Halo-formation and Equilibrium in High Intensity Hadron Rings : A Role of Nonlinear Parametric Resonances Excited by Intrinsic Beam-Core Oscillations

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Abstract

Halo formation under non-equilibrium state for a 2D Gaussian beam in a FODO lattice was examined. Nonlinear resonant-interactions between individual particles and intrinsic beam-core oscillations result in beam halo. Location of halo is analytically tractable using canonical equations derived from an isolated resonance Hamiltonian. Halo formation and achievement to equilibrium can be explained by transition of time-varying nonlinear resonances.

1 INTRODUCTION

One of major issues in high-power accelerators is activation of the environment surrounding accelerator due to beam loss. The beam loss must be reduced to a sufficiently low level to allow hands-on-maintenance. In order to produce an acceptable design, it is important to understand the mechanisms of emittance growth and halo formation that result in beam loss.

From this point of view, halo formation has been studied by simulation and theoretical analysis. Especially, particle-in-cell (PIC) simulation codes [1] and analysis using particle-core-models (PCM) [2] have greatly facilitated the understanding of space-charge effects for particle beams. In these studies, a resonant interaction between the individual particles and intrinsic beam-core oscillations is found as the driving mechanism of halo formation. However, the analysis using PCM has been made on an equilibrium state, where rms emittance is constant. Beam-property such as rms beam size in non-equilibrium seems to be different with that in equilibrium and to take a key role in the resonant interaction of injected beam. In non-equilibrium, PCM can not be adopted because rms emiitance grows. Furthermore, simulation analyses, such as FFT analysis and Poincaré map analysis, are not able to apply in non-equilibrium because these analyses need to track more than 100 turns but the non-equilibrium state finishes generally less than 50 turns.

The purpose of this paper is to examine halo formation for beam of a 2D Gaussian distribution under non-equilibrium in a circular accelerator. In this context, we have been developing a useful analytic model, which is based on Isolated Resonance Hamiltonian (IRH), capable of predicting the position of the halo as a function of the beam and machine parameters in even non-equilibrium. The theory has shown that halo formation and achievement to an equilibrium state can be explained by time-varying nonlinear resonances.

In this paper, the calculations were carried out for 2-D mismatched beams with Gaussian distribution in a typical FODO lattice. Most of the calculation parameters were taken from the 12GeV proton synchrotron in High Energy Accelerator Research Organization (KEK-PS) because of high tune shifts in contrast to low beam intensity, where the injection energy is 500MeV and C_0 = 340m is the circumference. In order to manifest the key role of the space-charge effects in halo formation, the acceleration was not taken account of and the momentum spread was assumed to be 0%. As the momentum spread is concerned, application of the developed analytic tool is straightforward for on-momentum. Furthermore, the bare tunes (ν_x, ν_y) were chosen from the operational parameter as (7.123,5.229) and (7.250,5.229). In the case of (7.123,5.229), a structure resonance in horizontal direction was shown by past simulation results, but any resonance was not in the case of (7.250,5.229) [3].

2 FORMALISM OF ISOLATED RESONANCE HAMILTONIAN FOR GAUSSIAN BEAM

Space charge potential originating from a beam with Gaussian distribution is written in the form of a Taylor expansion as

$$\varphi(x,y;s) = \frac{N}{4\pi\varepsilon_0}\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\sum_{r=0}^{n}\binom{n}{r} f_{n,r}(s)\, x^{2(n-r)} y^{2r} \quad (1)$$

$$f_{n,r}(s) = \int_0^{\infty}\frac{dt}{\left\{t+2\sigma_x(s)^2\right\}^{n-r+1/2}\left\{t+2\sigma_y(s)^2\right\}^{r+1/2}}$$

where N is the total number of particles per unit length, σ_x and σ_y are the rms beam size. The Hamiltonian equivalent to the betatron oscillation perturbed by the space charge effect is given as

$$H(x,y,p_x,p_y;s) = H_0(x,y,p_x,p_y;s) + \frac{e}{\gamma^2 p v}\varphi(x,y;s)$$

where H_0 is the unperturbed Hamiltonian, γ, p and v is the relativistic mass factor, the momentum and the velocity of the design particle, respectively. Here, action-angle variables (ψ_x, ψ_y, I_x, I_y) and a dependent variable $\theta = s / R_0$ are introduced [4], where

$$x = \sqrt{2\beta_x I_x}\cos\left(\psi_x + \psi_{0,x}\right),$$

$$y = \sqrt{2\beta_y I_y}\cos\left(\psi_y + \psi_{0,y}\right),$$

$R_0 = C_0 / 2\pi$ is the averaged orbit radius, β_x and β_y are Twiss parameter, $\psi_{0,x}$ and $\psi_{0,y}$ are the flutter of the phase with respect to the average phase advance of the unperturbed Betatron oscillation.

$\varphi(\psi_x, \psi_y, I_x, I_y; \theta)$ expanded by Fourier series can be separated into the oscillating terms with angle variable and the other oscillating term because of the flutter, rms beam size and Twiss parameter. The nonlinear resonances between the individual particles and intrinsic beam-core oscillations are excited in the case that the phase of φ slowly varies with θ. Because the past simulation results showed the nonlinear resonances in x direction [3], we chose the smallest slowly oscillating phase of φ as $2\delta\psi_x - \kappa\theta$, where δ and κ are integer. So, the slowly oscillating phase can be given by $i(2\delta\psi_x - \kappa\theta)$, where i is integer. The IRH can be given by averaging the Hamiltonian over many turns [5] because the rapidly oscillating terms disappear. Furthermore, a canonical transformation to ($\Psi_x = \psi_x - \kappa\theta / 2\delta$, ψ_y, I_x, I_y ; θ) is made to remove any time-dependence from the IRH. Finally, the IRH for nonlinear resonance between the Betatron oscillation and the oscillating space charge forces of 2D Gaussian beam is written as

$$H_{iso}\left(\Psi_x, I_x, I_y\right) = \left(\nu_x - \frac{\kappa}{2\delta}\right)I_x + \frac{eR_0}{\gamma^2 pv}\left\langle \varphi\left(\Psi_x, I_x, I_y\right)\right\rangle, \quad (2)$$

where $\langle\varphi(\Psi_x, I_x, I_y)\rangle$ is the time-averaged space charge potential. H_{iso} and I_y of Eq. (2) become constant of motion. The detail of Eq. (2) is given in Ref. [6].

In this paper, the position of the resonance islands was chosen as the measure of the relative strength of nonlinear resonances. The position of the resonance island for a structure resonance is given by $I_{x\max}$ and $I_{x\min}$, which are the maximum and minimum values of the action variable along the trajectory through the unstable fixed point. The stable and the unstable fixed point can be analytically evaluated from the canonical equations.

For numerical evaluation of Eq. (2), $n_{\max}$, which is the limitation of the summation about n of Eq. (1), and I_y were optimized by calculating $I_{x\max}$ and $I_{x\min}$ as functions of $n_{\max}$ and I_y, when a combination of (7.123,5.229) was chosen as the bare tune and the peak intensity of 8.5e11 particle per bunch (ppb) beam was assumed. $n_{\max} = 20$ that gives a saturation in the calculation result has been applied. A larger I_y indicated the smaller resonance islands because the depressed tune becomes closer to the bare tune. In order to manifest the key role of the space-charge effects in halo formation, the case of $I_y = 0$ has been considered. The result of the numerical calculation of H_{iso} using above parameters is shown in Fig. 1 for comparison with the simulation result in the case of (7.123, 5.22).

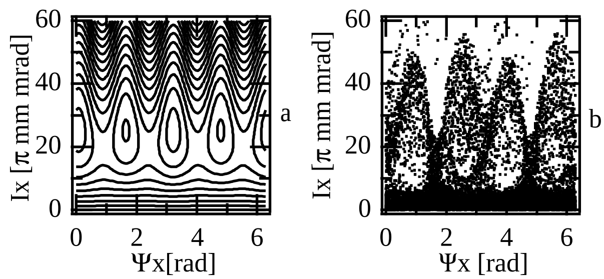

Fig. 1 The comparison of phase space map between (a) H_{iso} and (b) simulation. (10th turn)

In order to justify H_{iso}, the results of H_{iso} are compared with the simulation results by changing the intensity and measuring $I_{x\max}$ and $I_{x\min}$ in the case of (7.123, 5.229). As shown in Fig. 2, the results of H_{iso} are good agreement with the simulation results. Thus, H_{iso} of Eq. (2) has been confirmed to give a reasonable result.

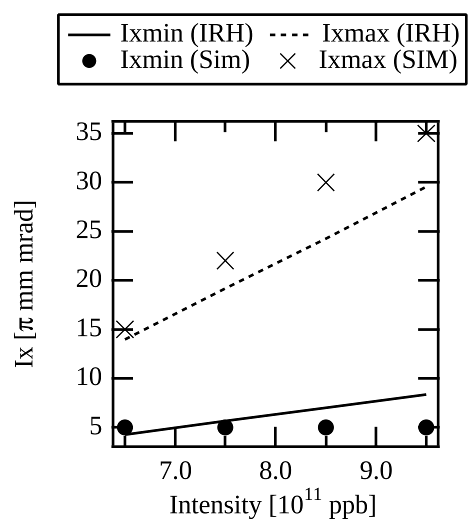

Fig. 2 Intensity dependence of the position of the resonance island. (10th turn)

3 NONLINEAR RESONANCE ANALYSIS FOR HALO FORMATION

In order to understand halo formation under a non-equilibrium state, the time varying H_{iso} for the Gaussian beam with (7.123,5.229) and (7.250,5.229) was examined.

The phase space structures in the case of (7.123,5.229) are shown in Fig. 3. The resonance caused by mismatching, where 2 resonance islands were made, was dominant at a few turns because the mismatching strongly remained. Furthermore, the nonlinear resonance was switched to the structure resonance, where the depressed tune in the horizontal plane is 7 and the beam core oscillates 28 times per 1 turn because of the lattice consisting of 28 cells at KEK-PS, after decaying of the mismatching due to the growth of the filamentation. It is understood that the particles moving to outer edge of the resonance island become halo. Thus the halo tends to grow in the tune pair of (7.123,5.229).

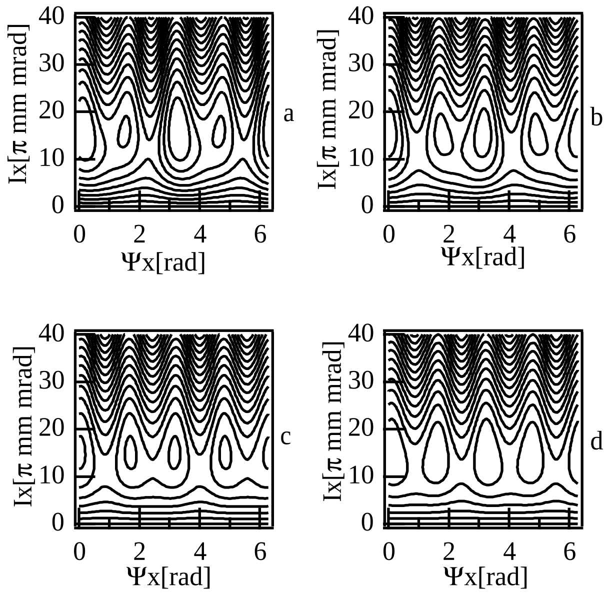

Fig. 3 Time varying of H_{iso}. (7.123, 5.229)
(a) 1st turn, (b) 3rd turn, (c) 5th turn and (d) 7th turn

The phase space structures in the case of (7.123,5.229) are shown in Fig. 4. The resonance caused by mismatching was dominant similar to the case of (7.123,5.229). However, the condition of the structure resonance was not satisfied because the depressed tune is far from 7, so the nonlinear resonance was lost after decaying of the mismatching. The particles moving to the resonance island caused by the mismatching are thought to be smeared out due to the nonlinear space charge fields. Therefore, the beam distribution seems to achieve an equilibrium state.

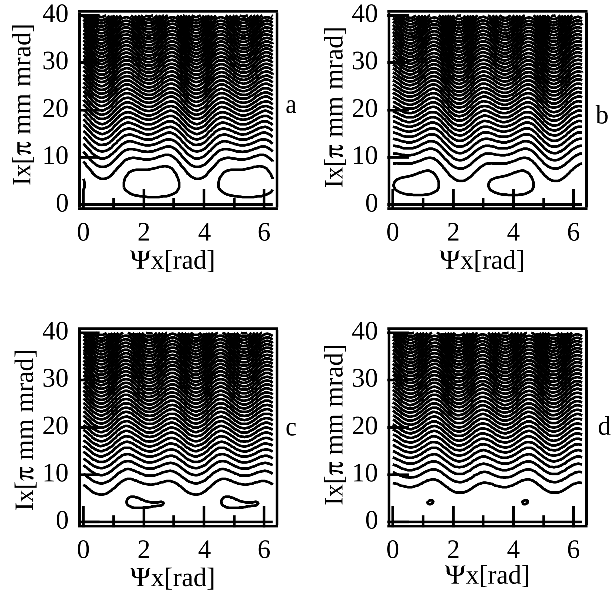

Fig. 4 Time varying of H_{iso}. (7.203, 5.229)
(a) 1st turn, (b) 3rd turn, (c) 5th turn and (d) 7th turn

4 CONCLUSION

The isolated resonance Hamiltonian theory which is capable of treating the intrinsic beam core oscillation has been established. The isolated resonance Hamiltonian is proved to be a useful tool to estimate the position and size of halo. It has been concluded that halo is driven by nonlinear resonances excited by the intrinsic beam core oscillation at the non-equilibrium state. The beam distribution seems to achieve the equilibrium state through the decay process of the nonlinear resonances.

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