ISBN: 978-3-95450-247-9 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2024-MOPS81

IMPLEMENTING BETATRON RADIATION FOR BEAM DIAGNOSTICS STUDIES

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Abstract

Betatron radiation is a form of synchrotron radiation emitted by relativistic electron or positron-like charged particles, due to their transverse oscillation in a nonlinear plasma ion column. As a valuable tool it can provide useful information about their trajectories, momentum and acceleration. Information about the properties of the beam is encoded in the betatron radiation, measurements of which can provide a non-invasive means to reconstruct beam parameters (energy, emittance, and divergence), offering insights into the dynamics of the plasma wakefield and facilitating advancements in particle accelerator technology.
One method of extracting this rich information about beam parameters from measurements of BR is- maximum Likelihood Estimation (MLE) technique, while machine learning (ML) approaches can then be applied to improve the accuracy of these measurements. Furthermore, a hybrid ML-MLE simulation approach was attempted in this work to obtain a finer insight, where ML and MLE individually have their limitations.

INTRODUCTION

Particle accelerators play a vital role in various scientific fields, from high-energy physics to medical applications. However, conventional accelerators suffer from limitations in size and achievable energies due to technological constraints. In recent time, Plasma Wakefield Acceleration (PWFA) has emerged as a promising alternative, offering the potential for compact and high-gradient accelerators [1, 2].

In plasma wakefield acceleration (PWFA), a dense drive beam is used to repel the plasma electrons, thus creating a linear focusing field, which is exactly what's needed to accelerate charged particles. Hence, a precisely timed injected secondary beam (e.g. of electrons), known as the witness beam, is accelerated to a very high energies over a very short distance, while also preserving the quality of the accelerated bunch [2]. As a consequence, electrons within the witness beam, experience transverse betatron oscillations at the plasma frequency. These oscillations, akin to a harmonic motion induced by the focusing force, result in the emission of betatron radiation. Intriguingly, betatron radiation (BR) encapsulates valuable information about the properties of the witness beam, effectively encoding details such as its energy, emittance, and divergence [3]. Hence, measurements of this radiation can provide a non-invasive means to reconstruct beam parameters, offering insights into the dynamics of the plasma wakefield and facilitating advancements in particle accelerator technology.

The goal of this work is to assess the ability of a hybrid ML-MLE simulation approach to test its superiority in accurately extracting one of the beam parameters, where ML and MLE individually might have their limitations [4].

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ANALYTICAL APPROACH

In order to test the simulation models in beam spot size measurement using the info out of the radiation spectra, first we needed to have the BR profile. PyWarpx is a popular Particle-in-Cell (PIC) code [5,6] used for simulating charged particle behavior in plasmas.

With PyWarpx, the motion of the particle beam was simulated for a wakefield scenario (i.e. a case, where a proton beam drives the wakefield in a plasma, and an electron beam is injected into the wakefield to undergo betatron oscillations). From the output (of the 3D PyWarpx simulation) consisting of field and particle data of trajectories and momentum, the corresponding BR for different chosen beam spot sizes were calculated without the explicit need of Liénard-Wiechert potential [6] method. The spectra from the electron witness beam with a proton driver beam with specific simulation parameters is shown in Fig. 1.

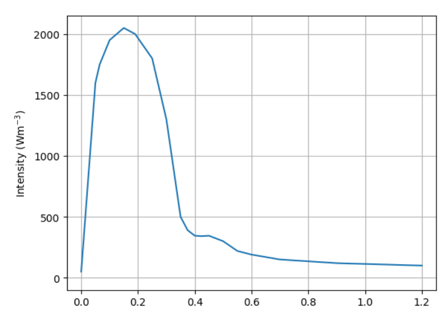

Figure 1: Betatron radiation spectra for the driver proton beam and trailing electron beam, as a function of the energy.

A comparison of the nature of radiation was tested for the case of the trailing electron beam while having a proton driver beam and an electron driver beam. While electrons are typically known for emitting more intense BR due to their lighter mass and higher achievable velocities, specific plasma conditions and interactions with proton driver beams can lead to unique observations of BR spectra. The higher observed radiation spectrum from an electron witness beam with a proton driver beam (compared to the case of electron driver beam) can be attributed to the complex dynamics

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of plasma wakefields under specific simulation parameters (Table 1), where the interactions between the proton beam and the plasma generate strong non-uniform/asymmetric wakefields that accelerate the witness electrons to energies resulting in significant radiation.

Statistically, beam distribution profiles are helpful in quantitative analysis by providing info on beam homogeneity, divergence and stability. The beam profile of the driver proton beam and of the trailing electron beam are shown in Fig. 2; initial beam distribution refers the case before acceleration and final is after the acceleration. While the driver produces the wakefields, it doesn't feel the linear focusing force, therefore the distribution remains less affected unlike the case of the trailing beam.

MAXIMUM LIKELIHOOD ESTIMATION TECHNIQUE AND ITS USE

Maximum likelihood estimation (MLE) is a statistical method [7], to estimate the parameters of a probability distribution based on observed data. It achieves this by finding the values of the parameters that make the observed data most likely to have occurred. The probability distribution function ($f(x|\theta)$, which specifies the probability of observing a data point x [where, $x = (x_1, ..., x_n)$], given the unknown parameter θ) is related to a likelihood function ($L(\theta|x)$) as the core of MLE. A maximum likelihood estimator is an extremum estimator obtained by maximizing, as a function of θ, the objective function $\hat{L}(\theta\,; x)$. If the data are independent and identically distributed, then we have-

$$\hat{L}(\theta\,; x) = \frac{1}{n}\sum_{i=1}^{n} \ln f(x_i \mid \theta) \tag{1}$$

this being the sample analogue of the expected log-likelihood $L(\theta) = \mathbb{E}[\,\ln f(x_i \mid \theta)\,]$, where this expectation is taken with respect to the true density.

Spot Size Identification using MLE

As mentioned before, primarily a beam's spot size from its radiation spectrum was correctly identified using MLE. Intensity spectra are then converted into a probability distribution dividing by sum of the counts, together forming a probability distribution function $f(x|\theta)$, where x here is the photon energy and θ represents beam spot-size.

Now, Eq. (1) helps us determining the likelihood of effective modelling of the test spectrum $f_{test}(x)$ for different values of θ by the probability distribution.

MLE algorithm was tested with a total of 400 training simulations and 100 test simulation runs, each with a spot size chosen randomly from a uniform distribution again between the values 0.5 μm and 10 μm. The overall results are displayed in the left most plot of Fig. 3, where “expected” green line represents perfect predictions. At a mean-squared error (MSE) of 0.232 μm^2, the prediction results appear reasonably fine, except in the regions around 4 μm and below 1.5 μm, where a few predictions are significantly off the actual spot sizes.

BEAM PARAMETER RECONSTRUCTION USING ML

While MLE is a powerful and widely used method for parameter estimation, its prediction ability can be limited under certain conditions, especially when extrapolating beyond the observed data range or when model assumptions are violated. Potentially using techniques like Bayesian inference, machine learning approaches can help improve the reliability of predictions made using MLE [8]. Here also despite the fact that the MLE method of beam parameter reconstruction was able to identify beam spot sizes at a basic level, it cannot predict parameter values outside the training data. Therefore, machine learning (ML) has also been explored as another alternative.

A Multilayer perception (MLP) method was attempted using the scikit-learn (sklearn) library with Tanh activation function and the Adam solver accompanied by hidden layer sizes: 500,100. Initially just like the MLE approach, 25 simulations were run for simplicity for limitation of computing power, which further extended to generate 400 training data sets and 100 test cases for different spot sizes ranging from 0.5 μm and 10 μm.

As can be seen in the middle plot of Fig. 3, the results for ML predictions indicate a slightly better predicted agreement than the MLE case, having a less prominent ”tail” around ∼ 1 μm; although, the overall predictions of the spot sizes for both the MLE and ML methods suggest that the simulations might not be highly sensitive for extremely small spot sizes.

BEAM PARAMETER RECONSTRUCTION USING A HYBRID ML-MLE APPROACH

A hybrid model was thus finally opted to capture the underlying statistical properties of the data for a better modelling and prediction results. Hybrid MLE-ML methods aim to combine the strengths of statistical estimation and machine learning to develop more effective and versatile models. Same method was used here as in the case for pure ML case, this time an extra incorporation of MLE to find maximum log likelihood estimation as true spot size value. The simulation with hybrid approach took less time than the pure ML case indicating that it is faster as the ML on the reduced array (for involving the MLE) is quicker.

However, this might be true only for large data sets because of the slowed algorithm by the reduced arrays, as data is harder to find patterns in; so there should be an optimal amount of data for ML in this case.

Rightmost plot in Fig. 3 shows a further improved predictions than the last two ones, with a better MSE value of 0.156 μm^2.

DIAGNOSTICS ASPECT: RADIATION DETECTION

On the other hand regarding diagnostic side, investigation is under way to incorporate BR aspect in case of the proton driven PWFA experiments [9]. The primary radiation of the

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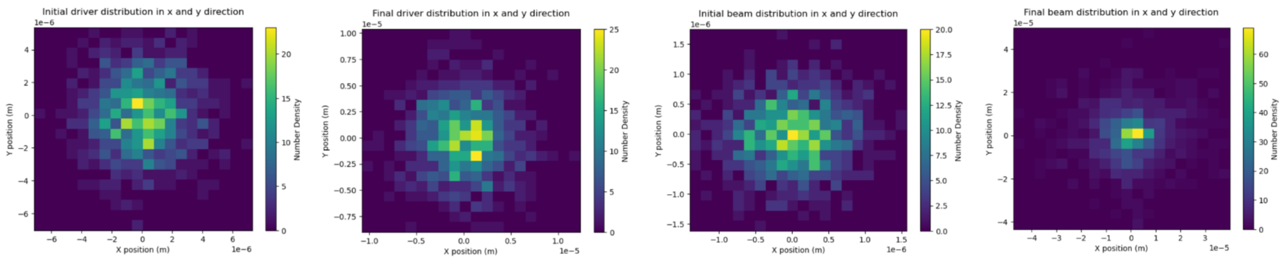

Figure 2: Beam distribution profile for proton driver beam and electron trailing beam.

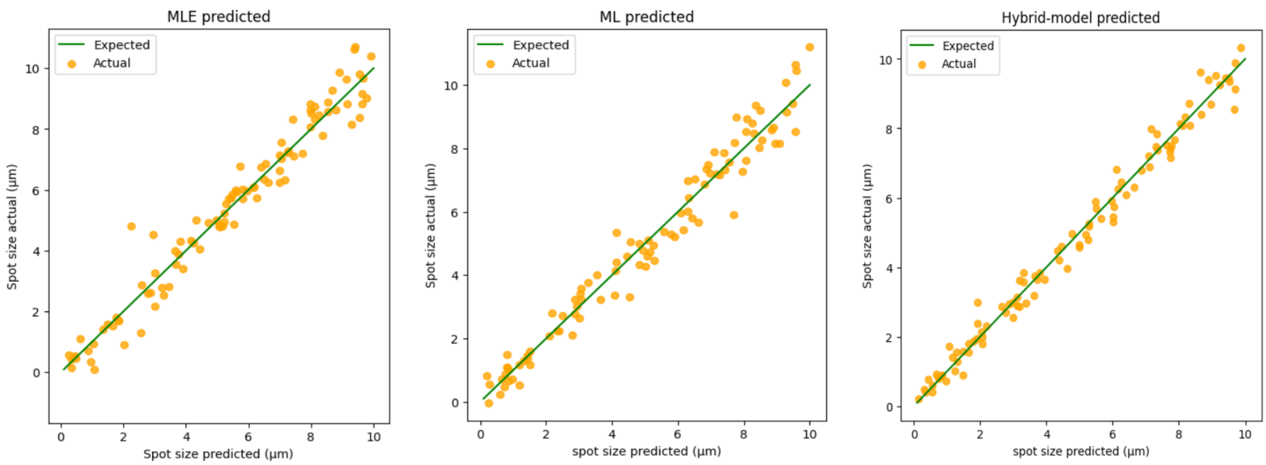

Figure 3: Comparison of spot-size prediction results with different simulation approaches.

Table 1: Simulation Parameters

Parameter	for driver beam	for trailing beam
$\sigma(x, y)$ (μm)	2	0.5 to 10
$\sigma(z)$ (μm)	4	1
Q (C)	1×10^{-9}	-5×10^{-10}
Density (m^{-3})	1×10^{23}	1×10^{22}
Energy (GeV)	1	0.01

witness beam has been identified to be in the X-ray region of the electromagnetic spectrum [10]. Based on various factors (listed in Table 2), a suitable X-ray detector needs to be finalized for procurement in the future.

CONCLUSION

Betatron diagnostics has the potential to be applied for the case of beam-driven PWFA experiments. Factors such as spectral coverage, energy resolution, spatial resolution, sensitivity will play a crucial role in selecting a suitable X-ray detector for the BR detection.

This work attempts to demonstrate the effectiveness of an hybrid model over the individual ML and MLE techniques to use BR as a tool for beam diagnostics. Here although the beam spot size was mainly tested, but the same technique can be applied and will be tested further to check accuracy in identifying other beam parameters.

Table 2: Specs. of the Chosen Detectors

Feature	Scintillator	Gas Detector	Solid-State Detector
Energy resolution	Moderate	Moderate to low	High
Efficiency	High for a wide range of X-rays	varies with the gas types	Decently High
Cost	Moderate	Low	High
Durability	Moderate	Can be fragile	High
Sensitivity	High	Moderate	High
Time resolution	Moderate	Fast	Fast

ACKNOWLEDGEMENTS

This work is supported by the AWAKE-UK phase II project funded by STFC under grant No. ST/X005208/1 and the STFC Cockcroft Institute core grant No. ST/V001612/1. We also thank Monika for the initial discussion related to the project.

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