

Combined Boson Cross Sections

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Abstract

Combining the muon and electron measurements gives $\sigma(Z^0 \rightarrow l^+ l^-) = 0.217 \pm 0.021$ nb. For the W boson we obtain $\sigma(W \rightarrow l\nu) = 2.23 \pm 0.20$ nb. The combined cross-section ratio is $R = \sigma(W \rightarrow l\nu)/\sigma(Z^0 \rightarrow l^+ l^-) = 9.98 \pm 0.74$, from which we obtain the combined branching ratio $\Gamma(W)/\Gamma(W \rightarrow l\nu) = 9.67 \pm 0.73$ and the combined ratio of the W and Z total widths $\Gamma(W)/\Gamma(Z) = 0.88 \pm 0.07$. A limit on the Top quark mass, independent of the assumed decay channel, of $M_t > 48(44)$ GeV/c² at the 90% (95%) confidence level is extracted. Combining the CDF results with the CERN values yields a world average for the inverse branching ratio of $\Gamma(W)/\Gamma(W \rightarrow l\nu) = 9.85 \pm 0.52$. The ratio of the muon and electron branching ratios $\sigma(W \rightarrow \mu\nu)/\sigma(W \rightarrow e\nu) = 1.04 \pm 0.08$ ($g_\mu/g_e = 1.02 \pm 0.04$) and the ratio of the muon and tau branching ratios $\sigma(W \rightarrow \tau\nu)/\sigma(W \rightarrow \mu\nu) = 0.9 \pm 0.13$ ($g_\tau/g_\mu = 0.95 \pm 0.07$) are consistent with unity, as was an earlier measurement, $g_\tau/g_e = 0.97 \pm 0.07$ [3], and confirm lepton universality in this new energy domain.

1 Introduction

Having measured the Z cross sections in the muon and electron decay channels [1,2], and the W cross section in the tau channel as well [3], we combine the results to obtain the 'best' CDF results for the rates of boson production and subsequent decay into leptons. Ratios of the $\sigma \cdot B$ measurements constrain the mass of the Top quark and the relative strengths of the charged lepton coupling constants.

To do this, we follow the method used by the Particle Data Group for combining results with a common systematic error [4]. The data are assumed to be in the form $A_i \pm \sigma_i \pm \Delta$, where Δ is the common systematic error. The weighted average is

$$A = \frac{1}{w} \sum_i w_i A_i \quad (1)$$

where $w_i = \frac{1}{\sigma_i^2}$ and $w = \sum w_i$. The variance is given by

$$\sigma^2 = \frac{1}{w} + \Delta^2. \quad (2)$$

This equation assumes that Δ , the correlated portion of the systematic error, has the same magnitude for the different measurements. It turns out, to a reasonable approximation, that the muon and electron results have the same percentage correlated error. This means that their absolute magnitudes are slightly different. So, we use the absolute value of the uncorrelated errors to get the combined mean and combined uncorrelated error. Then, we multiply the resulting combined mean by the uncorrelated percentage error, to get the combined uncorrelated error.

In this work, we take all variables to be either completely correlated, as, for example, the W acceptance A_W in the muon and electron analyses, or completely independent, for example for the number of observed $W \rightarrow \mu\nu$ and $W \rightarrow e\nu$ events.

In combining the muon results with the electron work, take note that there are two separate electron analyses. The R_e analysis calculates the ratio directly without calculating the cross sections. The luminosity for the no-jet sample was never accurately determined. A no-jet cut reduces their systematics but also changes some of the efficiency, etc., factors. Since the uncertainties are smaller we use the R_e results to find R_l . On the other hand, to get $\sigma(W \rightarrow l\nu)$ and $\sigma(Z^0 \rightarrow l^+l^-)$ we use the $\sigma(W \rightarrow e\nu)$ and $\sigma(Z^0 \rightarrow e^+e^-)$ results, which include jets and an accurate luminosity estimate. In both cases we use the same muon numbers.

2 Combining R and the Cross-Sections

2.1 W Bosons

We start with $\sigma(W \rightarrow l\nu)$. Table 1 lists the contributions to uncertainties in the electron and muon W cross-section calculations.

	Muons		Electrons		Comments
	pb	%	pb	%	
statistics	73.	3.2	44.	2.0	uncorrelated
A_W S.F.	75.	3.3	66.	3.0	correlated (3.3%)
A_W M_W	25.	1.1	17.5	0.8	correlated (1.1%)
A_W $W P_T$	5.0	0.22	22.	1.0	correlated (1.0%)
A_W H.O.	1.4	0.06	55.	2.5	correlated (2.5%)
$\epsilon_{\nu \text{ts}}$	-	-	11.	0.5	uncorrelated
ϵ_ν	19.	0.83	46.	2.1	uncorrelated
lept id eff	103.	4.5	79.	3.6	uncorrelated
QCD BCK	37.	1.6	44.	2.0	correlated (2.0%)
Top BCK	27.	1.2	28.	1.3	correlated (1.3%)
$Z^0 \rightarrow l^+l^-$ BCK	8.2	0.36	14.	0.62	uncorrelated
$Z^0 \rightarrow \tau\tau$	negligible		negligible		uncorrelated
$W \rightarrow \tau\nu$	16.	0.69	9.0	0.41	uncorrelated
Lum	156.	6.8	150.	6.8	correlated

Table 1: Error contributions to the W cross-section calculation.

To apply equation 1, we must first separate the systematic errors into correlated and uncorrelated pieces. We assume the W acceptance is correlated between the two analyses. However, the fractional error assigned to the W acceptance in each analysis is different. Therefore, we have broken this uncertainty down into each of its contributions. Both analyses have similar fractional errors on A_W from the structure functions and the W mass. We take the larger of the two for conservativeness. The muon analysis has significantly smaller errors from assumptions regarding the $W P_T$ spectrum and from the contributions of higher-order diagrams. We think these two uncertainties are actually correlated, and the difference in their magnitude comes from the different methods the two groups used to evaluate the error. The muon error was obtained always evaluating A_W in conjunction with ϵ_ν . For these two uncertainties, A_W is anti-correlated with ϵ_ν . As the $W P_T$ becomes larger, the W 's become more central, thus increasing the geometrical acceptance; however, the neutrino resolution becomes worse, thus decreasing ϵ_ν . The product of $\epsilon_\nu \cdot A_W$ varies less than the individual pieces. This is shown in table 2. The first column is the geometrical acceptance as a function

of P_T of the W , divided by the acceptance for the first entry (the mean acceptance for $0 < P_T < 8.75$ GeV). This gives the percentage change in the acceptance as a function of the P_T of the W . The second column is the same thing, except for the missing E_T efficiency. The third column is the product of the first two columns.¹ The electron analysis, on the other hand, got the uncertainty on A_W by looking only at how the geometrical acceptance changed as assumptions regarding the $W P_T$ spectrum were varied, not how the product of geometrical acceptance and missing E_T efficiency changed. There is no cancellation in this case, leading to larger errors. Since correlated errors only affect the magnitude of the error of the combined measurement, and not the mean value, we take the larger value for now, to be conservative.²

	Geometric	MET	product
0.0-8.75	1.00	1.00	1.0
8.75-17.5	1.00	0.980	0.980
17.5-26.25	1.02	0.945	0.964
26.25-35.0	1.05	0.899	0.944
35.0-43.75	1.08	0.874	0.944
43.75-52.5	1.07	0.863	0.923
52.5-61.25	1.10	0.855	0.940
61.25-70.0	1.19	0.858	1.02

Table 2: Fractional change in geometric acceptance, missing E_T efficiency, and the product, as a function of P_T of the W .

We take the uncertainty due to ϵ_ν to be uncorrelated, since it is dominated by Monte Carlo statistics and the CEM energy scale for the electron case, and by parameters in the underlying event model in the muon case. The electron analysis used data to get the efficiency of requiring $Z_{vtz} < 60$, while the muon analysis assumed the vertex distribution was gaussian, with $\sigma = 30$ cm. Thus, we take this uncertainty to be uncorrelated. The lepton identification efficiencies³ are assumed to be uncorrelated. We assume the QCD background uncertainty is correlated, as both analyses used the same method. We take the larger of the two errors as the uncertainty for conservativeness. We assume the Top background is correlated, as both analyses used similar Monte Carlo. The $Z \rightarrow l^+l^-$ background is assumed to be uncorrelated, because this uncertainty is dominated by Monte Carlo statistics for the $Z^0 \rightarrow e^+e^-$ case, while it is dominated by uncertainties in the structure functions and in the underlying event modeling for the $Z^0 \rightarrow \mu^+\mu^-$ case. The τ backgrounds are also assumed to be uncorrelated, as this error is dominated by Monte Carlo statistics in the muon case, and by the uncertainty in the branching ratio $BR(\tau \rightarrow e\nu_e\nu_\tau)$ and by systematics regarding the τ simulation for the electron case.

The correlated and uncorrelated errors are thus

$$\sigma(W \rightarrow \mu\nu) = 2287 \text{ pb} \pm 5.64\% \text{ (uncorr)} \pm 8.45\% \text{ (corr)}$$

$$\sigma(W \rightarrow e\nu) = 2190 \text{ pb} \pm 4.71\% \text{ (uncorr)} \pm 8.45\% \text{ (corr)}$$

or

$$\sigma(W \rightarrow \mu\nu) = 2287 \pm 129 \text{ (uncorr) pb} \pm 8.45\% \text{ (corr)}$$

$$\sigma(W \rightarrow e\nu) = 2190 \pm 103 \text{ (uncorr) pb} \pm 8.45\% \text{ (corr)}.$$

¹The values in this table are preliminary. They are just intended to show the correlation, and should not be used for any other purpose.

²However, if we choose the muon errors instead, the change in the final error (204 becomes 195) is insignificant.

³Many of these efficiencies have asymmetric errors. For the results in CDF-1349, for ϵ_{CMUO} , we used the smaller of the two errors. We have since decided to use the larger error bar, to be more conservative. This changes the systematic error on $\sigma(W \rightarrow \mu\nu)$ from 120 to 140 pb⁻¹, and the systematic error on $\sigma(Z^0 \rightarrow \mu^+\mu^-)$ from 9 to 10 pb⁻¹, but does not change the error on R .

Applying equation 1 gives

$$\sigma(W \rightarrow l\nu) = 2227 \pm 80 \text{ (uncorr)} \pm 188 \text{ (corr)} \text{ pb.}$$

Combining the statistical errors for the electron and muon cases gives an overall lepton statistical error of $(73^{-2} + 44^{-2})^{-1/2} = 38$. Subtracting this from the total error gives the systematic uncertainty, so

$$\sigma(W \rightarrow l\nu) = 2227 \pm 38 \text{ (stat)} \pm 201 \text{ (sys)} \text{ pb.}$$

It can be argued that the QCD backgrounds should be uncorrelated between the electron and muon sample, because even though the method used to evaluate the background is the same, the sources of the background are very different. If we make this assumption, the combined result becomes 2230 ± 202 . To the number of decimal points we quote in the abstract, this is the same result.

2.2 Z Bosons

Table 3 shows the error contributions from each factor that goes into the Z cross-section calculation, for e's and μ 's.

	Muons		Electrons		Comments
	pb	%	pb	%	
statistics	23.	9.7	13.	6.4	uncorrelated
A_Z	5.0	2.1	4.0	1.9	correlated (2.1%)
$\epsilon_{\nu_{tz}}$	-	-	1.0	0.5	uncorrelated
B_Z	-	-	2.6	1.25	uncorrelated
lept id eff	9.0	3.8	7.8	3.75	uncorrelated
Drell-Yan	-	-	2.1	1	correlated (1%)
Lum	16.	6.8	14.	6.8	correlated

Table 3: Error contributions to the Z cross-section calculation.

We take the Z acceptances to be completely correlated, as they were evaluated using very similar methods. We use the larger of the two fractional errors, to be conservative. As the background for the muon sample is negligible, while the background for the electron sample is about 2 %, we assume they must be uncorrelated. The lepton identification efficiencies are assumed to be uncorrelated. The luminosity is correlated. The muon analysis has not yet evaluated an uncertainty on the Drell-Yan correction. We assume that when they do, the result will be the same as the electron result.

Thus, breaking the muon and electron uncertainties into correlated and independent parts gives

$$\sigma(Z^0 \rightarrow \mu^+ \mu^-) = 238 \text{ pb} \pm 10.4\% \text{ (uncorr)} \pm 7.2\% \text{ (corr)}$$

$$\sigma(Z^0 \rightarrow e^+ e^-) = 209 \text{ pb} \pm 7.54\% \text{ (uncorr)} \pm 7.2\% \text{ (corr)}$$

or

$$\sigma(Z^0 \rightarrow \mu^+ \mu^-) = 238 \pm 24.8 \text{ (uncorr)} \text{ pb} \pm 7.2\% \text{ (corr)}$$

$$\sigma(Z^0 \rightarrow e^+ e^-) = 209 \pm 15.8 \text{ (uncorr)} \text{ pb} \pm 7.2\% \text{ (corr)}.$$

Applying equation 1 gives

$$\sigma(Z^0 \rightarrow l^+ l^-) = 217 \pm 13 \text{ (uncorr)} \pm 16 \text{ (corr)} \text{ pb.}$$

Combining the statistical uncertainties from the muon and electron analyses and subtracting the result (in quadrature) from the total uncertainty gives

$$\sigma(Z^0 \rightarrow l^+ l^-) = 217 \pm 11 \text{ (stat)} \pm 17 \text{ (sys)} \text{ pb.}$$

2.3 R Ratio

The fractional errors for the R analyses are shown in Table 4. The electron values are from the no-jet analysis.

	Muons		Electrons		Comments
	absolute	%	absolute	%	
statistics	1.1	11.5	0.8	7.8	uncorrelated
$\frac{A_Z}{A_W}$ (incl ϵ_ν)	0.31	3.2	0.31	3.0	correlated (3.2%)
B_{WQCD}	0.15	1.6	0.05	0.5	uncorrelated
$B_{WT_{op}}$	0.11	1.2	-	-	uncorrelated
$B_{WZ \rightarrow l^+ l^-}$	0.03	0.3	0.03	0.3	uncorrelated
$B_{WW\tau}$	0.07	0.69	0.035	0.34	uncorrelated
$B_{WZ\tau}$	negligible		negligible		uncorrelated
B_Z	-	-	0.16	1.6	uncorrelated
lept id eff	0.16	1.7	0.29	2.8	uncorrelated

Table 4: Error contributions to the R calculation.

This table is like the previous tables with respect to correlations, except for three entries. The electron analysis had a no-jet cut, while the muon analysis did not. Since most of the QCD background in the muon samples comes from events with jets, while the electron sample has a no-jet cut, we assume that the QCD background in this case is uncorrelated between the two samples. The Top background is assumed to be uncorrelated here, since there is no Top background when a no-jet cut is made.

Finally, much explanation is required regarding the $\frac{A_Z}{A_W}$ entry. For the electrons, this is just the number from the PRL. However, the muon analysis has not yet taken into account the fact that a large part of the structure function systematic cancels in the ratio. We take that into account here. From Table 3 in CDF 1349, we see

$$A_W = .1814 \pm 3.3\% \pm 1.4\%,$$

where the first uncertainty is from the structure functions, and the second uncertainty is from everything else. Likewise,

$$A_{00} + A_{0X} = .15 \pm 0.9\% \pm 1.7\%.$$

In the ratio, if you naively add the errors in quadrature, the error on A_W from the structure functions dominates.

However, Table 5 shows the value of the acceptance for each structure function, and the ratio. The error is taken to be the maximum value minus the minimum value over 2, and the fraction is the error divided by the MRSB value for the acceptance. Thus, for A_W , the fractional error is $(.1867 - .1749)/2/.1861 = 3.2\%$. For A_Z , the fractional error is $(.1499 - .1464)/2/.1499 = 1.1\%$. But, for A_W/A_Z , the fractional error is $(1.195 - 1.251)/2/1.241 = 2.3\%$. So we take the error on the ratio to be 2.3%, added in quadrature with the non-structure function errors on A_W and A_Z , or $\sqrt{2.3^2\% + 1.7^2\% + 1.4^2\%} = 3.2\%$.

Thus, breaking the electron and muon R results into correlated and uncorrelated errors yields

$$R_\mu = 9.6 \pm 12\% \text{ (uncorr.)} \pm 3.2\% \text{ (corr)}$$

$$R_e = 10.2 \pm 8.5\% \text{ (uncorr.)} \pm 3.2\% \text{ (corr)}$$

or

$$R_\mu = 9.6 \pm 1.15 \text{ (uncorr.)} \pm 3.2\% \text{ (corr)}$$

$$R_e = 10.2 \pm 0.87 \text{ (uncorr.)} \pm 3.2\% \text{ (corr).}$$

	A_W	$A_{00} + A_{0X}$	ratio
MRSE	.1828	.1496	1.222
MRSB	.1861	.1499	1.241
D01	.1785	.1478	1.208
D02	.1749	.1464	1.195
EHLQ	.1867	.1493	1.251

Table 5: Structure function dependence of the muon acceptances

Applying equation 1 gives

$$R_l = 9.98 \pm 0.67 \text{ (uncorr)} \pm 0.32 \text{ (corr)}.$$

We combine the statistical parts of the two measurements and find a combined statistical error of 0.65. We separate out the systematic part of the uncertainty, and get

$$R_l = 9.98 \pm 0.65 \text{ (stat)} \pm 0.36 \text{ (sys)}.$$

3 W width and model-independent Top quark mass limit

Table 8 shows the combined cross section and R results.

Figure 1 superimposes our cross-section results on the theoretical predictions [7]. If the Top quark is lighter than the W boson, the decay $W \rightarrow t\bar{b}$ is possible. The decay rate $\Gamma(W \rightarrow \mu\nu)$ depends on the Top mass, according to the phase space available to daughter Top quarks. The expression for R can be written as

$$\frac{\Gamma(W)}{\Gamma(W \rightarrow l\nu)} = \frac{1}{BR(W \rightarrow l\nu)} = \frac{1}{R} \times \frac{\sigma(W)}{\sigma(Z)} \times \frac{\Gamma(Z^\circ)}{\Gamma(Z^\circ \rightarrow l^+l^-)} \quad (3)$$

Theoretical uncertainties largely cancel in the total cross-section ratio, giving

$$\frac{\sigma(p\bar{p} \rightarrow WX)}{\sigma(p\bar{p} \rightarrow ZX)} = 3.23 \pm 0.03$$

at $\sqrt{s} = 1.8$ TeV [5]. The $\sim 1\%$ error is taken from reference [6]. The Z total and partial widths have been measured at LEP, $\Gamma(Z^\circ \rightarrow \mu^+\mu^-) = 83.37 \pm 0.84$ MeV, $\Gamma(Z^\circ \rightarrow e^+e^-) = 83.19 \pm 0.52$ MeV, $\Gamma(Z^\circ \rightarrow l^+l^-) = 83.22 \pm 0.40$ MeV, and $\Gamma(Z^\circ) = 2.487 \pm 0.010$ GeV [8]. Combining, we obtain

$$\frac{\Gamma(W)}{\Gamma(W \rightarrow l\nu)} = 9.67 \pm 0.73$$

which is superimposed on the Top mass dependent theoretical curve in Figure 2. At the 90% (95%) Confidence Level we place a limit on Top quark mass of 48 (44) GeV.

The expression for R can also be re-written

$$\frac{\Gamma(W)}{\Gamma(Z)} = \frac{1}{R} \times \frac{\sigma(W)}{\sigma(Z)} \times \frac{\Gamma(W \rightarrow l\nu)}{\Gamma(Z^\circ \rightarrow l^+l^-)}$$

where $\Gamma(W \rightarrow l\nu)/\Gamma(Z^\circ \rightarrow l^+l^-)$ can be calculated from theory as

$$\frac{\Gamma(W \rightarrow l\nu)}{\Gamma(Z^\circ \rightarrow l^+l^-)} = \left(\frac{M_W}{M_Z}\right)^3 \frac{2}{1 - 4 \sin^2 \theta_W + 8 \sin^4 \theta_W}.$$

Using $M_W/M_Z = 0.8791 \pm 0.0034$ [9] and $\sin^2 \theta_W = 0.2327 \pm 0.00085$ [10], we find $\Gamma(W)/\Gamma(Z) = 2.71 \pm 0.03$, and

$$\frac{\Gamma(W)}{\Gamma(Z)} = 0.88 \pm 0.07.$$

4 Lepton Universality

In this section, we calculate $\sigma(W \rightarrow \mu\nu) / \sigma(W \rightarrow e\nu)$ and $\sigma(W \rightarrow \mu\nu) / \sigma(W \rightarrow \tau\nu)$. From these, we calculate

$$\frac{g_\mu}{g_e} = \sqrt{\frac{\sigma(W \rightarrow \mu\nu)}{\sigma(W \rightarrow e\nu)}} \quad (4)$$

and

$$\frac{g_\tau}{g_\mu} = \sqrt{\frac{\sigma(W \rightarrow \tau\nu)}{\sigma(W \rightarrow \mu\nu)}} \quad (5)$$

at $\sqrt{S}=1.8$ TeV.

In the ratio of the cross-sections, the correlated uncertainties cancel. With these errors removed (see section 2),

$$\sigma(W \rightarrow \mu\nu) = 2287 \pm 129$$

and

$$\sigma(W \rightarrow e\nu) = 2190 \pm 103.$$

Taking the ratio gives

$$\frac{\sigma(W \rightarrow \mu\nu)}{\sigma(W \rightarrow e\nu)} = 1.04 \pm 0.08$$

or

$$\frac{g_\mu}{g_e} = 1.02 \pm 0.04.$$

CDF also has a low-statistics measurement of $\sigma(W \rightarrow \tau\nu)$, summarized in table 7 ⁴. This result was previously combined with the electron result to give

$$\frac{g_\tau}{g_e} = 0.97 \pm 0.07[3].$$

The data sample is the same as for the electron analysis, except for a few runs where the MET trigger was broken, so that the total luminosity is 4.015 pb^{-1} . Since the total uncertainty is large compared to the other studies, little is gained by combining $\sigma(W \rightarrow \tau\nu)$ with the electron and muon results for the total cross-section. Also, several uncertainties which were assumed to be completely correlated between the $\sigma(W \rightarrow \tau\nu)$ and $\sigma(W \rightarrow e\nu)$ analyses, such as the uncertainty in the geometric acceptance, would be necessary to do such a combination but were not calculated. So, instead we use it only to test $\tau - \mu$ universality. Here, we also consider the QCD and Top backgrounds for muons to be uncorrelated with the background for the taus. So,

$$\sigma(W \rightarrow \mu\nu) = 2287 \pm 136 \text{ pb}$$

and, from Table 7,

$$\sigma(W \rightarrow \tau\nu) = 2050 \pm 270 \text{ pb}.$$

Taking the ratio gives

$$\frac{\sigma(W \rightarrow \tau\nu)}{\sigma(W \rightarrow \mu\nu)} = 0.90 \pm 0.13$$

or

$$\frac{g_\tau}{g_\mu} = 0.95 \pm 0.07.$$

⁴Courtesy of A. Roodman, [3].

5 World Averages

The UA1 and UA2 experiments at CERN have also measured the boson production rates [11].⁵ The cross-section ratio depends weakly on \sqrt{s} due to non-linearity in the structure function x-dependence. Hence we choose to combine the inverse branching ratio (see equation 3), where the \sqrt{s} dependence is removed via the factor $\sigma(W)/\sigma(Z)$, from QCD calculations. Either $\Gamma(W)$ or $\Gamma(W)/\Gamma(Z)$ can easily be calculated using the LEP values in section 3 (see [8]), with their small (< 0.5%) errors.

To calculate the world average, we take the error on $\sigma(W)/\sigma(Z)$ and on $\Gamma(Z)/\Gamma(Z^0 \rightarrow l^+l^-)$ to be common to all three experiments. The values quoted in the references are

$$\sigma(W)/\sigma(Z) = 3.23 \pm 0.03 \text{ (CDF)}, \quad 3.23 \pm 0.05 \text{ (UA1)}, \quad 3.116 \pm 0.06 \text{ (UA2)}.$$

But $\frac{\sigma(W)}{\sigma(Z)}$ depends on the choice of $\sin^2\theta_W$ and M_W , and the above 3 values use different choices. For consistency, we use the MRSB4 values listed in reference [5], where $\sin^2\theta_W = 0.23$ and $M_W = 81 \text{ GeV}/c^2$.⁶ Then

$$\sigma(W)/\sigma(Z) = 3.23 \text{ (1.8 TeV)}, \quad 3.16 \text{ (0.63 TeV)}.$$

We take the uncertainty to be 0.03, since $\sin^2\theta_W$ and M_W precision has improved in the last year.

Using the values in section 3, $\Gamma(Z)/\Gamma(Z^0 \rightarrow l^+l^-) = 29.9 \pm 0.2$. Hence we take the common systematic uncertainty on the global average to be 1.2%. Next, we assume that the total errors on R from each of the 3 experiments are completely independent. Given that the measurements are dominated by statistics and that the geometry and efficiencies for the experiments are quite different, this is a good approximation. The values they report are

$$R = 9.5^{+1.1}_{-1.0} \text{ (UA1)}, \quad 9.38^{+0.82}_{-0.72} \pm 0.25 \text{ (UA2)}.$$

We treated the error bars as symmetric, using ± 1.05 for UA1 and ± 0.84 for UA2. The magnitude of the combined uncorrelated error is then 5.1%, and the total uncertainty for the world average is 5.24%. The UA1 uncertainty combines the statistical and systematic terms, but a close reading of the article shows that to a good approximation, it is all statistical. Hence the combined statistical error is 4.7%. Subtracting in quadrature gives a systematic error of 2.3%.

Using equation 3 and the values in the articles, we find

$$\frac{\Gamma(W)}{\Gamma(W \rightarrow l\nu)} = 9.67 \pm 0.73 \text{ (CDF)}, \quad 9.94 \pm 1.1 \text{ (UA1)}, \quad 10.07 \pm 0.89 \text{ (UA2)},$$

where the errors come only from the R measurements (i.e., the uncorrelated part of the uncertainty). Combining the above numbers gives

$$\frac{\Gamma(W)}{\Gamma(W \rightarrow l\nu)} = 9.85 \pm 0.46 \text{ (stat)} \pm 0.23 \text{ (sys)} \quad (\text{CDF} + \text{UA2}).$$

Figure 3 shows the results superimposed on the standard model prediction as a function of Top quark mass. At the 90% (95%) Confidence Level a Top quark with $M_{top} < 51$ (48) GeV/c^2 is excluded. This is the same world combined limit as found by UA1 using only the CDF electron results: the improved error bar from adding the muon results is offset by the slightly higher muon value. Table 9 summarizes the results.

Acknowledgements

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⁵UA2 results with more statistics will be presented at the upcoming Lepton-Photon conference.

⁶The subscript "4" refers to the Q_0^2 evolution. $M_W = 81$ was a reasonable value at the time the reference was written.

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Muon term	Value	$\delta x/x$	Electron term	Value	$\delta x/x$	ρ
$N_{W\mu}^{obs}$	1431 ± 38	2.6%	N_{We}^{obs}	2664 ± 52	1.9%	0
-	-	-	N_{W1}^{obs}	2308 ± 48	-	-
$N_{Z\mu}^{obs}$	108 ± 10.4	9.6%	N_{Ze}^{obs}	243 ± 15.6	6.4%	0
-	-	-	N_{Z1}^{obs}	201 ± 14	-	-
A_W^μ	$18.14 \pm 0.66\%$	3.6%	A_W^e	$35.2 \pm 1.5\%$	1.9%	1
A_{00}	$4.57 \pm 0.17\%$	3.7%	A_Z^e	$37.1 \pm 0.7\%$	1.9%	1
A_{0X}	$10.42 \pm 0.25\%$	2.4%	F_{ee}	0.40	-	-
			F_{cp}	0.47	-	-
			F_{cf}	0.13	-	-
$A_{W\tau}$	$0.73 \pm 0.13\%$	18%	$B(W \rightarrow \tau\nu)$	90 ± 10	11%	1
A_{ZW}	$19.98 \pm 0.49\%$	2.5%	$B(Z^0 \rightarrow e^+e^-)$	40 ± 15	37.5%	1
$A_{Z\tau}$	$0.424 \pm 0.13\%$	31%	$B(Z^0 \rightarrow \tau\tau)$	8 ± 4	50%	1
CF	$0.4 \pm 0.2\%$	50%	-	-	-	-
B_Q^μ	30 ± 20	67%	B_Q^e	100 ± 50	50%	1
B_{top}^μ	0_{-0}^{+15}	-	B_{top}^e	0_{-0}^{+31}	-	1
B_{ZQ}^μ	0_{-0}^{+1}	-	B_{ZQ}^e	5 ± 3	60%	1
B_{Wjet}^μ	0_{-0}^{+1}	-	B_{Wjet}^e	0	-	0
T	$91 \pm 2\%$	2.2%	-	-	-	-
ϵ_{cos}	99.7 ± 0.2	0.2%	-	-	-	-
ϵ_{CMUO}	$98.6_{-3.3}^{+1.2}\%$	3.3%	-	-	-	-
$\epsilon_1 = \epsilon_{da} \cdot \epsilon_{iso}$	$94.1 \pm 1.4\%$	1%	-	-	-	-
$\epsilon_2 = \epsilon_{mi} \cdot \epsilon_{trk}$	$97.4 \pm 1.0\%$	1%	-	-	-	-
DY	1.0128	-	K_{DY}	1.01 ± 0.01	1%	0
L_μ	3.54 ± 0.24	6.8%	L_e	4.05 ± 0.28	6.8%	1
-	-	-	c_1	$84 \pm 3\%$	3.6%	-
-	-	-	c_2	$93 \pm 3\%$	3.2%	-
-	-	-	p	$91 \pm 3\%$	3.3%	-
-	-	-	f	$91 \pm 4\%$	4.4%	-
-	-	-	ϵ_ν	$96 \pm 2\%$	2.1%	-
-	-	-	ϵ_{VTX}	$95.9 \pm 0.5\%$	0.5%	-
<i>Values from electron no-jet analysis</i>						
N'_W				$1727 \pm 43 \pm 12$	2.6%	
N'_Z				$187 \pm 14 \pm 3$	7.6%	
ϵ'_Z/ϵ'_W				1.04 ± 0.03	2.9%	
A'_Z/A'_W				1.065 ± 0.031	2.9%	

Table 6: Numbers used to calculate the boson cross sections.

	Met25	TauMet
Number of Events		
$W \rightarrow \tau\nu$ Data sample	207	77
QCD background	$63 \pm 3 \pm 8$	$26 \pm 2 \pm 4$
$Z \rightarrow \tau\tau$ background	7 ± 2	4 ± 1
$W \rightarrow e\nu$ background	5 ± 1	-
$N_{W \rightarrow \tau\nu}$	$132 \pm 14 \pm 8$	$47 \pm 9 \pm 4$
$A \times \epsilon(W \rightarrow \tau\nu)$		
$A(W \rightarrow \tau\nu)$.396	
$\tau \rightarrow \text{hadrons B.R.}$.639	
$\epsilon(W \rightarrow \tau\nu)$.0618	.0659
Correction $W \rightarrow e\nu$ fid. cuts	.99	
Correction $W \rightarrow e\nu$ bckgrd	1.04	
$A \times \epsilon(W \rightarrow \tau\nu)$.0161	.0172
Systematic Errors on $A \times \epsilon$		
M.C. statistics	± 0.005	± 0.005
Ntower, ΣP_T cuts	-	± 0.008
B.R. (correlated)	± 0.007	± 0.004
E-scale (correlated)	± 0.006	± 0.012
$\int \mathcal{L}$		
Integrated Luminosity	4.015 pb^{-1}	1.315 pb^{-1}
$\sigma \cdot B$		
$\sigma \cdot B(W \rightarrow \tau\nu)$ combined	$2.05 \pm 0.27 \text{ nb}$	
$\sigma \cdot B(W \rightarrow e\nu)$	$2.19 \pm 0.04 \text{ (stat)} \pm 0.11 \text{ (syst.) nb}$	
g_τ/g_e		
g_τ/g_e		0.97 ± 0.07

Table 7: Summary of the $W \rightarrow \tau\nu$ analysis

	Muons	Electrons	Combined
			Statistical, systematic, and luminosity errors.
$\sigma(W) \cdot B$ (nb)	$2.29 \pm 0.07 \pm 0.14 \pm 0.16$	$2.19 \pm 0.04 \pm 0.14 \pm 0.15$	$2.23 \pm 0.04 \pm 0.12 \pm 0.15$
$\sigma(Z) \cdot B$ (nb)	$0.238 \pm 0.023 \pm 0.010 \pm 0.016$	$0.209 \pm 0.013 \pm 0.009 \pm 0.014$	$0.217 \pm 0.011 \pm 0.008 \pm 0.015$
R	$9.6 \pm 1.1 \pm 0.5$	$10.2 \pm 0.8 \pm 0.4$	9.98 ± 0.74
$B.R.(W)^{-1}$	10.0 ± 1.2	9.47 ± 0.86	9.67 ± 0.73
M_{top} , 90% C.L.	$> 27 \text{ GeV}/c^2$	$> 48 \text{ GeV}/c^2$	$> 48 \text{ GeV}/c^2$
M_{top} , 95% C.L.	$> 12 \text{ GeV}/c^2$	$> 43 \text{ GeV}/c^2$	$> 44 \text{ GeV}/c^2$
$\frac{\Gamma(W)}{\Gamma(Z)}$			0.88 ± 0.07
g_μ/g_e			1.02 ± 0.03

Table 8: Muon, electron, and combined results.

	$\Gamma(W)/\Gamma(W \rightarrow l\nu)$	M_{top} (95% CL)	M_{top} (90% CL)
UA1	9.9 ± 1.1	-	$> 38 \text{ GeV}/c^2$
UA2	10.1 ± 0.9	-	-
CDF	9.67 ± 0.73	$> 44 \text{ GeV}/c^2$	$> 48 \text{ GeV}/c^2$
Combined	9.85 ± 0.52	$> 48 \text{ GeV}/c^2$	$> 51 \text{ GeV}/c^2$

Table 9: World averages combining CDF and UA_x results.

W CROSS SECTION, THEORY AND EXPERIMENT

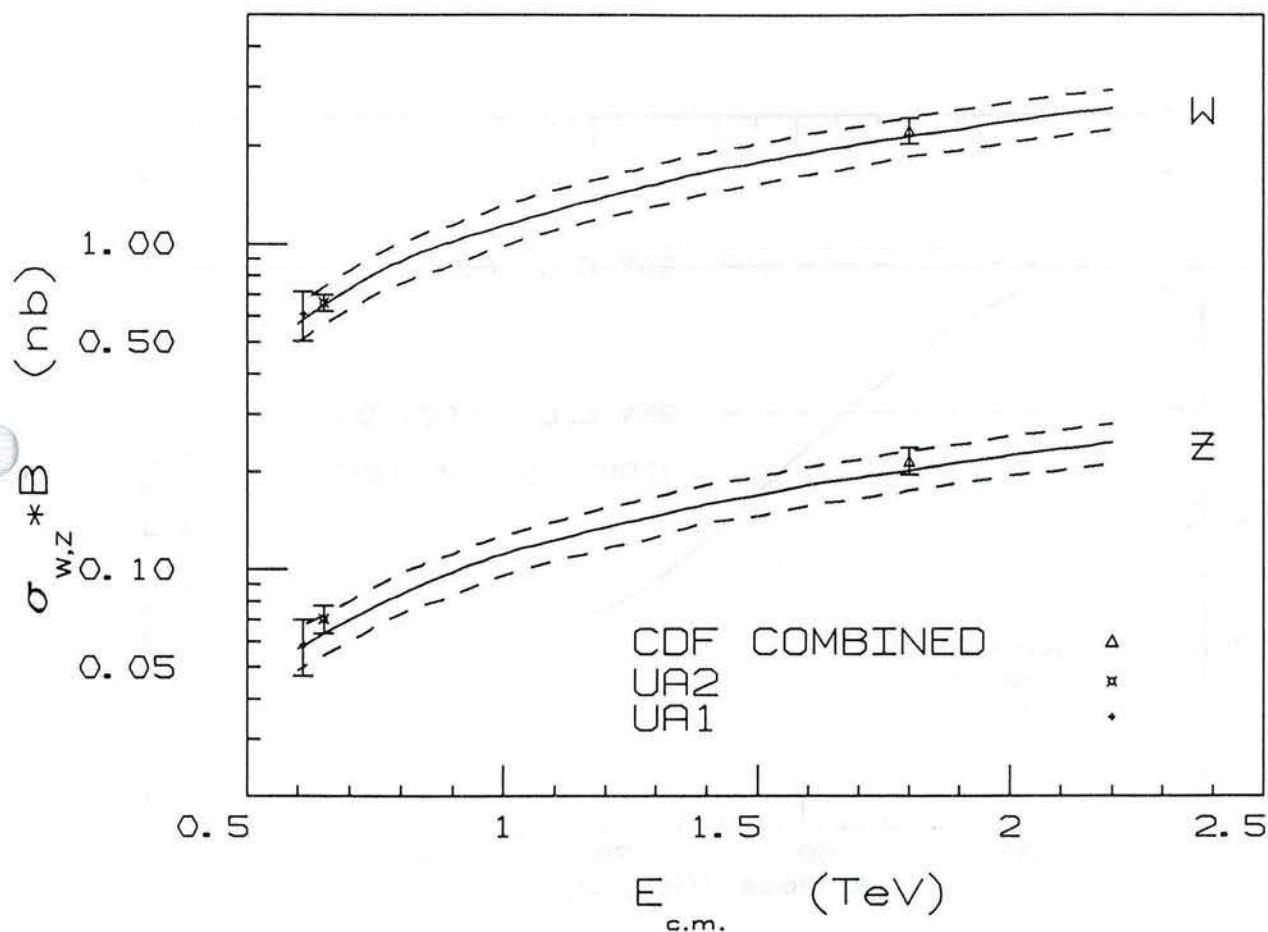


Figure 1: Partial cross-section versus c.m. energy. Curve is from Altarelli-Parisi, the dashed curves outline the 1σ band.

TOP QUARK MASS LIMIT FROM W BRANCHING RATIO

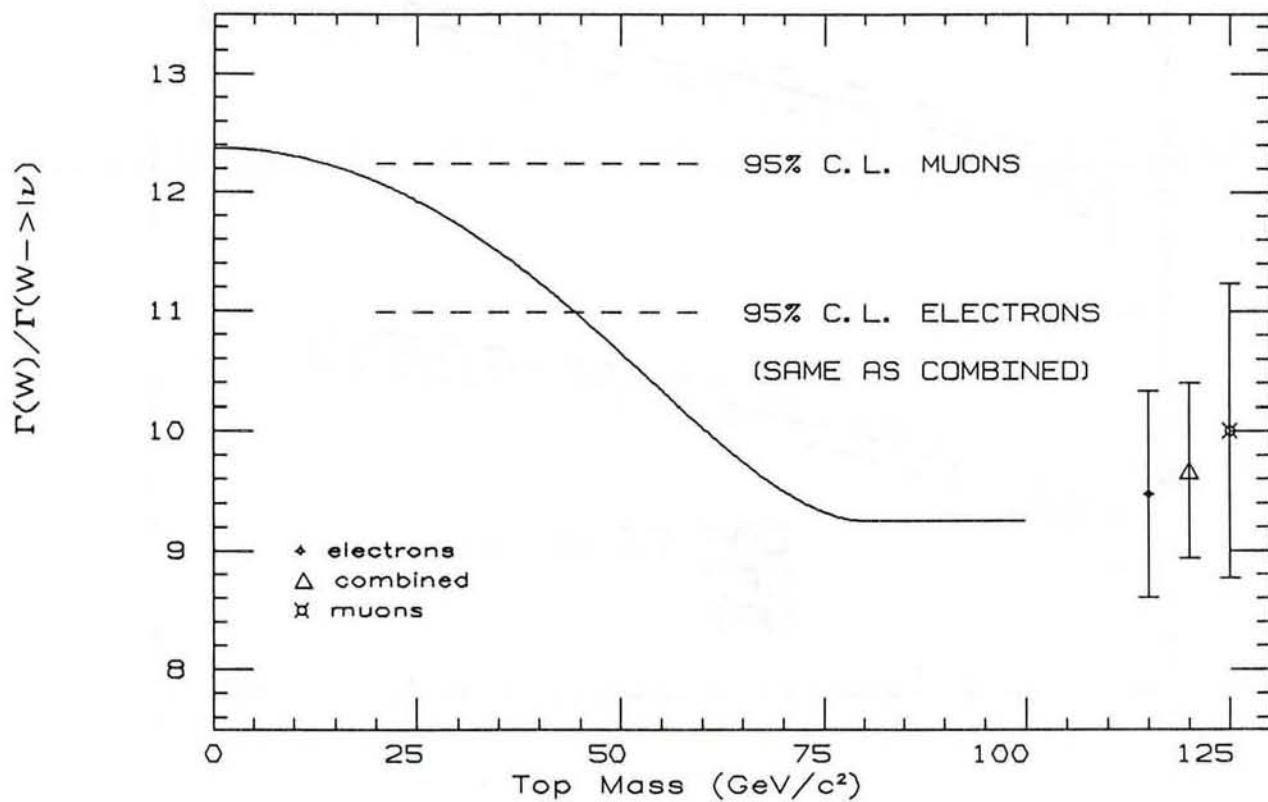


Figure 2: Theoretical prediction for the W Branching Ratio to leptons as a function of Top mass, along with the muon, electron, and combined measurement.

COMBINED 1/BR RESULTS

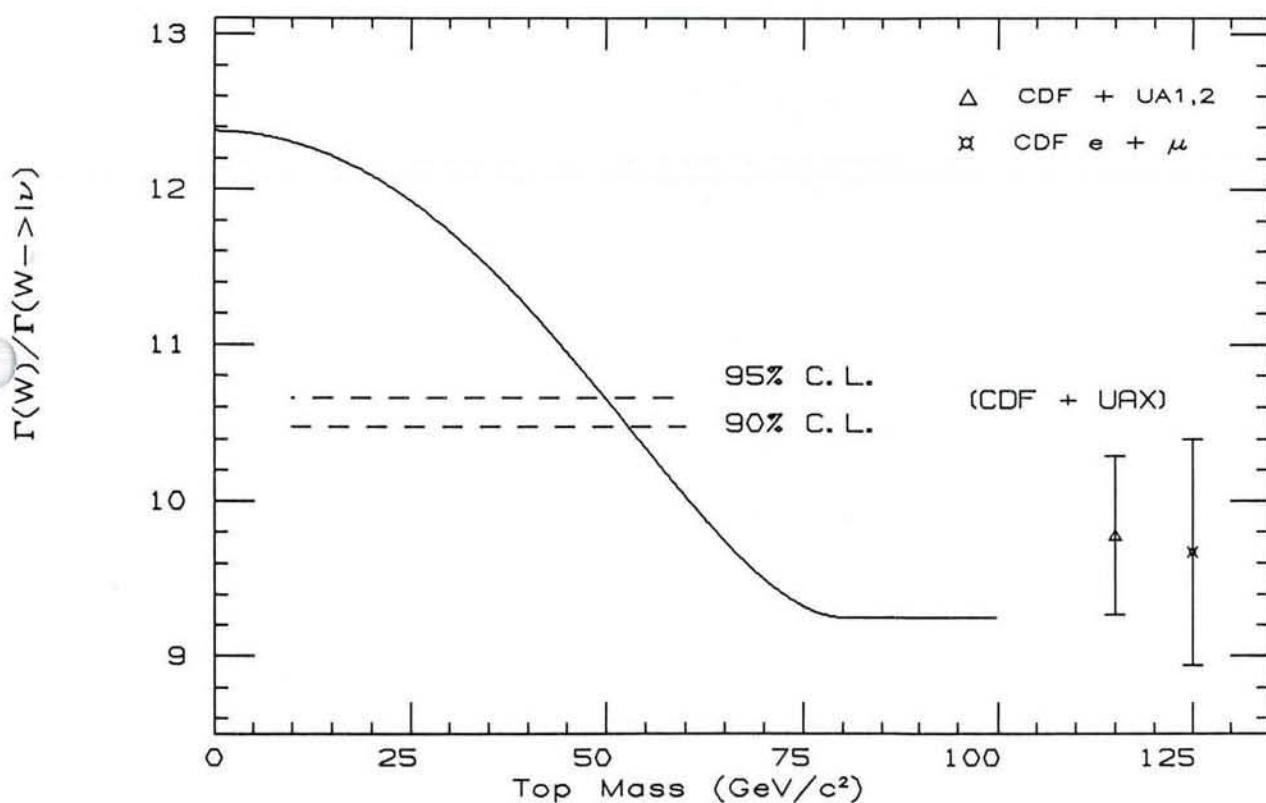


Figure 3: Theoretical prediction for the W Branching Ratio to leptons as a function of Top mass, along with the combined CDF and UA1,2 measurement.