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Simulations of black holes in compactified spacetimes

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Abstract. From the gauge/gravity duality to braneworld scenarios, black holes in compactified spacetimes play an important role in fundamental physics. Our current understanding of black hole solutions and their dynamics in such spacetimes is rather poor because analytical tools are capable of handling a limited class of idealized scenarios, only. Breakthroughs in numerical relativity in recent years, however, have opened up the study of such spacetimes to a computational treatment which facilitates accurate studies of a wider class of configurations. We here report on recent efforts of our group to perform numerical simulations of black holes in cylindrical spacetimes.

1. Introduction

Since the 2005 breakthroughs [1–3], the field of numerical relativity has seen remarkable progress. Major achievements range from the discovery of new phenomena—such as black holes (BHs) *kicks* [4–6] and zoom-whirl behaviour [7]—to simulations of high energy collisions of BHs [8–10].

A very interesting research direction, which has recently seen some pioneering works, concerns extending this field to the study of higher-dimensional systems [11–18].

In this work, we wish to study how the compactness of extra dimensions changes the dynamics of such higher-dimensional gravity scenarios. There is considerable literature on Kaluza-Klein BHs and BHs on cylinders [19–22], but, to the best of our knowledge, the full non-linear dynamics of BHs on such spacetimes remains unexplored.

2. Setup

For a five dimensional cylindrical Minkowski spacetime, $\mathbb{M}^{1,3}\times S^1$, the metric can be written as

$$ds^2 = \underbrace{-dt^2+dx^2+dy^2+y^2d\phi^2}_{\mathbb{M}^{1,3}}+\underbrace{dz^2}_{S^1} \ . \tag{1}$$

The S^1 direction is parameterised by z, which takes values in the interval $[-L,L]$, with the two endpoints identified and $L\in\mathbb{R}^+$. The coordinate ϕ also parameterises a circle, which is part of $\mathbb{M}^{1,3}$.

Following [15] we take our five dimensional metric ansatz to be

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + \lambda(x^\mu)d\phi^2 \ , \tag{2}$$

where $x^\mu=(t,x,y,z)$. We perform a dimensional reduction by isometry on ∂_ϕ and end up with a four dimensional model of gravity coupled to a scalar field. Performing the standard $3+1$ decomposition, the evolution equations of the resulting system are

$$\begin{aligned}
(\partial_t-\mathcal{L}_\beta)\,\gamma_{ij} &= -2\alpha K_{ij}\\
(\partial_t-\mathcal{L}_\beta)\,K_{ij} &= -D_i\partial_j\alpha+\alpha\left({}^{(3)}R_{ij}+KK_{ij}-2K_{ik}K^k{}_j\right)\\
&\quad -\alpha\frac{1}{2\lambda}\left(D_i\partial_j\lambda-2K_{ij}K_\lambda-\frac{1}{2\lambda}\partial_i\lambda\partial_j\lambda\right)\\
(\partial_t-\mathcal{L}_\beta)\,\lambda &= -2\alpha K_\lambda\\
\frac{1}{\alpha}(\partial_t-\mathcal{L}_\beta)\,K_\lambda &= -\frac{1}{2\alpha}\partial^i\lambda\partial_i\alpha+KK_\lambda-\frac{1}{\lambda}K_\lambda^2+\frac{1}{4\lambda}\partial^i\lambda\partial_i\lambda-\frac{1}{2}D^k\partial_k\lambda
\end{aligned} \tag{3}$$

For further details and numerical implementation see [15, 23]. We here use periodic boundary conditions along the z direction and Sommerfeld radiative boundary conditions along x and y.

3. Initial data

Following [15, 24], the four-dimensional Brill-Lindquist initial data appropriate to describe non-spinning, non-rotating BHs momentarily at rest, take the form

$$\gamma_{ij}dx^idx^j=\psi^2[dx^2+dy^2+dz^2] \ , \qquad \lambda=y^2\psi^2 \ , \qquad K_{ij}=0=K_\lambda \ .$$

In a spacetime with standard topology (wherein z parameterises a line), the initial data for two BHs with horizon radius $r_S^{1,2}$ and punctures placed at $(x,y,z)=(0,0,\pm a)$, takes the form

$$\psi = 1+\frac{(r_S^1)^2}{4[x^2+y^2+(z-a)^2]}+\frac{(r_S^2)^2}{4[x^2+y^2+(z+a)^2]} \ . \tag{4}$$

3.1. Head on collision of BHs

The appropriate initial data to describe a BH in S^1 can be viewed as having an infinite array of BHs, all with the same mass, separated by coordinate distance $\Delta z=2L$. Since the superposition of various BHs in a line is described by adding up the corresponding initial data, for the infinite array of two BHs in the circle located at $z=\pm a$ $(0<a<L)$ with horizon radii r_S^i, $i=1,2$ (or, equivalently, for two BHs in S^1) the initial data is given by

$$\begin{aligned}
\psi &= 1+\sum_{n=-\infty}^{+\infty}\frac{(r_S^1)^2}{4[x^2+y^2+(z-a-2Ln)^2]}+\sum_{n=-\infty}^{+\infty}\frac{(r_S^2)^2}{4[x^2+y^2+(z+a-2Ln)^2]}\\
&= 1+\frac{\pi(r_S^1)^2}{8L\rho}\frac{\sinh\frac{\pi\rho}{L}}{\cosh\frac{\pi\rho}{L}-\cos\frac{\pi(z-a)}{L}}+\frac{\pi(r_S^2)^2}{8L\rho}\frac{\sinh\frac{\pi\rho}{L}}{\cosh\frac{\pi\rho}{L}-\cos\frac{\pi(z+a)}{L}} \ .
\end{aligned} \tag{5}$$

where $\rho^2\equiv x^2+y^2$ and in the last equality we have used the result in [21].

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4. Results

We will now show some results obtained for a head-on collision (from rest) of BHs with an initial separation of $10.37\ r_S$, i.e., $a = 5.185\ r_S$. The z ($z \in [-L, L]$) coordinate has been compactified with $L/r_S = 64,\ 32,\ 16$. For comparison purposes, we have also performed a simulation with "standard" outgoing boundary conditions ($L \to \infty$), which will be here referred to as "outgoing".

All results will be presented in units of the Schwarzschild radius $r_S = r_{S,1} + r_{S,2}$.

4.1. Hamiltonian constraint

Figure 1 shows the Hamiltonian constraint along the x and z axis, respectively, for several time steps for the $L = 32$ case. As we can see, the constraint is indeed being satisfied with high accuracy.

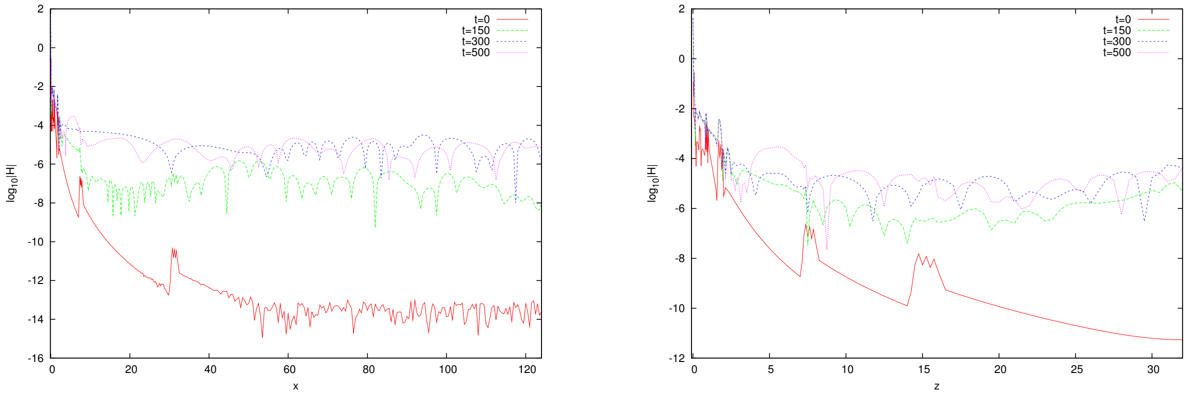

Figure 1. Hamiltonian constraint along the x and z axis, for the $L = 32$ case.

4.2. Collision time

Next, we studied the changes in collision time for different compactification radii. Whereas we don't observe any (noticable) difference for $L = 64$ and $L = 32$ as compared to the outgoing case, the case $L = 16$ shows already a noticable difference. The puncture trajectories for these cases are plotted in Figure 2.

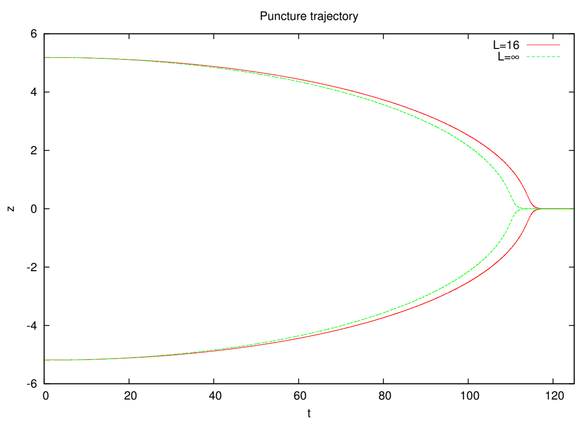

Figure 2. Puncture z coordinate as function of time for the outgoing and $L = 16$ cases.

Recall that one can think of a BH in a cylindrical space as an array of infinite BHs. Therefore, for a head-on collision on such a cylindrical space, each BH will also feel the gravitational pull of all the other BHs. Naïvely, one thus expects that for this cylindrical case it will take longer for the BHs to collide, which is what we observe in Figure 2.

5. Final remarks

Using the formalism introduced in [15], we were able to reduce the head-on collision of (non-spinning) BHs on cylindrical spacetimes (in any dimension) to an effective $3+1$ system with a scalar field, and used this procedure to successfully evolve a head-on collision of two BHs on a five-dimensional cylindrical spacetime.

Further issues that we wish to investigate include monitoring the deformation of the BHs' apparent horizon and computing the energy radiated (along the lines of [17]). We further plan to perform simulations with smaller compactification radii and the equivalent six-dimensional system.

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