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Nonorthogonality of the Long- and Short-lived Neutral Kaon States and Phenomenological Analysis of Experiments on CP Violation in K^0 Decay

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Nonorthogonality of the Long- and Short-lived Neutral Kaon States and Phenomenological Analysis of Experiments on CP Violation in K^o Decay *

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ABSTRACT

It is noted that the non-zero charge asymmetry in the $K_{L_{e3}}$ decay demonstrates the nonorthogonality of the K_L^o and K_S^o states. The result of the foregoing letter on the charge asymmetry is then combined with other measurements relevant to CP violation in K decay. It is found that the experimental results are consistent with a phenomenogical analysis. The pion-pion scattering phase shift which follows from this analysis is consistent with other indirect observations only if the mass difference $m_L - m_S$ is negative.

The result on the charge asymmetry reported in the preceding letter can be related to other properties of neutral kaon decay. Here we wish to call attention to two essentially separate connections, one to the nonorthogonality of the long- and short-lived K^0 states, the other to the CP violating amplitudes in the two-pion decay.

We assume CPT symmetry in the absence of any evidence to the contrary. It was shown by Lee, Oehme and Yang[1] that the long- and short-lived states can then be written in terms of the eigenstates of hypercharge, K and $\bar{K}$:

$$|L\rangle = p|K\rangle + q|\bar{K}\rangle \qquad (1a)$$

$$|S\rangle = p|K\rangle - q|\bar{K}\rangle \qquad (1b)$$

$$|p|^2 + |q|^2 = 1,$$

where p and q are as yet undetermined coefficients, which are equal in the case of CP conservation. If CP symmetry is violated, $|L\rangle$ and $|S\rangle$ are in general not orthogonal. Let α be the overlap:

$$\alpha = |\langle L|S\rangle| = |p|^2 - |q|^2.$$

Consider now the four leptonic decay amplitudes:

$$K^o \rightarrow \pi^- e^+ \nu \quad \ldots \quad f \qquad (2a)$$

$$\bar{K}^o \rightarrow \pi^+ e^- \bar{\nu} \quad \ldots \quad f^* \qquad (2b)$$

$$K^o \rightarrow \pi^+ e^- \bar{\nu} \quad \ldots \quad g^* \qquad (2c)$$

$$\bar{K}^o \rightarrow \pi^- e^+ \nu \quad \ldots \quad g \qquad (2d)$$

It is a consequence of CPT symmetry that two independent amplitudes suffice for the description of the two reactions. The f amplitude (reactions 1 and 2) corresponds to $\Delta S = \Delta Q$, the g amplitude (reactions 3 and 4) to $\Delta S = -\Delta Q$. Let $x = g/f$ be the ratio of $\Delta S = -\Delta Q = +\Delta Q$ amplitudes, and δ be the charge asymmetry in K_L decay as defined in the foregoing letter. Then

$$\delta = \frac{\Gamma_{e^+} - \Gamma_{e^-}}{\Gamma_{e^+} + \Gamma_{e^-}} = \alpha(1 - |x|^2)/|1 + x|^2. \qquad (3)$$

A non-zero δ implies that the long-lived and short-lived K states are not orthogonal. The most direct result of the foregoing experiment, is, therefore, that <u>K_L</u> and K_S <u>are not orthogonal</u>.

In order to proceed it is necessary to know x. From experiment (see, for instance, the summary[2] prepared for the Oxford Conference, 1965) only an upper limit of ~ 0.2 can be put on $|x|$. In the following we assume that $x = 0$. This is attractive from a theoretical point of view, and in any case, no gross error is likely. We then have the second result:

$$\alpha = |p|^2 - |q|^2 = \delta = (2.24 \pm 0.36) \times 10^{-3} .$$

It has been shown by Wu and Yang[3] that this result can be related to the two-pion decay amplitudes. The analysis has been applied by Gaillard et al[4] to the data existing at that time.

Let

$$\eta_{+-} = \frac{\langle \pi^+, \pi^- | T | L\rangle}{\langle \pi^+, \pi^- | T | S\rangle}$$

$$\eta_{oo} = \frac{\langle \pi^o \pi^o | T | L\rangle}{\langle \pi^o \pi^o | T | S\rangle}$$

$$\epsilon = \frac{p - q}{p + q}$$

and

$$\epsilon' = \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \mathrm{Im} \frac{\langle 2 | T | K\rangle}{\langle 0 | T | K\rangle}$$

where δ_0 and δ_2 are the pion-pion scattering phases in the isospin 0 and 2 states, respectively. Using the phase convention $\mathrm{Im}\langle 0 | T | K\rangle = 0$, and assuming CPT symmetry the following phenomenological relations can be obtained (see Ref. 2 or 3):

$$\eta_{+-} = \epsilon + \epsilon' \tag{4a}$$

$$\eta_{oo} = \epsilon - 2\epsilon' \quad . \tag{4b}$$

If, in addition, the CP violating amplitudes in decay modes other than the two-pion mode can be neglected (see, for instance, Ref. 2 or 3), the unitary requirement yields

$$\mathrm{Im}\ \epsilon / \mathrm{Re}\ \epsilon = 2\Delta m / \Gamma_S \tag{4c}$$

where $\Delta m = m_L - m_S$.

From the definitions of ϵ and α, and from the fact that they are small, it follows that

$2\ \mathrm{Re}\ \epsilon = \alpha$.

For the following analysis it is further assumed that $x \simeq 0$, that is, that the $\Delta S = -\Delta Q$ amplitude is negligible, so that $\delta = \alpha$. This gives the fourth of the relations (4):

$$\delta = 2 \text{ Re } \epsilon \ . \tag{4d}$$

We summarize: The relations (4a) - (4d) are derived with the assumption of CPT invariance, smallness of CP violation in other than the 2π decay mode, and $\Delta S = \Delta Q$. They have as consequences that the five measurable parameters $|\eta_{+-}|$, phase of η_{+-}, $|\eta_{oo}|$, phase of η_{oo}, and δ, are expressible in terms of three parameters $|\epsilon|$, $|\epsilon'|$, and phase of ϵ'.

Relevant existing measurements are given in Table I, and shown in Fig. 1.

The relations (4a) to (4d) imply that it is possible in Fig. 1 to draw a straight line, the line $-3\epsilon'$, from η_{+-} through ϵ to η_{oo}, such that the length from η_{+-} to ϵ is one-half that from ϵ to η_{oo}. This line is also shown in Fig. 1 and it can be seen that a good fit is possible. On the basis of this fit:

$$|\epsilon'| = (1.69 \pm 0.2) \times 10^{-3}$$

$$\phi_{\epsilon'} = (147 \pm 4)^{\circ} = (2.56 \pm 0.07) \text{ rad.}$$

$$\phi_{\eta_{oo}} = (-9 \pm 5)^{\circ} = (-0.16 \pm 0.09) \text{ rad.}$$

and

$$\delta_2 - \delta_0 = (57 \pm 4)^{\circ} = (1.0 \pm 0.07) \text{ rad.}$$

The errors are approximate only.

The above solution of Eqs. (4a) - (4d) which is also that shown in Fig. 1, is based on $m_L > m_S$, since there is mounting evidence [9-11] for this sign of Δm. The resultant phase for pion-pion scattering, $\delta_2 - \delta_0$, has then however the opposite sign of that which follows from the analysis of pion production in pion-nucleon collisions, although the magnitude, $(57 \pm 4)^{\circ}$ is quite close to the magnitude of 53° reported by Walker et al[12] in a recent analysis of pion production experiments.

Another solution to the CP violation data is possible, however, based on negative Δm. The solution is obtained by changing the sign of the imaginary axis in Fig. 1. All magnitudes remain the same, and all phases change sign. The $\delta_2-\delta_o$ phase shift obtained on the basis of $m_L < m_S$ agrees with the results of the analysis of the pion production experiments.

TABLE I *

Experimental Data Relevant to the
Phenomenological Relations (4a) - (4d)

Quantity	Value	Reference
$\lvert\eta_{+-}\rvert$	$(1.96 \pm 0.09) \times 10^{-3}$	a
$\lvert\eta_{oo}\rvert$	$(4.3\ ^{+1.1}_{-0.8}) \times 10^{-3}$	b
	$(4.17 \pm 0.30) \times 10^{-3}$	c
ϕ_{+-}	1.47 ± 0.3	d
$\Delta m/\Gamma_S$	0.44 ± 0.028	e
	0.48 ± 0.026	f
δ	$(2.24 \pm 0.36) \times 10^{-3}$	g

* The phase of ϕ_{+-} is based on positive Δm. For negative Δm the sign of ϕ_{+-} reverses and the signs of all other angles in Fig. 1 reverse as well.

a) J. Cronin, Compilation presented at Rochester Conference, August, 1967.
b) Ref. 4.
c) Ref. 5 and private communication by J. Cronin. We wish to thank Prof. Cronin for making the more recent results available to us prior to publication.
d) Ref. 6.
e) Ref. 7 and Ref. 6.
f) Ref. 8.
g) Preceding letter.

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Fig. 1 Experimental results for η_{+-}, η_{oo}, ϵ and ϵ' in the complex plane.

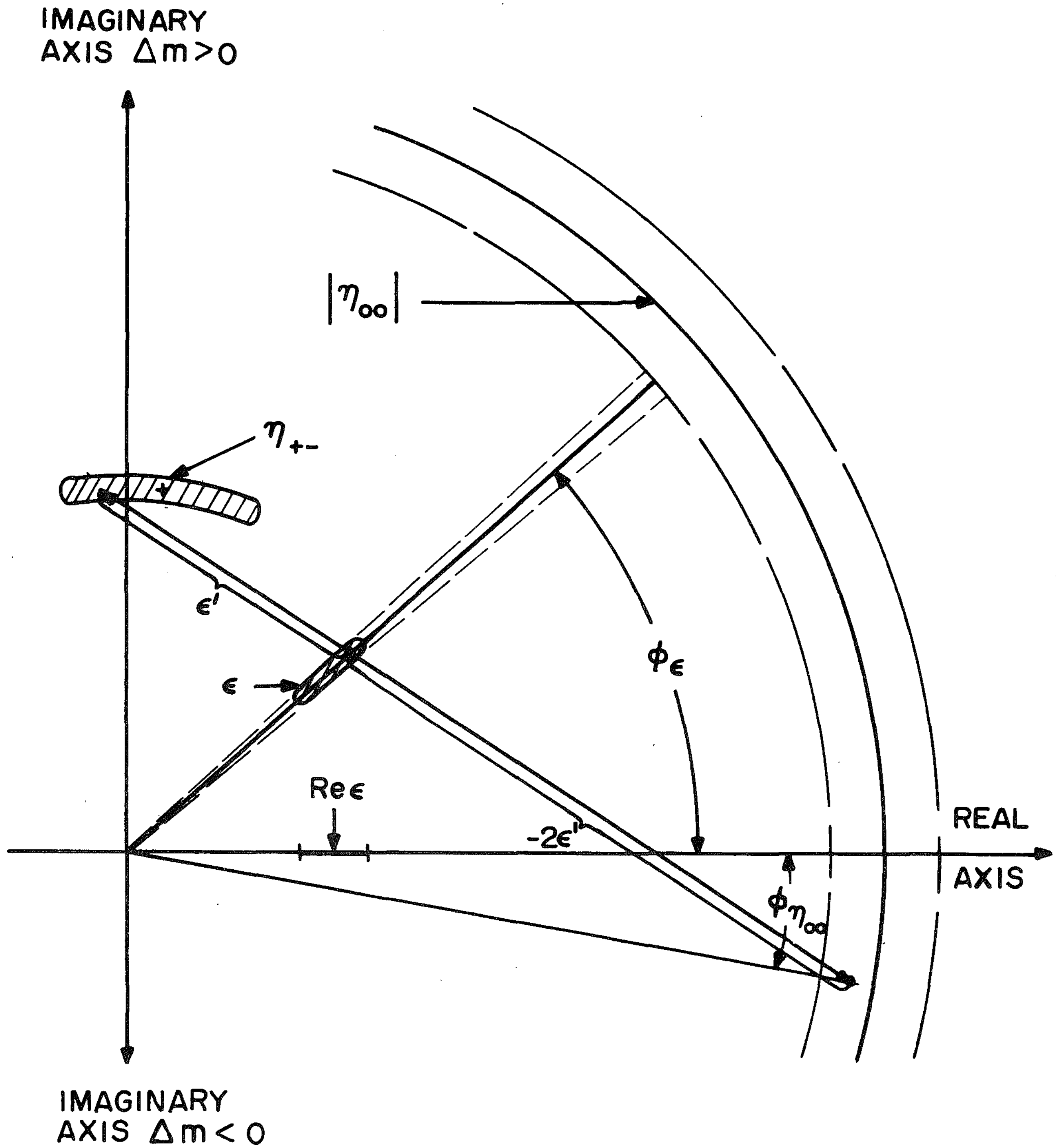
IMAGINARY
AXIS Δm>0
|η00|
η+-
ε'
ε
φε
Reε
-2ε''
REAL
AXIS
φη00
IMAGINARY
AXIS Δm<0

Fig. 1