Comparative study of Dirac and Majorana ultrahigh-energy neutrino oscillations in an interstellar magnetic field

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Abstract. We examine the ultrahigh-energy neutrino propagation in interstellar space in the Dirac and Majorana cases. Employing the two-neutrino mixing approximation and using the astrophysical constraints on neutrino magnetic moments, we show that both the flavor and the spin oscillations of the Dirac and Majorana neutrinos exhibit qualitatively different behaviors in an interstellar magnetic field for neutrino-energy values characteristic of, respectively, the cosmogenic neutrinos and well above the Greisen-Zatsepin-Kuz'min cutoff.

1. Introduction

The ultrahigh-energy (UHE) cosmic neutrinos (even above PeV–EeV energies) are believed to be produced by reactions of UHE cosmic rays composed of protons and nuclei and are expected to provide information about cosmic accelerators and the high-energy, distant universe. From the neutrino massiveness it follows that neutrinos should have nonzero electromagnetic characteristics [1]: even though neutrinos are generally believed to be electrically neutral particles they can still have nonzero magnetic moments. This means that the propagation of the UHE cosmic neutrinos can be influenced by the presence of magnetic fields due to the effect of spin oscillations [2]. In this work we focus on the difference between the Dirac and Majorana neutrino propagation in an interstellar magnetic field. Specifically, we examine the differences in the flavor and spin oscillation patterns of the Dirac and Majorana neutrinos.

2. The Dirac neutrino case

We limit ourselves to the case of two Dirac neutrino physical states, ν_1 and ν_2, with masses m_1 and m_2. For treating neutrino evolution in the presence of a uniform magnetic field $\mathbf{B}$ and homogeneous matter, we employ a four-component basis of the helicity states $\nu_{1,s=\pm1}$ and

$\nu_{2,s=\pm1}$. The Schrödinger-like evolution equation is then given by:

$$i\frac{d}{dt}\begin{pmatrix}\nu_{1,s=1}\\ \nu_{1,s=-1}\\ \nu_{2,s=1}\\ \nu_{2,s=-1}\end{pmatrix} = H_{eff}\begin{pmatrix}\nu_{1,s=1}\\ \nu_{1,s=-1}\\ \nu_{2,s=1}\\ \nu_{2,s=-1}\end{pmatrix}, \qquad H_{eff} = H_{vac} + H_{mat} + H_B. \tag{1}$$

The effective Hamiltonian H_{eff} consists of the vacuum part H_{vac} and the interaction part H_B, corresponding to the neutrino interaction with a magnetic field (since the matter density in the interstellar space is $n_e \lesssim 10^6$ cm^{-3}, we neglect the neutrino-matter interaction part H_{mat}). In the mass representation, the vacuum Hamiltonian and the Hamiltonian of the neutrino interaction with a magnetic field acquire the forms:

$$H^m_{vac} = \omega\begin{pmatrix}-1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}, \qquad H^m_B = \begin{pmatrix}-\mu_{11}\frac{B_\parallel}{\gamma_{11}} & \mu_{11}B_\perp & -\mu_{12}\frac{B_\parallel}{\gamma_{12}} & \mu_{12}B_\perp\\ \mu_{11}B_\perp & \mu_{11}\frac{B_\parallel}{\gamma_{11}} & \mu_{12}B_\perp & \mu_{12}\frac{B_\parallel}{\gamma_{12}}\\ -\mu_{12}\frac{B_\parallel}{\gamma_{12}} & \mu_{12}B_\perp & -\mu_{22}\frac{B_\parallel}{\gamma_{22}} & \mu_{22}B_\perp\\ \mu_{12}B_\perp & \mu_{12}\frac{B_\parallel}{\gamma_{12}} & \mu_{22}B_\perp & \mu_{22}\frac{B_\parallel}{\gamma_{22}}\end{pmatrix}, \tag{2}$$

where $\omega = \frac{\Delta m^2}{4E_\nu}$, $\Delta m^2 = m_2^2 - m_1^2$, E_ν is the neutrino energy, $B_\parallel$ and $B_\perp$ are the parallel and transverse magnetic-field components with respect to the neutrino velocity, and μ_{jk} $(j, k = 1, 2)$ are magnetic moments in the mass representation. Here γ_1 and γ_2 are the Lorentz factors of the massive neutrinos, and $\frac{1}{\gamma_{12}} = \frac{1}{2}\left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right)$. The evolution equation (1) in the masss basis leads to a homogeneous system of first-order linear differential equations, which is equivalent to a fourth-order homogeneous linear differential equation and can be solved analytically [2].

We neglect the neutrino interaction with the longitudinal magnetic-field component, setting $(\mu/\gamma)B_\parallel = 0$. The latter is justified by large γ values for UHE neutrinos. Also in this study, the transition magnetic moments of Dirac neutrinos are zeroed, i.e. $\mu_{12} = \mu_{21} = 0$. If the initial neutrino state is $\nu^f(0) = \nu^L_\mu$, the flavor-change probability $P^D_{\nu^L_\mu\to\nu^L_e}$ for the Dirac neutrino propagating in vacuum is determined by the following formula:

$$P^D_{\nu^L_\mu\to\nu^L_e} = \sin^2 2\theta \sin^2\left(\frac{\pi x}{L_{vac}}\right)\cos^2\left(\frac{\pi x}{L_B}\right), \tag{3}$$

where x is the neutrino propagation distance, $L_{vac} = \frac{4\pi E_\nu}{\Delta m^2}$ and $L_B = \frac{\pi}{\mu_\nu B}$ are vacuum and magnetic oscillation lengths respectively. In the case of unequal diagonal magnetic moments $\mu_{11} \neq \mu_{22}$ the probability takes the form:

$$P^D_{\nu^L_\mu\to\nu^L_e} = \frac{1}{4}\sin^2 2\theta\left[\cos^2\left(\frac{\pi x}{L_{B_1}}\right) + \cos^2\left(\frac{\pi x}{L_{B_2}}\right) - 2\cos\left(\frac{\pi x}{L_{B_1}}\right)\cos\left(\frac{\pi x}{L_{B_2}}\right)\cos\left(\frac{2\pi x}{L_{vac}}\right)\right], \tag{4}$$

where L_{B_1} and L_{B_2} are the magnetic oscillation lengths for neutrinos with different magnetic moments μ_{11} and μ_{22}. For spin oscillations, the transition probability $P_{\nu^L\to\nu^R}$ in the case of equal magnetic moments is determined by $P^D_{\nu^L\to\nu^R} = \sin^2\left(\frac{\pi x}{L_B}\right)$. In the case of different magnetic moments $\mu_{11} \neq \mu_{22}$ one has:

$$\begin{aligned}P^D_{\nu^L\to\nu^R} &= \sin^2\left(\frac{\pi x}{L_{vac}}\right)\left[\cos^2\theta\sin\left(\frac{\pi x}{L_{B_1}}\right) - \sin^2\theta\sin\left(\frac{\pi x}{L_{B_2}}\right)\right]^2\\ &+ \cos^2\left(\frac{\pi x}{L_{vac}}\right)\left[\sin^2\theta\sin\left(\frac{\pi x}{L_{B_2}}\right) + \cos^2\theta\sin\left(\frac{\pi x}{L_{B_1}}\right)\right]^2 + \frac{1}{4}\sin^2 2\theta\\ &\times\left[\sin^2\left(\frac{\pi x}{L_{B_1}}\right) + \sin^2\left(\frac{\pi x}{L_{B_2}}\right) - 2\sin\left(\frac{\pi x}{L_{B_1}}\right)\sin\left(\frac{\pi x}{L_{B_2}}\right)\cos\left(\frac{2\pi x}{L_{vac}}\right)\right].\end{aligned} \tag{5}$$

3. The Majorana neutrino case

When considering Majorana neutrinos, the main difference in comparison to Dirac neutrinos consists in the general properties of their magnetic moments μ_{ij}^M. The matrix of magnetic moments for Majorana neutrinos is antisymmetric and hermitian, so that in the discussed case of two neutrino mixing, the diagonal magnetic moments vanish $\mu_{11}^M = \mu_{22}^M = 0$, and the transition magnetic moments are opposite in sign and are purely imaginary: $\mu_{12}^M = -\mu_{21}^M = -(\mu_{12}^M)^*$. Since the magnetic moments take purely imaginary values, they can be parameterized using a putative magnetic moment μ_ν as follows: $\mu_{12}^M = \pm i\mu_\nu$. The probabilities of flavor and spin oscillations for Majorana neutrinos are then given by:

$$P^M_{\nu_\mu^L \to \nu_e^L} = \left(\frac{\tilde{L}}{L_{vac}}\right)^2 \sin^2 2\theta \sin^2\left(\frac{\pi x}{\tilde{L}}\right), \qquad P^M_{\nu^L \to \nu^R} = \left(\frac{\tilde{L}}{L_B}\right)^2 \sin^2\left(\frac{\pi x}{\tilde{L}}\right), \tag{6}$$

where $\tilde{L} = \frac{L_{vac}L_B}{\sqrt{L_{vac}^2 + L_B^2}}$. In contrast to the Dirac case, both probabilities oscillate with the same period.

4. Results and discussion

We present the numerical results for the probabilities of flavor and spin oscillations of Dirac and Majorana neutrinos at two different energies: $E_\nu = 10^{18}$ eV and $E_\nu = 10^{22}$ eV. The magnetic field strength is set to the value $B = 2.93\,\mu$G [3]. The putative magnetic moment value is chosen to be $\mu_\nu = 2.2 \times 10^{-12}\mu_B$ that corresponds to the upper astrophysical limit obtained from observations of red giants CL (90 %) [4]. The squared mass difference is taken from solar neutrino measurements, $\Delta m^2 = \Delta m^2_{sol} = 7.53 \times 10^{-5}$ eV [5]. All numerical calculations were performed in the case when the initial state of the neutrino is $\nu^f(0) = \nu_\mu^L$.

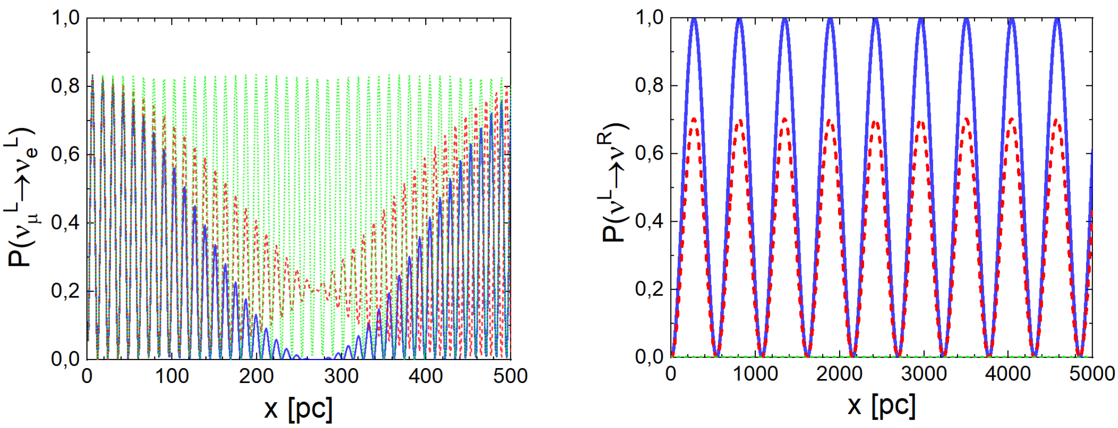

Figure 1. The neutrino flavor-change (left panel) and spin-flip (right panel) probabilities as functions of the distance x travelled by an 1-EeV neutrino interacting with an interstellar magnetic field. The Dirac neutrino case is represented by the blue solid ($\mu_{11} = \mu_{22} = 2.2 \times 10^{-12}\mu_B$) and red dashed ($\mu_{11} = 2.2 \times 10^{-12}\mu_B$, $\mu_{22} = 0$) curves, while the green dots represent the Majorana neutrino case $|\mu_{12}^M| = 2.2 \times 10^{-12}\mu_B$.

The behavior of flavor-change probability for Dirac neutrinos is governed by the L_B to L_{vac} relation. If the neutrino energy is $E_\nu = 10^{18}$ eV (see fig. 1), for Dirac neutrinos the length of the vacuum oscillations is much less than the length of the magnetic oscillations $L_B \gg L_{vac}$, and

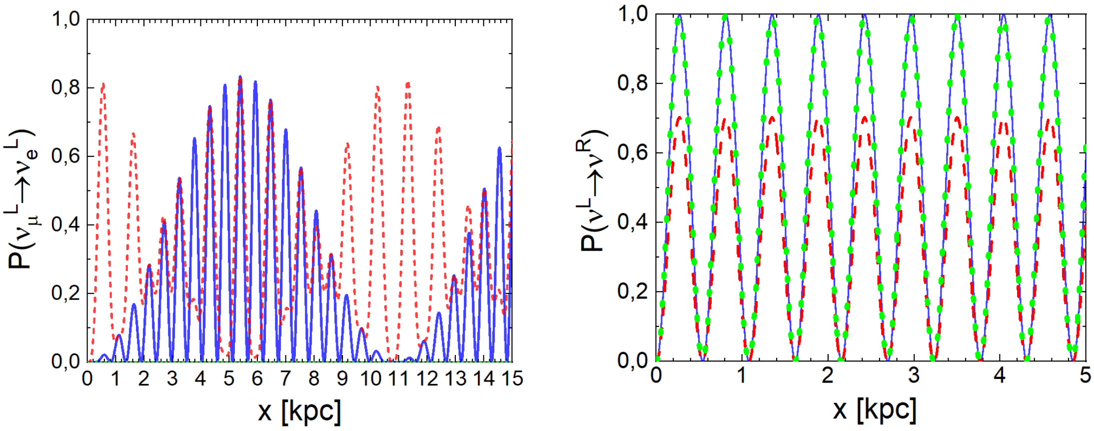

Figure 2. The same as in fig. 1 but for $E_\nu = 10\,\mathrm{ZeV}$.

therefore the probability exhibits fast oscillations with the period L_{vac} which are modulated by a slowly changing envelope curve that depends on the μ_ν value. The spin-flip probability for Dirac neutrinos is affected only by the value of L_B.

For Majorana neutrinos both flavor-change and spin-flip probabilities shown in figs. 1 and 2 oscillate with the same period $\tilde{L}$, but are proportional to the $\frac{\tilde{L}}{L_{vac}}$ and $\frac{\tilde{L}}{L_B}$ ratios respectively. Thus, for neutrino energies of the order of $E_\nu \sim 1$ EeV ($E_\nu \sim 10$ ZeV), the probability of spin (flavor) oscillations practically vanishes.

5. Summary

We have considered the effective Schrödinger equation for both Dirac and Majorana neutrinos with a nonzero magnetic moment propagating in an interstellar magnetic field. The probabilities of neutrino spin and flavor oscillations have been derived. The numerical results for these probabilities have been presented at neutrino energies below and above the Greisen-Zatsepin-Kuzmin limit ($E_\nu = 10^{18}$ eV and $E_\nu = 10^{22}$ eV respectively). The marked differences between the Dirac and Majorana cases have been outlined. In particular, it has been shown that in contrast to the Dirac neutrino case the spin-flip (flavor-change) probability for Majorana neutrinos is practically zero when $L_B \gg L_{vac}$ ($L_{vac} \gg L_B$).

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