n_M = 1, \quad (\text{I.2})$$

$$2(2n_M) n_M = 1, \quad (\text{I.3})$$

$$n_M^2 = \frac{1}{4}, \quad (\text{I.4})$$

$$n_M = \frac{1}{2}. \quad (\text{I.5})$$

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Such that $n_M = 1/2$ and Eq. (B.7), then

$$2n_{Lr} \frac{1}{2} n_L = n_T, \quad (\text{I.6})$$

$$n_{Lr} = \frac{n_T}{n_L} = 1, \quad (\text{I.7})$$

$$n_{Lr} = n_L = n_T = 1. \quad (\text{I.8})$$

Appendix J. Discreteness of Measure

Not measurable

$$5.25l_f - 5.00l_f = 0.25l_f. \quad (\text{J.1})$$

Not measurable

$$10.25l_f - 5.00l_f = 5.25l_f, \quad (\text{J.2})$$

$$5.25l_f - 5.00l_f = 0.25l_f. \quad (\text{J.3})$$

Measurable

$$10.00l_f - 5.00l_f = 5.00l_f. \quad (\text{J.4})$$

Appendix K. Measurement Frameworks

The three frameworks are as follows:

Reference Framework — This is the framework of the observer where properties of the reference $\overline{AB}$ are observed. With respect to the standard understanding, this framework differs only in that measure is a count function of discrete length measures equal to one.

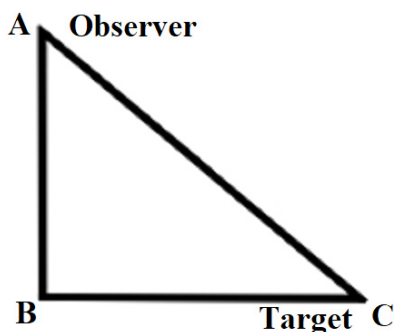


Fig. K.1. Count of distance measures along $\overline{AC}$

Measurement Framework — This framework shares properties with the Reference Framework. It is characterized as some known count of the reference describing where count properties of the reference $\overline{BC}$ are observed.

Target Framework — This framework is characterized by the property of measure of non-discreteness, that being the framework of the universe that contains the phenomenon $\overline{AC}$.

Appendix L. Precise Description of $\hbar$

Consider the Planck Length

$$\frac{c^3}{G} \approx \frac{\hbar}{l_f^2}. \quad (\text{L.1})$$

Using $G = c^3 t_f / m_f$ (Eq. (F.2)), then

$$\frac{Q_L c^3}{r \theta_{si}} \approx \frac{G}{r^2}, \quad (\text{L.2})$$

$$\frac{Q_L n_{Lr} l_f c^3}{\theta_{si}} \approx c^3 \frac{t_f}{m_f}, \quad (\text{L.3})$$

$$\theta_{si} \approx Q_L n_{Lr} l_f \left(\frac{m_f}{t_f} \right), \quad (\text{L.4})$$

$$\theta_{si} \approx Q_L n_{Lr} l_f \left(\frac{c^3}{G} \right) = Q_L n_{Lr} l_f \left(\frac{\hbar}{l_f^2} \right) = Q_L n_{Lr} \frac{\hbar}{l_f}, \quad (\text{L.5})$$

$$\hbar = \frac{\theta_{si} l_f}{Q_L n_{Lr}}, \quad (\text{L.6})$$

$$\hbar = \lim_{n_{Lr} \rightarrow \infty} \frac{\theta_{si} l_f}{Q_L n_{Lr}} = 2 \theta_{si} l_f = 1.05453 \cdot 10^{-34} \text{ Js}. \quad (\text{L.7})$$

Appendix M. Precise Description of G

Using discrete gravity (Eq. (C.3)), then

$$\frac{Q_L c^3}{r \theta_{si}} \approx \frac{G}{r^2}, \quad (\text{M.1})$$

$$G = \frac{Q_L n_{Lr} l_f c^3}{\theta_{si}}, \quad (\text{M.2})$$

Table L.1. Difference between $\hbar$ and $\hbar$ relative to $\lim_{n_{Lr} \rightarrow 84.600549677554} f(\hbar)$.

	$84.6005 l_f$	$200 l_f$	$500 l_f$	$1,000 l_f$	$10,000 l_f$	13.794 Gly
Dif	$6.1599 \cdot 10^{-10} \%$	$2.86768 \cdot 10^{-5} \%$	$3.39266 \cdot 10^{-5} \%$	$3.46766 \cdot 10^{-5} \%$	$3.49240 \cdot 10^{-5} \%$	$3.49265 \cdot 10^{-5} \%$

Table M.1. Difference between $Q_L c^3 / r \theta_{si}$ and G / r^2 with respect to $\lim_{nLr \rightarrow \infty} f(G)$.

	$84.6005 l_f$	$500 l_f$	$10,000 l_f$	a_0^a	1 m	13.794 Gly
Difference	$3.49 \cdot 10^{-5} \%$	$1.00 \cdot 10^{-6} \%$	$2.5 \cdot 10^{-9} \%$	$0.0 \cdot 10^{-30b}$	$0.0 \cdot 10^{-30b}$	$0.0 \cdot 10^{-30b}$

^aDescribes the distance associated with the ground state orbital of an atom.

^bExceeds the physical significance of the source measures.

$$G = 2Q_L n_{Lr} c^3 \frac{t_f}{m_f}, \quad (\text{M.3})$$

$$G = \lim_{n_{Lr} \rightarrow \infty} 2Q_L n_{Lr} c^3 \frac{t_f}{m_f} = c^3 \frac{t_f}{m_f} = 6.67408 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}. \quad (\text{M.4})$$

Appendix N. Solutions to G Using the CODATA

Using Planck's Unit expressions solved for G , we consider CODATA values from 2010 to 2018.

Notably, the geometry associated with the *Informativity differential* is easily exposed at the mid-point of the 2010 and 2018 results for G , giving us the 2014 selection.

$$G_{\text{avg}} = \frac{6.6743015 \cdot 10^{-11} + 6.6738480 \cdot 10^{-11}}{2} = 6.67407(48) \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}. \quad (\text{N.1})$$

Table N.1. Calculations of G using Planck's unit expressions.

CODATA ^a	2010 ($10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$)	2014 ($10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$)	2018 ($10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$)
$G = l_p^2 c^3 / \hbar$	6.673845(89)	6.674088(32)	6.674301(29)
$G = t_p^2 c^5 / \hbar$	6.673846(87)	6.674089(29)	6.674302(86)
$G = \hbar c / m_p^2$	6.67382(75)	6.674078(07)	6.674300(63)

^aPlanck's Unit expressions are resolved using those values provided in CODATA publications for 2010, 2014 and 2018 [1–3].

Table N.2. Matching CODATA values for $\hbar$, G and the Planck units with MQ calculated values.

Year	$\hbar$ (10^{-34} Js)	l_p (10^{-35} m)	t_p (10^{-44} s)	m_p (10^{-8} kg)	(10^{-11} $G \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$)
2010 ^a	1.054571726(47)	1.616199(97)	5.39106(32)	2.17651(13)	6.67384(80)
2014 ^a	1.054571800(13)	1.616229(38)	5.39116(13)	2.176470(51)	6.67408(31)
2018 ^a	1.054571817	1.616255(18)	5.391247(60)	2.176434(24)	6.67430(15)
$f(\hbar, G)^b$	1.0545349844(45)	1.6161716871(50)	5.3909684650(90)	2.1764705491(26)	6.6738448362(53)
$f(\hbar, G)^b$	1.0545349844(45)	1.6161999120(35)	5.3910626131(72)	2.1764325398(24)	6.6740779428(56)
$f(\hbar, G)^b$	1.0545718176(46)	1.6162281374(12)	5.3911567628(97)	2.1764705491(26)	6.6740779428(56)
$f(\hbar, G)^b$	1.0545718176(46)	1.6161999120(35)	5.3910626131(72)	2.1765085590(91)	6.6738448362(53)

^aCODATA publications for 2010, 2014 and 2018 [1–3].

^bUsing precise G and $\hbar$, we account for the *Informativity differential* at the *blackbody demarcation* $n_{Lr} = 84.6005496647(07)$ (i.e. italicized), the upper count limit (i.e. $\lim_{nLr \rightarrow \infty}$) (not italicized), or a mix of the two.

Appendix O. Bounds to Measure

Such that $c = l_f/t_f = 2.9979245810^8 \text{ m s}^{-1}$ and such that $n_L = n_T = 2n_M$ (Eq. (G.3)), then

$$n_L = 2.99792458 \cdot 10^8 / l_f = 1.8549218804(41) \cdot 10^{43} \text{ units s}^{-1}. \quad (\text{O.1})$$

$$n_M = 4.037112339423(07) \cdot 10^{35} / m_f = 1.8549218804(41) \cdot 10^{43} \text{ units s}^{-1}. \quad (\text{O.2})$$

$$n_T = 1/t_f = 1.8549218804(41) \cdot 10^{43} \text{ units s}^{-1}. \quad (\text{O.3})$$

Count per second describes the length frequency, the mass frequency, and the Planck frequency. The bound to the mass-to-length frequency is (Eq. (G.3))

$$\frac{n_M}{n_{Lr}} < \frac{1}{2}. \quad (\text{O.4})$$

Notably, $2n_M/n_{Lr}$ equals v^2/c^2 .

Appendix P. Electromagnetic Description of θ_{si}

Such that $\theta_{si} = Q_L r c^3 / G$, (Eq. (C.3)), Planck's expression for length $(c^3/G) = (\hbar/l_f^2)$ and $\lim_{n_{Lr} \rightarrow \infty} (Q_L n_{Lr}) = 1/2$ (Eq. (D.12)), then the electromagnetic description of θ is

$$\theta_{si} = \frac{Q_L r c^3}{G} = \left(\frac{c^3}{G} \right) Q_L n_{Lr} l_f = \frac{c^3}{G} \frac{l_f}{2} = \frac{\hbar}{l_f^2} \frac{l_f}{2} = \frac{\hbar}{2l_f} = 3.26239 \text{ kg m s}^{-1}. \quad (\text{P.1})$$

Appendix Q. Momentum Description of θ_{si}

Such that $\theta_{si} = Q_L r c^3 / G$, (Eq. C.3), $G = (l_f/t_f)^3 (t_f/m_f)$ (Eq. (F.2)) and $\lim_{n_{Lr} \rightarrow \infty} (Q_L n_{Lr}) = 1/2$ (Eq. (D.12)), then the momentum description of θ is

$$\theta_{si} = \frac{Q_L r c^3}{G} = \left(\frac{c^3}{G} \right) Q_L n_{Lr} l_f = \frac{l_f c^3}{2G} = \frac{l_f m_f}{2t_f} = 3.26239 \text{ kg m s}^{-1}. \quad (\text{Q.1})$$

Table P.1. MQ predicted radian measures of the k vectors of the pump, signal, and idler for the maximally entangled Bell state of X-rays at the degenerate frequency.^a

Angle	θ_p	Pump (radians)	θ_s	Signal (radians)	θ_i	Idler (radians)
$\pi - \theta_{\text{Max}}$	$(\hbar/2l_f) - \pi$	0.1207976503 (35)	$\pi - (\hbar/2l_f)$	-0.1207976503 (35)	$\pi - (\hbar/2l_f)$	-0.1207976503 (35)
θ_{Max}	$2\pi - (\hbar/2l_f)$	3.0207950032 (55)	$(\hbar/2l_f)$	3.2623903039 (25)	$(\hbar/2l_f)$	3.2623903039 (25)

^aCorresponds to measurements by Schwartz *et al.* [17–19].

Table Q.1. MQ predicted radian measures of the k vectors of the pump, signal, and idler for the maximally entangled Bell state of X-rays at the degenerate frequency.^a

Angle	θ_p	Pump (radians)	θ_s	Signal (radians)	θ_i	Idler (radians)
$\pi - \theta_{\text{Max}}$	$(l_f c^3/2G) - \pi$	0.1207976503 (35)	$\pi - (l_f c^3/2G)$	-0.1207976503 (35)	$\pi - (l_f c^3/2G)$	-0.1207976503 (35)
θ_{Max}	$2\pi - (l_f c^3/2G)$	3.0207950032 (55)	$(l_f c^3/2G)$	3.2623903039 (25)	$(l_f c^3/2G)$	3.2623903039 (25)

^aCorresponds to measurements by Schwartz *et al.* [17–19].

Appendix R. Angular Interpretation of $\hbar$

$$L = r\theta = l_f \left(\frac{\hbar}{2l_f} \right) = \frac{\hbar}{2}, \quad (\text{R.1})$$

$$\theta = \frac{\hbar}{2l_f} = 3.26239 \text{ radians}, \quad (\text{R.2})$$

$$\theta_{si} = \frac{Q_L r c^3}{G} = Q_L n_{Lr} l_f \frac{c^3}{G} = \frac{l_f c^3}{2G}, \quad (\text{R.3})$$

$$\theta_{si} = \frac{l_f}{2} \left(\frac{c^3}{G} \right) = \frac{l_f}{2} \left(\frac{\hbar}{l_f^2} \right) = \frac{\hbar}{2l_f}, \quad (\text{R.4})$$

$$\theta_{si} = \theta = 3.26239 \text{ radians}. \quad (\text{R.5})$$

Appendix S. Correlating Angular Measure and Momentum at the Bound

Angular interpretation of $\hbar$.

$$\theta = \frac{\hbar}{2l_f} = 3.26239 \text{ radians}. \quad (\text{S.1})$$

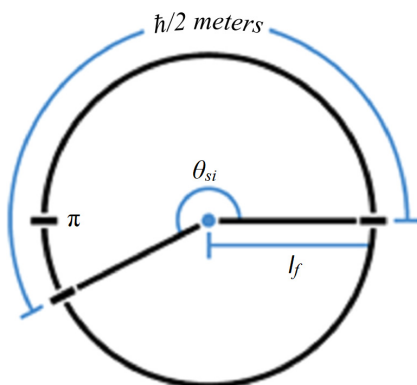


Fig. R.1. Arc length of a circle of radius l_f subtending angle θ radians. Adapted from [9]

Table S.1. Measures of θ_{si} at the Planck scale.

Dimension	Expression	Results	Units
Angle	$(n_{sx} \cos(\theta), n_{sy} \sin(\theta))$ $+ (n_{ix} \cos(\theta), n_{iy} \sin(\theta))$ $= (n_{px} \cos(2\pi - \theta),$ $n_{py} \sin(2\pi - \theta)) + \vec{G}$	3.26239 ± 0.000002^a	rad
Momentum	$\theta_{si} = \frac{l_f m_f}{2t_f}$	$3.26239030392(48)^b$	kg m/s
Non-dimensional	$((\frac{1}{2\theta_{si}})^{1/3})^2 + (\frac{n_L}{n_{Le}})^2 = 1$	$3.26239030392(48)$	—

^a θ_{si} can be measured as the polarization angle of quantum entangled X-rays at the degenerate frequency [17–19] or calculated as a function of the fine structure constant. With respect to the Target Frame θ_{si} is non-dimensional.

^bCalculated as a function of the 2018 value for the fine structure constant. [3]

Using discrete gravity to resolve momentum with respect to the Measurement Frame.

$$\frac{Q_L c^3}{rp} = \frac{G}{r^2}, \quad (\text{S.2})$$

$$p = \frac{Q_L r c^3}{G} = Q_L n_{Lr} l_f \frac{c^3}{G} = \frac{l_f c^3}{2 G}. \quad (\text{S.3})$$

Such that $G = c^3 t_f / m_f$ (Appendix F), $l_f m_f = 2\theta_{si} t_f$ (Appendix V) and $\hbar = 2\theta_{si} l_f$ (above), then

$$p = \frac{l_f}{2} \left(\frac{c^3}{1} \right) \left(\frac{m_f}{c^3 t_f} \right) = \frac{2l_f}{4} \left(\frac{m_f}{t_f} \right) \left(\frac{l_f}{l_f} \right), \quad (\text{S.4})$$

$$p = \frac{2l_f}{2l_f} \left(\frac{l_f m_f}{2t_f} \right) = \frac{\hbar}{2l_f}. \quad (\text{S.5})$$

Comparing results such that each equals $\hbar/(2l_f)$, then

$$\theta \text{ radians} = p \text{ kg m s}^{-1}. \quad (\text{S.6})$$

Appendix T. Metric Differential

1. Gravitational Curvature: $c/\theta_{si} = m_f/2$

$$\frac{Q_L l_f}{n_{Lr} t_f^2}, \quad (\text{T.1})$$

$$\frac{Q_L l_f}{n_{Lr} t_f^2} \left(\frac{c}{\theta_{si}} \right), \quad (\text{T.2})$$

$$\frac{Q_L c^3}{r \theta_{si}} = \frac{G}{r^2}. \quad (\text{T.3})$$

2. Fine Structure Constant: $42\theta_{si}$ —RND($42\theta_{si}$)

$$\alpha_f^{-1} = 42\theta_{si}, \quad (\text{T.4})$$

$$\alpha_p^{-1} = \alpha_f^{-1} + Q_{n\theta} = 42\theta_{si} + (42\theta_{si} - \text{RND}(42\theta_{si})), \quad (\text{T.5})$$

$$\alpha_p^{-1} = 84\theta_{si} - \text{RND}(42\theta_{si}). \quad (\text{T.6})$$

3. Elementary Charge: $41\theta_{si}$ /RND($41\theta_{si}$)

$$e_f = \frac{1}{4(\theta_{si} - 1)} l_f c^2, \quad (\text{T.7})$$

$$r_{q41} = \frac{(n_\theta - 1)\theta_{si}}{\text{RND}((n_\theta - 1)\theta_{si})}, \quad (\text{T.8})$$

$$e_p = r_{q41} e_f. \quad (\text{T.9})$$

4. Scalar Counts of the Fundamental Measures θ_{si} and $l_f : 1/2$

$$n_\theta = \text{RND}(n_L/2). \quad (\text{T.10})$$

5. Orbital Star Velocities: θ_{si}

$$v_{bc} = c\sqrt{2m_f}. \quad (\text{T.11})$$

$$v_b = \theta_{si} c\sqrt{2m_f}. \quad (\text{T.12})$$

6. Observational Domains: θ_{si}

$$\Delta_{mdif} = \left(\frac{\pi}{\theta_{si}} \right)^{2/3}. \quad (\text{T.13})$$

Notably, all dimensional transforms reflect a multiple of kilograms while the expression for orbital velocity has m_f , meant to be a non-dimensional count.

Table T.1. Unit analysis of the metric differential between the Target and Measurement Frames.^a

Dimension	Target Frame	Measurement Frame	Dif	Metric Differential
Gravitation	m s^{-2}	kg m s^{-2}	kg	$m_f/2$
Fine Structure	non-dimensional	non-dimensional	—	$42\theta_{si}$ —RND($42\theta_{si}$)
Elementary Charge	$\text{m}^3 \text{s}^{-2}$	$\text{C (kg m}^3 \text{s}^{-2})^b$	kg	$41\theta_{si}$ /RND($41\theta_{si}$)
Counts n_θ to n_L	non-dimensional	non-dimensional	—	$n_\theta = \text{RND}(n_L/2)$
Orbital Velocity	m s^{-1}	m s^{-1}	—	θ_{si}
Observational Domains	non-dimensional	non-dimensional	—	$(\pi/\theta_{si})^{2/3}$

^a θ_{si} can be measured as the polarization angle of quantum entangled X-rays at the degenerate frequency [17–19] or calculated as a function of the fine structure constant. With respect to the Target Frame, θ_{si} is non-dimensional.

^bThe square of the Coulomb is measured in esu.

Appendix U. The Metric Offset

$$r_q = \frac{(n_\theta - 1)\theta_{si}}{\text{RND}((n_\theta - 1)\theta_{si})}, \quad (\text{U.1})$$

$$r_q = \frac{(42 - 1)\theta_{si}}{\text{RND}((42 - 1)\theta_{si})} = \frac{41\theta_{si}}{\text{RND}(41\theta_{si})} = 0.998194048216. \quad (\text{U.2})$$

Appendix V. Fundamental Expression

Such that half the Planck momentum is defined as $p = l_f m_f / t_f$ and where this momentum p and angular measure θ_{si} are shown to be equal in value using the Shwartz and Harris experiments measure of θ for the signal and idler of polarization entangled photons at the degenerate frequency, we use the expression for discrete gravity (Appendix C) and reduce. This is accomplished where $Q_L n_{Lr} = 1/2$ at the upper count limit, Eq. (D.12).

$$G = \frac{Q_L r c^3}{\theta_{si}} = \frac{Q_L n_{Lr} l_f c^3}{\theta_{si}} = Q_L n_{Lr} l_f c^3 \frac{2t_f}{l_f m_f}, \quad (\text{V.1})$$

$$G = \left(\frac{l_f}{t_f}\right)^3 \left(\frac{t_f}{m_f}\right). \quad (\text{V.2})$$

Then,

$$\theta_{si} = \frac{Q_L r c^3}{G} = \frac{l_f c^3}{2 G} = \frac{l_f c^3}{2 \frac{1}{1} \left(\frac{m_f}{c^3 t_f}\right)} = \frac{l_f}{2} \left(\frac{m_f}{t_f}\right). \quad (\text{V.3})$$

$$l_f m_f = 2\theta_{si} t_f. \quad (\text{V.4})$$

This is physically confirmed by the count solution (Appendix I) using Heisenberg's uncertainty principle such that $n_M = 1/2$, $n_L = n_T = 1$.

Appendix W. Fundamental Measures

$$\frac{Q_L c^3}{r\theta_{si}} = \frac{G}{r^2}, \quad (\text{W.1})$$

$$l_f = \frac{G\theta_{si}}{Q_L n_{Lr} c^3} = \frac{2G\theta_{si}}{c^3}. \quad (\text{W.2})$$

Using G and c from the 2014 CODATA [2].

$$l_f = \frac{2G\theta_{si}}{c^3} = \frac{2 \cdot 6.67408 \cdot 10^{-11} \cdot 3.26239}{(299792458)^3} = 1.61620 \cdot 10^{-35} \text{ m}, \quad (\text{W.3})$$

$$t_f = \frac{l_f}{c} = \frac{2G\theta_{si}}{c^4} = \frac{2 \cdot 6.67408 \cdot 10^{-11} \cdot 3.26239}{(299792458)^4} = 5.39106 \cdot 10^{-44} \text{ s}, \quad (\text{W.4})$$

$$m_f = \frac{2\theta_{si}}{c} = t_f \frac{c^3}{G} = \frac{2 \cdot 3.26239}{299792458} = 2.17643 \cdot 10^{-8} \text{ kg}. \quad (\text{W.5})$$

Appendix X. Correlating the Planck Units with the Fundamental Measures

$$\frac{2G\theta_{si}}{c^3} \approx \left(\frac{\hbar G}{c^3}\right)^{1/2} (\text{m}), \quad (\text{X.1})$$

$$\frac{2G\theta_{si}}{c^4} \approx \left(\frac{\hbar G}{c^5}\right)^{1/2} (\text{s}), \quad (\text{X.2})$$

$$\frac{2\theta_{si}}{c} \approx \left(\frac{\hbar c}{G}\right)^{1/2} (\text{kg}). \quad (\text{X.3})$$

Such that $\hbar \approx \theta_{si} l_f / Q_L n_{Lr}$ and $G \approx 2Q_L n_{Lr} c^3 t_f / m_f$, we can validate the correlation.

$$\frac{2G\theta_{si}}{c^3} \approx \left(\frac{\hbar G}{c^3}\right)^{1/2}, \quad (\text{X.4})$$

$$\hbar \approx \frac{4\theta_{si}^2}{c^3} G, \quad (\text{X.5})$$

$$\frac{\theta_{si} l_f}{Q_L n_{Lr}} \approx \frac{4\theta_{si}^2}{c^3} 2Q_L n_{Lr} c^3 \frac{t_f}{m_f}, \quad (\text{X.6})$$

$$\theta_{si} l_f \approx 8\theta_{si}^2 \frac{t_f}{m_f} (Q_L n_{Lr})^2, \quad (\text{X.7})$$

$$l_f m_f \approx 2\theta_{si} t_f (2Q_L n_{Lr})^2, \quad (\text{X.8})$$

$$\lim_{n_{Lr} \rightarrow \infty} (Q_L n_{Lr}) = \frac{1}{2}. \quad (\text{X.9})$$

Appendix Y. Correlating the Newton and MQ Expressions for Gravitational Curvature

$$\frac{Q_L c^3}{r\theta_{si}} \approx \frac{G}{r^2}, \quad (\text{Y.1})$$

$$\frac{Q_L}{\theta_{si}} \approx \frac{1}{n_{Lr} l_f} \frac{t_f}{m_f}, \quad (\text{Y.2})$$

$$Q_L n_{Lr} \approx \theta_{si} \frac{1}{l_f} \frac{t_f}{m_f} = \frac{1}{2}, \quad (\text{Y.3})$$

$$\lim_{n_{Lr} \rightarrow \infty} (Q_L n_{Lr}) = \frac{1}{2}. \quad (\text{Y.4})$$

Appendix Z. Energy Relation of Baryonic and Electromagnetic Phenomena

$$\hbar = \lim_{n_{Lr} \rightarrow \infty} \frac{l_f \theta_{si}}{Q_L n_{Lr}} = 2\theta_{si} l_f, \quad (\text{Z.1})$$

$$m_f = \frac{2\theta_{si}}{c} = \frac{\theta_{si}}{Q_L n_{Lr} c}, \quad (\text{Z.2})$$

$$E_m = mc^2 = n_M \left(\frac{\theta_{si}}{Q_L n_{Lr} c} \right) c^2 = n_M \left(\frac{\theta_{si} c}{Q_L n_{Lr}} \right), \quad (\text{Z.3})$$

$$E_l = n_M \hbar v = n_M \left(\frac{4\pi \theta_{si} l_f}{t_f} \right) = n_M 4\pi \theta_{si} c, \quad (\text{Z.4})$$

$$\frac{E_l}{E_m} = \frac{n_M 4\pi \theta_{si} c}{n_M (\theta_{si} c / Q_L n_{Lr})} = 4\pi Q_L n_{Lr} = 2\pi, \quad (\text{Z.5})$$

$$E_m = \frac{E_l}{2\pi} = \frac{n_M \hbar v}{2\pi} = \left(\frac{n_M}{2\pi} \right) \hbar v. \quad (\text{Z.6})$$

Appendix AA. Fine Structure Constant

$$\alpha_f^{-1} = 42\theta_{si} = 137.020, \quad (\text{AA.1})$$

$$Q_{n\theta} = 42\theta_{si} - \text{RND}(42\theta_{si}), \quad (\text{AA.2})$$

$$\alpha_p^{-1} = \alpha_f^{-1} + Q_{n\theta} = 42\theta_{si} + (42\theta_{si} - \text{RND}(42\theta_{si})), \quad (\text{AA.3})$$

$$\alpha_p^{-1} = 84\theta_{si} - \text{RND}(42\theta_{si}) = 137.041, \quad (\text{AA.4})$$

$$n_{Lr} = 276/\theta_{si} = 84.6005, \quad (\text{AA.5})$$

$$Q_L n_{Lr} = n_{Lr} ((1 + n_{Lr}^2)^{1/2} - n_{Lr}) = 0.499983, \quad (\text{AA.6})$$

$$\Delta_{(P-C)} = \alpha_p^{-1} (1 - 2Q_L n_{Lr}), \quad (\text{AA.7})$$

$$\alpha_c^{-1} = \alpha_p^{-1} - \Delta_{(P-C)}, \quad (\text{AA.8})$$

$$\alpha_c^{-1} = \alpha_p^{-1} - \alpha_p^{-1} (1 - 2Q_L n_{Lr}) = 2Q_L n_{Lr} \alpha_p^{-1}, \quad (\text{AA.9})$$

$$\alpha_c^{-1} = 2Q_L n_{Lr} (84\theta_{si} - \text{RND}(42\theta_{si})) = 137.036. \quad (\text{AA.10})$$

Table AA.1. Fundamental, Planck-like and classical solutions to the inverse fine structure constant.

Measure	$\alpha_f^{-1} = 42\theta_{si}$	$\alpha_p^{-1} = m_e a_0 / m_f l_f$	$\alpha_c^{-1} = 4\pi \epsilon_0 \hbar c / e^2$
CODATA ^a	—	137.04078552(97)	137.035999084(21)
MQ(α)	137.020392764(84)	137.040785529(68)	137.035999083(49)
S&H	137.02039	137.04078	137.03600
Difference	—	0.00000000(00)	0.000000000(72)

^a2018 CODATA publication [3].

Table AB.1. Planck's unit expressions calculated using $\hbar, \hbar^a$

Unit	n_{Lr}	$\theta(\text{radians})$	Length (10^{-35} m)	Time (10^{-44} s)	Mass (10^{-8} kg)
$\hbar$	84.6005496647 (07)	3.26250425394 (01)	1.6162281374 (12)	5.3911567628 (97)	2.1764705491 (26)
$\hbar$	∞	3.26239030392 (48)	1.6161999120 (35)	5.3910626131 (72)	2.1764325398 (24)
CODATA ^b			1.616199 (97)	5.39106 (32)	2.17651 (13)

^aPlanck units intentionally calculated with $\lim_{n_{Lr} \rightarrow \infty} f(G)$ for all results. As treated today, this mixes frames, which causes the discrepancy between $\hbar$ and $\hbar$ results.

^b2010 CODATA publication [1].

Appendix AB. Blackbody Demarcation

$$\theta_{si} = \frac{Q_L r c^3}{G} = \left(\frac{c^3}{G} \right) Q_L n_{Lr} l_f = \frac{c^3 l_f}{2G}, \quad (\text{AB.1})$$

$$Q_L = (1 + n_{Lr}^2)^{1/2} - n_{Lr}, \quad (\text{AB.2})$$

$$\theta_{si} = \left(\frac{c^3}{G} \right) Q_L n_{Lr} l_f = \left(\frac{\hbar}{l_f^2} \right) Q_L n_{Lr} l_f, \quad (\text{AB.3})$$

$$n_{Lr} = \frac{\theta_{si} l_f}{\hbar Q_L} = \frac{\theta_{si} l_f}{\hbar ((1 + n_{Lr}^2)^{1/2} - n_{Lr})}, \quad (\text{AB.4})$$

$$(n_{Lr}^2 + n_{Lr}^4)^{1/2} - n_{Lr}^2 = \frac{\theta_{si} l_f}{\hbar}, \quad (\text{AB.5})$$

$$n_{Lr}^2 + n_{Lr}^4 = \left(\frac{\theta_{si} l_f}{\hbar} + n_{Lr}^2 \right)^2 = \frac{\theta_{si}^2 l_f^2}{\hbar^2} + \frac{2\theta_{si} l_f n_{Lr}^2}{\hbar} + n_{Lr}^4, \quad (\text{AB.6})$$

$$n_{Lr}^2 \left(1 - \frac{2\theta_{si} l_f}{\hbar} \right) = \frac{\theta_{si}^2 l_f^2}{\hbar^2}, \quad (\text{AB.7})$$

$$n_{Lr} = \sqrt{\frac{\theta_{si}^2 l_f^2}{\hbar^2 (1 - (2\theta_{si} l_f / \hbar))}} = \theta_{si} l_f \sqrt{\frac{1}{\hbar(\hbar - 2\theta_{si} l_f)}}, \quad (\text{AB.8})$$

$$n_{Lr} = 84.6005496647(07), \quad (\text{AB.9})$$

$$n_{Lr} l_f = 1.36731400926(18) \cdot 10^{-33} \text{ m}, \quad (\text{AB.10})$$

$$Q_L n_{Lr} = 0.499982536418(89). \quad (\text{AB.11})$$

Appendix AC. Quantization Ratios

$$r_q = \frac{n_L \theta_{si}}{\text{RND}(n_L \theta_{si})}. \quad (\text{AC.1})$$

Table AC.1. Quantization ratios r_q for n_{Lr} with respect to the measurement frame.

Corresponding Count n_{Lr}	Parameters		
	r_q	$n_{Lr}\theta_{si}$	n_θ
82.7614034026(45)	0.9981940482(16)	270	41
83.0679271189(51)	1.0001488522(98)	271	42
83.3744508352(57)	1.0001488522(98)	272	42
83.6809745515(63)	1.0001488522(98)	273	42
83.9874982678(69)	1.0001488522(98)	274	42
84.2940219841(76)	1.0001488522(98)	275	42
84.6005457004(82)	1.0001488522(98)	276	42
84.9070694167(88)	1.0001488522(98)	277	42
85.2135931330(94)	1.0020198790(63)	278	43

Using blackbody radiation as a guide, then

$$r_q = \frac{84.600545700482\theta_{si}}{\text{RND}(84.600545700482\theta_{si})} = 1.0000000000(00). \tag{AC.2}$$

Also, the *fundamental expression* resolves the count relation between frames as

$$(n_L)l_f\frac{m_f}{t_f} = (2n_\theta)\theta_{si}. \tag{AC.3}$$

Table AC.2. Repeating *quantization ratios* for counts n_θ with respect to the Target Frame.

Beginning n_{Lr}	Ending n_{Lr}	Repeat Count
3	6	2
4	40	10
7	21	3
11	44	4
15	105	7
19	627	33
23	322	14
26	52	2
27	135	5
31	93	3
34	204	6
35	70	2
39	78	2
41	82	2
42	1050	25
49	147	3
50	200	4
53	265	5
58	116	2
61	6893	113
64	128	2

This gives us a Target Frame count equal to

$$n_\theta = \text{RND}(n_L/2) = \text{RND}(84.600545700482/2) = 42. \quad (\text{AC.4})$$

Combining all results, the solution must conform with a Shwartz *et al.* measure of $\theta_{si} = 3.26239 \text{ rad} \pm 2 \mu\text{rad}$ [17], a count $n_\theta = 42$ and a solution to the fine structure constant having 12 digits of correspondence (see the following section). All three criteria are met only where

$$n_L \theta_{si} = 276. \quad (\text{AC.5})$$

Discrete solutions for *quantization ratios* are presented for the first 64 integers. Notably, our universe corresponds to 42, this describing the fundamental expression for the inverse fine structure constant and with that establishing the ground state orbital of an atom (Table AC.2).

Appendix AD. θ_{si} As Measured with the Fine Structure Constant

Using the Pythagorean Theorem, we resolve an expression for Q_L from discrete gravity. To solve for θ_{si} , we multiply all instances of n_{Lr} by θ_{si} such that $n_{Lr}\theta_{si} = 276$ until only θ_{si} remains.

$$Q_L = (1 + n_{Lr}^2)^{1/2} - n_{Lr}, \quad (\text{AD.1})$$

$$Q_L \theta_{si} = (\theta_{si}^2 + (\theta_{si} n_{Lr})^2)^{1/2} - n_{Lr} \theta_{si}, \quad (\text{AD.2})$$

$$Q_L = \frac{(\theta_{si}^2 + (n_{Lr} \theta_{si})^2)^{1/2} - n_{Lr} \theta_{si}}{\theta_{si}}. \quad (\text{AD.3})$$

Using the expression for α_c , then

$$\frac{1}{\alpha_c} = 2Q_L n_{Lr} (84\theta_{si} - \text{RND}(42\theta_{si})), \quad (\text{AD.4})$$

$$2\alpha_c = \frac{1}{n_{Lr}} \frac{\theta_{si}}{\theta_{si}} \frac{1}{Q_L (84\theta_{si} - \text{RND}(42\theta_{si}))}, \quad (\text{AD.5})$$

$$2\alpha_c = \frac{1}{n_{Lr} \theta_{si}} \frac{1}{Q_L} \frac{\theta_{si}}{(84\theta_{si} - \text{RND}(42\theta_{si}))}. \quad (\text{AD.6})$$

And such that $n_{Lr}\theta_{si} = 276$ and $\alpha_c = 7.2973525693(11)10^{-3}$ (CODATA), then

$$2\alpha_c = \frac{1}{n_{Lr} \theta_{si}} \frac{\theta_{si}}{(\theta_{si}^2 + (n_{Lr} \theta_{si})^2)^{1/2} - n_{Lr} \theta_{si}} \frac{\theta_{si}}{(84\theta_{si} - \text{RND}(42\theta_{si}))}, \quad (\text{AD.7})$$

$$\theta_{si} = 3.26239030392(48).^* \quad (\text{AD.8})$$

*Units are a function of physical context and frame of reference, either radians, momentum or non-dimensional.

Appendix AE. Charge Coupling Demarcation

Such that $n_{Lr}\theta_{si} = 276$ describes α_f^{-1} and such that $\theta_{si} = 3.26239030392(48) \text{ kg m s}^{-1}$, then

$$n_{Lr} = 276/\theta_{si} = 84.6005457004(82), \quad (\text{AE.1})$$

$$n_{Lr}l_f = 1.36731394519(20) \cdot 10^{-33} \text{ m}, \quad (\text{AE.2})$$

$$Q_L n_{Lr} = 0.499982536417(25). \quad (\text{AE.3})$$

Appendix AF. Upper Count Bound to Fundamental Mass

$$v = \sqrt{2\frac{GM}{R}} = \sqrt{2\frac{l_f^3}{t_f^3} \frac{t_f}{m_f} \frac{n_M m_f}{n_{Lr} l_f}} = \sqrt{2\frac{l_f^2}{t_f^2} \frac{n_M}{n_{Lr}}} = c\sqrt{2\frac{n_M}{n_{Lr}}}. \quad (\text{AF.1})$$

The smallest count of m_f with respect to l_f corresponds to the inverse of $m_f = 2.17647x10^{-8} \text{ kg}$.

$$\frac{2.17643 \cdot 10^{-8} \text{ units } m_f}{1 \text{ unit } l_f} = \frac{1 \text{ unit } m_f}{4.59467491733 \cdot 10^7 \text{ units } l_f} = \frac{1}{1/m_f} = n_{Mb}. \quad (\text{AF.2})$$

Such that m_f is dimensionless and $1/(1/m_f) = n_{Mb}$ at the bound, then

$$v_{bc} = c\sqrt{2\frac{n_M}{n_{Lr}}} = c\sqrt{2m_f}. \quad (\text{AF.3})$$

The effective classical bound velocity v_b localized to the Target Frame, θ_{si} is then

$$v_b = \theta_{si} c\sqrt{2m_f} = 204.054 \text{ km s}^{-1}. \quad (\text{AF.4})$$

Appendix AG. Observable Mass Bound

1. Calculations

$$\theta_{si} c\sqrt{2m_f} = \sqrt{\frac{GM_b}{R}}, \quad (\text{AG.1})$$

$$M_b = 2\theta_{si}^2 R c^2 m_f \frac{1}{G} = 2\theta_{si}^2 R c^2 m_f \frac{m_f}{c^3 t_f}, \quad (\text{AG.2})$$

$$M_b = 2\theta_{si}^2 R \frac{m_f^2}{l_f} = 2\theta_{si} R \frac{m_f l_f}{2t_f} \frac{m_f^2}{l_f}, \quad (\text{AG.3})$$

$$M_b = \theta_{si} R \frac{m_f^3}{t_f}. \quad (\text{AG.4})$$

2. Dimensional Analysis

$$M_b = 2n_{Lr} \frac{l_f^2 m_f^4}{4t_f^2} = 2n_{Lr} \left(\frac{l_f m_f^2}{2t_f} \right)^2, \quad (\text{AG.5})$$

$$M_b = R\theta_{si} \frac{m_f^3}{t_f} = n_{Lr} l_f \theta_{si} \frac{m_f^3}{t_f}, \quad (\text{AG.6})$$

$$M_b = 2n_{Lr} \theta_{si}^2 m_f^2. \quad (\text{AG.7})$$

And such that one unit of m_f is dimensionless where $v_{bc} = \theta_{si} c (2m_f)^{1/2}$ and such that θ_{si} is dimensionless when defined with respect to a system bound (Appendix AQ), then we are left with the units of mass in kilograms.

Appendix AH. Effective Mass of a Galaxy

$$v_b = \theta_{si} c \sqrt{2m_f}, \quad (\text{AH.1})$$

$$\Delta_{f(v_m)} = \frac{(v_m - v_b)}{v_b} = \frac{v_m}{v_b} - 1, \quad (\text{AH.2})$$

$$M_e = M_b (2\Delta_{f(v_m)} + 1) = M_b \left(2 \frac{v_e}{v_b} - 2 + 1 \right), \quad (\text{AH.3})$$

$$M_e = M_b \left(2 \frac{v_e}{v_b} - 1 \right), \quad (\text{AH.4})$$

$$M_e = \theta_{si} R \frac{m_f^3}{t_f} \left(2 \frac{v_e}{\theta_{si} c \sqrt{2m_f}} - 1 \right), \quad (\text{AH.5})$$

$$M_e = R \frac{m_f^3}{t_f} \left(2 \frac{v_e}{c \sqrt{2m_f}} - \theta_{si} \right), \quad (\text{AH.6})$$

$$v_e = \frac{v_b}{2} \left(\frac{M_e}{M_b} + 1 \right). \quad (\text{AH.7})$$

Appendix AI. Effective Velocity of a Star

Such that $v_e = v_b$ for effective and bound velocities with increasing radial distance, then

$$v_e = \left(\frac{GM_e}{R} \right)^{1/2} = \left(\frac{GM_b}{R} \left(2 \frac{v_e}{v_b} - 1 \right) \right)^{1/2}, \quad (\text{AI.1})$$

$$v_e = \left(\frac{c^3 t_f}{m_f} \frac{1}{R} \left(\theta_{si} R \frac{m_f^3}{t_f} \right) (2(1) - 1) \right)^{1/2}, \quad (\text{AI.2})$$

$$v_e = (2\theta_{si}^2 m_f c^2)^{1/2}, \quad (\text{AI.3})$$

$$v_e = (2\theta_{si}^2 E_f)^{1/2}, \quad (\text{AI.4})$$

$$v_e = \theta_{si} \sqrt{2E_f}. \quad (\text{AI.5})$$

Appendix AJ. Correlating Star Velocity to Kinetic Energy

$$v_e = \sqrt{2E_f} = \sqrt{4\theta_{si}c} = c\sqrt{\frac{4\theta_{si}}{c}} = c\sqrt{2m_f} \quad (\text{AJ.1})$$

$$c\sqrt{2m_f} = c\sqrt{\frac{2}{1/m_f}} = c\sqrt{\frac{2}{n_{Mb}}} = \sqrt{\frac{2c^2}{n_{Mb}}}. \quad (\text{AJ.2})$$

Such that $1 = (2\theta_{si}/cm_f)$ where $E_f = 2\theta_{si}c$ (Appendix Y), E represents some count $2\theta_{si}c$ of E_f and any count n_{Mb} of m_f , then

$$v = \sqrt{\left(\frac{2\theta_{si}}{c} \frac{1}{m_f}\right) \frac{2c^2}{n_{Mb}}} = \sqrt{\frac{4\theta_{si}c}{n_{Mb}m_f}} = \sqrt{\frac{2E}{m}}, \quad (\text{AJ.3})$$

$$E = \frac{mv^2}{2}. \quad (\text{AJ.4})$$

Appendix AK. Galactic Rotation

1. Definitions

$$v_{bc} = c\sqrt{\frac{n_M}{n_{Lr}}} = c\sqrt{2m_f}, \quad (\text{AK.1})$$

$$v_b = \theta_{si}c\sqrt{2m_f}, \quad (\text{AK.2})$$

$$M_b = R\theta_{si} \frac{m_f^3}{t_f}. \quad (\text{AK.3})$$

2. Calculations

$$\Delta_{f(v_m)} = \frac{(v_m - v_b)}{v_b} = \frac{v_m}{v_b} - 1, \quad (\text{AK.4})$$

$$M_e = M_b(2\Delta_{f(v_m)} + 1) = M_b \left(2\frac{v_m}{v_b} - 2 + 1 \right), \quad (\text{AK.5})$$

$$M_e = M_b \left(2\frac{v_m}{v_b} - 1 \right), \quad (\text{AK.6})$$

$$M_e = R\theta_{si} \frac{m_f^3}{t_f} \left(2\frac{v_m}{\theta_{si}c\sqrt{2m_f}} - 1 \right). \quad (\text{AK.7})$$

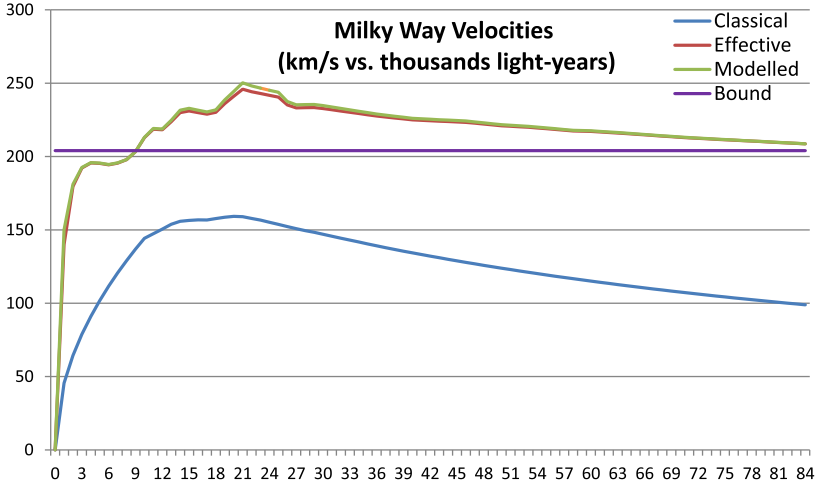


Fig. AK.1. (Color online) Milky Way star velocities (green) [26], effective velocity (red), bound velocity unadjusted for mass distribution (purple) and classical velocity (blue).

The effective velocity v_e is

$$v_e = \left(\frac{GM_e}{R} \right)^{1/2} = \left(\frac{GM_b}{R} \left(2 \frac{v_m}{v_b} - 1 \right) \right)^{1/2}, \quad (\text{AK.8})$$

$$v_e = \left(\left(c^3 \frac{t_f}{m_f} \right) \left(R \theta_{si} \frac{m_f^3}{t_f} \right) \frac{1}{R} \left(2 \frac{v_m}{(\theta_{si} c \sqrt{2m_f})} - 1 \right) \right)^{1/2}, \quad (\text{AK.9})$$

$$v_e = \left(c^2 m_f^2 \left(2 \frac{v_m}{\sqrt{2m_f}} - c \theta_{si} \right) \right)^{1/2}, \quad (\text{AK.10})$$

$$v_e^2 = 4 \theta_{si}^2 \left(2 \frac{v_m}{\sqrt{2m_f}} - c \theta_{si} \right). \quad (\text{AK.11})$$

Appendix AL. Quantization Crossover

$$M_e = R \frac{m_f^3}{t_f} \left(2 \frac{v_m}{c \sqrt{2m_f}} - \theta_{si} \right), \quad (\text{AL.1})$$

$$v_e^2 = 4 \theta_{si}^2 \left(2 \frac{v_m}{\sqrt{2m_f}} - c \theta_{si} \right). \quad (\text{AL.2})$$

To demonstrate equilibrium where classical and non-classical behaviors meet, we resolve v_m at the velocity bound v_b and substitute $v_b = \theta_{si} c (2m_f)^{1/2}$.

$$v_e^2 = 4 \theta_{si}^2 \left(2 \frac{v_b}{\sqrt{2m_f}} - c \theta_{si} \right), \quad (\text{AL.3})$$

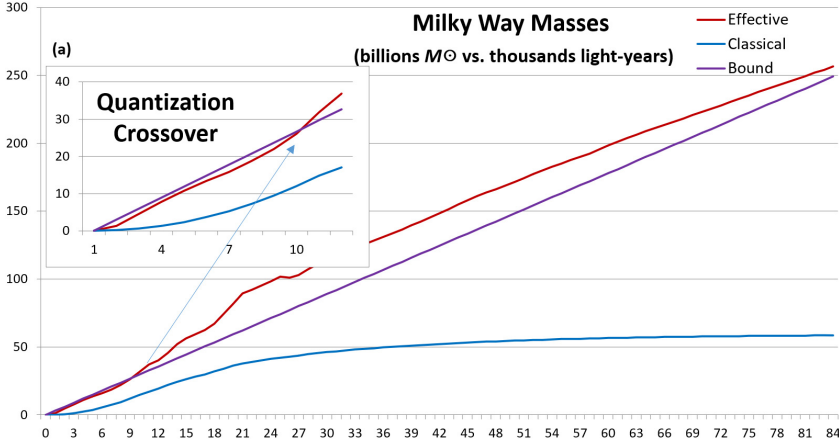


Fig. AL.1. (Color online) Milky Way mass: classical (blue), effective (red) [27, 28] and bound (purple).

$$v_e^2 = 8\theta_{si}^2 \frac{\theta_{si}c\sqrt{2m_f}}{\sqrt{2m_f}} - 4\theta_{si}^3c, \quad (\text{AL.4})$$

$$v_e^2 = 8\theta_{si}^3c - 4\theta_{si}^3c = 4\theta_{si}^3c, \quad (\text{AL.5})$$

$$v_e = 2\theta_{si}\sqrt{\theta_{si}c}. \quad (\text{AL.6})$$

$$v_e = \theta_{si}\sqrt{2E_f}. \quad (\text{AL.7})$$

Changing from Target to Measurement frame, (i.e. remove θ_{si}), we resolve $v_e = (2E_f)^{1/2}$. This reduces to the expression for kinetic energy, demonstrating that the crossover occurs where $v_e = v_b$ at $9.072419 \cdot 10^3$ light-years (2.779712 kpc).

Appendix AM. Elementary Charge

Defining Two Frames at the Bound

$$(\text{definition}) \frac{b}{d} = b - d. \quad (\text{AM.1})$$

Correlating Frames to Physical Values

Using the Shwartz experiments for reference, the base b is the elementary charge e_f as a function of the fundamental mass $f(m_f)$ relative to its quantum of angular momentum $\hbar$ (i.e. $f(m_f)/\hbar$). And we know that the Planck momentum relative to theta is $\theta_{si} = (1/2)l_fm_f/t_f$ (Appendix Q) thus, indicating that we divide $\hbar$ by two. And we know that $f(m_f)$ — the *metric differential* associated with m_f between frames — is the square of m_f when coupled with θ_{si} , $M_b = 2n_{Lr}(\theta_{si}m_f)^2$

(i.e. $l_f(m_fm_f)/2t_f$), Eq. (AG.6).

$$\frac{b}{d} = e_f \left(\frac{f(m_f)}{\hbar/2} \right) = e_f \frac{m_f^2}{(2\theta_{si}l_f)/2} = e_f \frac{m_f^2}{\theta_{si}l_f}. \quad (\text{AM.2})$$

Such that $d = \theta_{si}$, $b = e_fm_f^2/l_f$ and such that $b/d = b - d$, then

$$\frac{e_fm_f^2}{\theta_{si}l_f} = \frac{e_fm_f^2}{l_f} - \theta_{si}, \quad (\text{AM.3})$$

$$e_f(m_f^2\theta_{si} - m_f^2) = l_f\theta_{si}^2, \quad (\text{AM.4})$$

$$e_f = \frac{l_f\theta_{si}^2}{m_f^2(\theta_{si} - 1)} = \frac{l_f^3}{4t_f^2(\theta_{si} - 1)}, \quad (\text{AM.5})$$

$$e_f = \frac{1}{4(\theta_{si} - 1)}l_fc^2 = 1.60512538253(81) \cdot 10^{-19}. \quad (\text{AM.6})$$

Applying the Metric Differential

$$n_\theta = n_{Lr@41} - 1 + ((2Q_L n_{Lr} n_{Lr})_{f(41)} - (2Q_L n_{Lr} n_{Lr})_{f(42)}), \quad (\text{AM.7})$$

$$n_\theta = 40 + (40.993904252021 - 41.994049305633), \quad (\text{AM.8})$$

$$n_\theta = 41.000145053612. \quad (\text{AM.9})$$

$$r_{q41} = \frac{(n_\theta - 1)\theta_{si}}{\text{RND}((n_\theta - 1)\theta_{si})} = 0.998197579719(51), \quad (\text{AM.10})$$

$$e_p = r_{q41}e_f = 1.60223227199(59) \cdot 10^{-19}. \quad (\text{AM.11})$$

Applying the Informativity Differential

$$Q_L n_{Lr} = n_{Lr}((1 + n_{Lr}^2)^{1/2} - n_{Lr}), \quad (\text{AM.12})$$

$$\Delta_{(P-C)} = e_p(1 - 2Q_L n_{Lr}), \quad (\text{AM.13})$$

$$e_c = e_p - \Delta_{(P-C)}, \quad (\text{AM.14})$$

$$e_c = e_p - e_p(1 - 2Q_L n_{Lr}) = 2Q_L n_{Lr}e_p = 2Q_L n_{Lr}r_{q41}e_f, \quad (\text{AM.15})$$

$$e_c = 1.602176310(70) \cdot 10^{-19} \text{ C}. \quad (\text{AM.16})$$

Appendix AN. Elementary Charge Demarcation

1. Definitions

$$\gamma = \frac{2(\theta_{si} - 1)^2}{(Q_L n_{Lr})^4(84\theta_{si} - \text{RND}(42\theta_{si}))r_{q41}^2}, \quad (\text{AN.1})$$

$$r_{q41} = \frac{(\theta_{si} - 1)}{(Q_L n_{Lr})^2} \left(\frac{2}{84\theta_{si} - \text{RND}(42\theta_{si})\gamma} \right)^{1/2}. \quad (\text{AN.2})$$

2. Calculations

$$e_c = 2Q_L n_{Lr} e_p = 2Q_L n_{Lr} r_{q41} e_f, \quad (\text{AN.3})$$

$$e_c = 2Q_L n_{Lr} \frac{(\theta_{si} - 1)}{(Q_L n_{Lr})^2} \left(\frac{2}{(84\theta_{si} - \text{RND}(42\theta_{si}))\gamma} \right)^{1/2} \times \frac{1}{4(\theta_{si} - 1)} l_f c^2, \quad (\text{AN.4})$$

$$e_c = \frac{l_f c^2}{2Q_L n_{Lr}} \left(\frac{2}{(84\theta_{si} - \text{RND}(42\theta_{si}))\gamma} \right)^{1/2}, \quad (\text{AN.5})$$

$$Q_L n_{Lr} = \frac{l_f c^2}{2e_c} \left(\frac{2}{(84\theta_{si} - \text{RND}(42\theta_{si}))\gamma} \right)^{1/2}, \quad (\text{AN.6})$$

$$n_{Lr}((1 + n_{Lr}^2)^{1/2} - n_{Lr}) = \frac{l_f c^2}{2e_c} \left(\frac{2}{(84\theta_{si} - \text{RND}(42\theta_{si}))\gamma} \right)^{1/2}, \quad (\text{AN.7})$$

$$n_{Lr} = 84.6005499213(87), \quad (\text{AN.8})$$

$$n_{Lr} l_f = 1.36731401341(03) \cdot 10^{-33} \text{ m}, \quad (\text{AN.9})$$

$$Q_L n_{Lr} = 0.499982536418(99). \quad (\text{AN.10})$$

Appendix AO. Electric Constant

1. Definitions

$$\gamma = \frac{2(\theta_{si} - 1)^2}{(Q_L n_{Lr})^4 (84\theta_{si} - \text{RND}(42\theta_{si})) r_{q41}^2}, \quad (\text{AO.1})$$

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = \frac{e_c^2}{2\hbar \alpha_c c} = \frac{1}{\hbar} \frac{e_c^2}{4\pi \alpha_c c} = \frac{Q_L n_{Lr}}{\theta_{si} l_f} \frac{e_c^2}{4\pi \alpha_c c}. \quad (\text{AO.2})$$

2. Calculations

$$\varepsilon_0 = 2Q_L n_{Lr} (84\theta_{si} - \text{RND}(42\theta_{si})) \frac{Q_L n_{Lr}}{\theta_{si} l_f} \frac{e_c^2}{4\pi c}, \quad (\text{AO.3})$$

$$\varepsilon_0 = \frac{(Q_L n_{Lr})^2 (84\theta_{si} - \text{RND}(42\theta_{si}))}{2\pi \theta_{si} l_f c} e_c^2, \quad (\text{AO.4})$$

$$\varepsilon_0 = \frac{(Q_L n_{Lr})^2 (84\theta_{si} - \text{RND}(42\theta_{si}))}{2\pi \theta_{si} l_f c} \left(2Q_L n_{Lr} \frac{l_f \theta_{si}^2 r_{q41}}{m_f^2 (\theta_{si} - 1)} \right)^2, \quad (\text{AO.5})$$

$$\varepsilon_0 = \frac{(Q_L n_{Lr})^4 (84\theta_{si} - \text{RND}(42\theta_{si})) l_f c^2 r_{q41}^2}{4\pi m_f (\theta_{si} - 1)^2}, \quad (\text{AO.6})$$

$$\varepsilon_0 = \frac{c^2}{2\pi \gamma} \frac{l_f}{m_f} = 8.85418 \cdot 10^{-12} \text{ F m}^{-1}. \quad (\text{AO.7})$$

3. Extending Precision with $\gamma = f(\mu_0)$ at the Blackbody Demarcation (Appendix AR)

$$\varepsilon_0 = \frac{c^2}{2\pi\gamma} \frac{l_f}{m_f} = 8.85418781278(70)10^{-12} \text{ F m}^{-1}, \quad (\text{AO.8})$$

$$2\pi\gamma\varepsilon_0 m_f = c^2 l_f, \quad (\text{AO.9})$$

$$G = 2\pi\varepsilon_0\gamma. \quad (\text{AO.10})$$

Appendix AP. Magnetic Constant

1. Definitions

$$\gamma = \frac{2(\theta_{si} - 1)^2}{(Q_L n_{Lr})^4 (84\theta_{si} - \text{RND}(42\theta_{si})) r_{q41}^2}, \quad (\text{AP.1})$$

$$\varepsilon_0 = \frac{c^2}{2\pi\gamma} \frac{l_f}{m_f} = 8.85418 \cdot 10^{-12} \text{ F m}^{-1}. \quad (\text{AP.2})$$

2. Calculations

$$\mu_0 = \frac{1}{\varepsilon_0 c^2} = \frac{4\pi m_f (\theta_{si} - 1)^2}{(Q_L n_{Lr})^4 (84\theta_{si} - \text{RND}(42\theta_{si})) l_f c^4 r_{q41}^2}, \quad (\text{AP.3})$$

$$\mu_0 = \frac{2\pi\gamma}{c^4} \frac{m_f}{l_f} = 1.256638 \cdot 10^{-6} \text{ H m}^{-1}. \quad (\text{AP.4})$$

3. Extending Precision with $\gamma = f(\varepsilon_0)$ at the Blackbody Demarcation (Appendix AR)

$$\mu_0 = \frac{2\pi\gamma}{c^4} \frac{m_f}{l_f} = 1.25663706211(82) \cdot 10^{-6} \text{ H m}^{-1}, \quad (\text{AP.5})$$

$$\gamma = \left(\frac{\mu_0}{2\pi} \right) \text{ Gc}^2. \quad (\text{AP.6})$$

Appendix AQ. Coulomb's Constant

1. Definitions

$$\gamma = \frac{2(\theta_{si} - 1)^2}{(Q_L n_{Lr})^4 (84\theta_{si} - \text{RND}(42\theta_{si})) r_{q41}^2}, \quad (\text{AQ.1})$$

$$\varepsilon_0 = \frac{c^2}{2\pi\gamma} \frac{l_f}{m_f} = 8.85412 \cdot 10^{-12} \text{ F m}^{-1}. \quad (\text{AQ.2})$$

2. Calculations

$$k_e = \frac{1}{4\pi\epsilon_0} = \frac{m_f(\theta_{si} - 1)^2}{(Q_L n_{Lr})^4 (84\theta_{si} - \text{RND}(42\theta_{si})) l_f c^2 r_{q41}^2}, \quad (\text{AQ.3})$$

$$k_e = \frac{\gamma}{2c^2} \frac{m_f}{l_f} = 8.987555 \cdot 10^9 \text{ Nm}^2 \text{ C}^{-2}. \quad (\text{AQ.4})$$

Extending Precision with $\gamma = f(\mu_0)$ at the Blackbody Demarcation (Appendix AR)

$$k_e = \frac{\gamma}{2c^2} \frac{m_f}{l_f} = 8.98755179227(44) \cdot 10^9 \text{ Nm}^2 \text{ C}^{-2}, \quad (\text{AQ.5})$$

$$\gamma = 2k_e c^3 \frac{t_f}{m_f}, \quad (\text{AQ.6})$$

$$\gamma = 2k_e G. \quad (\text{AQ.7})$$

Appendix AR. Gamma — The Target to Measurement Frame Transform

$$\gamma = 2k_e G = 2k_e c^3 \frac{t_f}{m_f} = \frac{\text{Nm}^2}{\text{C}} \frac{\text{m}^3}{\text{s}^2 \text{kg}} = \frac{\text{kgm}}{\text{s}^2} \frac{\text{m}^5}{\text{Cs}^2 \text{kg}} = \frac{\text{m}^6}{\text{Cs}^4}, \quad (\text{AR.1})$$

$$\gamma = 2k_e G = 1.1996724235(38) \text{m}^6 \text{ C}^{-1} \text{s}^{-4a}. \quad (\text{AR.2})$$

$$\gamma = \frac{1}{2\pi\epsilon_0} G = 1.1996724235(38) \text{m}^6 \text{ C}^{-1} \text{s}^{-4a}. \quad (\text{AR.3})$$

$$\gamma = \left(\frac{\mu_0}{2\pi} \right) G c^2 = 1.19967242354(18) \text{m}^6 \text{ C}^{-1} \text{s}^{-4a}. \quad (\text{AR.4})$$

$$4\pi\epsilon_0 k_e = 1, \quad (\text{AR.5})$$

$$4\pi k_e = \mu_0 c^2, \quad (\text{AR.6})$$

$$\epsilon_0 \mu_0 c^2 = 1. \quad (\text{AR.7})$$

*Precision is extended not as a function of G , but $c^3 t_f / m_f$. Selecting a non-related source measurement (k_e , u_0 or ϵ_0) for γ assures physical independence.

Table AR.1. Electromagnetic constants as a function of the fundamental measures and γ , adapted from [14].

Physical Constants	Gamma Source Measures			
	e (10^{-19} C)	ϵ_0 (10^{-12} F m $^{-1}$)	μ_0 (10^{-6} H m $^{-1}$)	k_e (10^9 N m ^{2} C $^{-2}$)
MQ	1.60217064225(03) ^a	8.85418781281(30)	1.25663706211(82)	8.98755179227(44)
CODATA [3]	1.602176634	8.8541878128(13)	1.25663706212(19)	8.9875517923(14)

^aResolves the value of e at the *charge coupling demarcation*. The remaining values are calculated using the gamma approach.

Table AS.1. Elementary charge as a function of demarcation n_{Lr} and γ .

Source	Gamma source measures				
	Constants	n_{Lr} of source	ε_0 (10^{-19} C)	μ_0 (10^{-19} C)	k_e (10^{-19} C)
α		84.6005457004(82)	1.60217663400(74)	1.60217663400(51)	1.60217663400(74)
$\hbar$		84.6005496647(07)	1.60217663400(22)	1.60217663399(98)	1.60217663400(22)
e^a		84.6005499213(87)	1.60217663400(18)	1.60217663399(94)	1.60217663400(18)

^aAssociated n_{Lr} is solved as a function of the fine structure constant and the defined value for e [3].

Appendix AS. Solutions to Elementary Charge Using the Gamma Approach

Gamma resolved as a function of the electromagnetic constants.

$$\gamma = \frac{1}{2\pi\varepsilon_0}G = 1.1996724235(38) \text{ m}^6 \text{ C}^{-1} \text{ s}^{-4}, \quad (\text{AS.1})$$

$$\gamma = \left(\frac{\mu_0}{2\pi}\right) Gc^2 = 1.19967242354(18) \text{ m}^6 \text{ C}^{-1} \text{ s}^{-4}, \quad (\text{AS.2})$$

$$\gamma = 2k_e G = 1.1996724235(38) \text{ m}^6 \text{ C}^{-1} \text{ s}^{-4}. \quad (\text{AS.3})$$

Definition of gamma.

$$(\text{definition})\gamma = \frac{2(\theta_{si} - 1)^2}{(Q_L n_{Lr})^4 (84\theta_{si} - \text{RND}(42\theta_{si})) r_{q41}^2}, \quad (\text{AS.4})$$

$$r_{q41} = \frac{(\theta_{si} - 1)}{(Q_L n_{Lr})^2} \left(\frac{2}{84\theta_{si} - \text{RND}(42\theta_{si})\gamma} \right)^{1/2}. \quad (\text{AS.5})$$

With these, we can resolve elementary charge.

$$e_c = 2Q_L n_{Lr} r_{q41} e_f, \quad (\text{AS.6})$$

$$e_c = 2Q_L n_{Lr} \frac{(\theta_{si} - 1)}{(Q_L n_{Lr})^2} \left(\frac{2}{(84\theta_{si} - \text{RND}(42\theta_{si}))\gamma} \right)^{1/2} \frac{1}{4(\theta_{si} - 1)} l_f c^2, \quad (\text{AS.7})$$

$$e_c = \frac{1}{2Q_L n_{Lr}} \left(\frac{2}{(84\theta_{si} - \text{RND}(42\theta_{si}))\gamma} \right)^{1/2} l_f c^2. \quad (\text{AS.8})$$

Appendix AT. Unification

$$G = \frac{Q_L c^3 r}{\theta_{si}} = \frac{Q_L l_f^3}{\theta_{si} t_f^3} n_{Lr} l_f = \frac{1}{\theta_{si}} \frac{Q_L n_{Lr} l_f^4}{t_f^3}, \quad (\text{AT.1})$$

$$G = \frac{t_f}{l_f m_f Q_L n_{Lr}} \frac{Q_L n_{Lr} l_f^4}{t_f^3} = \left(\frac{l_f c^2}{m_f} \right), \quad (\text{AT.2})$$

$$\varepsilon_0 = \frac{(Q_L n_{Lr})^4 (84\theta_{si} - \text{RND}(42\theta_{si})) r_{q41}^2}{4\pi(\theta_{si} - 1)^2} \left(\frac{l_f c^2}{m_f} \right), \quad (\text{AT.3})$$

$$\varepsilon_0 = \frac{(Q_L n_{Lr})^4 (84\theta_{si} - \text{RND}(42\theta_{si})) r_{q41}^2}{4\pi(\theta_{si} - 1)^2} G, \quad (\text{AT.4})$$

$$\gamma = \frac{2(\theta_{si} - 1)^2}{(Q_L n_{Lr})^4 (84\theta_{si} - \text{RND}(42\theta_{si})) r_{q41}^2}, \quad (\text{AT.5})$$

$$\frac{\varepsilon_0}{G} = \frac{1}{2\pi} \frac{1}{\gamma}, \quad (\text{AT.6})$$

$$G = 2\pi\varepsilon_0\gamma. \quad (\text{AT.7})$$

Appendix AU. Contraction and Dilation with Inertial Mass

$$a^2 + b^2 = c^2, \quad (\text{AU.1})$$

$$a_{\max}^2 + b_{\min}^2 = c_{\text{bound}}^2, \quad (\text{AU.2})$$

$$(an_{\max})^2 + (bn_{\min})^2 = n_{\text{bound}}^2, \quad (\text{AU.3})$$

$$\left(\frac{a}{n_{\max}} \right)^2 + \left(\frac{b}{n_{\min}} \right)^2 = n_{\text{bound}}^2. \quad (\text{AU.4})$$

Testing each condition, we find $n_{\text{bound}} = 1$.

$$n_{\text{bound}} = \left(\left(\frac{0}{n_{\max}} \right)^2 + \left(\frac{n_{\min}}{n_{\min}} \right)^2 \right)^{1/2} = 1, \quad (\text{AU.5})$$

$$n_{\text{bound}} = \left(\left(\frac{n_{\max}}{n_{\max}} \right)^2 + \left(\frac{0}{n_{\min}} \right)^2 \right)^{1/2} = 1. \quad (\text{AU.6})$$

Thus,

$$\left(\frac{a}{n_{\max}} \right)^2 + \left(\frac{b}{n_{\min}} \right)^2 = 1, \quad (\text{AU.7})$$

$$\left(\frac{a}{n_{Lc}} \right)^2 + \left(\frac{b}{n_{Ll}} \right)^2 = 1, \quad (\text{AU.8})$$

$$\left(\frac{n_L}{n_{Lc}} \right)^2 + \left(\frac{n_{Lo}}{n_{Ll}} \right)^2 = 1, \quad (\text{AU.9})$$

$$\frac{n_{Lo}^2}{n_{Ll}^2} = \left(1 - \frac{n_L^2}{n_{Lc}^2} \right), \quad (\text{AU.10})$$

$$n_{Lo} = n_{Ll} \left(1 - \frac{n_L^2}{n_{Lc}^2} \right)^{1/2}. \quad (\text{AU.11})$$

For clarity, beta is

$$\frac{n_L^2}{n_{Lc}^2} = \left(\frac{n_L l_f}{t_f} \right)^2 \left(\frac{t_f}{n_{Lc} l_f} \right)^2 = \frac{v^2}{c^2}. \quad (\text{AU.12})$$

Applying $c = l_f/t_f$, we resolve the expression for time. Applying the *fundamental expression* $l_f = 2\theta_{si} t_f/m_f$ to length, we resolve the expression for mass.

$$l_o = l_l \left(1 - \frac{n_L^2}{n_{Lc}^2} \right)^{1/2}, \quad (\text{AU.13})$$

$$\frac{2\theta_{si} t_f}{m_o} = \frac{2\theta_{si} t_f}{m_l} \left(1 - \frac{n_L^2}{n_{Lc}^2} \right)^{1/2}, \quad (\text{AU.14})$$

$$m_l = m_o \left(1 - \frac{n_L^2}{n_{Lc}^2} \right)^{1/2}. \quad (\text{AU.15})$$

The contraction and dilation expressions for each are then

$$l_o = l_l \left(1 - \frac{n_L^2}{n_{Lc}^2} \right)^{1/2}, \quad (\text{AU.16})$$

$$m_o = m_l / \left(1 - \frac{n_L^2}{n_{Lc}^2} \right)^{1/2}, \quad (\text{AU.17})$$

$$t_o = t_l \left(1 - \frac{n_L^2}{n_{Lc}^2} \right)^{1/2}. \quad (\text{AU.18})$$

Appendix AV. Contraction and Dilation with Gravitation Mass

$$v = \left(\frac{2Gm_f}{l_f} \right)^{1/2}, \quad (\text{AV.1})$$

$$v^2 = \frac{2Gn_M m_f}{n_{Lr} l_f} = n_M m_f \frac{2}{n_{Lr} l_f} \frac{t_f c^3}{m_f}, \quad (\text{AV.2})$$

$$v^2 = \frac{2n_M t_f c^3}{n_{Lr} l_f} = \frac{2n_M c^3}{n_{Lr} c} = \frac{2n_M c^2}{n_{Lr}}, \quad (\text{AV.3})$$

$$\frac{v^2}{c^2} = 2 \frac{n_M}{n_{Lr}}. \quad (\text{AV.4})$$

Such that the expressions for motion and inertial mass both resolve to v^2/c^2 , we resolve equivalence

$$\frac{n_L^2}{n_{Lc}^2} = 2 \frac{n_M}{n_{Lr}}. \quad (\text{AV.5})$$

Thus,

$$l_o = l_l \left(1 - 2 \frac{n_M}{n_{Lr}} \right)^{1/2}, \quad (\text{AV.6})$$

$$m_o = m_l \left/ \left(1 - 2 \frac{n_M}{n_{Lr}} \right)^{1/2} \right., \quad (\text{AV.7})$$

$$t_o = t_l \left(1 - 2 \frac{n_M}{n_{Lr}} \right)^{1/2}. \quad (\text{AV.8})$$

Appendix AW. Equivalence Between Inertial & Gravitational Mass

Such that expressions for motion and inertial mass both resolve a v^2/c^2 component, we can describe their equivalence

$$\frac{n_L^2}{n_{Lc}^2} = 2 \frac{n_M}{n_{Lr}}. \quad (\text{AW.1})$$

Appendix AX. Unity Expression

Expansion of space as a count n_L of l_f at $t = 0$ of the expansionary epoch.

$$n_{To} = n_{Tl} \left(1 - \frac{n_L^2}{n_{Lc}^2} \right)^{1/2}, \quad (\text{AX.1})$$

$$A_{ee} = A_{qe} (2\theta_{si})^{1/3}, \quad (\text{AX.2})$$

$$n_{Tl} = n_{To} (2\theta_{si})^{1/3}, \quad (\text{AX.3})$$

$$\frac{n_{To}}{n_{Tl}} = \frac{1}{(2\theta_{si})^{1/3}}, \quad (\text{AX.4})$$

$$\frac{1}{(2\theta_{si})^{1/3}} = \left(1 - \frac{n_L^2}{n_{Lc}^2} \right)^{1/2}, \quad (\text{AX.5})$$

$$\left(\frac{1}{2\theta_{si}} \right)^{2/3} + \frac{n_L^2}{n_{Lc}^2} = 1, \quad (\text{AX.6})$$

$$\left(\left(\frac{t_f}{l_f m_f} \right)^{1/3} \right)^2 + \left(\frac{n_L}{n_{Lc}} \right)^2 = 1. \quad (\text{AX.7})$$

Appendix AY. Physical Significance of the Fundamental Expression for all Applications Throughout the Measurement Domain

Comparing dimension-dependent properties of the universe with their Measurement Frame measures today establishes the physical significance of the *fundamental expression* across the entire measurement domain.

Table AY.1. Tests extending the *fundamental expression* across the measurement domain.

Target frame variances from local measurement				
	Variance Test	Target	Measured	Difference
M_{CMB} $= f(n_L l_f)$	$A_{qe} \theta_{si}^4 / n_L l_f$	$1.50160 \cdot 10^{50} \text{ kg}$	$1.50149 \cdot 10^{50} \pm 2.9 \cdot 10^{35} \text{ kg}^a$	$0.00011 \cdot 10^{50} \text{ kg}$
M_{vis} $= f(n_{Tu})$	$M_{vis} = \Omega_{vis} n_{Tu} M_{acr}$	$1.4794 \cdot 10^{53} \text{ kg}$	$(1.48 \pm 0.01) \cdot 10^{53} \text{ kg}^b$	$0.00 \cdot 10^{53} \text{ kg}$
T_{CMB} $= f(n_M m_f)$	$((n_M m_f)_{qe} c^3 / 4\sigma V_{qe})^{1/4}$	2.72553 K	$2.72548 \pm 0.00057 \text{ K}^a$	0.00005 K
Ω_{vis} $= f(\theta_{si})$	$\Omega_{vis} = 2/\theta_{si}(\theta_{si}^2 + 2)$	4.8488348953%	$4.97 \pm 0.82\%^c$	0.12%

^aA consolidated study of CMB temperature measurements prior to 2009 [24].

^bAstrophysical constants [25].

^c2018 Planck Collaboration Results [26].

Appendix AZ. Correlating Diameter, Age of Universe to Fundamental Mass

We begin with the *fundamental expression* and expand to include counts. Such that n_L and n_T are representative of the age $A_U = n_T t_f$ and diameter $D_U = n_L l_f$ of the universe, then

$$l_f m_f = 2\theta_{si} t_f, \quad (\text{AZ.1})$$

$$n_L l_f n_M m_f = 2\theta_{si} n_T t_f, \quad (\text{AZ.2})$$

$$(n_L l_f) n_M m_f = 2\theta_{si} (n_T t_f), \quad (\text{AZ.3})$$

$$D_U (n_M m_f) = 2\theta_{si} A_U. \quad (\text{AZ.4})$$

We anchor the relation to the Target Frame using the mass frequency

$$n_M = 1/m_f = 4.59467491733(49) \cdot 10^7 \text{ units kg}^{-1}. \quad (\text{AZ.5})$$

And where the speed of light constrains $n_L = n_T$ for all frames, then

$$n_M m_f = (4.59467491733(49) \cdot 10^7)(2.17643253982(38) \cdot 10^{-8}) = 1. \quad (\text{AZ.6})$$

It follows, from the perspective of an observer with respect to the Measurement Frame

$$D_U = 2\theta_{si} A_U = 23.2623913.794 = 90.003 \text{ Gly}, \quad (\text{AZ.7})$$

$$m_f = \frac{D_U}{A_U c} = \frac{90.003}{13.794 \times 299792458} = 2.1764 \cdot 10^{-8} \text{ kg}. \quad (\text{AZ.8})$$

Appendix BA. Hubble's Constant (For The Expansion of Space)

$$H = \frac{\text{km/Mpc}}{A_U} = \frac{3.08567758 \cdot 10^{19} \text{ km/Mpc}}{13.794 \cdot 10^9 3.15576 \cdot 10^7 \text{ s/y}} = 70.885 \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad (\text{BA.1})$$

$$H_f = \frac{70.885 \text{ km s}^{-1} \text{ Mpc}^{-1}}{3.08567758 \cdot 10^{19} \text{ km Mpc}^{-1}} = 2.2972 \cdot 10^{-18} \text{ s}^{-1}. \quad (\text{BA.2})$$

Appendix BB. Physically Significant Domains of the Observable Universe

Definitions

$$\Omega_{tot} = \Omega_{obs} + \Omega_{dk}, \quad (\text{BB.1})$$

$$\Omega_{dk} + \Omega_{obs} = 1, \quad (\text{BB.2})$$

$$\Omega_{obs} = 2\theta_{si}\Omega_{vis}, \quad (\text{BB.3})$$

$$\Omega_{uobs} = \Omega_{obs} - \Omega_{vis}. \quad (\text{BB.4})$$

Calculations

$$2\theta_{si}\Omega_{vis} = \Omega_{obs}, \quad (\text{BB.5})$$

$$\frac{2}{\theta_{si}^2} = 4 \frac{\Omega_{vis}}{\theta_{si}\Omega_{obs}} = \Omega_f, \quad (\text{BB.6})$$

$$\Omega_f = \frac{2}{\theta_{si}^2} = 18.7913577316(20)\%. \quad (\text{BB.7})$$

$$2\theta_{si}\Omega_{vis} = \Omega_{obs}, \quad (\text{BB.8})$$

$$2\theta_{si}\Omega_{vis} \frac{\theta_{si}^2}{2} - 1 = \Omega_{obs} \frac{\theta_{si}^2}{2} - 1, \quad (\text{BB.9})$$

$$\theta_{si}^3\Omega_{vis} - 1 = \Omega_{obs} \frac{\theta_{si}^2}{2} - 1. \quad (\text{BB.10})$$

Given the dark equality above and such that $\Omega_{dk} + \Omega_{obs} = \Omega_{tot}$, then we replace the Ω_{obs} to show that

$$\Omega_{dk} = (\Omega_{tot} - \Omega_{dk}) \frac{\theta_{si}^2}{2} - 1 = (1 - \Omega_{dk}) \frac{\theta_{si}^2}{2} - 1, \quad (\text{BB.11})$$

$$\Omega_{dk} = (1 - \Omega_{dk}) \frac{\theta_{si}^2}{2} - 1, \quad (\text{BB.12})$$

$$\Omega_{dk} = \frac{\theta_{si}^2 - 2}{\theta_{si}^2 + 2} = 68.3624161042(52)\%. \quad (\text{BB.13})$$

Such that the dark is the total (i.e. 1) minus the observable, then

$$1 - \Omega_{obs} = \Omega_{obs} \frac{\theta_{si}^2}{2} - 1, \quad (\text{BB.14})$$

$$2 = \Omega_{obs} \frac{\theta_{si}^2}{2} + \Omega_{obs}, \quad (\text{BB.15})$$

$$\Omega_{obs} = \frac{4}{\theta_{si}^2 + 2} = 31.6375838957(48)\%, \quad (\text{BB.16})$$

Table BB.1. Comparison of CMB Planck survey results with MQ calculations.

Phenomenon / MQ Term	Domain		
	Λ CDM % ^a	(MQ) %	(MQ) kg
Dark energy Ω_Λ /Dark Ω_{dk}	$68.34 \pm 0.84\%$	68.3624161042(52)	$2.0857 \cdot 10^{54}$
Dark Matter Ω_c /Unobserved Ω_{uobs}	$26.73 \pm 0.82\%$	26.7887490004(13)	$8.1733 \cdot 10^{53}$
$\Omega_c + \Omega_b$ /Observable Ω_{obs}	$31.66 \pm 0.84\%$	31.6375838957(48)	$9.6526 \cdot 10^{53}$
Baryonic Ω_b /Visible Ω_{vis}	$4.97 \pm 0.82\%$	4.84883489533(52)	$1.4794 \cdot 10^{53}$
—/Fundamental Ω_f	—	18.7913577316(20)	$5.7333 \cdot 10^{53}$

^a2018 Planck collaboration results using TT,TE,EE+lowE [26].

$$\Omega_{obs} = 2\theta_{si}\Omega_{vis} = \frac{4}{\theta_{si}^2 + 2}, \quad (\text{BB.17})$$

$$\Omega_{vis} = \frac{2}{\theta_{si}(\theta_{si}^2 + 2)} = 4.84883489533(53)\%, \quad (\text{BB.18})$$

$$\Omega_{uobs} = \Omega_{obs} - \Omega_{vis} = 26.7887490004(13)\%. \quad (\text{BB.19})$$

Appendix BC. Alternative Domain Relations

Dark

$$\Omega_{obs} = \frac{4}{\theta_{si}^2 + 2}, \quad (\text{BC.1})$$

$$\Omega_{obs} \left(\frac{\theta_{si}^2 + 2}{2} \right) = 2, \quad (\text{BC.2})$$

$$\Omega_{obs} \left(\frac{\theta_{si}^2}{2} + 1 \right) = 2, \quad (\text{BC.3})$$

$$1 - \Omega_{obs} = \frac{\theta_{si}^2}{2} \Omega_{obs} - 1, \quad (\text{BC.4})$$

$$\Omega_{dk} = \frac{2\theta_{si}}{\Omega_{obs}} \frac{\theta_{si}\Omega_{obs}}{4} \Omega_{obs} - 1, \quad (\text{BC.5})$$

$$\Omega_{dk} = \frac{\theta_{si}\Omega_{obs}}{4\Omega_{vis}} \Omega_{obs} - 1, \quad (\text{BC.6})$$

$$\Omega_{dk} = \frac{\Omega_{obs}}{\Omega_f} - 1. \quad (\text{BC.7})$$

Observable

$$M_f = A_U \theta_{si} \frac{m_f}{t_f}, \quad (\text{BC.8})$$

$$\Omega_{dk} = 1 - \frac{\Omega_{obs}}{\Omega_{tot}}, \quad (\text{BC.9})$$

$$\Omega_{dk} = \frac{\Omega_{obs} - \Omega_f}{\Omega_f}, \quad (\text{BC.10})$$

$$1 - \frac{\Omega_{obs}}{\Omega_{tot}} = \frac{\Omega_{obs} - \Omega_f}{\Omega_f}, \quad (\text{BC.11})$$

$$1 = \left(\frac{\Omega_{obs} - \Omega_f}{\Omega_f} + \frac{\Omega_{obs}}{\Omega_{tot}} \right) = \left(\frac{\Omega_{obs}}{\Omega_f} - 1 + \frac{\Omega_{obs}}{\Omega_{tot}} \right), \quad (\text{BC.12})$$

$$\frac{\Omega_{obs}}{\Omega_f} + \frac{\Omega_{obs}}{\Omega_{tot}} = 2, \quad (\text{BC.13})$$

$$\Omega_{tot}\Omega_{obs} + \Omega_f\Omega_{obs} = 2\Omega_f\Omega_{tot}, \quad (\text{BC.14})$$

$$\Omega_{tot}\Omega_{obs} = \Omega_f(2\Omega_{tot} - \Omega_{obs}), \quad (\text{BC.15})$$

$$\Omega_{obs} = \Omega_f \left(2 - \frac{\Omega_{obs}}{\Omega_{tot}} \right), \quad (\text{BC.16})$$

$$\Omega_{obs} = \Omega_f(2 - \Omega_{obs}), \quad (\text{BC.17})$$

$$\Omega_{obs}(1 + \Omega_f) = 2\Omega_f, \quad (\text{BC.18})$$

$$\Omega_{obs} = 2 \frac{\Omega_f}{\Omega_f + 1}. \quad (\text{BC.19})$$

Appendix BD. Curvature of Universe: Flat (13, Eq. (97))

$$100\pi \frac{\theta_{si}^2 - 2}{\theta_{si}^2 + 2} = 214.766864214(77). \quad (\text{BD.1})$$

Appendix BE. Mass Accretion in the Universe

Definitions

$$M_{dk} = \frac{M_{obs}}{M_f} - 1, \quad (\text{BE.1})$$

$$M_{dk} = 1 - M_{obs}, \quad (\text{BE.2})$$

$$M_f = n_{Tu}m_f\theta_{si}. \quad (\text{BE.3})$$

Calculations

$$\frac{M_{obs}}{M_f} - 1 = 1 - M_{obs}, \quad (\text{BE.4})$$

$$M_{tot} = M_f \left(\frac{2}{M_{obs}} - 1 \right), \quad (\text{BE.5})$$

$$M_{tot} = n_{Tu}m_f\theta_{si} \left(\frac{2\theta_{si}^2 + 2}{1} - 1 \right) = n_{Tu}m_f\frac{\theta_{si}^3}{2}. \quad (\text{BE.6})$$

$$M_{\text{acr}} = \frac{n_{Mu}}{n_{Tu}} = \frac{\theta_{si}^3}{2} = 17.3611\,207370(89) \text{ units } m_f \text{ per unit } t_f. \quad (\text{BE.7})$$

$$M_{\text{acr}} = \frac{\theta_{si}^4}{l_f} = 7.00887947539(16) \cdot 10^{36} \text{ kg s}^{-1}. \quad (\text{BE.8})$$

Appendix BF. Quantum Referencing

Until the count n_{Lr} of l_f representing the radius of a discrete measurement frame is greater than $n = \sqrt{3}$, points external to the system cannot be referenced. The universe cannot expand at the speed of light.

$$\sqrt{n_a^2 + n_{bn}^2} = \sqrt{n_a^2 + \left(\sqrt{n_a^2 + n_b^2}\right)^2} =, \quad (\text{BF.1})$$

$$\sqrt{n_a^2 + n_a^2 + n_b^2} = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}. \quad (\text{BF.2})$$

Appendix BG. Quantum Epoch

$$\frac{n_{Lu} l_f}{n_{Tu} t_f} = \frac{2\theta_{si}}{n_{Mu} m_f}, \quad (\text{BG.1})$$

$$\frac{n_{Lu'} l_f}{n_{Tu} t_f} = \frac{2\theta_{si}}{n_{Mu} m_f}, \quad (\text{BG.2})$$

$$v' = \frac{n_{Lu'} l_f}{n_{Tu} t_f} = \frac{2\theta_{si}}{n_{Mu} m_f} = \frac{2\theta_{si}}{M_U} = \frac{2\theta_{si}}{V_U \rho_c} = \frac{2\theta_{si}}{(4/3)\pi c^3 \theta_{si}^3 A_{qe}^3 \rho_c}, \quad (\text{BG.3})$$

$$v' = \frac{3}{2\pi c^3 \theta_{si}^2 A_{qe}^3} \frac{1}{\rho_c} = \frac{3}{2\pi c^3 \theta_{si}^2 A_{qe}^3} \frac{8\pi G}{3H_f^2} = \frac{4}{c^3 \theta_{si}^2 A_{qe}^3 H_f^2} \frac{l_f c^3}{2\theta_{si}}, \quad (\text{BG.4})$$

$$\begin{aligned} v' &= \frac{1}{H_f^2} \frac{2l_f}{\theta_{si}^3 A_{qe}^3} = A_{qe}^2 \frac{2l_f}{\theta_{si}^3 A_{qe}^3} = \frac{2l_f}{\theta_{si}^3 A_{qe}} \\ &= 8.1195732436(21)^{-50} \text{ m s}^{-1}, \end{aligned} \quad (\text{BG.5})$$

$$R_{qe} = \frac{2l_f}{\theta_{si}^3} \int \left(\frac{1}{A_{qe}} \right) dt = \frac{2l_f}{\theta_{si}^3} \ln(A_{qe}), \quad (\text{BG.6})$$

$$\begin{aligned} A_{qe} &= e^{\sqrt{3}\theta_{si}^3/2} = 1.14652655409(32) \cdot 10^{13} \text{ s} \\ &= 363,312.341272(21)y. \end{aligned} \quad (\text{BG.7})$$

Appendix BH. Quantity, Age, Density, Temperature of CMB

$$R_U = \frac{2l_f}{\theta_{si}^3} \int \left(\frac{1}{A_U} \right) dt = \frac{2l_f}{\theta_{si}^3} \ln(A_U), \quad (\text{BH.1})$$

Table BH.1. Comparison of MQ predicted CMB values with measurements.

	Q_{dk}	A_{CMB}	ρ_{CMB}	T_{CMB}
MQ	214.767	363,312.34127(22) y	$4.1750 \cdot 10^{-14} \text{ J m}^{-3}$	2.7255 K
Measurements	216.4 ± 1.3^b	379,000 y ^b	$4.17 \cdot 10^{-14} \text{ J m}^{-3c}$	$2.72548 \pm 0.00057 \text{ K}^a$

^aA consolodated study of CMB temperature measurements prior to 2009 [24].

^bMicrowave (WMAP) All-Sky Survey [29].

^cCorresponds to about 411 photons/cm³ [30].

$$A_{qe} = e^{\sqrt{3}\theta_{si}^3/2} = 1.14652655409(32) \cdot 10^{13} \text{ s}, \quad (\text{BH.2})$$

$$\begin{aligned} A_{ee} &= A_{qe}(2\theta_{si})^{1/3} = 2.14242727551(18) \cdot 10^{13} \text{ s} \\ &= 678,894.236415(88) \text{ y}. \end{aligned} \quad (\text{BH.3})$$

$$M_{CMB} = n_{Tu} m_f \frac{\theta_{si}^3}{2} = 1.5016 \cdot 10^{50} \text{ kg}, \quad (\text{BH.4})$$

$$\rho = \frac{M_{CMB} c^2}{V_U} = 4.1750 \cdot 10^{-14} \text{ J m}^{-3}, \quad (\text{BH.5})$$

$$a = \frac{4\sigma}{c} = \frac{8\pi^5 k_B^4}{15c^3 h^3} = 7.565733250 \cdot 10^{-16} \text{ J m}^{-3} \text{ K}^{-4}, \quad (\text{BH.6})$$

$$T = \left(\frac{\rho}{a}\right)^{1/4} = 2.7255 \text{ K}. \quad (\text{BH.7})$$

Appendix BI. Age of Universe as a Function of CMB Temperature

$$T_{CMB} = 2.7255 \text{ K [24]}, \quad (\text{BI.1})$$

$$k = 1.380649 \cdot 10^{-23} \text{ J K}^{-1}(\text{exact}) \text{ [25]}, \quad (\text{BI.2})$$

$$a = \frac{8\pi^5 k^4}{15c^3 h^3} = 7.565733250 \cdot 10^{-16} \text{ J m}^{-3} \text{ K}^{-4} \text{ [25]}, \quad (\text{BI.3})$$

$$R_U = A_U \theta_{si} c, \quad (\text{BI.4})$$

$$V_U = \frac{4}{3} \pi R_U^3, \quad (\text{BI.5})$$

$$E_{\rho:cmb} = T_{CMB}^4 a, \quad (\text{BI.6})$$

$$E_{CMB} = E_{\rho:cmb} V_U, \quad (\text{BI.7})$$

$$M_{CMB} = \frac{E_{CMB}}{c^2}, \quad (\text{BI.8})$$

$$M_{CMB} = \frac{4T_{CMB}^4 a \pi (A_U \theta_{si} c)^3}{3c^2}. \quad (\text{BI.9})$$

$$A_{qe} = e^{(\sqrt{3}\theta_{si})/2}, \quad (\text{BI.10})$$

$$A_{ee} = A_{qe}(2\theta_{si})^{1/3}, \quad (\text{BI.11})$$

$$M_{CMB} = A_{ee} \frac{\theta_{si}^4}{l_f}, \quad (\text{BI.12})$$

$$M_{CMB} = e^{(\sqrt{3}\theta_{si})/2} (2\theta_{si})^{1/3} \frac{\theta_{si}^4}{l_f}. \quad (\text{BI.13})$$

$$\frac{4T_{CMB}^4 a\pi (A_U \theta_{si} c)^3}{3c^2} = e^{(\sqrt{3}\theta_{si})/2} (2\theta_{si})^{1/3} \frac{\theta_{si}^4}{l_f}, \quad (\text{BI.14})$$

$$4T_{CMB}^4 a\pi A_U^3 \theta_{si}^3 c^3 l_f = 3e^{(\sqrt{3}\theta_{si})/2} (2\theta_{si})^{1/3} c^2 \theta_{si}^4, \quad (\text{BI.15})$$

$$A_U = \left(\frac{3\theta_{si} (2\theta_{si})^{1/3} e^{(\sqrt{3}\theta_{si})/2}}{4\pi a c l_f T_{CMB}^4} \right)^{1/3} = 13.794 \text{ billion years.} \quad (\text{BI.16})$$

Appendix BJ. Mass of the Universe as a Function of CMB Temperature

$$M_{acr} = \frac{\theta_{si}^4}{l_f} = 7.0088794753(92) 10^{36} \text{ kg s}^{-1}, \quad (\text{BJ.1})$$

$$A_U = \left(\frac{3\theta_{si} (2\theta_{si})^{1/3} e^{(\sqrt{3}\theta_{si})/2}}{4\pi a c l_f T_{CMB}^4} \right)^{1/3} = 13.794 \text{ billion years,} \quad (\text{BJ.2})$$

$$M_U = M_{acr} \left(A_U \frac{\text{bly}}{3.15576 \cdot 10^{16} \text{ s}} \right)^{-1} = 3.0510 \cdot 10^{54} \text{ kg,} \quad (\text{BJ.3})$$

$$M_f = \Omega_f M_U = 5.7333 \cdot 10^{53} \text{ kg,} \quad (\text{BJ.4})$$

$$M_{vis} = \Omega_{vis} M_U = 1.4794 \cdot 10^{53} \text{ kg,} \quad (\text{BJ.5})$$

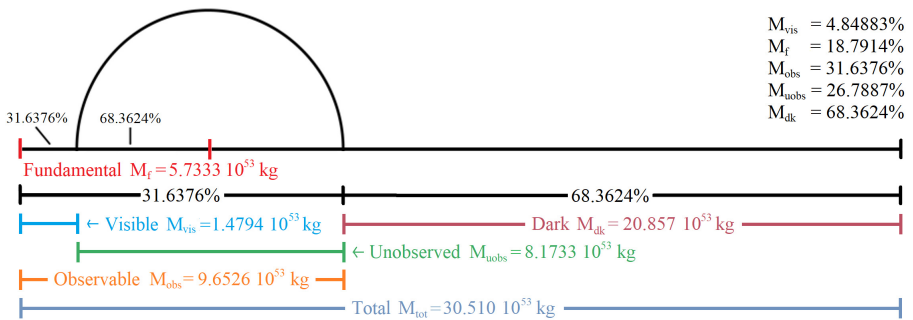


Fig. BJ.1. Mass in the universe. Adapted from [2]

$$M_{obs} = \Omega_{obs} M_U = 9.6526 \cdot 10^{53} \text{ kg}, \quad (\text{BJ.6})$$

$$M_{uobs} = \Omega_{uobs} M_U = 8.1733 \cdot 10^{53} \text{ kg}, \quad (\text{BJ.7})$$

$$M_{dk} = \Omega_{dk} M_U = 2.0857 \cdot 10^{54} \text{ kg}, \quad (\text{BJ.8})$$

Appendix BK. Time Dilation/Length Contraction between Epochs

$$\frac{(4/3)\pi(A_{ee}\theta_{si}c)^3}{(4/3)\pi(A_{qe}\theta_{si}c)^3} = 2\theta_{si}, \quad (\text{BK.1})$$

$$A_{ee} = A_{qe}(2\theta_{si})^{1/3} = 2.14242727551(18) \cdot 10^{13} \text{ s}, \quad (\text{BK.2})$$

$$n_{Tl} = n_{To}(2\theta_{si})^{1/3}, \quad (\text{BK.3})$$

$$\frac{n_{To}}{n_{Tl}} = \frac{1}{(2\theta_{si})^{1/3}}, \quad (\text{BK.4})$$

$$\frac{n_{To}}{n_{Tl}} = \left(1 - \frac{n_L^2}{n_{Lc}^2}\right)^{1/2}, \quad (\text{BK.5})$$

$$\frac{1}{(2\theta_{si})^{1/3}} = \left(1 - \frac{n_L^2}{n_{Lc}^2}\right)^{1/2}, \quad (\text{BK.6})$$

$$n_L^2 = n_{Lc}^2 - \frac{n_{Lc}^2}{(2\theta_{si})^{2/3}}, \quad (\text{BK.7})$$

$$n_L = n_{Lc} \sqrt{1 - \frac{1}{(2\theta_{si})^{2/3}}}. \quad (\text{BK.8})$$

Appendix BL. Ground State Orbital of an Atom

Using Planck's expression for the ground state orbital of an electron, we replace $l_f m_f$ with the *fundamental expression* and then expand to incorporate the *Informativity differential*.

$$\alpha_p^{-1} = \frac{a_0 m_e}{m_f l_f} = a_0 m_e \frac{Q_L n_{Lr}}{\theta_{si} t_f}, \quad (\text{BL.1})$$

$$a_0 = 2 \frac{\theta_{si} t_f}{\alpha_p m_e}, \quad (\text{BL.2})$$

$$a_0 = \frac{1}{\alpha_c m_e} \frac{\theta_{si} t_f}{Q_L n_{Lr}}. \quad (\text{BL.3})$$

Appendix BM. Extending the Physical Constants

$$m_f = \frac{2\theta_{si}}{c} = 2.17643253982(38) \cdot 10^{-8} \text{ kg}, \quad (\text{BM.1})$$

$$\alpha_p^{-1} = 84\theta_{si} - \text{RND}(42\theta_{si}) = 137.040785529(68), \quad (\text{BM.2})$$

$$l_f = \frac{m_e a_0 \alpha_p}{m_f} = 1.6161999120(35) \cdot 10^{-35} \text{ m}, \quad (\text{BM.3})$$

$$t_f = \frac{l_f}{c} = 5.3910626131(72) \cdot 10^{-44} \text{ s}. \quad (\text{BM.4})$$

$$G = c^2 \frac{l_f}{m_f} = 6.6740779428(56) \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}, \quad (\text{BM.5})$$

$$\hbar = 2\theta_{si} l_f = 1.0545349844(45) \cdot 10^{-34} \text{ J s}, \quad (\text{BM.6})$$

$$\hbar = \frac{\theta_{si} l_f}{Q_L n_{Lr}} = 1.0545718176(46) \cdot 10^{-34} \text{ J s} \quad (\text{BM.7})$$

Notably, $\hbar$ is a function of its associated demarcation. To avoid feedback of the source measure, we use the demarcation associated with elementary charge as an example count n_{Lr} for this measure.

Calculations of domain distributions are a percent of the whole.

$$\Omega_f = \frac{2}{\theta_{si}^2} = 18.7913577316(20)\%, \quad (\text{BM.8})$$

$$\Omega_{dk} = \frac{\theta_{si}^2 - 2}{\theta_{si}^2 + 2} = 68.3624161042(52)\%, \quad (\text{BM.9})$$

$$\Omega_{obs} = \frac{4}{\theta_{si}^2 + 2} = 31.6375838957(48)\%, \quad (\text{BM.10})$$

$$\Omega_{vis} = \frac{\Omega_{obs}}{2\theta_{si}} = 4.84883489533(52)\%, \quad (\text{BM.11})$$

$$\Omega_{uobs} = \Omega_{obs} - \Omega_{vis} = 26.7887490004(13)\%. \quad (\text{BM.12})$$

Extending the electromagnetic constants as a function of γ .

$$\gamma = \frac{1}{2\pi\epsilon_0} G = 1.1996724235(38) \text{ m}^6 \text{ C}^{-1} \text{ s}^{-4}, \quad (\text{BM.13})$$

$$\gamma = \left(\frac{\mu_0}{2\pi}\right) G c^2 = 1.19967242354(18) \text{ m}^6 \text{ C}^{-1} \text{ s}^{-4}, \quad (\text{BM.14})$$

$$\gamma = 2k_e G = 1.1996724235(38) \text{ m}^6 \text{ C}^{-1} \text{ s}^{-4}, \quad (\text{BM.15})$$

$$\varepsilon_0 = \frac{G}{2\pi\gamma} = 8.85418781281(30) \cdot 10^{-12} \text{ F m}^{-1}, \quad (\text{BM.16})$$

$$\mu_0 = \frac{2\pi\gamma}{Gc^2} = 1.25663706211(82) \cdot 10^{-6} \text{ H m}^{-1}, \quad (\text{BM.17})$$

$$k_e = \frac{\gamma}{2G} = 8.98755179227(44) \cdot 10^9 \text{ Nm}^2 \text{ C}^{-2}. \quad (\text{BM.18})$$

Appendix BN. How to Derive the Planck Expressions

Such that $G=c^3 t_f / m_f$ (Appendix F) $\hbar = 2\theta_{si} l_f$ (Appendix R) and the *fundamental expression* $l_f m_f = 2\theta_{si} t_f$ (Appendix V), then

Length

$$l_f m_f = 2\theta_{si} t_f, \quad (\text{BN.1})$$

$$l_f m_f = (2\theta_{si} l_f) \frac{G}{l_f G} t_f = \hbar \frac{G}{l_f G} t_f, \quad (\text{BN.2})$$

$$l_f m_f \left(c^3 \frac{t_f}{m_f} \right) = \hbar \frac{G}{l_f} t_f, \quad (\text{BN.3})$$

$$l_f c^4 t_f = \hbar G, \quad (\text{BN.4})$$

$$l_f = \left(\frac{\hbar G}{c^3} \right)^{1/2}. \quad (\text{BN.5})$$

Time

$$l_f \frac{c}{c} = \left(\frac{\hbar G}{c^3} \right)^{1/2}, \quad (\text{BN.6})$$

$$t_f = \left(\frac{\hbar G}{c^5} \right)^{1/2}. \quad (\text{BN.7})$$

Mass

$$t_f = \left(\frac{\hbar G^2}{c^5 G} \right)^{1/2}, \quad (\text{BN.8})$$

$$t_f = \frac{G}{c^3} \left(\frac{\hbar c}{G} \right)^{1/2} = c^3 \frac{t_f}{m_f} \frac{1}{c^3} \left(\frac{\hbar c}{G} \right)^{1/2}, \quad (\text{BN.9})$$

$$m_f = \left(\frac{\hbar c}{G} \right)^{1/2}. \quad (\text{BN.10})$$

Appendix BO. Physical Support for Measurement Quantization

Table BO.1. Comparison of MQ predicted values to the measurement data.

Measure	MQ Expression	MQ	Measured
Fundamental Constant θ_{si}	$2\alpha_c = \frac{1}{n_{L,r}\theta_{si}} \frac{\theta_{si}}{(\theta_{si}^2 + (n_{L,r}\theta_{si}/2)^2)^{1/2} - n_{L,r}\theta_{si}}$	$3.26239030392(48) \text{ kg m s}^{-1}$	$3.26239 \text{ rad} \pm 2 \text{ } \mu\text{rad}^a$
Gravitational Constant G	$\frac{(84\theta_{si} - \text{RND}(42\theta_{si}))}{l_f c^3 / 2\theta_{si}}$	$6.6740779428(56) \text{ } 10^{-11} \text{ m}^2 \text{ kg}^{-1} \text{ s}^{-2}$	$6.67408(31) \text{ } 10^{-11} \text{ m}^2 \text{ kg}^{-1} \text{ s}^{-2b}$
Fundamental Length l_f	$2G\theta_{si}/c^3$	$1.6161999120(35) \text{ } 10^{-35} \text{ m}$	$1.616199(97) \text{ } 10^{-35} \text{ m}^b$
Fundamental Time t_f	l_f/c	$5.39106426131(72) \text{ } 10^{-44} \text{ s}$	$5.39106(32) \text{ } 10^{-44} \text{ s}^b$
Fundamental Mass m_f	$2\theta_{si}/c$	$2.17643253982(38) \text{ } 10^{-8} \text{ kg}$	$2.17651(13) \text{ } 10^{-8} \text{ kg}^b$
Reduced Planck Constant $\hbar$	$l_f \theta_{si} / Q_L n_{L,r}$	$1.05457181764(63) \text{ } 10^{-34} \text{ Js}$	$1.054571817 \text{ } 10^{-34} \text{ Js}^c$
Speed of Light c	$\frac{c}{l_f \theta_{si} / Q_L n_{L,r}}$	$299792458 \text{ ms}^{-1}$	$299792458 \text{ ms}^{-1c}$
Fine Structure Constant (Planck) α_p^{-1}	$84\theta_{si} - \text{RND}(42\theta_{si})$	$137.040785529(69)$	137.040785532^c
Fine Structure Constant (EM) α_c^{-1}	$2Q_L n_{L,r}(84\theta_{si} - \text{RND}(42\theta_{si}))$	$137.035999083(49)$	$137.035999084(21)^c$
Elementary Charge e	$e_c = 2Q_L n_{L,r} r_{q41} e f$	$1.6021763105(70) \text{ } 10^{-19} \text{ C}$	$1.602176634 \text{ } 10^{-19} \text{ C}^c$
Gamma (MQ)	$\frac{2(\theta_{si}-1)^2}{(Q_L n_{L,r})^4 (84\theta_{si} - \text{RND}(42\theta_{si})) r_{q41}^2}$	$1.19967290789(52)$	—
Gamma f (u_0)	$\mu_0 G c^2 / 2\pi$	$1.19967242354(18)$	—
Electric Constant ϵ_0	$c^2 l_f / 2\pi \gamma m_f$	$8.85418781278(70) \text{ } 10^{-12} \text{ Fm}^{-1}$	$8.85418781280 \text{ } 10^{-12} \text{ Fm}^{-1c}$
Magnetic Constant w_0	$2\pi \gamma m_f / c^4 l_f$	$1.2566370621(18) \text{ } 10^{-6} \text{ Hm}^{-1}$	$1.25663706212(19) \text{ } 10^{-6} \text{ Hm}^{-1c}$

^a θ_{si} can be measured as the polarization angle of quantum entangled X-rays at the degenerate frequency [17–19] or calculated as a function of the fine structure constant. With respect to the Target Frame θ_{si} is non-dimensional [2].

^b2014 CODATA publication [2].

^c2018 CODATA publication [3].

^dPlanck collaboration results using TT,TE,EE+lowE [26, 31].

^eA consolidated study of CMB temperature measurements prior to 2009 [24].

Table BO.1. (*Continued*)

Measure	MQ Expression	MQ	Measured
Coulomb's Constant k_e	$\gamma m_f / 2c^2 l_f$	$8.98755179227(44) \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$	$8.9875517923(14) \times 10^9 \text{ Nm}^2 \text{ C}^{-2c}$
Blackbody demarcation	$n_{L,r} = \theta_{si} l_f \sqrt{1/\hbar(\hbar - 2\theta_{si} l_f)}$	84.6005496647(07)	—
Charge coupling demarcation	$n_{L,r} = 276/\theta_{si}$	84.6005457004(82)	—
Elementary charge demarcation	$n_{L,r}((1 + n_{L,r}^2)^{1/2} - n_{L,r}) = \frac{e_c}{2\epsilon_p}$	84.6005499213 (87)	—
Bohr Radius a_0	$\theta_{si} t_f / \alpha_c m_e Q_L n_{L,r}$	$5.29177210904 \times 10^{-11} \text{ m}$	$5.29177210903 \times 10^{-11} \text{ m}^c$
Dark energy Ω_Λ /Dark Ω_{dk}	$(\theta_{si}^2 - 2)/(\theta_{si}^2 + 2)$	$68.3624161042(52)\%$	$68.34 \pm 0.84\%^d$
Dark Matter	$\Omega_{obs} - \Omega_{vis}$	$26.7887490004(13)\%$	$26.73 \pm 0.82\%^d$
Ω_c /Unobserved Ω_{uobs}	$4/(\theta_{si}^2 + 2)$	$31.6375838957(48)\%$	$31.66 \pm 0.84\%^d$
$\Omega_c + \Omega_b$ /Observable Ω_{obs}	$\Omega_{obs}/2\theta_{si}$	$4.84883489533(52)\%$	$4.97 \pm 0.82\%^d$
Baryonic Ω_b /Visible Ω_{vis}	$\Omega_f = 2/\theta_{si}^2$	$18.7913577316(20)\%$	—
-/Fundamental Ω_f	$100\pi(\theta_{si}^2 - 2t)/(\theta_{si}^2 + 2)$	$214.766864214(77)$	216.4 ± 1.3^d
Multipole Moment of Ω_{dk}	$e\sqrt{3}\theta_{si}^{3/2}$	363.312 y	$378,000 \text{ y}^d$
CMB Age A_{qe}	$n_{qe} m_f \theta_{si}^3/2$	$1.5016 \times 10^{50} \text{ kg}$	—
CMB Mass Equivalent M_{qe}	$M_{qe} c^2/V_{qe}$	$4.1750 \times 10^{-14} \text{ Jm}^{-3}$	$4.005 \times 10^{-14} \text{ Jm}^{-3d}$
CMB Density	$(\rho c/4\sigma)^{1/4}$	2.72553 K	$2.72548 \pm 0.00057 \text{ K}^e$
CMB Temperature	$f(4\theta_{si}^2((2v_m/\sqrt{2mf}) - c\theta_{si}))^{1/2}$	$\sigma = 1.394 \text{ km s}^{-1}$	—
Effective Star Velocity v_e (84k ly)	$(\frac{3\theta_{si}(2\theta_{si})^{1/3}e(\sqrt{3}\theta_{si})/2}{4\pi a_{cl} f T_{CMB}^4})^{1/3}$	$13.794 \times 10^9 \text{ y}$	$13.787 \pm 0.020 \times 10^9 \text{ y}^d$
Age of Universe A_U	$2\theta_{si} = (D_U/A_U) = (l_f m_f/t_f)$	$70.885 \text{ km s}^{-1} \text{ Mpc}^{-1}$	$67.66\text{--}74.03 \text{ km s}^{-1} \text{ Mpc}^{-1c}$
Hubble Constant H_U			

Appendix BP. Symbol Definitions

- θ_{si} can be measured as the polarization angle of quantum entangled X-rays at the degenerate frequency of a maximal Bell state. As an angle $\theta_{si}=3.26239 \text{ rad} \pm 2 \text{ } \mu\text{rad}$ [17–19]; as a momentum $\theta_{si} = 3.26239030392(48)\text{kg m s}^{-1}$; and with respect to the Target Frame, θ_{si} has no units. The relation of angle and mass is mathematically demonstrated, as well, by Chen *et al.* [22].
- c is the speed of light which may also be written as $c = n_L l_f / n_T t_f = 299,792,458 \text{ m/s}$ [3] such that $n_L = n_T = 1$ is physically significant.
- G is the gravitational constant, $6.6740779428(56) 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ such that its value considers the effects of length contraction associated with discrete measure at the upper count limit. Italicized G identifies a measure not at the limit (e.g. $G = 6.6738448362(53) 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ at the *blackbody demarcation*).
- l_f, m_f and t_f are the fundamental measures, more precise expressions for Planck's units — length, mass, and time — that consider the effects of length contraction associated with discrete measure.
- l_p, m_p and t_p are Planck's units for length, mass, and time.
- n_L, n_M and n_T are physically significant discrete counts of l_f, m_f and t_f , respectively.
- n_L describes the count of l_f representative of a change in position of an observable measured with respect to the observer's frame of reference.
- n_{Lr} describes the count of l_f representative of the position of an observable with respect to the frame of a center of mass.
- n_{Lc} describes the count of l_f representative of a change in position of light measured with respect to the observer's frame of reference.
- n_θ is the Target Frame count of θ_{si} resolved as a function of the Measurement Frame demarcation adjusted for the *metric differential* (i.e. $\text{RND}(84.60055/2)=42$).
- Q_L is the fractional portion of a count of l_f when engaging in a more precise calculation.
- $Q_L n_{Lr}$, also known as the *Informativity differential* describes the length contraction associated with discrete measure.
- h is Planck's constant.
- $\hbar$ is the reduced Planck constant, $1.054571817 10^{-34} \text{ m}^2 \text{ kg s}^{-1}$. When accounting for the *Informativity differential* at the upper count bound, this term is not italicized (i.e. $\hbar = 1.0545349844(45) 10^{-34} \text{ m}^2 \text{ kg s}^{-1}$).
- A_{ee} is the dilated age of the universe as measured from the current expansionary epoch. Its value is a function of expansion with respect to the total elapsed time representing the quantum epoch.
- A_{qe} is the non-dilated age of the universe corresponding to the quantum epoch.
- Ω_{vis} is that domain of the universe that is presently visible.
- Ω_{obs} is that domain of the universe that is presently visible or will be visible given infinite elapsed time.

- Ω_{dk} is that domain of the universe that will never be visible.
- Ω_{uobs} is that domain of the universe that will eventually be visible but is not presently visible.
- Ω_{tot} is the entire domain that makes up the universe (i.e. 1 or 100%).
- Ω_f is that domain of the universe that is fundamental. Its physical significance describes a pivot point with respect to the remaining domains.
- M_{tot} is the total mass of the universe, including all forms of energy.
- M_{acr} is the rate of mass accretion in the universe.
- M_e is the effective mass which represents the gravitational mass measurable by an observer.
- M_b is the bound mass which represents the upper count bound of m_f that can be measured per t_f in a homogenous mass distribution.
- V_U is the volume of the universe.
- A_U is the age of the universe.
- R_U is the radius of the universe.
- D_U is the diameter of the universe.
- H is the value of H_0 in $\text{m s}^{-1}\text{Mpc}^{-1}$ when resolved with respect to *universal expansion* (i.e. the Target Frame).
- H_U is the rate of universal expansion with respect to the diameter of the universe (per D_U). This differs from stellar expansion (i.e. Hubble's description).
- H_f is the Hubble frequency.
- n_{Tu} is a count of t_f equal to the age of the universe.
- n_{To} is the count of t_f observed with respect to the observer's frame of reference.
- n_{Tl} is the count of t_f measured with respect to the observer's frame of reference.
- n_{Lu} is a count of l_f equal to the diameter of the universe.
- n_{Lo} is the count of l_f observed with respect to the observer's frame of reference.
- n_{Ll} is the count of l_f measured with respect to the observer's frame of reference.
- n_{Mu} is a count of m_f equal to the total of mass/energy in the universe.
- n_{Mo} is the count of m_f observed with respect to the observer's frame of reference.
- n_{Ml} is the count of m_f measured with respect to the observer's frame of reference.
- n_{Mb} is a count of m_f representing the mass upper bound of a gravitational field.
- n_{Tc} is a count of t_f corresponding to the elapsed time in the local frame of the propagation of light.
- l_l is a measure of length with respect to the observer's measurement frame.
- l_o is the observed length subject to the effects of motion and/or gravitation.
- m_l is a measure of mass with respect to the observer's frame of reference.
- m_o is the observed mass subject to the effects of motion and/or gravitation.
- t_l is a measure of time in the local frame of reference.
- t_o is the observed time subject to the effects of motion and/or gravitation.
- t_c is a measure of elapsed time with respect to the propagation of light.
- n_{min} is the minimum physically significant count term of some fundamental measure.

- n_{max} is the maximum physically significant count term of some fundamental measure.
- Δl is the observed change in length n_L due to motion or gravitation.
- E_f is the energy of one unit of fundamental mass m_f .
- v_b is the bound velocity that corresponds to the bound mass constrained by the Planck frequency.
- v_m is the modeled velocity best reflecting the measurement data.
- v_e is the escape velocity associated with a gravitational mass.
- v_e is the effective velocity, a function of the effective mass — that count of m_f constrained by the Planck frequency — and reflecting the mass profile of a system.
- R is the distance between a center of mass and an inertial frame.
- F is force with units of kg m s^{-2} .
- S is the unknown constant when resolving a description of gravity, latter replaced with θ_{si} .
- v is velocity between an observer and a target.
- r is the distance between an observer and a target.
- M is all the mass in a considered system.
- σ_x is a description of the uncertainty in the position of a particle.
- σ_p is a description of the uncertainty in the momentum of a particle.
- E_{CMB} is the total energy associated with the CMB.
- $E_{\rho:CMB}$ is the energy density associated with the CMB.
- k is the Boltzmann constant.
- ρ is the energy density accumulated at a given age of the universe.
- a is the total energy radiated as described with respect to blackbody radiation (i.e. the Stefan–Boltzmann law).
- T_{CMB} is the temperature of the Cosmic Microwave Background.
- Q_{dk} is the monopole moment associated with dark energy.
- L is the arclength of a circle of a given radius subtended by a given angle.
- π , is a numerical value describing the arc-distance of a circle of radius one.
- E_l is the energy of one quantum of electromagnetic radiation.
- E_m is the energy of half a fundamental unit of mass m_f .
- Λ describes Einstein’s cosmological constant.
- α_f^{-1} is the fundamental form of the inverse fine structure constant defined with respect to the Target Frame.
- α_p^{-1} is the Planck-like form of the inverse fine structure constant defined with respect to the Measurement Frame.
- α_c^{-1} is the classical form of the inverse fine structure constant as adopted by the CODATA collaboration. This term accounts for length contraction associated with discrete measure.
- ε_0 is the electric constant.
- u_0 is the magnetic constant.
- $Q_{n\theta}$ is the *metric differential* $42\theta_{si}\text{-RND}(42\theta_{si})$; describes the difference at the demarcation count $n = 42$ of θ_{si} between the discrete and non-discrete frames.

- Δ_{fr} is the *metric count offset* $(n_\theta - 1)\theta_{si}/\text{RND}((n_\theta - 1)\theta_{si})$; describes the quotient offset between the discrete and non-discrete frames relative to the reference count $n = 1$ of θ_{si} .
- γ is a collection of terms that describe the geometry between the Target and Measurement Frames.
- m_e is the mass of an electron.
- a_0 is the ground state orbital of an atom.
- k_e is the Coulomb's constant.
- e is the elementary charge.
- e_f is the fundamental form of elementary charge, defined with respect to the Target Frame before adjusting for the *metric* and *Informativity differentials*.
- e_p is the Planck form of elementary charge, defined with respect to the Measurement Frame, accounts for the *metric differential* between frames.
- e_c is the classical form of elementary charge, accounts for the *metric* and *Informativity differential*.
- r_q is the quantization ratio $(n_\theta\theta_{si})/\text{RND}(n_\theta\theta_{si})$. It describes the relation between the discrete and non-discrete frames of reference.
- $\text{RND}(n)$ is a function; rounded to the nearest whole-unit value.

Appendix BQ. Term Definitions

Count Frequencies

The frequency expressions refer to the relation of counts but are often described using only the reference measures. They are length frequency l_f/t_f , mass frequency m_f/t_f , length-to-mass frequency l_f/m_f and frequency $1/t_f$. The inverse of each is referred to with the same name.

Fundamental Expression

The simplest expression correlating the three fundamental measures, $l_fm_f = 2\theta_{si}t_f$.

Fundamental Mass

The fundamental mass of the universe distinguishes a specific amount of mass whereby from a point in space-time additional mass would cause overlapping mass events that could not be distinguished due to physically significant bounds to the measure of fundamental units of mass. Understanding and resolving fundamental mass in turn allows one to solve for all the mass distributions presently understood only with ΛCDM .

Fundamental Measure

One of the measures length l_f , mass m_f , and time t_f along with their correlation called the fundamental expression. Using measurement data from the Shwartz and Harris experiments in combination with Heisenberg's Uncertainty Principle, each is macroscopically defined and physically significant.

Informativity Differential

The *Informativity differential* $2Q_L n_{Lr}$ describes a new form of length contraction associated with discrete measure. The loss of immeasurable space at each increment of t_f describes gravity.

Observational Domains

Fundamental

Describes a mathematical inflection between the observable and the dark.

Visible

Describes that which can and has been observed.

Observable

Describes that which has, can and will be observed given infinite elapsed time.

Unobserved

Describes the difference between the visible and the observable.

Dark

Describes that which can never be observed due to the expansion of space.

Total

Describes the sum of the observable and the dark.

Measurement Distortion

A short-hand notation for the contraction and dilation of measure.

Reference Frames

Reference Framework

This is the framework of the observer where properties of the reference AB are observed. With respect to the standard understanding, this framework differs only in that measure is a count function of discrete length measures equal to one.

Measurement Framework

This framework shares properties with the Reference Framework. If BC is characterized as some known count of the reference describing where count properties of the reference are observed.

Target Framework

This framework is characterized by the property of measure of non-discreteness, that being the framework of the universe that contains the phenomenon AC.

Self-referencing

An expression defined with respect to the frame of the observer.

Self-defining

An expression defined with respect to the frame of the universe.

System Parameters

Any fixed value associated with a self-defining expression.

Unity Expression

A self-defining Pythagorean expression with a hypotenuse equal to 1. Often used specifically with reference to Appendix AX.

Appendix BR. List of Appendices: Applications of MQ

- A. Angular Description of θ_{si}
- B. Heisenberg's Uncertainty Principle
- C. Discrete Gravity
- D. Length Contraction (*Informativity differential*)
- E. Classical Solution to G
- F. Discrete Solution to G
- G. Escape Velocity as a Function of Classical G
- H. Escape Velocity as a Function of Discrete G
- I. Solving the Minimum Measure Counts
- J. Discreteness of Measure
- K. Measurement Frameworks
- L. Precise Description of $\hbar$
- M. Precise Description of G
- N. Solutions to G using the CODATA
- O. Bounds to Measure
- P. Electromagnetic Description of θ_{si}
- Q. Momentum Description of θ_{si}
- R. Angular Interpretation of $\hbar$
- S. Correlation Angular Measure and Momentum at the Bound
- T. Metric Differential
- U. The Metric Offset
- V. Fundamental Expression
- W. Fundamental Measures
- X. Correlating the Planck Units with the Fundamental Measures
- Y. Correlating the Newton and MQ expressions for Gravitational Curvature
- Z. Energy Relation of Baryonic and Electromagnetic Phenomena
- AA. Fine Structure Constant
- AB. Blackbody Demarcation
- AC. Quantization Ratios
- AD. θ_{si} as Measured with the Fine Structure Constant
- AE. Charge Coupling Demarcation
- AF. Upper Count Bound to Fundamental Mass
- AG. Observable Mass Bound
- AH. Effective Mass of a Galaxy
- AI. Effective Velocity of a Star
- AJ. Correlating Star Velocity to Kinetic Energy
- AK. Galactic Rotation

- AL. Quantization Crossover
- AM. Elementary Charge
- AN. Solutions to Elementary Charge Using the Gamma Approach
- AO. Elementary Charge Demarcation
- AP. Electric Constant
- AQ. Magnetic Constant
- AR. Coulomb's Constant
- AS. Gamma — Target to Measurement Frame Transform
- AT. Unification
- AU. Contraction and Dilation with Inertial Mass
- AV. Contraction and Dilation with Gravitational Mass
- AW. Equivalence between Inertial and Gravitational Mass
- AX. Unity Expression
- AY. Physical Significance of the Fundamental Expression for All Applications
Throughout the Measurement Domain
- AZ. Correlating Diameter, Age of Universe to Fundamental Mass
- BA. Hubble's Constant (for the Expansion of Space)
- BB. Physically Significant Domains of the Observable Universe
- BC. Alternative Domain Relations
- BD. Curvature of Universe: Flat
- BE. Mass Accretion in the Universe
- BF. Quantum Referencing
- BG. Quantum Epoch
- BH. Quantity, Age, Density, Temperature of the CMB
 - BI. Age of Universe as a Function of CMB Temperature
 - BJ. Mass of the Universe as a Function of CMB Temperature
- BK. Time Dilation/Length Contraction Between Epochs
- BL. Ground State Orbital of an Atom
- BM. Extending the Physical Constants
- BN. How to Derive the Planck Expressions
- BO. Physical Support for Measurement Quantization
- BP. Symbol Definitions
- BQ. Term Definitions
- BR. List of Appendices: Applications of MQ

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