

# INVERSE INFERENCE OF INITIAL BEAM PROFILE AND KEY PARAMETERS BASED ON AUTOMATIC DIFFERENTIATION METHOD\*

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## Abstract

For experiments requiring the longitudinal shaping of the beam at the exit of an electron linear accelerators, it is crucial to infer the initial beam profile at the entrance of the linear accelerator and key parameters. After passing through the dispersion section of beam bunch compressor, and the high-frequency system, the electron beam will undergo modulation on the longitudinal density. Based on the longitudinal dynamic process, this paper proposes to use automatic differentiation to provide the design of beam initial conditions and key parameters corresponding to a specific longitudinal profile of the beam at the exit of the linear accelerator. Finally, we implemented this method on a section of linear accelerator consisting of two L-band accelerating cavities, one S-band accelerating cavity, and a bunch compressor.

## INTRODUCTION

The advancement of accelerator technologies, including free-electron lasers, terahertz radiation sources, and plasma accelerators, underscores the critical need for precise control over the temporal characteristics of electron beams [1]. These applications benefit from customized current profiles, which are vital for achieving optimal performance. For example, in plasma acceleration, specialized electron beam profiles play a key role in achieving high transformer ratios. This often involves using an asymmetric driver beam with a triangular shape, characterized by a gradual increase in density at the front and a sharp decrease at the rear to minimize the decelerating field experienced by the driver beam. Furthermore, utilizing a trapezoidal distribution for the beam to be accelerated results in a flatter longitudinal wakefield at the accelerated beam's location, facilitating the production of a beam with low energy spread for efficient acceleration [2]. Additionally, the elimination of current fluctuations is crucial for enhancing the operation of free-electron lasers, underscoring the significance of advanced beam shaping techniques in accelerator physics for designing optimal current profiles for various FEL applications. To address these requirements, shaping the temporal profiles of electron beams is indispensable, emphasizing the importance of advanced beam shaping techniques in accelerator physics.

Several reliable methods have been developed to shape the beam profiles. One common method involves directing an electron beam with a particular energy spread through an energy dispersion section, such as a bunch compressor with multiple bending magnets, and possibly additional

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sextupoles and octupoles [3]. These components offer a more flexible combination of dispersion terms. By selecting the appropriate combination of parameters in the compressor and an initial beam with a particular energy chirp, it is possible to achieve the desired beam profiles. Furthermore, understanding the impact of the RF system on the energy chirp, and anticipating the longitudinal distribution of the beam in advance, will be a crucial step. This is also the main issue that this paper work aims to address.

With the application of automatic differentiation in the reconstruction of beam phase space, more and more attention is being focused on using automatic differentiation to solve problems with high complexity, high dimensionality, and difficulties in convergence. R. Roussel and A. Edelen's team proposed using automatic differentiation techniques to rapidly reconstruct the 6-dimensional information of the initial beam based on neural network structures, utilizing the 6-dimensional information of the beam images observed at the diagnostic screen [4]. This laid the foundation for the application of automatic differentiation techniques in solving inverse problems.

Based on this method, we analyzed the longitudinal dynamics of the beam and provide the relationship between the longitudinal particle coordinates of the beam at the exit and the initial RF system parameters, chicane parameters, and initial particle longitudinal coordinates. By using automatic differentiation methods, the initial beam profile and the parameter design of the linear accelerator can be rapidly determined.

## LONGITUDINAL DYNAMICS

For generality, we discuss the longitudinal dynamics process of an electron beam passing through an RF system consisting of two L-band traveling wave tubes and one S-band traveling wave tube, as well as a chicane design similar to that described in Ref. [3] (comprising 4 dipoles and 2 quadrupoles, 2 sextupoles, and one octupole, providing a more flexible combination of dispersion terms) as shown in Fig 1. Assuming the initial electron beam is monoenergetic with an initial energy of 200 MeV, the electron energy (charge  $e$ ) at the chicane after passing through the three RF systems is

$$E_f \approx E_0 + eV_1 k_L \cos(\varphi_1 + k_L Z_0) + eV_2 k_L \cos(\varphi_2 + k_L Z_0) + eV_3 k_s \cos(\varphi_3 + k_s Z_0) \quad (1)$$

where  $k_{L,S} = (2\pi/\lambda_{L,S})$  is the RF wavenumber,  $z_0$  is the longitudinal position of the electron with respect to the reference particle (bunch center), and the bunch head is at

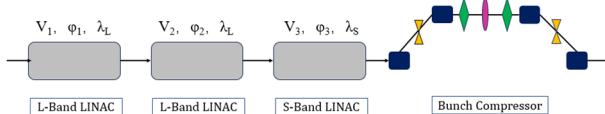


Figure 1: Schematic of two linac segments followed by harmonic RF and bunch compressor.

$z_0 < 0$ . The RF phase,  $\varphi$ , is defined to be zero at accelerating crest and a phase in the interval  $[-\pi, 0]$  will accelerate the bunch head less than it will the bunch tail.

For higher harmonic compensation, the phase of the S-band cavity is typically set near  $\pm\pi$ . Here, to explore obtaining a more diverse energy chirp shape, the phase of the S-band cavity will also be considered as one of the optimization parameters in this study. After an electron beam passing through this RF system, we can obtain the energy deviation relative to the reference particle, to second order in the longitudinal coordinate  $z_0$ , is then given by.

$$\delta = \frac{E_f - E_{f0}}{E_{f0}} \approx \left( \frac{-eV_1 k_L \sin \varphi_1 - eV_2 k_L \sin \varphi_2 - eV_3 k_s \sin \varphi_3}{E_{f0}} \right) z_0 + \left( \frac{-eV_1 k_L^2 \cos \varphi_1 - eV_2 k_L^2 \cos \varphi_2 - eV_3 k_s^2 \cos \varphi_3}{2E_{f0}} \right) z_0^2 \quad (2)$$

Similar to Ref. [5], the incoming uncorrelated energy spread is ignored here for simplicity, but it does not alter the arguments presented below. Equation (3), which represents the relative energy deviation, is abbreviated.

$$\delta(z_0) = \Delta E/E_{f0} \approx az_0 + bz_0^2 \quad (3)$$

After specifying the cavity voltage and frequency of the given RF system, the parameters  $a$  and  $b$  represent the phase functions of the three RF systems. The choice of phase directly influences the relative energy deviation with respect to the reference particle, thereby resulting in different energy chirp.

For a bunch compressor, an electron beam with small emittance and large energy spread, a Taylor map that represents the final longitudinal coordinate  $z_f$  of a charged particle as a Taylor series of the initial coordinates can be simplified as [6]:

$$z_f = z_0 + R_{56} \delta(z_0) + T_{566} \delta(z_0)^2 + U_{5666} \delta(z_0)^3 + \dots \quad (4)$$

where  $R_{56}$ ,  $T_{566}$ , and  $U_{5666}$  are the first-, second- and third-order longitudinal dispersion terms respectively. It can be observed that by controlling the initial energy chirps of the beam and the dispersion terms of the bunch compressor, we can achieve arbitrary temporal profile of beams at the exit of the chicane. Therefore, we consider utilizing different initial energy chirps designed by the RF system, combined with the appropriate dispersion term combination provided by the chicane, to achieve a specific longitudinal profile of the beam at the chicane exit.

## AUTOMATIC DIFFERENTIATION METHODS

In Ref. [4], they have demonstrated the iterative inclusion of neural network parameters with the designed loss

function, which can reconstruct the desired 6D phase space information of the initial beam. Therefore, we consider using a neural network to generate the longitudinal distribution of the initial beam. The input data for the neural network is one-dimensional random noise, with the data distribution following a standard Gaussian distribution. The amount of data is equal to the number of particles. The output is the longitudinal coordinate relative to the reference particle within the beam. Due to the powerful mapping capability of neural networks, we expect to accurately characterize the appropriate longitudinal profile of the initial beam by adjusting the parameters of the network, such as the weights and biases of each node. With the requirement for the differentiability of the calculation process in neural network gradient backpropagation, we use the root mean square error (RMSE) between the two profiles as the loss function to characterize the proximity of the longitudinal profile at the exit of the chicane to the target profile.

Neural network module in PyTorch provides us with a framework for parameter inference based on automatic differentiation [7]. By defining differentiable loss functions and computation processes, after obtaining the longitudinal coordinates of beam particles at the exit of the chicane (relative to the reference particle at  $z_0=0$ ), we can calculate the gradients of the loss function with respect to the neural network parameters, RF system, and chicane parameters using the chain rule based on the root mean square error (RMSE) between the target profile and the obtained longitude profile. By dynamically adjusting parameter values using gradients through multiple rounds of iterative adjustments, the parameters are continuously fine-tuned until convergence is achieved.

## RESULTS

In the analysis of beamline beam dynamics, it is essential to not only ascertain the chicane parameters but also gather additional beam information, such as the energy chirp (which can be controlled jointly by the initial beam profile and the RF system). Below, we present the initial beam profile and energy chirp results corresponding to the triangular target distribution. Assuming the initial beam energy is 200 MeV with zero energy spread, neglecting the influence of transverse phase space motion, and considering only longitudinal dynamics processes.

Figure 2(a) displays the probability density curve of the target distribution, with a distribution range of  $\pm 0.5$  ps. Figure 2(b) shows the probability density curve of the corresponding initial beam longitudinal profile obtained through reverse solving. It can be observed that, in order to meet the characteristic of higher peak beam intensity downstream of the chicane, the initial beam will generate a peak intensity within beam. Subsequently, after energy modulation by the RF system and density modulation by the dispersion term of the chicane, the longitude profile will be adjusted to closely match the target beam profile. Figure 2(c) displays the probability density curve of the beam at the downstream exit of the chicane. There is still a certain deviation from the expected target distribution, which

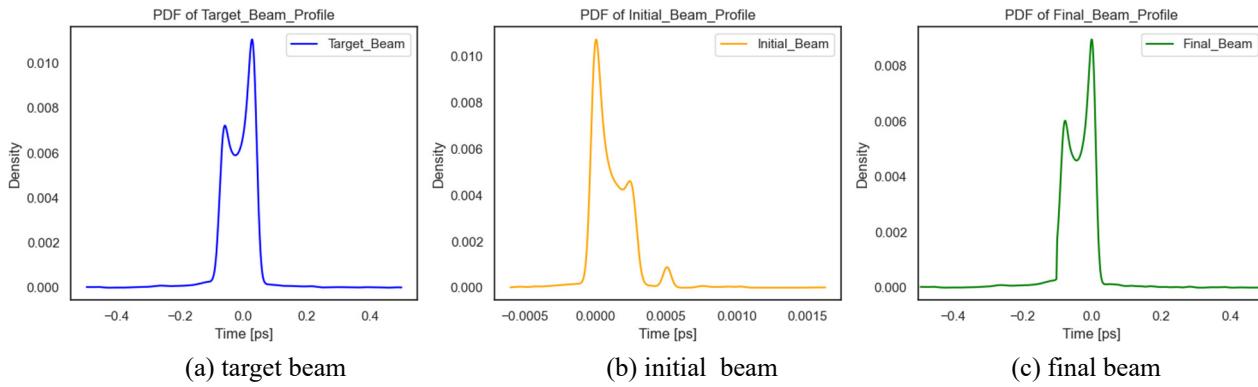


Figure 2: The probability density curves of three beam profiles.

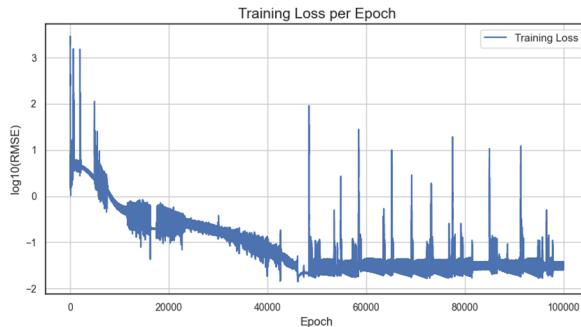


Figure 3: Evolution of training loss error with iteration number.

is related to the design of the algorithm itself. The global search capability of the gradient-based algorithm still needs improvement. However, this result has already demonstrated the potential of automatic differentiation-based methods in parameter inference.

Figure 3 illustrates the evolution of the loss function with the iterations. It can be observed that the RMSE value decreases over iterations and converges to a small value with oscillations near this minimum. This phenomenon is likely due to the sensitivity of the loss function of the training network to the learning rate. The convergence of the loss function also indicates that the longitudinal profile of the beam at the downstream exit of the chicane, calculated through the initial beam generated by the neural network, gradually approaches the target profile. The process shows a decreasing difference between them.

## CONCLUSION

In this paper, we utilize automatic differentiation for parameter inference and optimization by designing the neural network structure and loss function. Based on the calculation results of the loss function, the algorithm adjusts the neural network parameters using gradient information to generate the desired initial beam longitudinal distribution. Simultaneously, it adjusts the parameters of the RF system and chicane to ensure that the final beam profile closely matches the target distribution. Through multiple iterative, we can obtain the required initial beam profile and the corresponding RF system and chicane parameter values. This method combines the mapping capability of neural

networks and the advantages of automatic differentiation, providing an efficient and flexible approach for beam dynamics research and optimization.

However, this method also has limitations. Because it is based on gradient-based reverse solving of initial parameters, it cannot provide multiple optimal solutions for one-to-many problems, and it is prone to getting stuck in local optima during the solving process, leading to solution failure. Additionally, the longitudinal dynamics in this paper currently do not consider nonlinear effects such as CSR effects and space charge effects. The presence of CSR effects will result in more complex physical processes, which will be further addressed in future work.

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