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Review

Christoph Greub's impact and legacy

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Abstract As Christoph Greub embarks on a well-deserved retirement, we take this opportunity to celebrate his remarkable career and lasting contributions. Over the years, Christoph has been a pillar of excellence, known for his unwavering commitment and expertise in B physics. This tribute reflects on his professional journey, the impact he has had on colleagues, young collaborators and the legacy he leaves behind. With deep appreciation, we honour Christoph's accomplishments and wish him the best in this exciting new chapter of life.

1 Preface

Since the establishment of the Standard Model of Particle Physics in the 1970s, the High-Energy Physics community embarked on a new quest: to understand its apparent deficiencies by testing its implications conceptually, theoretically and experimentally to higher and finer levels of precision. This new field was called "Physics Beyond the Standard Model", or *BSM Physics* for short.

Many avenues branched from here, each of them pushed forward by brave physicists, leaders of their own quest, creators of their own branch. One such avenue was the study of BSM physics via flavour-changing transitions in particle physics experiments. And one of the leaders of this branch was Christoph Greub.

Christoph Greub was born on October 21st, 1959 in the city of Bern. He studied physics at the University of Bern, after which he wrote an "Einleitender" on the semileptonic top quark decay, in December 1984, and a master thesis ("Lizenziatsarbeit") on the QCD corrections to W production in $p\bar{p}$ scattering in July 1984. From 1985 to 1989, Christoph did his doctoral studies under the supervision of Prof. Peter Minkowski on lepton spectra in W-production at the Sp$\bar{\text{p}}$S collider. After his PhD, he was a postdoctoral researcher at DESY (1989–1991), the University of Zürich (1991–1994), and SLAC (1994–1996). Christoph then returned to the University of Bern in 1997 where he did his habilitation in 1999 and became a professor in the year 2000.

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During his career as postdoc, and later as a professor in Bern, Christoph contributed very important results on the physics of flavour transitions and their implications for BSM physics.[1] He also trained numerous PhD students and postdoctoral researchers, and transferred his knowledge, his swift tricks and his particular philosophy of doing science to those lucky enough to collaborate with him.

In order to honour Christoph on his 65th birthday and to celebrate his long and influential career as a physics professor and researcher, a one day workshop was organised at the University of Bern on October 25th, 2024, with the title[2]

"Christophest: Precision Predictions for FCNC Processes"

The participants of this workshop were long-term collaborators of Christoph, established researchers that were postdocs or PhD students of Christoph at some stage in their careers, current or recent students, and his colleagues at the physics department. A common feature in all of them was their personal and professional respect for Christoph.

The workshop featured talks on the issues of matching and running in EFTs (Jason Aebischer), status on the theory calculations of $b \to s\gamma$ (Mikolaj Misiak), non-local effects in $b \to s\ell\ell$ (Javier Virto), QCD corrections to $B - \bar{B}$ mixing (Matthias Steinhauser), progress on the calculation of the inclusive $B \to X_s \ell\ell$ rate (Tobias Hurth), and QCD corrections to semileptonic and non-leptonic B decays (Matteo Fael). There was plenty of discussion on the various topics and a very inspiring atmosphere.

In preparation for the workshop, a document was prepared with a collection of contributions from some of Christoph's collaborators, on the topics of his lifetime research. This document was printed, dedicated to him and signed by the participants, and handed over to Christoph as a gift after the workshop.

This is the document that we present here, after some corrections and the addition of this preface and of Sects. 11 and 12. We believe the content presented here may be of interest to both the flavour physics community and the wider High-Energy Physics community, and not only as document with historical value but also as a snapshot of the physics issues and techniques in the field as of 2025.

Christoph has been a mentor, a model, and an inspiration to all of us, not only professionally but also at the personal level. Throughout the following pages, the contributors have expressed this view in their own words. We all carry within us many lessons learned from him, and thus Christoph has made us better; better researchers, better mentors, better people. As a community, we feel we owe him much, and we sincerely wish him great success and fortune for the future (Figs. 1, 2).

J. Aebischer, T. Becher, M. Fael, J. Virto,
(the editors),
March 2025.

2 Radiative B decays: the drosophila of particle physics

Authors: F. Borzumati and D. Wyler
Milano and Zürich

2.1 Prologue

.... Christoph Greub is one of the great masters of these decays (A. Buras)

There are various ways to do physics and many different types of physicists who do it. Christoph Greub, who turns 65 this year, is a very special one. Everyone who has the privilege to know him would agree to this. Rarely have we seen a less prejudiced, less conceited physicist and at the same time a more independent, sincere, tolerant and cooperative colleague. On top of this all, his strength and determination to overcome the most difficult problems are remarkable and not easy to match. We hope to convey some of these thoughts in the next pages.

The discovery of asymptotic freedom in the mid-1970s opened the possibility to carry out reliable calculations at energy scales below M_W. This allowed to turn flavour physics, in particular B physics, into a laboratory for testing the Standard Model (SM) with high precision and for searching beyond it.

While progress has been fast in the beginning and comparatively easy (from today's viewpoint), the last 20 years were and are characterised by hard calculational work to arrive at ever higher precision needed to match the parallel improvements in experiment.

Among the various B processes, the radiative B decays have proved to be particularly interesting for both objectives, testing the SM and paving the way to glean new physics. We therefore often called them the drosophila of particle physics.

[1]See https://inspirehep.net/authors/1007583 for Christoph's inspire record.
[2]This is a clickable link. See https://indico.global/event/9558/.

Fig. 1 Master and Ph.D. theses written by Christoph as a student (1984–1989), and his habilitation thesis of 1998

Fig. 2 Above: Christoph at the *Christophest* workshop. Below: Participants of the workshop. First row, left to right: Alexander Lenz, Javier Virto, Gilberto Colangelo, Mikolaj Misiak, Christoph Greub, Martin Hoferichter, Urs Wenger, Simon Holz, Gogniat Joël. Middle row, left to right: Juerg Gasser, Francesco Saturnino, Uwe-Jens Wiese, Tobias Hurth. Back row, left to right: Michael Gerber, Dario Müller, Julian Eicher, Hrachya H. Asatryan, Daniel Wyler, Lukas Born, Kay Bieri, Jason Aebischer, Matteo Fael, Matthias Steinhauser, Enrico Lunghi, Maximilian Zillinger, Gabriele Levati, Ahmed Ali. The person taking the picture and not appearing in it is Thomas Becher

With this analogue in mind, Christoph got hooked to studying B processes, in particular $\overline{B} \to X_s\gamma$, corresponding to $b \to s\gamma$ at the quark level, to which he returned again and again, always with the goal of providing a new missing part of the most precise calculation possible. It took the dogged determination of people like Christoph to perform these calculations, in which he is deeply engaged even today, when the physics community is preparing to fill the last holes. As we know him for over 30 years, we have no doubt that he will eventually close these holes (we hope he will be the first one to do it correctly).

To appreciate his (and other's) work, we take a quick look at the situation in flavour physics 50 years ago. After the discovery of asymptotic freedom [1, 2], it was realised that perturbative QCD would be the tool of choice to calculate the effects of the strong interactions on the decays of the heavier mesons and baryons. Before the b quark was discovered, these were applied to Kaon physics. (See pioneering work in Refs. [3, 4].)

The methodology for treating strong interactions in flavour physics [5–7] was picked up by Frederick Gilman and Mark Wise [8]. In parallel, the importance of asymptotic freedom for flavour physics had also been recognised by the then flourishing soviet school of theoretical physics that produced early on several important papers [9, 10] and in particular Ref. [11]. Furthermore, this school developed many mathematical tools that would be of importance in the following. See for example Refs. [12, 13] and later also [14–16].

At the same time, the potential of radiative decays of mesons (and baryons) was beginning to be recognised in Refs. [17–20], precursors in many ways to the activities of Christoph and others. These papers (some of them at least) showed that QCD corrections could alter both, the strength and the form of the weak interactions. In the language of effective Hamiltonians, they showed how a contribution to the $\mathcal{O}_7$ operator could come from the operator $\mathcal{O}_2$ once QCD interactions were switched on (see Sect. 1.2). Since the SM was somehow a "young model", some of the above papers had been written in the hope that strong interactions would be able to magnify the effect of hypothetical right currents to be added to the SM [21–23].

Even if several years had elapsed since the discovery of the b quark, practically none of these investigations were applied to the physics of the B meson, which somehow seemed to be at a scale too high for QCD effects to be of any significance. They were centred on the physics of the K and Σ hadrons.

From the mid-1970s to the mid-1980s, however, the SM started becoming more and more what appears to be even today: a granitic theory with some unknown input parameters. In the expectation of upcoming experimental results from ARGUS and CLEO, one of the main issue seemed to be how well the SM could predict the branching ratios of heavy-meson decays, including now also the B meson.

It is in this atmosphere of expectation that, at the end of 1986, before the ARGUS results on the B-$\overline{B}$ mixing [24] (which hinted at a rather heavy t-quark mass), two interesting papers appeared [25, 26]. Based on the calculation of Ref. [11], they highlighted the fact that not only QCD corrections could be very relevant at the B scale, but that the radiative decay $\overline{B} \to X_s\gamma$ was actually a tale of two interactions, with the QCD corrections possibly even providing the bulk of the branching ratio, depending on the value of the t-quark mass.

This observation and the prospect of B meson factories helped moving B decays into a focus of interest in the particle physics community: radiative decays seemed to offer a rare window for detecting new degrees of freedom beyond the Standard Model (BSM) as well as an interesting playground to test QCD.

On the theory side, the methodology for their calculation was addressed once more in Ref. [27, 28] and the need for precision in such calculation was advocated. Calculating these decay rates grew into a field in itself. It became important to scrutinise all aspects of the field theoretic calculations, regularisation schemes, gamma matrices algebra, loop calculations, etc. [29]. To be able to reduce spurious scale uncertainties it appeared mandatory to have QCD corrections performed up to the highest possible order. There were also more phenomenological issues to be considered in order to identify how the calculated branching ratio could be measured in the new and upcoming machines, such as ARGUS, CLEO, LEPII, Belle and Babar, as well as the SSC/LHC.

A new field of research had opened up, still vigorous today and developing to very high levels of sophistication. Christoph contributed to all facets of it. To appreciate the importance of his papers (true milestones in this field), we provide in Sect. 1.2 a minimal technical vocabulary of terms and notions needed for our discussion. In Sect. 1.3, the evolution of the calculation of the $b \to s\gamma$ rate and of Christoph's journey is traced. Then, in Sect. 1.4, we put in context and discuss Christoph's contribution towards an evaluation of $b \to s\gamma$ as precise as possible even in Supersymmetry (SUSY). We conclude in Sect. 1.5.

2.2 Effective Hamiltonian and $b \to s\gamma$

Charmless inclusive $b \to s$ decays, such as $\overline{B} \to X_s\gamma$, where X_s denotes any state with strangeness +1, are particularly well suited to yield new information on the SM and beyond. This is because they can be well approximated by the quark level transitions $b \to x_s\gamma$ (with x_s different from X_s as it includes, for example, also Bremsstrahlung photons needed for consistency). These transitions can be accurately calculated by perturbative QCD within the effective-Hamiltonian formalism.

The SM contains a handful of "heavy" particles ($W^\pm$ and Z bosons, the t quark and the Higgs boson). Everything else, the other quarks, all leptons as well as photon and gluons are very light or massless and their physics, including that of the b quark, is dealt with the formalism of effective Hamiltonians. These are made of effective operators of light particles, while the details and impact of the heavy ones is relegated into the form and strength of the operators.

At the tree level, only current–current four-fermion operators can be generated by the SM, whereas at loop levels, the number of effective operators increases substantially. For processes involving b quarks, the effective

Hamiltonian is

$$\mathcal{H}_{eff}^{W} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^{*} \sum_i C_i(\mu)\mathcal{O}_i(\mu)\,, \tag{1}$$

where G_F is the Fermi constant, V_{tb} and V_{ts} are elements of the Cabibbo–Kobayashi–Maskawa matrix, μ the "scale" at which this Hamiltonian is applied ($\mu \simeq m_b$), and $C_i(\mu)$ the Wilson coefficients.

For the present discussion, we only need the current–current operator $\mathcal{O}_2$ and the magnetic, chromomagnetic ones $\mathcal{O}_7$, $\mathcal{O}_8$:

$$\begin{aligned}\mathcal{O}_2 =&(\bar{s}\gamma_\mu P_L c)\,(\bar{c}\gamma^\mu P_L b),\\ \mathcal{O}_7 =&\frac{e}{16\pi^2}\,\overline{m}_b(\mu)\,(\bar{s}\sigma^{\mu\nu}P_R b)\,F_{\mu\nu},\\ \mathcal{O}_8 =&\frac{g_S}{16\pi^2}\,\overline{m}_b(\mu)\,(\bar{s}\sigma^{\mu\nu}P_R b)\,G_{\mu\nu},\end{aligned} \tag{2}$$

with e, g_S the electromagnetic, strong couplings, and $\overline{m}_b(\mu)$ the running mass of the b quark field in the modified minimal subtraction scheme ($\overline{\mathrm{MS}}$). The complete list of operators can be found, for example, in Ref. [30]. The operators in Eq. (2), as those in $\mathcal{H}_{eff}^{W}$, have all dimension *six*.

The quark-level amplitude for $b \to s\gamma$ can be written as[3]

$$\mathcal{A}(b \to s\gamma) = -\frac{4G_F}{\sqrt{2}}(V_{tb}V_{ts}^{*})\sum_i C_i(\mu)\,\langle s\gamma|\mathcal{O}_i(\mu)|b\rangle_{\mathrm{tree}}\,. \tag{3}$$

In the SM, QCD corrections are due to the exchange of gluons. These bring in powers of the large logarithm $L = \log(m_b^2/m_W^2)$ (weighted by factors of the strong coupling α_S). Since L is a large number, these corrections need to be resummed. Leading-log-order (LO) corrections contain the powers of $\alpha_S L$, $(\alpha_S L)^n$, while next-to-leading logs (NLO) (next-to-next-to-leading logs (NNLO)) ones contain the powers $\alpha_S(\alpha_S L)^n$ ($\alpha_S^2(\alpha_S L)^n$). Adding more and more orders of corrections should progressively reduce the scale dependence of $\mathcal{A}(b \to s\gamma)$ in Eq. (3).

The procedure for a consistent SM calculation at orders LO (NLO, NNLO) precision requires then three steps:

1) a matching calculation of the full SM with the effective theory at the scale $\mu = M_W$ to order α_S^0 (α_S^1, α_S^2) for the Wilson coefficients;
2) a renormalisation group treatment of the Wilson coefficients using the anomalous dimension matrix to order α_S^1 (α_S^2, α_S^3);
3) a calculation of the operator matrix elements at the scale $\mu = m_b$ to order α_S^0 (α_S^1, α_S^2).

Depending on the process, not all of these contributions are present. Thus, in some cases, the effort to reach a certain accuracy level is reduced.

In the following, we shall describe the progress made for the decay $b \to s\gamma$, from the early 1990s to today, from LO to NNLO (soon to be reached). Such a big calculation requires many people and indeed many have contributed apart from [JA] besides? Christoph. In the words of Andrzej Buras [29], while discussing radiative and semileptonic B decays:

"In particular, Christoph Greub is one of the great masters of these decays and his group made important contributions here both in the SM and beyond it.".

Maybe a typical Burasian exaggeration, but certainly not undeserved!

2.3 The path to the NNLO corrections to $b \to s\gamma$ and Christoph

After his PhD thesis with Peter Minkowski on lepton spectra from W production, Christoph, in 1989, went to DESY. At the time, the potential of B physics had been made clear. It had also become possible to produce B mesons in large quantities to allow precision measurements and this made the case for more extensive theoretical work. Encouraged by Ahmed Ali, Christoph turned to B-meson decays. His early work in B physics, often together with Ali, was largely devoted to phenomenological studies, that were important for understanding the experimental results. These are highly cited and are described in Ali's contribution.

We believe, though, that the real calling came to Christoph at SLAC, around 1995. It was then and there that he started to look into the detailed calculations for the quark transition $b \to s\gamma$, laid out in Refs. [8, 11, 27, 28],

[3] The extension from s to x_s is usually included at the end of the calculation.

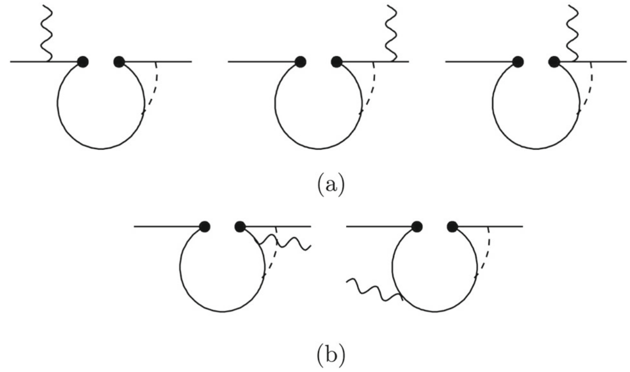

Fig. 3 Typical two-loop diagrams contributing to the mixing of the operator $\mathcal{O}_2$ into $\mathcal{O}_7$. Wavy lines denote photons, solid lines the quarks b, s and c, and dashed lines the gluons g. From Ref. [33]

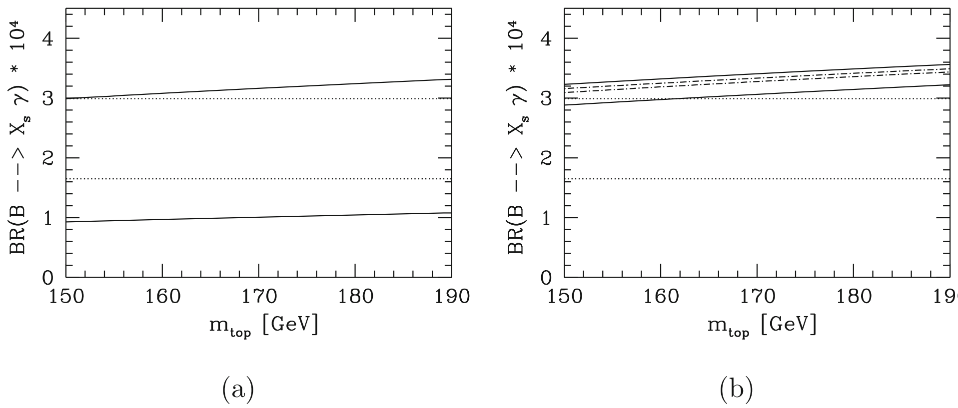

Fig. 4 Reduction of the scale dependence in the branching ratio. The horizontal band is the experimental range in 1996. The solid band in figure **a** shows the LO result for a variation of the scale μ between $m_b/2$ and $2m_b$ and in figure **b** after the NLO calculation. From Ref. [33], where also the dashed–dotted line is explained

where the importance of the effective-Hamiltonian method was highlighted. By then, only the LO corrections were known, and some first work on the NLO had just begun. In 1995, the inclusive branching ratio for $\overline{B} \to X_s\gamma$ was measured to be $2.32 \pm 0.67 \times 10^{-4}$ by CLEO [31] at Cornell.[4] The expected increased experimental precision made it clear that NLO calculations were necessary.

Having in mind the systematic treatment outlined in the previous section, Christoph realised that several elements of the NLO calculation were missing, in particular the order α_S matrix element of the current–current operator O_2. See Fig. 3. The two-loop calculation with their complicated denominator's structure and the finite mass of the charm quark in the loop presented a real calculational challenge. But Christoph was determined to solve it.

At the time, 1995, Christoph and one of us (Daniel), both at SLAC, were looking for ways to calculate these diagrams. With its generous hospitality, lush gardens and sunny weather, SLAC was a wonderful place to think about real problems. Discussions were held mostly outside, with Christoph disappearing from time to time into the library to search for ideas that would bring us further. At one point he came out proclaiming: "I have it!". He had found some work by soviet physicists who had themselves discovered that massive integrals could be replaced by a series of massless integrals by means of Mellin–Barnes transformations (MBt).

Early applications of the MBt transformation to Feynman integrals were pioneered, as mentioned already, by Natalia I. Usyukina (already in 1975), Eduard E. Boos, Andrei I. Davydychev in the early 1990s and Vladimir A. Smirnov [12–16]. Although well known to many people, the application of the MBt seemed to have been overlooked in the type of calculations we were interested in. To the best of our knowledge, Christoph was the first to use it in the context of B-meson decays. Since then, this has become a standard technique, although the newest works, also by Christoph, use different methods.

The result of his work, published in 1996 [33], was a crucial ingredient in the program for the NLO calculations, reducing drastically the scale dependence and laying the ground for the next level of precision (Fig. 4). It took however still sometime to have all NLO corrections assembled, which happened in 2002 [34].[5]

This work established Christoph as one of the prominent people in the field of radiative B decays. It has indeed become Christoph's trademark to pursue a difficult calculation when he recognises its importance in moving the

[4] The current experimental average is $(3.49 \pm 0.19) \times 10^{-4}$ [32].

[5] As noted in Ref. [29], additional (small) contributions need to be added to strictly justify the label "all NLO".

field forward. In this sense, many of his papers provided valuable, often central inputs that others could rely on. He is most happy when he can provide such a puzzle piece.

While the NLO calculations were being completed, and the experimental results were becoming more precise, people started to work on the next level, the NNLO. In fact, the much quoted updates of 2007 and 2015 [35, 36] contain already some NNLO corrections.

Christoph continued to make valuable contributions towards the complete NNLO calculation. Together with Patrick Liniger, he calculated the NLO contribution to the decay $b \to sg$ [37]. Afterwards, with Matthias Steinhauser and Kay Bieri, he computed a class of NNLO contributions in Ref. [38].

From then on, many NNLO calculations followed, often in collaboration with Hrachia M. Asatrian (and/or Hrachia's son, Hayk, as well as grandson, Hrachya), Artyom Hovhannisyan, Tobias Hurth, Vahagan Poghosyan, and Hayk Gabrielyan. In Refs. [39, 40]), the dependence on the quark mass renormalisation scheme could be reduced. Some of the contributions at the NNLO to the dipole operator were provided in Ref. [41], while the completion of the order α_S^2 correction to the magnetic and chromomagnetic operators $\mathcal{O}_7$ and $\mathcal{O}_8$ was presented in Ref. [42]. This paper was written in collaboration with Hrachia Asatrian, Thorsten Ewerth, Andrea Ferroglia and Giovanni Ossola. Furthermore, in a series of papers, together with Hrachia Asatrian, Hayk H. Asatryan (Hrachia's son)), Hovhannisyan, Bieri, and Matthew Walker, the decays $b \to s\ell^+\ell^-$ and $b \to d\ell^+\ell^-$ were calculated to order α_S [43–45], thereby providing again a solid basis to subsequent experimental and theoretical work.

Despite all these efforts, however, a major uncertainty due to the dependence on the charm quark mass, still remained. Its reduction required a three-loop calculation of the matrix element of the current–current operators. The situation was indeed similar to that before the 1996 paper mentioned above, just at one more loop. In 2022, Christoph and his collaborators began this calculation, using new methods.

In spring 2023, together with Hrachia Asatrian, Francesco Saturnino, and Christoph Wiegand, he published a first paper presenting a part of the two-loop calculation [46], and then began working on the remaining graphs. Sometime afterwards, the group of Steinhauser published the complete calculation [47, 48]. Christoph, in collaboration with Hrachia Asatrian, Hrachya H. Asatryan (Hrachia's grandson), Lukas Born and Julian Eicher, completed his work most recently in July 2024 [49].

The gap in the works needed for the completion of the NNLO evaluation is being closed right now, with Christoph playing an important role also at the finishing line.

2.4 Excursions into BSM (SUSY)

As mentioned already, towards the end of the 1980s, the machines already built, CLEO and ARGUS, to be soon operational, LEP, the commissioned B factories, Babar and Belle, and the proposed hadron colliders SSC and LHC, were expected to test the SM and search for new physics.

Theorists and experimentalists were sifting through all sorts of processes, including the B-meson decays and mixings, to single out those that could help to reach these goals. The radiative B decays seemed to have accessible rates only for large values of the t-quark mass, of the order of several multiples of the mass of the $W^\pm$ boson [50]. The narrative changed when it was realised that QCD corrections to these decays could enhance their rates quite significantly [25, 26]. This observation, together with the fact that exclusive radiative decays, such as $\overline{B} \to K^*\gamma$, were expected to provide rather clean signatures (with nearly monoenergetic photons), immediately prioritised their searches.

In parallel, theorists were trying to identify all possible extensions of the SM, with enlargements of the particle content (through an increase of the number of fermion generations, or of the number of Higgs multiplets, or through the introduction of other more complicated objects like leptoquarks), and/or with enlargements of the number of interactions (with the local ones requiring new gauge bosons), etc. As for more complex and complete models underlying the SM, the diatribe between strong- vs. weak-interaction aficionados was in full swing: it was not clear whether the SM had to be considered a low-energy limit of technicolour or supersymmetric theories. In this last case, actually, the low-energy limit is not really to the SM, but to the SM with an additional Higgs, that is, a two-Higgs-doublet model. (See, for example, Ref. [51].)

Phenomenological SUSY models [52–54], derived from spontaneously broken $N = 1$ supergravity, emerged as models with many virtues, the most important being the fact that local SUSY can cure the problem of quadratic divergences [55–57]. It protects scalar fields from jumping to higher scales, the Planck mass $M_{\rm P}$ or, possibly, a Grand Unification (GUT) scale. The spontaneous breaking of local SUSY does not spoil this feature. Thus, once local SUSY is decoupled, in the limit $M_{\rm P} \to \infty$, one is left with a globally supersymmetric theory, explicitly broken by some "soft-mass terms". Depending on how local SUSY is broken, and by which mediators this breaking is transmitted to the visible sector, the parameters describing these soft terms can vary, giving rise to the SUSY parameter space of the various phenomenological class of models studied at length in the past decades.

Some of the intrinsic features of the inherited global SUSY, together with the freedom in the soft-term parameters, are at the origin of another of the virtues of these models, sometime later often viewed as one of their shortcomings: they provide, in general, sources for FCNC and CP violation effects not present in the SM. They

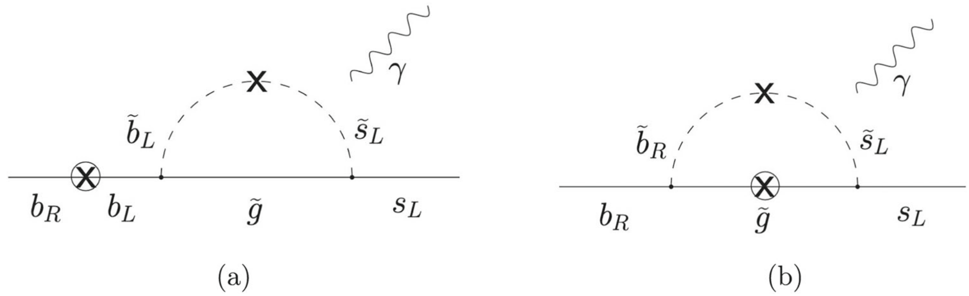

Fig. 5 **a** Gluino contribution diagram with chirality flip in the external b quark, as in the SM; **b** diagram in which the required chirality violation is induced by the gluino mass together with the trilinear mixing in the squark line. In both the cases, the flavour violation is realised by the squark trilinear mixing term. It is understood that the photon line is attached in all possible ways. From Ref. [30]

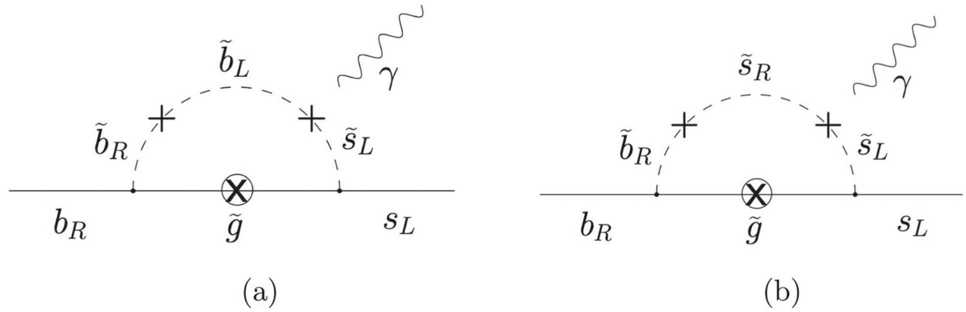

Fig. 6 The flavour and handedness violation in the squark exchanged in the gluino loop can be realised with a single trilinear term in the off-diagonal entry of the squark mass matrix (if such a term exists) as in Fig. 5b, or with a double violation, as shown here in the (**a**) and (**b**) frames. Again, the photon line is assumed to be attached in all possible ways. From Ref. [30]

have, in general, flavour-violating mass terms for squarks, flavour-violating trilinear terms mixing squarks of different handedness (or "chirality"), and can induce a flavour nondiagonal vertex gluino–squark–quark [58].

The magnetic operator $\mathcal{O}_7$, responsible for $b \to s\gamma$, is characterised by both, a flavour and a chirality violation. It gets SUSY contributions through the virtual exchange of: 1) charginos $\widetilde{\chi}^{\pm}$ together with up-type squarks; 2) neutralinos $\widetilde{\chi}^0$ with down-type squarks; 3) charged Higgs boson $H^{\pm}$ with up-type quarks; 4) gluinos $\widetilde{g}$ with down-type squarks. The last one, often dubbed the SUSY-QCD contribution is shown diagrammatically in Fig. 5.

All the superpartners realising these contributions were assumed at that time (end of the 1980s) to be roughly at the electroweak scale $M_{\rm weak}$. Thus, since the coupling gluino–quark–squark is weighted by the strong coupling, the gluino contribution seemed to induce an enhancement factor $O(\alpha_S^2/\alpha_W^2)$ with respect to the SM and the other SUSY contributions. Whether this would be reduced by various numerical factors, due to i) the actual size of the superpartner masses, ii) the GIM mechanism, or whatever replaces it in SUSY models (the interference of the different contributions due to different down squarks exchanged); or whether it could be even further enhanced by the fact that iii) the chirality flip in the gluino contribution can be realised in different ways (see Fig. 5) was checked in Refs. [59, 60].[6] It was there concluded that all the above features were conjuring to give a gluino contribution considerably larger than the SM one, and the radiative decays were immediately singled out as probably the most promising B decay.

Clearly, a more precise assessment of the role that SUSY can play for these decays, one that would take into account all contributions and the large QCD corrections (due to gluon exchange) was still missing. This is a formidable (if not altogether impossible) task: in general SUSY models can have a number of parameters in the low hundreds! Only a specific type of models capable of reducing drastically this number and with some simplifying assumptions (request of minimal flavour violation [61]) allows to perform such a task.

This calculation was done in Ref. [62] using MSUGRA as model (with a minimal number of SUSY parameters) in the mass-eigenstate formalism. As for QCD corrections, however, it was assumed that gluinos (as all other

[6] The cross in the gluino line in the frame b) of Fig. 5 denotes simply that the gluino mass $m_{\widetilde{g}}$ is taken in the gluino propagator. This mass has the task of restoring two units of the $U(1)_{\rm R}$ charge lacking in the trilinear soft-mass term with respect to the fermion mass term that realises the chirality flip in frame a).

superpartners), at the scale $M_{\rm weak}$, could be linked to the $W^{\pm}$ boson, and the program of inclusion of QCD corrections could follow that of the SM, outlined in Sect. 1.2. It is easy to see that this was a faulty assumption. Because of the strong gluino coupling, the loops corresponding to the diagrams of Figs. 5, 6, or SUSY-QCD contributions, are of strength α_S. According to Sect. 1.2 they would then contribute at NLO and not at LO as was assumed in Ref. [62], this being an oddity, as the gluino contribution could turn out (at least at that time) to be numerically larger than the SM LO contribution. Clearly this was calling for a systematic organisations of the various contributions in terms of α_S, and consistent with that of the SM. Times, however, were still not mature enough to address this problem. It would take the stubbornness, dedication, and competence of a group of physicists based mainly in Switzerland, including Christoph, to tackle and solve it.

For a few years, Ref. [62] represented the state of the art for B processes in SUSY. Several other studies appeared (see for example Refs. [63–67]), some checking Ref. [62], some with more extensive analyses of the SUSY parameter space, and/or simply addressing other specific phenomenological issues.

In the meantime, the SM calculation of the $b \to s\gamma$ rate was making great strides towards the completion of the NLO corrections, as outlined in Sect. 1.3. (For a detailed list of papers, see Ref. [29].)

Roughly at the same time, almost a decade after the announcement of Refs. [25, 26], measurements of the decay $b \to s\gamma$ were made available by the CLEO collaboration [31], and preliminary results were also reported a little later by the ALEPH collaboration at LEP [68]. The measured rates were compatible with the value calculated for the SM. Needless to say, whether some SUSY contribution could still be allowed, or had to be already excluded at least for some specific models, was clearly requiring a more precise SUSY calculation. And Christoph put himself to work on the subject.

He addressed the issue of the organisation of QCD correction for the gluino contribution in SUSY models in Ref. [30] (see also [69]). One of the problems with this contribution is that the diagrams in Figs. 5, 6 that realise it, give rise to different magnetic operators, of type $e\, g_S^2\, \overline{m}_b\, (\bar{s}\sigma^{\mu\nu} P_R b)\, F_{\mu\nu}$, of dimension *six*, and $e\, g_S^2\, (\bar{s}\sigma^{\mu\nu} P_R b)\, F_{\mu\nu}$, of dimension *five*. Moreover, it is not clear whether the strong coupling α_S in this contribution should be included in the definition of the gluino-induced operators or in the corresponding Wilson coefficients. Generally, both choices are acceptable.

This conundrum was solved by simultaneously enlarging the effective-Hamiltonian basis to include the gluino-induced operators, and attaching the coupling α_S to the operators themselves. The effective Hamiltonians is now split as

$$\mathcal{H}_{eff} = \mathcal{H}_{eff}^{W} + \mathcal{H}_{eff}^{\tilde{g}}\,, \tag{4}$$

where $\mathcal{H}_{eff}^{W}$ is the SM effective Hamiltonian in Eq. (1) and $\mathcal{H}_{eff}^{\tilde{g}}$ originates after integrating out squarks and gluinos. The enlargement of the operator basis in Eq. (4) is quite substantial. The counterpart of $\mathcal{O}_7$, for example, is now given by

$$\begin{aligned} \mathcal{O}_{7,\,b,\,\widetilde{g}} &= e\, g_S^2(\mu)\, \overline{m}_b(\mu)\, (\bar{s}\sigma^{\mu\nu} P_R b)\, F_{\mu\nu}, \\ \mathcal{O}_{7,\,\widetilde{g},\,\widetilde{g}} &= e\, g_S^2(\mu)\, (\bar{s}\sigma^{\mu\nu} P_R b)\, F_{\mu\nu}, \\ \mathcal{O}_{7,\,c,\,\widetilde{g}} &= e\, g_S^2(\mu)\, \overline{m}_c(\mu)\, (\bar{s}\sigma^{\mu\nu} P_R b)\, F_{\mu\nu}. \end{aligned} \tag{5}$$

(For the need to introduce the third of these operators, see Ref. [30].) Moreover, a substantial new set of four-quark operators has to be added.

With this strategy, gluino exchanges induce terms of the type $\alpha_S(\alpha_S L)^n$ at LO in QCD, and terms $\alpha_S^2(\alpha_S L)^n$ at NLO.

Having found a way to organise the towers of QCD corrections, order by order, the LO was calculated. Later on, in Ref. [70], together with T. Hurth, Volker Pilipp, Christof Schupbach, and M. Steinhauser, Christoph went on to calculate the NLO contribution to $b \to s\gamma$ in a model containing only the SM, gluinos and down squarks.

Paper [30] was not the first (neither the only) foray Christoph made BSM, after his very first papers on heavy Majorana neutrinos. It was not even his only one in B physics BSM. (See contribution to this volume by Andreas Crivellin.) His first step in this direction was the NLO evaluation of the $b \to s\gamma$ decay rate in a two-Higgs-doublet model (2HDM).

While they had been considered as suitable extensions of the SM on their own right, 2HDMs attracted the attention of the authors of Ref. [73] as they could be considered the limit to which SUSY tends when sfermions and gauginos have mass much larger than $M_{\rm weak}$.

As mentioned already, two-Higgs doublets are needed for SUSY models to be consistently realised [57].[7] In a 2HDM, the charged state $H^{\pm}$ contributes to the $b \to s\gamma$ decay pretty much like the longitudinal component of

[7] Refs. [71, 72] had already considered B processes in these models and had claimed for the first time that interesting limits for the mass of charged Higgs, which these models contain, could be obtained from forthcoming measurements.

$W^{\pm}$ does, although with different couplings. The expertise accumulated in the corresponding calculation in the SM made possible a speedy calculation also in 2HDMs.

Christoph came out with the first correct result for the calculation of the $b \to s\gamma$ decay at the NLO in QCD in Refs., [73, 74]. See also [75] and [76].

The presentation of the results in Ref. [73] is a witness to Christoph's commitment to be of service for the community: formulae were given, as much as possible, factorising out couplings encapsulating all details of the specific 2HDM used, making them adaptable also to analyses in Higgs-doublet models different than those considered in [73], without having to redo the lengthy two-loop calculation. Indeed, this paper is widely cited even today.

Later, following the lead of the authors of Ref. [77], it was soon realised that when $\tan\beta$ is large there is a class of SUSY-QCD corrections, as for example to some quark mass, or corrections to some Higgs-quark vertices, characterised by the combination $\alpha_S \tan\beta$.[8] This can be rather large, even larger than the combination $\alpha_S L$ discussed so far. When appearing in contributions to $b \to s\gamma$, as in the charged Higgs contributions, these corrections need to be resummed. What is interesting is that these mass/Yukawa coupling SUSY-QCD corrections are of non-decoupling nature in the limit of heavy superpartners, and have to be considered even when the contributions from the diagrams of Figs. 5 and 6 tend to vanish in this limit. These interesting observations were checked/corrected in Refs. [78, 79]. Also Christoph analysed this issue and clarified for which mass range of the charged Higgs the inclusion of these corrections becomes important [80].

Unfortunately, with the passing of time, and the accumulation of large quantities of data from colliders, it started becoming painfully clear that weak-scale SUSY had some difficulty in keeping reconciling with experiments. This happened already by the end of LEPII [81] and later, more clearly, in the first phase of LHC. It was realised that SUSY particles, if they exist, were probably rather heavy, say around some value $M_{\rm hSUSY}$, considerably larger than $M_{\rm weak}$. It was however considered still possible that just some of the superpartners escape experimental exclusion. The correct picture at the electroweak scale could then be obtained by integrating out the heavy SUSY particles at their scale $M_{\rm hSUSY}$. This would give rise to a new effective theory of "partial SUSY", to be matched with the full SUSY theory at $M_{\rm hSUSY}$. A set of RGEs can allow then to bring the sector of lighter SUSY particles from $M_{\rm hSUSY}$ down to $M_{\rm weak}$. These RGEs are clearly different from that of full SUSY model and are to be derived from scratch. Clearly, the conventional relations between SUSY parameters are also modified.

Never one to shy away in front of a challenge, Christoph, together with his collaborators Jason Aebischer, Andreas Crivellin and Youichi Yamada, derived such a new picture of partial SUSY [82]. In the full SUSY model they assumed heavy gluinos, heavy squarks of first and second generation, and also a heavy right-handed sbottom, say with mass $M_{\rm hSUSY}$ in the few-TeV range, and other parameters considerably lighter (not exceeding 300 GeV). They showed how to integrate out the heavy parameters and how to calculate to order $O(\alpha_S,\, Y_{t,b}^2)$ (with Y_b and Y_t the Yukawa coupling of the b and t quarks, respectively) the interactions of this Lagrangian at $M_{\rm weak}$. The procedure described in this paper may turn out to be useful in the coming years.

All in all, Christoph's papers involving BSM models amount to a considerable accomplishment for anybody, particularly so for a person somehow perceived as a SM physicist !!!

2.5 Personal reflections

We have reviewed Christoph's journey throughout B physics. In this field he has provided many lasting contributions, thereby helping understanding the role that B physics plays in unravelling the secrets of physics. A brief excursion from him into charm physics [83] produced also some results regarding flavour-violating charm decays, to become important if future machines will produce large quantities of them. He also looked into other areas where to extend our knowledge. He explored the physics of exotic neutrinos with Wilfried Buchmuller [84–88].

His achievements, however, are not limited to research successes. There is a lot more to discover in the way he does physics. In Christoph's publication list, there are several papers with physicists from Armenia, three of them by the name of Asatrian/Asatryan. As is well known, the dissolution of the Soviet Union left many scientists without support, including those in Yerevan. This city is home to an important and (previously) well supported institute of physics, proof of the high intellectual culture in Armenia. It was Ali who realised the high quality of the work done there and a collaboration involving Ali, Hrachhia Asatrian and Christoph took off [89]. This evolved later on into a collaboration of Christoph with various members of the Yerevan group, in particular Hrachia Asatrian, his son Hyak, and later his grandson Hrachya. He kept up this highly achieving collaboration for over 25 years, enabling his Armenian friends to work at the forefront in our field despite their difficult situation. Christoph proved to be a loyal friend, without compromising on quality. His true concern for physicists outside our well-padded western community is remarkable.

Similarly, he proved to be an unusually supportive and dedicated advisor to his students. He worked intensively with them, giving them credit and always putting his own person behind theirs, selflessly promoting them to be co-authors and to become part of the international community.

[8] Large values of $\tan\beta$ are typically obtained in SUSY GUT models with b-t Yukawa unification.

We can now complete the words by Buras in Sect. 1.1:
Christoph is a superb physicist, a master in his field AND a wonderful person.

3 Rare B-meson decays in the standard model

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3.1 Getting to know Christoph

Christoph joined the theoretical physics group at DESY, Hamburg, in the autumn of 1989 on a 2-year post-doctoral position. This was his first assignment, having just completed his Ph.D. under the supervision of Peter Minkowski [90]. I still recall with delight our first meeting. Christoph had the boyish looks of a teenager and resembled our undergrad summer students, who spend typically several weeks in our group, then return to their classes to resume their studies. Hard to believe that it was 35 years ago! He has not changed all that much in these years. Fig. 7 shows a recent photograph, taken in 2022, while I was visiting Christoph in Bern.

The decade of the 1980s was the high energy epoch dominated by the UA1 and UA2 collaborations at the CERN proton–antiproton collider, which had found the electroweak $W^{\pm}$ and Z bosons, predicted in the Standard Model (SM). These experiments were also at the forefront of heavy quark physics, their production and decays. In particular, very impressive measurements of the process $p\bar{p} \to b\bar{b} + X$ were reported at large momentum transfer, leading to, among other final states, same- and opposite-sign dileptons $p\bar{p} \to \ell^{\pm}\ell^{\pm} + X$ [91]. I participated in the theoretical studies related to the UA1 heavy quark data [92]. Just after that, ARGUS (A Russian–German–United States–Swedish) collaboration at the electron–positron collider ring DORIS II at DESY, had observed large mixing of the neutral B^0 meson into its antiparticle $\bar{B}^0$ meson [24], which is one of the highlights of experimental discoveries at DESY. This led to the inference that the top quark, which in the SM is responsible for driving the weak mixing to the observed level but was not yet discovered, had a mass well in excess of 100 GeV.

The large B^0–$\bar{B}^0$ mixing observed by ARGUS had many implications, in particular, it provided a strong case to carry out CP-asymmetry measurements in the mixing-induced decays of the B^0–$\bar{B}^0$ system, which were undertaken at the so-called B-meson factories, at KEK and SLAC. The inferred large top quark mass also implied that a lot of flavour-changing-neutral-current transitions, involving, in particular, electroweak penguin diagrams, which are

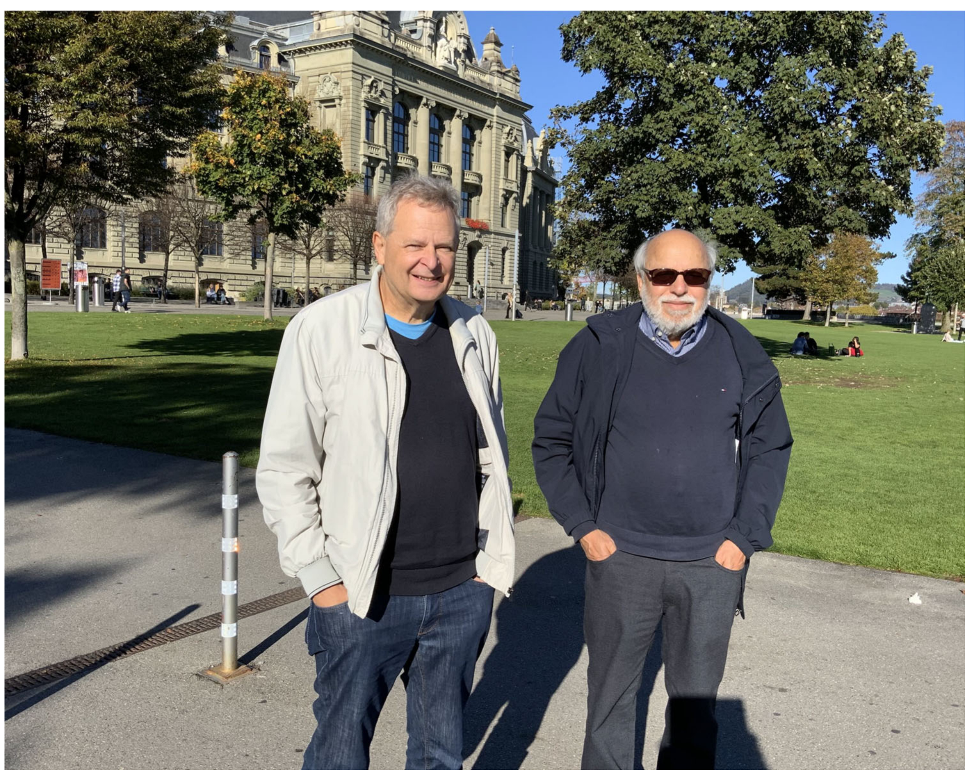

Fig. 7 With Christoph Greub at his Alma Mater in 2022

governed by the GIM mechanism [93] in the SM, were within reach of the ongoing experiment CLEO at Cornell, and at the planned next generation B-meson experiments Babar and Belle.

I convinced Christoph to work on rare B-decays. We started with the radiative decays $B \to X_s\gamma$ and $B \to K^*\gamma$, and their CKM-suppressed counterparts[9] $B \to X_d\gamma$ and $B \to (\rho,\,\omega)\gamma$. Later, as the B factories started to gather steam, the interest in B physics grew substantially worldwide. At DESY, we wrote several papers on the semileptonic decays $B \to X_s\ell^+\ell^-$ and $B \to (K,\,K^*)\ell^+\ell^-$, co-authored with Patricia Ball, Laksana Handoko, Gudrun Hiller, Enrico Lunghi, Thomas Mannel and Sasha Parkhomenko. Christoph participated in this endeavour with great dedication. That he brought in this collaboration a sound calculational expertise, in particular, in perturbative QCD, was a huge plus. Here, due to space limitations, I will concentrate on some selected research papers which we co-authored, and which, in my opinion, had an impact on the interpretation of data from a number of key experiments.

3.2 The radiative decays $B \to X_s\gamma$ and $B \to K^*\gamma$

In our first paper [96], we calculated the inclusive decay rate for $B \to X_s\gamma$, where X_s is a bunch of hadrons with an overall strangeness quantum number $S = -1$. The calculations were carried out in the SM using the effective-Hamiltonian approach, which included dimension-6 operators. The large renormalisation effects were included in the leading-order QCD. At the parton level, the diagrams that we calculated were $b \to sg$ and $b \to sg\gamma$, where g is a gluon. This enabled us to calculate the photon energy spectrum and the decay rate in this process. This paper was closely followed by another [97], in which a profile of the final states in the inclusive decay $B \to X_s\gamma$ was presented based on perturbative QCD and adopting a phenomenological wave-function for the B-meson encoding the Fermi motion of the b-quark [98]. We estimated the inclusive branching ratio $\mathcal{B}(B \to X_s\gamma) = (3-4) \times 10^{-4}$ for the top quark mass range $m_t =$ 100-200 GeV. The estimated branching ratio $\mathcal{B}(B \to K^*\gamma) = (5^{+3}_{-2}) \times 10^{-5}$ was about a factor of 5 above the experimental upper limit at that time.

As the analysis and interpretation of the CLEO data showed, our papers [96, 97] provided a target for the experiments to reach. In particular, they gave the experimental collaborations a fairly accurate description of the photon energy spectrum and the invariant hadronic mass distributions in $B \to X_s\gamma$. In the CLEO $B \to X_s\gamma$ discovery paper [31], the X_s mass distribution calculated by us played a key role in modelling their Monte Carlo, enabling the CLEO collaboration to establish a signal and claim a significant first measurement of the inclusive radiative penguin decay in B physics. The CLEO measurement $\mathcal{B}(B \to X_s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$ [31] is in agreement with our SM-based estimates, within errors. Soon thereafter, CLEO published the evidence for penguin-diagram decays and the first observation of the $B \to K^*(892)\gamma$ decay [99]. Their reported branching ratio $\mathcal{B}(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ is in good agreement with what we calculated.

With consolidated data and detailed systematic studies, CLEO published their improved branching fraction and photon energy spectrum for $B \to X_s\gamma$ in 2001 [100], obtaining $\mathcal{B}(B \to X_s\gamma) = (3.21 \pm 0.43 \pm 0.27^{+18}_{-10}) \times 10^{-4}$, where the errors are statistical, systematic, and from theory, respectively. The photon energy spectrum in the decay $B \to X_s\gamma$ measured by the CLEO collaboration is shown in Fig. 8, and compared with the spectator-model spectrum [97] with the model parameters fixed at $m_b = 4.690\,\mathrm{GeV}$ and $p_F = 410\,\mathrm{MeV}$, which is a measure of the b-quark Fermi motion in the B meson, providing a good fit of the data. The decay rates $\Gamma(B \to X_s\gamma)$ and $\Gamma(B \to K^*\gamma)$ are sensitive to the CKM-matrix elements $|V_{tb}V_{ts}|$, and the importance of the radiative decays for the CKM phenomenology was pointed out. Using the experimental data in the early epoch of these measurements, a 95% C.L. bound $0.50 \leqslant |V_{ts}|/|V_{cb}| \leqslant 1.67$ was derived [101].

Since then, a lot of progress has been made in theory and experiment, as discussed briefly below, and the radiative decay $B \to X_s\gamma$ has become a standard candle to search for physics beyond the SM. In particular, Christoph has worked with great dedication in completing the $\mathcal{O}(\alpha_s^2)$ calculations of the inclusive decay $B \to X_s\gamma$. This is a very involved task, requiring the calculation of a large number of three-loop on-shell and four-loop tadpole Feynman diagrams, in addition to the Wilson coefficients to the desired level of theoretical accuracy. Pieces of these calculations were undertaken by several groups, with the final result distilled in a joint paper in 2007 [35], yielding $\mathcal{B}(B \to X_s\gamma) = (3.15 \pm 0.23) \times 10^{-4}$, for $E_\gamma > 1.6\,\mathrm{GeV}$ in the B-meson rest frame. This result has been further improved in the meanwhile, with Christoph on board, by taking into account some non-perturbative effects. The updated CP- and isospin-averaged branching ratio [36]

$$\mathcal{B}(B \to X_s\gamma) = (3.36 \pm 0.23) \times 10^{-4}\,, \tag{6}$$

is in excellent agreement with the current world average [102]

$$\mathcal{B}(B \to X_s\gamma) = (3.49 \pm 0.19) \times 10^{-4}\,. \tag{7}$$

[9] Here, CKM stands for the weak mixing matrix, named after N. Cabibbo [94], and M. Kobayashi and T. Maskawa [95].

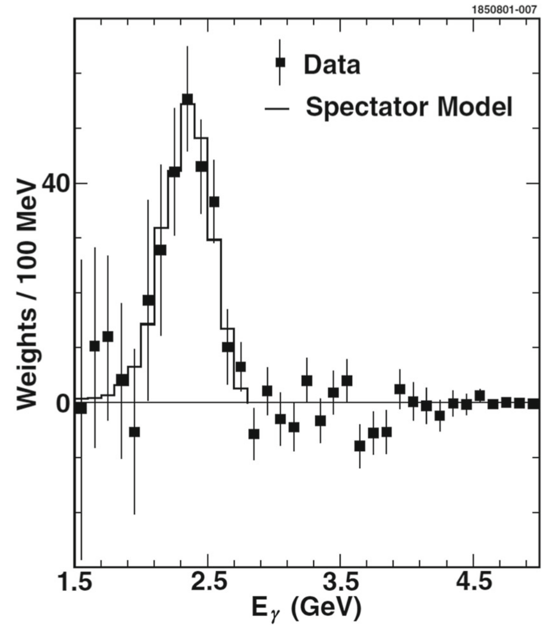

Fig. 8 Observed laboratory frame photon energy spectrum from the $b \to s\gamma$ and $b \to d\gamma$ signal. Also shown is the spectrum from Monte Carlo simulation of the Ali-Greub spectrum model. (From Chen et al. (CLEO collaboration) Phys. Rev. Lett. 87, No. 25 (2001) 251807)

The measured ratio of $\mathcal{B}(B \to X_s\gamma)$ and the semileptonic branching ratio $\mathcal{B}(B \to X_c e\bar{\nu}_e)$, yield $|V_{ts}|/|V_{cb}|= 0.98 \pm 0.04$ [102].

Theoretical progress in B decays owes itself to a large extent to the advent of heavy quark effective theory [103]. In particular, it was shown that in the heavy quark limit, a factorisation framework can be applied to two-body non-leptonic decays, such as $B \to \pi\pi$ [104] and radiative $B \to V\gamma$ decays, where $V = \rho, \omega, \phi, K^*$ [105]. The matrix element of a given operator in the effective weak Hamiltonian can be written in the form

$$\langle V\gamma|Q_i|B\rangle = F^{B\to V_\perp} T_i^I + \int dw\; du \phi_+^B(\omega)\, \phi_\perp^V(u)\, T_i^{II}. \tag{8}$$

The non-perturbative effects are contained in $F^{B\to V_\perp}$, the $B \to V$ transition form factor at $q^2 = 0$, and $\phi_+^B(\omega)$ and $\phi_\perp^V(u)$, the leading-twist light-cone distribution amplitudes. The hard-scattering kernel T_i^I and T_i^{II} are perturbatively calculable. With the advent of the soft-collinear-effective-theory (SCET) [106], the derivation of the factorisation formula from a two-step matching procedure has provided additional insight into its structure [107]. In the SCET approach, the factorisation formula is written as

$$\langle V\gamma|Q_i|B\rangle = \Delta_i C^A \zeta_{V_\perp} + \frac{\sqrt{m_B} F f_{V_\perp}}{4} \int dw\; du \phi_+^B(\omega)\, \phi_\perp^V(u)\, t_i^{II}, \tag{9}$$

where F and $f_{V_\perp}$ are meson decay constants. The SCET form factor is related to the QCD form factor through perturbative and power corrections. In SCET, the perturbative hard-scattering kernels are the matching coefficients $\Delta_i C^A$ and t_i^{II}. They are known to NLO accuracy in the renormalisation group improved perturbation theory (Becher et al in Ref. [107]).

In the paper written with Christoph and Ben Pecjak [108], important steps for a complete NNLO description were worked out. The results for the dipole operators O_7 and O_8 were complete to NNLO accuracy, and for the four-quark operator $O_1 = (\bar{q}c)_{V-A}(\bar{c}b)_{V-A}$, they were partial, obtained in the large-β_0 limit. In addition, corrections to the so-called spectator scattering were neglected. They are, however, not anticipated to be numerically significant. Phenomenological analysis led to the estimates of the branching ratios in (partial) NNLO accuracy, yielding $\mathcal{B}(B^+ \to K^{*+}\gamma) = (4.6 \pm 1.4) \times 10^{-5}$ and $\mathcal{B}(B^0 \to K^{*0}\gamma) = (4.3 \pm 1.4) \times 10^{-5}$, with $\mathcal{B}(B_s^0 \to \phi\gamma) \simeq \mathcal{B}(B^0 \to K^{*0}\gamma)$, where the error is dominated by the form factors. These estimates are in agreement with the current experimental averages [102]:

$$\begin{aligned} &\mathcal{B}(B^+ \to K^{*+}\gamma) = (3.92 \pm 0.22) \times 10^{-5}, \\ &\mathcal{B}(B^0 \to K^{*0}\gamma) = (4.18 \pm 0.25) \times 10^{-5}, \\ &\mathcal{B}(B_s^0 \to \phi\gamma) = (3.4 \pm 0.4) \times 10^{-5}. \end{aligned} \tag{10}$$

3.3 The CKM-suppressed decay $B \to X_d\gamma$

The inclusive decay $B \to X_d\gamma$ is CKM-suppressed, compared to the decay $B \to X_s\gamma$ by $\mathcal{O}(\lambda^2) = 0.05$. They are of great theoretical interest due to their impact on the CKM unitarity tests and also due to the anticipated large CP violation in the decay rates, reflecting the specific CKM-matrix element dependence of the tree and penguin amplitudes. The calculations for the $B \to X_d\gamma$ decay were done in the effective-Hamiltonian approach, with dimension-6 operators, which were defined in an operator basis similar to the $B \to X_s\gamma$, so that the matching conditions, $C_i(M_W)$, and the Wilson coefficients at the scale $\mu = m_b$, i.e. $C_i(m_b)$, are exactly the same. The decay rates and CP asymmetry were worked out in partial NLL approximation in [109], and in the NLL accuracy, including leading power corrections in [89]. The ratio of the charge-conjugate averaged branching ratios $R(d\gamma/s\gamma) \equiv \langle\mathcal{B}(B \to X_d\gamma)\rangle/\langle\mathcal{B}(B \to X_s\gamma)\rangle$ and the direct CP asymmetry $a_{\rm CP}(B \to X_d\gamma)$ are expressed as [89]

$$\begin{aligned} R(d\gamma/s\gamma) &= \frac{|\xi_t|^2}{|\lambda_t|^2} + \frac{D_u}{D_t}\frac{|\xi_u|^2}{|\lambda_t|^2} + \frac{D_r}{D_t}\frac{\mathrm{Re}(\xi_t^*\xi_u)}{|\lambda_t|^2}, \\ a_{\rm CP}(B \to X_d\gamma) &= -\frac{\mathrm{Im}(\xi_t^*\xi_u)D_i}{|\lambda_t|^2 D_t^{(0)}}. \end{aligned} \tag{11}$$

Here $\lambda_t = V_{ts}V_{tb}$, $\xi_u = A\lambda^3(\bar\rho - i\bar\eta)$ and $\xi_t = A\lambda^3(1-\bar\rho+i\bar\eta)$, where A, λ, $\bar\rho$, $\bar\eta$ are the Wolfenstein parameters of the CKM matrix. The leading term for $R(d\gamma/s\gamma)$ in Eq. (11) is independent of dynamical details, and the subleading terms are small, with typically $D_u/D_t \simeq 0.07$, $D_r/D_t \simeq -0.14$. The coefficient appearing in $a_{\rm CP}(B \to X_d\gamma)$ has the value $D_i/D_t^{(o)} \simeq 0.37$. The charged-conjugate averaged branching ratio $\langle\mathcal{B}(B \to X_d\gamma)\rangle$, the ratio $R(d\gamma/s\gamma)$, and the CP asymmetry were numerically estimated to have the central values [89]:

$$\begin{aligned} &\langle\mathcal{B}(B \to X_d\gamma)\rangle \simeq 1.52 \times 10^{-5}, \qquad R(d\gamma/s\gamma) \simeq 0.046, \\ &a_{\rm CP}(B \to X_d\gamma) \simeq 0.16. \end{aligned}$$

Measurements of the fully inclusive decay rates for $B \to X_d\gamma$ have yet to be carried out. This is due to the small branching ratio, but also the background, contributed mostly by the dominant $B \to X_s\gamma$ decay. The closest that one has come so far is a semi-inclusive measurement by BaBar [110], based on 7 decay modes contributing to $B \to X_d\gamma$: $B^0 \to \pi^+\pi^-\gamma$, $\pi^+\pi^-\pi^0\gamma$, $\pi^+\pi^-\pi^+\pi^-\gamma$ and $B^+ \to \pi^+\pi^0\gamma$, $\pi^+\pi^-\pi^+\gamma$, $\pi^+\pi^-\pi^+\pi^0\gamma$, $\pi^+\eta\gamma$, with a cut-off on the hadronic mass $m(X_d) \leqslant 2.0\,\mathrm{GeV}$. This has yielded a branching ratio

$$\langle\mathcal{B}(B \to X_d\gamma)\rangle \simeq (9.2 \pm 2.0 \pm 2.3) \times 10^{-6}.$$

Building up the decay $B \to X_s\gamma$ also in 7 decay modes, replacing a π^+ by K^+, with the hadronic mass $m(X_s)$ up to 2.0 GeV, led to the ratio

$$R(d\gamma/s\gamma) \simeq 0.040 \pm 0.009 \pm 0.010,$$

in good agreement with the value predicted in [89]. This, in turn, leads to a determination of the CKM-matrix element ratio [110] $|V_{td}/V_{ts}| = 0.199 \pm 0.022(\mathrm{stat.}) \pm 0.024(\mathrm{syst.}) \pm 0.002\,(\mathrm{model})$ from radiative penguin decays, in agreement with the more accurate determination of the same from the $B^0_{(s)} - \bar B^0_{(s)}$ mixing [102]: $|V_{td}/V_{ts}| = 0.207 \pm 0.001 \pm 0.003$.

The only currently active experiment that can carry out an inclusive measurement of $\mathcal{B}(B \to X_d\gamma)$ and $a_{\rm CP}(B \to X_d\gamma)$ is Belle II. The projected combined (statistical and systematic) uncertainty on this branching ratio is 14% for an assumed integrated luminosity of $50\,\mathrm{ab}^{-1}$ [111]. However, theoretical progress, in particular, in the Lattice-QCD based estimates of the form factors, as well as in QCD sum rules, allow to carry out very stringent tests of the SM in exclusive radiative B-decays, such as $B \to K^*\gamma$, $\rho\gamma$, $\omega\gamma$, which are already at hand from Belle [112] and Babar [113], and which have good prospects of substantial improvement at Belle II.

3.4 The semileptonic decays $B \to X_s\ell^+\ell^-$, $(K,\, K^*)\ell^+\ell^-$

The next topic which I would like to discuss are the semileptonic decays $B \to X_s\ell^+\ell^-$, $(K,\, K^*)\ell^+\ell^-$, where $\ell^\pm = e^\pm$, $\mu^\pm$. By the advent of this century, statistical power of the B-factory experiments, BaBar and Belle, had reached the level to measure these decays. They offer to test the SM in much more detail through a number of angular correlations, in addition to the dilepton invariant mass spectrum. Theoretically, a lot of work had already been done on these decays in the context of the SM and some extensions of it [114]. Partial results in NNLO to the differential decay rate $d\Gamma(B \to X_s\ell^+\ell^-)/ds$, where s is the square of the dilepton invariant mass, were available,

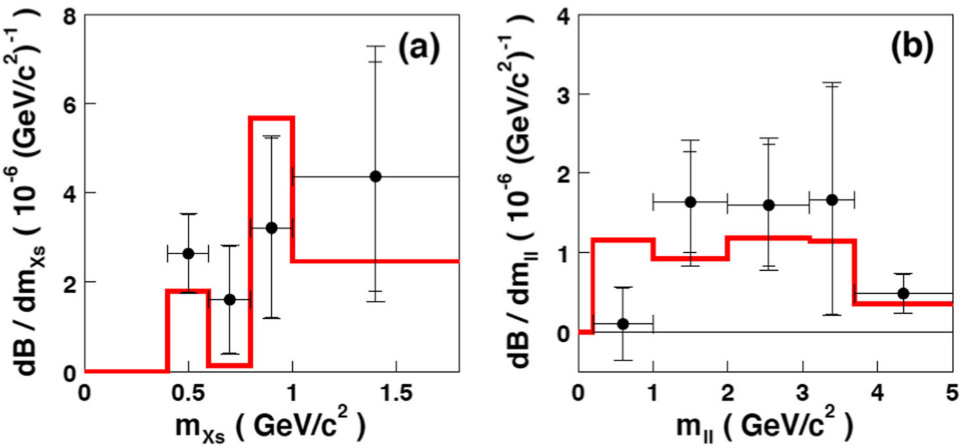

Fig. 9 Differential branching ratio for $B \to X_s\ell^+\ell^-$ as a function of **a** hadron mass and **b** dilepton mass, data (points) compared to the signal Monte Carlo (histograms) discussed in the text. (From Aubert et al. (Babar collaboration) Phys. Rev. Lett. 93, 081802 (2004))

worked out by Christoph and his collaborators [43, 115]. Likewise, power corrections of $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ [116] and $\mathcal{O}(\Lambda_{\rm QCD}/m_c)$ [117] for the differential decay rates were also known. For the exclusive decays, $B \to (K, K^*)\ell^+\ell^-$, form factors were calculated in the QCD sum rule approach [118], written in a paper co-authored with Patricia Ball, Gudrun Hiller and Laksana Handoko. The main aim of the paper [119] was to carry out a state-of-the-art estimate of the inclusive and exclusive decays, involving $b \to s$ radiative and semileptonic transitions, both in the SM, and in supersymmetric theories.

In the NNLO approximation, the invariant dilepton mass distribution for the inclusive decay $d\Gamma(B \to X_s\ell^+\ell^-)/d\hat{s}$, where $\hat{s} = s/m_b^2$, is written as [119]

$$\frac{d\Gamma(B \to X_s\ell^+\ell^-)}{\hat{s}} = \left(\frac{\alpha}{4\pi}\right)^2 \frac{G_F^2 m_b^5 |V_{ts}^* V_{tb}|^2}{48\pi^3}(1-\hat{s})^2\Big((1+2\hat{s})(|\tilde{C}_9^{\rm eff}|^2 + |\tilde{C}_{10}^{\rm eff}|^2) + 4(1+2/\hat{s})|\tilde{C}_7^{\rm eff}|^2 + 12{\rm Re}(\tilde{C}_7^{\rm eff}\tilde{C}_9^{\rm eff*})\Big), \quad (12)$$

where G_F is the Fermi coupling constant, and the various $\tilde{C}_i^{\rm eff}$ ($i = 7, 9, 10$) are the effective Wilson coefficients [43, 115, 120]. In extensions of the SM, their values differ from their corresponding SM values.

Estimating the parametric theoretical uncertainties, residing in the scale, $\delta\mathcal{B}(\mu)$, top quark mass, $\delta\mathcal{B}(m_t)$ and quark mass ratio $\delta\mathcal{B}(m_c/m_b)$, we estimated $\mathcal{B}(B \to X_s\ell^+\ell^-) = (4.2 \pm 0.7) \times 10^{-6}$ (for $M_{\ell^+\ell^-} > 0.2\,$GeV) in the SM. Like the $B \to X_s\gamma$ case, a fully inclusive measurement of the decay $B \to X_s\ell^+\ell^-$ is not yet at hand. Instead, BaBar studied this decay mode by measuring a subset of all possible final states, where the hadronic system X_s consists of one $K^\pm$ or a K_s^0, and up to two pions, with at most one π^0, and estimated the missing one using the X_s-hadronisation model, based on [119] and the Fermi motion model [98]. Their measured value is [121]: $\mathcal{B}(B \to X_s\ell^+\ell^-) = (5.6 \pm 1.5({\rm stat.}) \pm 0.6({\rm syst.}) \pm 1.1({\rm model})) \times 10^{-6}$. The Babar m_{X_s}- and $M_{\ell^+\ell^-}$-distributions are shown in Fig. 9. A similar analysis was undertaken by the Belle collaboration, which measured a statistically improved branching ratio [122] $\mathcal{B}(B \to X_s\ell^+\ell^-) = (4.11 \pm 0.83({\rm stat.})^{+0.85}_{-0.81}) \times 10^{-6}$, which is consistent with the Babar result [121] and the SM [119].

Subsequently, Belle [123] and Babar [124] published their results on the measurements of the exclusive decays $B \to (K, K^*)\ell^+\ell^-$ based on their consolidated data. Measurements from the CDF collaboration [125] at Fermilab and the LHCb collaboration [126] at CERN were also reported. A comparison of the Babar, Belle and CDF data is shown in Fig. 10, which are compared with the SM estimates [119] and found in agreement. Tests of the lepton universality in $b \to s\ell^+\ell^-$ were also reported by the Belle Collaboration [123]. Defining the ratios $R_K \equiv \mathcal{B}(B \to K\mu^+\mu^-)/\mathcal{B}(B \to Ke^+e^-)$ and $R_{K^*} \equiv \mathcal{B}(B \to K^*\mu^+\mu^-)/\mathcal{B}(B \to K^*e^+e^-)$, their measured values $R_K = 0.83 \pm 0.17 \pm 0.08$ and $R_{K^*} = 1.03 \pm 0.19 \pm 0.6$ are consistent with the lepton flavour universality. In the meanwhile, the LHCb collaboration [127] has measured these ratios with great accuracy, and found agreement with the SM.

An observable, the lepton forward–backward asymmetry in $B \to X_s\ell^+\ell^-$, was suggested long ago as yet another precision test of the SM [129]. Defined as

$$\mathcal{A}_{\rm FB}(q^2_{\rm min}, q^2_{\rm max}) \equiv \frac{\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \int_{-1}^{1} d\cos\theta \;\; {\rm sgn}(\cos\theta) \frac{d^2\Gamma}{dq^2 d\cos\theta}}{\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \int_{-1}^{1} d\cos\theta \frac{d^2\Gamma}{dq^2 d\cos\theta}}\,, \quad (13)$$

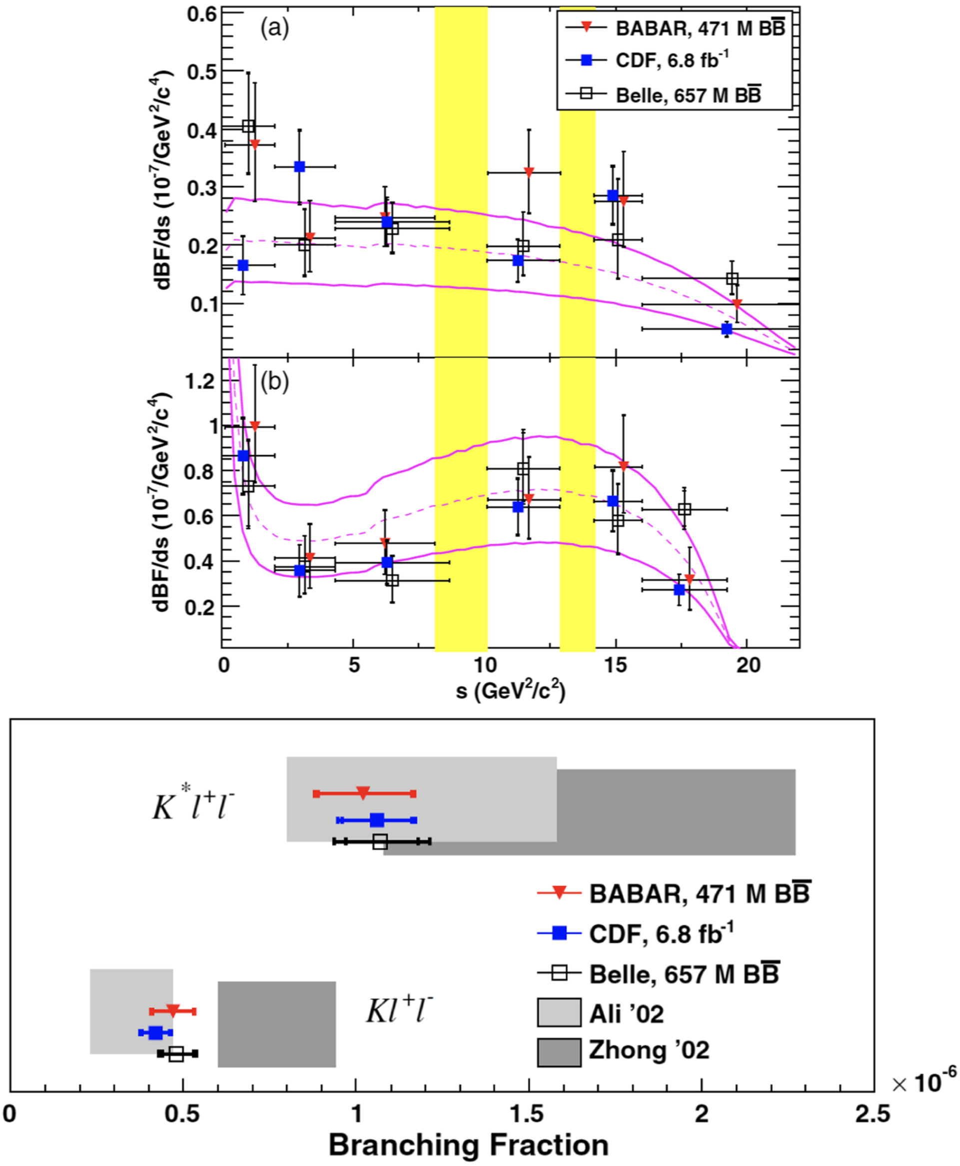

Fig. 10 (Up): Partial branching fractions for the **a** $B \to K\ell^+\ell^-$ and **b** $B \to K^*\ell^+\ell^-$ as function of s showing BABAR measurements (red triangle) and the SM prediction from the Ali et al. model [119]. (Down): Total branching fraction for the $B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$ from Babar (red triangle), compared with Belle (open squares) [123] and CDF (blue solid) [125] measurements and with predictions of Ali et al. model [119] and Zhong, Wu and Wang [128]. (From Lees et al. (BABAR collaboration) Phys Rev. D86, 032012 (2012))

this provides a different constraint on the effective Wilson coefficients C_i ($i = 7$, 9, 10) than the decay rate. This observable has also been calculated at NNLO already a while ago, in which Christoph was also involved [44, 130].

A representative effect of possible beyond-the SM physics in $A_{\rm FB}(q^2)$ in $B \to X_s\ell^+\ell^-$ is exemplified in Fig. 11 (left panel) from [119]. This figure was obtained by working out the allowed ranges of the effective Wilson coefficients $C_7(\mu_W)$, $C_8(\mu_W)$, $C_9^{\rm NP}(\mu_W)$, and $C_{10}^{\rm NP}(\mu_W)$ from data on the branching ratios of $B \to X_s\gamma$, $B \to (K, K^*)\ell^+\ell^-$. Available data allowed considerable room for beyond-the-SM effects and some specific cases were worked out in the supersymmetric case. The resulting profile of $A_{\rm FB}(\hat{s})$, where $\hat{s} = q^2/m_b^2$, is shown for the SM. The curves labelled as 1, 2, and 3 represent various BSM scenarios. On the right panel in this figure the measured $A_{\rm FB}(q^2)$ from Belle [131] is shown, compared with the SM calculation [119] with the indicated ranges of the parameters.

3.5 Reflections

My active collaboration with Christoph spanned the years 1989–2008, almost 2 decades. This period saw a rapid development of flavour physics, transforming from the pass time of a few into the main frame of high energy physics. Experimental progress has been spearheaded by ARGUS, CLEO, Babar and Belle at the electron–positron colliders, and CDF, D0, ATLAS, CMS and LHCb at the hadron colliders, of which the last three at the LHC, together with Belle II at the KEK, are currently at the forefront of precision flavour physics. Prominent among their achievements is a precise determination of the CKM matrix elements. In particular, lepton flavour universality, a linchpin of the SM, is quantitatively verified. Equally important are precision tests of the GIM mechanism, which

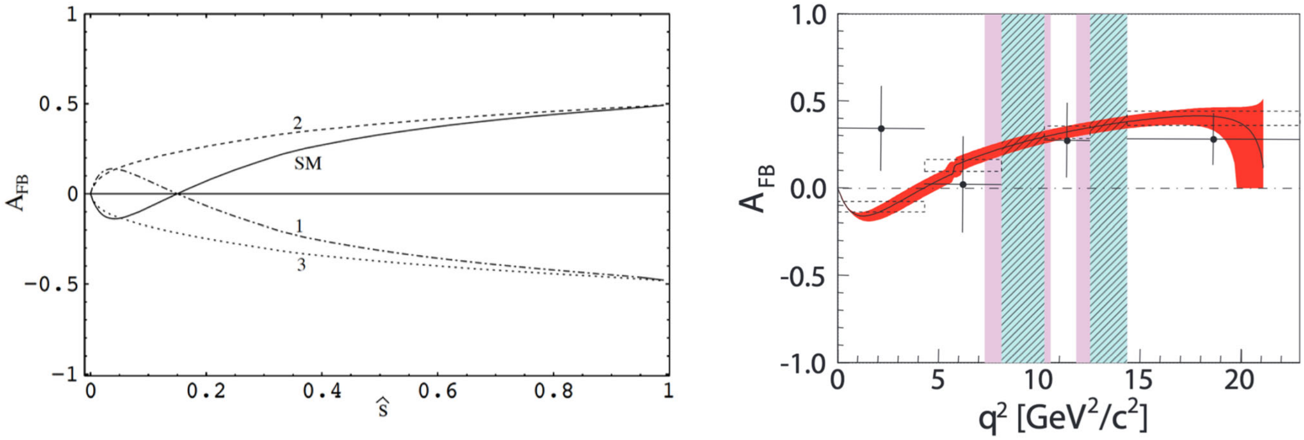

Fig. 11 (Left): $A_{\rm FB}(\hat{s})$ as a function of $\hat{s} = q^2/m_b^2$ in the SM and in Beyond-the-SM theories (from [119]). (Right): Measured Forward–backward asymmetry as a function of q^2 and compared with the SM-based theoretical calculation [119]. The uncertainty on the SM prediction is estimated by varying the b-quark mass, $m_b = (4.80 \pm 0.15)$ GeV, the strange-quark mass $m_s = (0.20 \pm 0.10)$ GeV, and the scale $\mu = 2.5$ and 5.0 GeV. (From Sato et al. (Belle collaboration) Phys. Rev. D93, 032008 (2016))

governs the flavour-changing-neutral-current transitions in the SM. A representative sampling of rare B-decays is discussed here in the form of radiative and semileptonic B-decays, $b \to s\gamma$ and $b \to s\ell^+\ell^-$, and some exclusive decays.

Central to the theoretical progress is the asymptotic nature of QCD, enabling a perturbative treatment of a large number of high energy phenomena. This, in turn, led to the development of theoretical techniques, foremost among them Lattice-QCD, which has become a completely quantitative tool, and to some extent the QCD sum rules. The discovery of the heavy quark symmetry has greatly simplified the calculational framework. Among other applications, it has enabled us to study factorisation properties of entangled decay amplitudes. Likewise, the emergence of effective theories, Heavy Quark Effective Theory (HQET) and the Soft-Collinear-Effective Theory (SCET), in particular, has led to deep insights in the dynamics of heavy quarks. Some of these techniques, rather their applications, are discussed here, selecting those processes in whose studies Christoph was involved.

In retrospect, we benefitted greatly from these developments, but our main focus has remained on the experiments. That our contribution was helpful in the interpretation of data in flavour physics is a source of some satisfaction. In this endeavour, Christoph has been an equal partner, a remarkably competent theoretical physicist, and someone who remained steadfast against the temptation of overinterpreting data. For me, what remains now is to thank Christoph for the very pleasant collaboration and to wish him many active and productive post-retirement years.

4 Inclusive $\bar{B} \to X_s\gamma$ in the SM and beyond

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4.1 Starting flavour physics at two loops with Christoph

Christoph and I met in Zurich when Christoph moved from Hamburg to the University of Zurich for his second postdoc position and when I started my PhD at the same time. We talked more about non-physical topics during these 3 years together in Zurich because at that time Christoph had already been working on rare decays in flavour physics since his postdoc in Hamburg while I was more interested in theoretical questions in quantum field theory. A scientific collaboration between Christoph and me only came about when Daniel Wyler and Christoph were looking for a partner for a two-loop calculation of the inclusive $\bar{B} \to X_s\gamma$ decay rate to next-to-leading logarithmic (NLL) precision. For me, this was a leap in the dark, but also a great opportunity from which I benefited greatly and for which I am still very grateful to both of them. In addition, Christoph was no longer in Zurich, but already started a new postdoc at SLAC in California when we started the project in the fall of 1995.

The puzzle of different results at leading-order (LL) precision in earlier analyses on this decay had just been solved by the observation that the anomalous dimension matrix is scheme dependent and that it is matched by the scheme dependence of some matrix elements of four-quark operators [132, 133]. It was also shown that the NLL contributions would reduce the perturbative scale dependence from 25% to about 10% [28].

Therefore, the next challenge was the calculation of the NLL contributions. The inclusive modes are known to be theoretically clean. The heavy quark effective theory tells us that the decay width $\Gamma(B \to X_s\gamma)$ is well approximated by the partonic decay rate $\Gamma(b \to X_s\gamma)$ which can be analysed in renormalisation group improved perturbation theory. At that time, only the class of non-perturbative effects which scales like $1/m_b^2$ were known and estimated to be well below 10% [134]. The QCD corrections enhance the partonic decay rate $\Gamma(b \to s\gamma)$ by more than a factor of two. These QCD effects can be attributed to logarithms of the form $\alpha_s^n(m_b)\,\log^m(m_b/M)$, where $M = m_t$ or $M = M_W$ and $m \leq n$ (with $n = 0, 1, 2, ...$). In order to get a reasonable result at all, one has to resum at least the leading-log (LL) series ($m = n$). Working to next-to-leading-log (NLL) precision means that one is also resumming all the terms of the form $\alpha_s(m_b)\,(\alpha_s^n(m_b)\,\ln^n(m_b/M))$.

An appropriate framework to achieve the necessary resummations is an effective low-energy theory, obtained by integrating out the heavy particles which in the SM are for example the top quark and the Z and W boson. The effective Hamiltonian relevant for $b \to s\gamma$ has the form

$$H_{eff}(b \to s\gamma) = -\frac{4G_F}{\sqrt{2}}\,\lambda_t\,\sum_{i=1}^{8} C_i(\mu)\,O_i(\mu)\,, \tag{14}$$

where $O_i(\mu)$ are the relevant operators, $C_i(\mu)$ are the corresponding Wilson coefficients, which contain the complete top and W mass dependence, and $\lambda_t = V_{tb}V_{ts}^*$ with V_{ij} being the CKM-matrix elements. Then the perturbative QCD corrections for the $b \to s\gamma$ decay rate are twofold, namely one needs the corrections to the matrix elements of the operators O_i at the low-energy scale $\mu \approx m_b$ and also the corrections to the Wilson coefficients $C_i(\mu)$ at the scale $\mu \approx m_b$. Only the sum of the two contributions is renormalisation scheme independent and in fact, from the μ-independence of the effective Hamiltonian, one can derive a renormalisation group equation (RGE) for the Wilson coefficients $C_i(\mu)$:

$$\mu\frac{d}{d\mu}C_i(\mu) = \gamma_{ji}\,C_j(\mu)\,, \tag{15}$$

where the 8×8 matrix γ is the anomalous dimension matrix of the operators O_i.

In our first NLL project, we focussed on the calculation of the matrix elements of the operators at the scale $\mu \approx m_b$. We used the Mellin–Barnes technique to solve the two-loop integrals which were non-trivial at that time. After inserting the Mellin Barnes representations of the propagators one finds well-known Euler–Beta functions and the remaining complex integral naturally leads to an expansion in the ratio $z = (m_c/m_b)^2$. We could write the result of the 2-loop diagrams in the form

$$M = c_0 + \sum_{n,\,m} c_{nm}\left(\frac{m_c^2}{m_b^2}\right)^n \log^m\left(\frac{m_c^2}{m_b^2}\right), \tag{16}$$

with $n = 1, 2, 3, 4, ..$ and $m = 0, 1, 2, 3, ...$ The coefficients c_0 and c_{nm} are pure numbers, i.e. independent of any parameters. We found that there is no naked $\log(m_c^2/m_b^2)$ term present in Eq. (16), so the limit $m_c \to 0$ of M exists.

Of course, Christoph, as an experienced calculator, finished the calculation earlier than I did, but he showed great patience until I was able to confirm his results. In the meanwhile, he partially calculated the diagrams in the famous t' Hooft–Veltman scheme; those partial results occurred in the appendix of our joint papers [33, 135]. The main result of our analysis which was published in February 1996 consisted in a drastic reduction of the renormalisation scale uncertainty.

While the μ dependence was about $\pm 25\%$ in the leading logarithmic calculation (varying μ between $m_b/2$ and $2m_b$), it got reduced to $\pm 6\%$ when taking systematically into account the virtual corrections to the matrix elements we calculated. The term $\sim \alpha_s log(m_b^2/\mu^2)$, which caused the large scale dependence of the leading logarithmic result, is cancelled by the $O(\alpha_s)$ virtual corrections to the matrix element. In Fig. 12, the remaining μ-dependence as a function of the top quark mass m_t is shown. Moreover, Fig. 12 indicates that the central value was shifted outside the 1σ bound of the CLEO measurement [31] which reads

$$\mathcal{B}(B \to X_s\gamma)_{\text{CLEO}} = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}\,. \tag{17}$$

The first error is statistical and the second is systematic (including model dependence). However, at that time, the essential coefficient $C_7(\mu_b)$ was only known to leading-log precision. It was therefore unclear how much the overall normalisation would be changed when the NLL value for $C_7(\mu_b)$ was used.

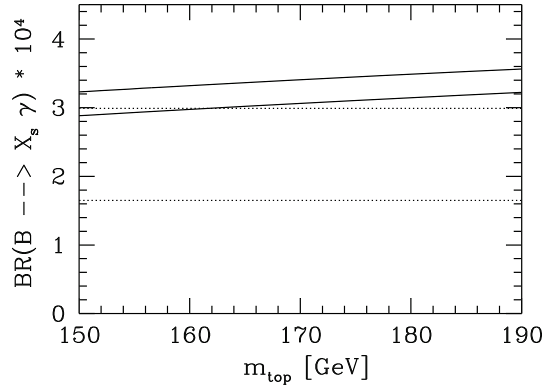

Fig. 12 Branching ratio for $B \to X_s\gamma$ as a function of m_t. The upper (lower) solid curve is for $\mu = m_b/2$ ($\mu = 2m_b$). The dotted curves show the CLEO $1-\sigma$ bounds [31]. The other input parameters are taken at their central values

Already a couple of months afterwards, in December 1996, also the NLL corrections to the Wilson coefficients were established. The improved Wilson coefficients are obtained in two steps: first, the matching at the scale $\mu = M_W$ has to be calculated including order α_s corrections. At that scale, the matrix elements of the operators in the effective theory lead to the same logarithms as the ones in the full theory (SM). Consequently, the Wilson coefficients $C_i(\mu = M_W)$ only pick up small QCD corrections, which can be calculated in fixed order perturbation theory. This step was calculated by Kassa Adel and York-Peng Yao already in 1993 [136]. The second step down to the scale $\mu \approx m_b$ has to be done using the order α_s^2 anomalous dimension matrix $\gamma^{(1)}$. Effectively, the large logarithms get summed up in the Wilson coefficients using the RG equation given in Eq. (15). This second step is the hardest one, because some entries of the anomalous dimension matrix (like γ_{27}) have to be extracted from 3-loop diagrams. This step was finalised in the winter 1996 by Konstantin Chetyrkin, Mikolaj Misiak and Manfred Munz [137]. It turned out that the crucial Wilson coefficient $C_7(\mu = m_b)$ gets only very small NLL corrections at the 6% level in the NDR scheme.

Combining all the NLL calculations the first complete theoretical prediction to NLL precision for the $\bar{B} \to X_s\gamma$ branching ratio was presented in Ref. [137]:

$$\mathcal{B}(\bar{B} \to X_s\gamma) = (3.28 \pm 0.33) \times 10^{-4}\,. \tag{18}$$

The theoretical error had two dominant sources: The μ dependence was reduced to 5% as mentioned above. Another 5% uncertainty stems from the m_c/m_b dependence. The theoretical error was half of the uncertainty in the previous LL result. And the NNL prediction was still in agreement with the CLEO measurement at the 2σ level.

Our next NLL project was the matching of the Wilson coefficients at the electroweak scale—crosschecking the result presented by Kassa Adel and York-Peng Yao in 1993 [136]. It is standard that such important calculations get checked by an independent group; however, the result achieved by Kassa Adel and York-Peng Yao was even somehow questioned in the literature [138]. Nevertheless, the outcome of our project [139] was the exact confirmation of the original result. We used the method of asymptotic expansions using dimensional regularisation based on the heavy mass expansion [140–142], but used these expansions only as a method for working out the dimensionally regularised two-loop Feynman graphs and not to get directly renormalised quantities. The idea of deriving operator product expansions using subtractions of leading asymptotics goes back to Wolfhart Zimmermann [143]. In Refs. [144, 145], an explicit method was presented how the Wilson coefficients can be directly calculated from the Feynman diagrams using the Zimmermann rearrangement. Actually, the calculation of Kassa Adel and York-Peng Yao [136] was based on the latter method.

It was only later that we realised that at least two other groups were working on this calculation and that we had emerged as the winner of this race. This is due to Christoph's determination, which was also demonstrated by the fact that he did not want to accept an invitation for a plenary talk at a major international flavour conference in Hawaii (!) before this project was completed.

During that period it came to the famous "Voloshin shock". Actually, we called it this way for a good reason: Voloshin found a -20% non-perturbative correction to the $\bar{B} \to X_s\gamma$ rate [146] which heavily questioned the dominance of the perturbative contributions, which we had just spent all our time calculating. Christoph wrote me an email to Stony Brook about it in December 1996, but he already had the right intuition when he questioned the size of the effect. Unfortunately, I cannot find the email to quote his reasoning. And indeed, further scientific work [117, 147, 148] from the spring of 1997 showed that the effect is more of the order of $+3\%$ —reassuring the importance of all our NLL efforts.

We also used our NLL calculation of the matrix elements in the context of the decay $c \to u\gamma$ [83]. As flavour-changing neutral current (FCNC), it does not occur at the tree level in the SM either. Moreover, it is strongly

GIM-suppressed at one loop. The leading QCD logarithms were known to enhance the one-loop amplitude by more than one order of magnitude. However, we showed that the amplitude increases further by two orders of magnitude after including the NLL QCD corrections. Thus, the $c \to u\gamma$ process is completely dominated by a two-loop term which had not been considered in the perturbation analysis of charm decays. Nevertheless, $\Delta S = 0$ radiative decays of charmed hadrons remain dominated by the $c \to d\bar{d}u\gamma$ and $c \to s\bar{s}u\gamma$ quark subprocesses.

4.2 The dream of discovering supersymmetry in flavour observables

In 2001, CLEO presented an improved measurement [100]. The relative error dropped significantly (see previous CLEO measurement in Eq. (17)):

$$\mathcal{B}(B \to X_s\gamma) = (3.21 \pm 0.43 \pm 0.27^{+0.18}_{-0.10}) \times 10^{-4} \, . \tag{19}$$

The first error is statistical, the second is systematic, and the third error represents the model dependence. Thus, the SM prediction and the new CLEO measurement were highly consistent with each other. This feature was and still is somehow unexpected because in principle FCNC processes like $\bar{B} \to X_s\gamma$ offer high sensitivity to new physics (NP). Additional contributions to the decay rate, in which SM particles in the loops are replaced by new particles, are not suppressed by the loop factor $\alpha/4\pi$ relative to the SM contribution. Thus, FCNC decays provide information about the SM and its extensions via virtual effects to scales presently not accessible otherwise. This approach is complementary to the direct production of new particles at collider experiments. In consequence, the $\bar{B} \to X_s\gamma$ decay rate leads to very strong bounds on the parameter space of models beyond the SM. This decay had already earned a reputation as a "new physics killer" back then.

In the pre-LHC era, supersymmetry was *the* paradigm for physics beyond the SM. From today's perspective, this mono-culture is incomprehensible, but back then it was common sense. Supersymmetry was regarded as a very attractive solution to the hierarchy problem because in a supersymmetric version of the SM, the so-called Minimal Supersymmetric Standard Model (MSSM), the sensitivity to the highest scale in the theory is eliminated by the corresponding supersymmetric partners and, thus, the low-energy theory is stabilised. There are more features in supersymmetric theories which are promising like the unification of the gauge couplings and the existence of a dark matter candidate. And supersymmetry also represents the unique extension of Poincaré symmetry. The precise mechanism of the necessary breaking of supersymmetry is unknown. A reasonable approach to this problem is the inclusion of the most general soft breaking term consistent with the SM gauge symmetries in the MSSM. This leads to a proliferation of free parameters in the theory. In fact, the decay $\bar{B} \to X_s\gamma$ is sensitive to the mechanism of supersymmetry breaking because, in the limit of exact supersymmetry, the decay rate would be just zero:

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{Exact\, SUSY} = 0. \tag{20}$$

This follows from an argument first given by Sergio Ferrara and Ettore Remiddi in 1974 [149]. In that work, the absence of the anomalous magnetic moment in a supersymmetric abelian gauge theory was shown.

The first systematic MSSM analysis of the decay $\bar{B} \to X_s\gamma$ was worked out already in 1991 [62]. Since then many constraints based on LL or NLL QCD calculations within various supersymmetric extensions of the SM were performed. And also Christoph and I worked on various analyses on supersymmetry, together but also in other collaborations. Christoph started a project on supersymmetry with Francesca Borzumati, Daniel Wyler and myself in 1999. In the MSSM, there are two kinds of new contributions to FCNC processes. The first class results from flavour mixing in the mass matrices of the sfermions, the superpartners of the fermions. There are also CKM-induced contributions from charged Higgs and chargino exchanges. We focussed on the gluino-induced contributions and first analysed how the operator basis of the SM gets enlarged and showed that LL QCD corrections are important in order to extract reliable bounds on the off-diagonal elements of the squark mass matrices [30].

Christoph's field of research can be described as precision flavour physics. In fact, Christoph is a precision calculator, which means in his case that he does not make any mistakes in his calculations. In this sense, our gluino project with Francesca and Daniel was special because I was able to correct Christoph—once in my lifetime as far as I remember—but it was only a factor in some terms.

A consistent analysis of the bounds on the sfermion mass matrix should also include interference effects between the various contributions. Together with Thomas Besmer, a PhD student in Zurich, Christoph and I analysed the interplay between the various sources of flavour violation and the interference effects of the SM, gluino, chargino, neutralino and charged Higgs–boson contributions. New bounds on simple combinations of elements of the soft part of the squark mass matrices were found to be, in general, one order of magnitude weaker than the bound on a single off-diagonal element. Our bounds were based on a systematic LL QCD analysis [150]. Surprisingly, such an analysis did not exist by then.

The next logical step was the calculation of NLL gluino corrections. This project was done by Christoph, two members of his research group in Bern, Volker Pilipp and Christof Schüpbach, as well as Matthias Steinhauser and myself sometime later. The most difficult part of this calculation was the evaluation of the two-loop diagrams in the full theory with one gluino and a virtual gluon, with two gluinos or with one gluino and a four-squark vertex [70]. From today's perspective, this high precision appears to be an overkill, since supersymmetry at the electroweak scale had and still has not yet been found. Nevertheless, we have learned all the details of the renormalisation of the MSSM to two-loop and, as it is well known, only at this level all the non-trivial features of the renormalisation procedure emerge. Therefore, at least for me, it was the project on supersymmetry with Christoph in which I learned the most about quantum field theory.

4.3 The charm mass calls for more precision

Some years after the completion of the NLL calculations of the $\bar{B} \to X_s\gamma$ decay rate in 1996, an analysis by Paolo Gambino and Mikolaj Misiak noticed that the NLL calculation has a large charm mass renormalisation scale dependence [151]. The reason is that the matrix elements of the four-quark operators through which the charm quark mass dependence dominantly enters, vanish at the lowest order (LL) and, as a consequence, the charm quark mass does not get renormalised in a NLL calculation. This means that the symbol m_c can be identified for example with $m_{c,\,\text{pole}}$ or with the $\overline{\text{MS}}$ mass $\bar{m}_c(\mu_c)$ at some scale μ_c. Changing from one to the other leads to an 11% enhancement of the branching ratio. This called for a NNLL QCD calculation.

Christoph further investigated the effect of a NLLL calculation on this problem with his colleagues from Yeravan, Armenia, Hrachia Asatrian, Artyon Hovhannisyan, and Vahagn Poghosyan and also asked me to take part, because the problem was directly connected to our first joint project on the NLL matrix elements. In 2005, we calculated those NNLL terms that are induced by renormalising the charm quark mass in the NLL expressions, i.e. those terms that are sensitive to the definition of the charm quark mass. These terms correspond to δm_c insertions in the diagrams related to the NLL order matrix elements of the four-quark operators. This way we further motivated the NNLL project because we were able to show that the error due to the charm mass renormalisation at the NNLL level would get reduced by a factor of 2 [39].

The same team, extended with Christoph's postdoc Thorsten Ewerth, took part in the global effort of the NNLL calculation of the $\bar{B} \to X_s\gamma$ decay rate. Several collaborations participated in this endeavour led by Mikolay Misiak. We calculated the NNLL QCD corrections of the matrix elements of the electromagnetic dipole operator at the low scale by directly calculating the individual cut contributions [41]. This method is extendable to all interference terms of the electromagnetic and the chromomagnetic operators. The analysis included two-, three- and also four-particle cut contributions. Following a method proposed by Charalampos Anastasiou and Kirill Melnikov [152], the phase-space representations of the delta functions was used to convert phase space integrations into loop integrations so that the standard Laporta algorithm was applicable.

Christoph had already started an intensive collaboration with the Physics Institute in Yerevan, Armenia, for his work on semileptonic penguins. Since then, Hrachia Asatrian has been a regular guest in Bern, and Christoph has never regarded this collaboration as a one-way street, but has also travelled to Yerevan on a regular basis. Hrachia told me almost every time when I met him in Bern that he and his colleagues really appreciated this.

All different NNLL corrections from various groups were then combined in September 2006 for the first estimation of the branching ratio of $\bar{B} \to X_s\gamma$ at the NNLL level [35]—10 years after the finalisation of the NLL result:

$$\mathcal{B}(\bar{B} \to X_s\gamma) = (3.15 \pm 0.23) \times 10^{-4}\,, \tag{21}$$

for $E_\gamma > 1.6$ GeV in the $\bar{B}$-meson rest frame. The four types of uncertainties: non-perturbative (5%), parametric (3%), higher order (3%) and m_c-interpolation ambiguity (3%) are added in quadrature in Eq. (21).

The renormalisation scale dependences were significantly reduced by the NNLL calculation as it is shown in Fig. 13. At that time, Belle and Babar had already measured the $\bar{B} \to X_s\gamma$ branching ratio [153, 154]. The world average performed by the Heavy Flavour Averaging Group [155] for $E_\gamma > 1.6$ GeV was

$$\mathcal{B}(\bar{B} \to X_s\gamma) = \left(3.55 \pm 0.24\,{}^{+0.09}_{-0.10} \pm 0.03\right) \times 10^{-4}\,. \tag{22}$$

Thus, it turns out that the theory estimate was about 1σ lower than the experimental average.

The theory estimate was not yet called a prediction at that time because the three-loop matrix elements of the four-quark operators have been found using an interpolation in the charm quark mass, which introduces uncertainties that are difficult to quantify. In fact, these contributions were obtained via the interpolation in the charm mass between the large-m_c asymptotic expansion and the $m_c = 0$ boundary condition that was estimated using the Brodsky–Lepage–Mackenzie (BLM) approximation.

An update of this estimate was then presented almost another decade later in 2015 [36], again by a large group of scientists including Christoph. That analysis took into account non-local power corrections—so-called resolved

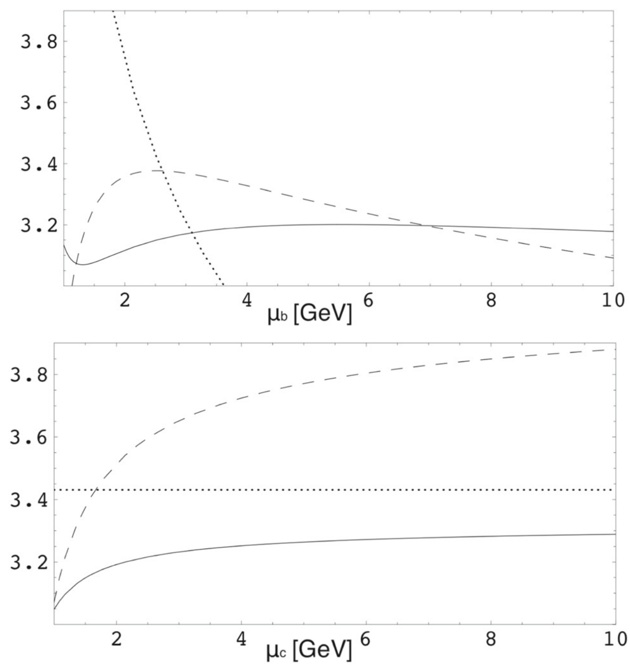

Fig. 13 Renormalisation-scale dependence of $\mathcal{B}(B \to X_s\gamma)$ in units 10^{-4} at leading log (dotted lines), next-to-leading log (dashed lines) and next-to-next-to-leading log (solid lines). The plots describe the dependence on (top) the low-energy scale μ_b and (bottom) the charm mass renormalisation scale μ_c, from [35]

contributions—which were systematically analysed and estimated to be of order $\pm 5\%$ in the meanwhile [156]. And among many other improvements, the BLM approximation was replaced by a complete calculation of the three-loop matrix elements of the four-quark operators at $m_c = 0$. The theory estimate changed to

$$\mathcal{B}(\bar{B} \to X_s\gamma) = (3.36 \pm 0.23) \times 10^{-4}, \tag{23}$$

for $E_\gamma > 1.6$ GeV. The uncertainty consists of non-perturbative ($\pm 5\%$), higher order ($\pm 3\%$), interpolation ($\pm 3\%$) and parametric ($\pm 2\%$) errors.

The measurements of the branching ratio by CLEO [100], Belle [157, 158], and Babar [159, 160] were combined by the Heavy Flavour Average Group [161] to

$$\mathcal{B}(\bar{B} \to X_s\gamma) = (3.43 \pm 0.21. \pm 0.07) \times 10^{-4}. \tag{24}$$

Obviously these new results have further sharpened the role of the decay $\bar{B} \to X_s\gamma$ in the new physics search.

Also today—another decade later—, the inclusive decay $\bar{B} \to X_s\gamma$ is still highly topical; the charm dependence at NNLL will soon be known without any interpolations and α_s corrections of the non-local (resolved) power corrections are calculated. Christoph has made many important contributions to the analysis of the inclusive decay $\bar{B} \to X_s\gamma$ over the years. I have only reviewed the projects on which I had the privilege to work with him. He is an excellent expert in his field, but remains modest. It was always a pleasure to discuss with Christoph, and I have always appreciated his openness. He is a colleague you can talk to and rely on. In the last 13 years, when we have not worked together, he has always been more than helpful with advise and comparisons of results. I hope that Christoph will stay in touch with the physics community and wish him all the best for this new phase, good health, happiness and time for all the activities he enjoys.

5 Weak radiative decays of charm and bottom quarks

Authors: Mikołaj Misiak
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A milestone in the determination of Next-to-Leading Order (NLO) QCD corrections to $b \to s\gamma$ was the evaluation of two-loop matrix element of the four-quark operator $(\bar{s}_L\gamma^\alpha c_L)(\bar{c}_L\gamma_\alpha b_L)$ by Christoph Greub, Tobias Hurth and Daniel Wyler [33, 135]. Soon after, it was observed by the same authors and myself [83] that analogous two-loop diagrams in the $c \to u\gamma$ case give the dominant contribution to the perturbative decay rate, not just a correction. In this section, I begin with recalling the arguments why this is the case, and to what extent this observation might

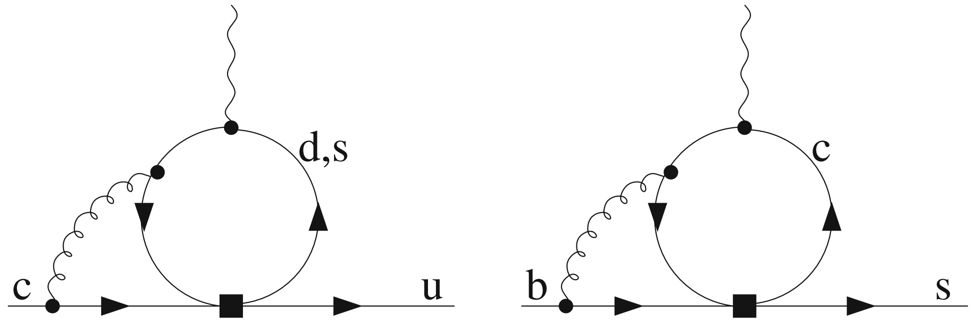


Fig. 14 Sample two-loop WET contributions to $c \to u\gamma$ from $Q^{d,s}_{1,2}$ (left), and to $b \to s\gamma$ from $Q^{c}_{1,2}$ (right)

be relevant for phenomenology. Next, I give a brief summary of Christoph's contributions to the never-ending calculation of Next-to-Next-to-Leading Order (NNLO) QCD corrections to $b \to s\gamma$ in the Standard Model (SM).

The $c \to u\gamma$ and $b \to s\gamma$ transitions are most conveniently studied in the framework of the Weak Effective Theory (WET) obtained from the SM via decoupling of the W boson and all the heavier particles. The most important WET operators for the considered processes read

$$\begin{array}{l|l}
\qquad\quad c \to u\gamma & \qquad\quad b \to s\gamma \\
Q_1^d = (\bar{u}_L\gamma^\alpha T^a d_L)(\bar{d}_L\gamma_\alpha T^a c_L) & Q_1^c = (\bar{s}_L\gamma^\alpha T^a c_L)(\bar{c}_L\gamma_\alpha T^a b_L) \\
Q_2^d = (\bar{u}_L\gamma^\alpha d_L)(\bar{d}_L\gamma_\alpha c_L) & Q_2^c = (\bar{s}_L\gamma^\alpha c_L)(\bar{c}_L\gamma_\alpha b_L) \\
Q_1^s = (\bar{u}_L\gamma^\alpha T^a s_L)(\bar{s}_L\gamma_\alpha T^a c_L) & Q_7 = \frac{e m_b}{16\pi^2}(\bar{s}_L\sigma^{\alpha\beta} b_R)F_{\alpha\beta} \\
Q_2^s = (\bar{u}_L\gamma^\alpha s_L)(\bar{s}_L\gamma_\alpha c_L) & Q_8 = \frac{g_s m_b}{16\pi^2}(\bar{s}_L\sigma^{\alpha\beta} T^a b_R)G^a_{\alpha\beta}
\end{array}$$

where T^a stand for the $SU(3)_c$ generators. The dominant contribution to $b \to s\gamma$ comes from Q_7. Its Wilson coefficient at the electroweak scale receives the Leading Order (LO) contribution only at the one-loop level in the SM. Therefore, it is sensitive to beyond-SM physics even in models with SM-like flavour violation. As pointed out in Ref. [83], the analogous operator in the $c \to u\gamma$ case is strongly suppressed in the SM. At the electroweak scale, the suppression factors come from the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements, as well as from ratios of the light quark masses to the W-boson mass. The non-CKM suppression factors are no longer powerlike once the two-loop renormalisation group evolution down to the charm scale is taken into account, which enhances the corresponding amplitude by more than an order of magnitude. Nevertheless, the dominant contributions to $c \to u\gamma$ come from CKM-unsuppressed two-loop on-shell matrix elements of $Q^{d,s}_{1,2}$, given by diagrams like the one depicted in Fig. 14 (left).

The diagrams with the down- and strange-quark loops come with CKM factors that are almost equal in magnitude but have opposite signs. In effect, the total perturbative amplitude vanishes in the limit $m_d = m_s$. Its exact dependence on the quark masses can easily be extracted from the analogous calculation performed in Refs. [33, 135] in the $b \to s\gamma$ case. The dominant terms in this amplitude behave like

$$z_s \log^n(z_s) - z_d \log^n(z_d),$$

where $z_q = \frac{m_q^2}{m_c^2}$, and $n = 0, 1, 2, 3$. Despite the suppression by z_q, the considered amplitude is more than two orders of magnitude larger than the seemingly LO one coming from CKM-suppressed diagrams—see Ref. [83] for more details.

Such major differences between the perturbative $c \to u\gamma$ and $b \to s\gamma$ results imply that the role of non-perturbative effects is very different, too. In the $b \to s\gamma$ case, the dominant contribution to the inclusive hadronic decay $\bar{B} \to X_s\gamma$ originates from the dipole operator Q_7, in which case the operator product expansion for inclusive heavy-meson decays is well understood, and non-perturbative effects give only corrections of order Λ^2/m_b^2. The same is true for short-distance contributions from the charm loops like the one in Fig. 14 (right), while the so-called resolved photon effects need to be treated in the framework of Soft-Collinear Effective Theory (SCET) [156]. A similar SCET treatment in the case of inclusive $D \to X\gamma$ would come with considerably larger uncertainties. Moreover, such an inclusive decay receives important contributions from tree-level quark subprocesses that screen the loop-generated $c \to u\gamma$ contribution. Consequently, one might hope for constraining beyond-SM physics with $c \to u\gamma$ only in models with non-SM-like flavour violation, where no CKM-suppression of the dipole operator contribution takes place.

At the time when Ref. [83] was still under review in the Physics Letters B, I drove overnight from the Sequoia National Park down to SLAC, to give a talk on yet unpublished three-loop anomalous dimensions for $b \to s\gamma$. Christoph, who worked then as a postdoc at SLAC, arranged accommodation for me at the place where his former landlady lived. In the morning before my talk, he arranged a tour of the SLAC accelerator complex and the BABAR construction site for just the two of us, guided by Helen Quinn. She gently tested our knowledge on applications of classical electrodynamics to linear accelerator technology. Christoph was always successful in responses, which allowed me to just nod my head and avoid uncovering my ignorance. During lunch, the SLAC theory group members asked so many questions concerning the topic of my coming talk that they could safely fall asleep during the actual seminar. Christoph was one of the very few people in the audience who remained awake until the very end. In the afternoon, he drove me around San Francisco and surroundings, including the Golden Gate bridge. We ended up for dinner in a restaurant that was famous for ribs—his beloved dish.

Since then, a noticeable fraction of his and my research programme was devoted to calculating various NNLO perturbative contributions to $\bar{B} \to X_s\gamma$. We worked in separate groups but often in a coordinated manner. Every 9 years, the results were published in common summary papers [35, 36, 163]. Most recently, the very NLO calculation of Refs. [33, 135] has been extended to the NNLO level. It required calculating four-loop propagator diagrams with two-, three- and four-body unitarity cuts for the physical value of the charm quark mass, to avoid the previously applied interpolation in m_c between the $m_c = 0$ and $m_c \gg m_b$ limits. The two-body cut contributions evaluated independently in Refs. [46, 49], in Ref. [47], and in Ref. [48] were found to be in perfect agreement. Evaluation of the three- and four-body contributions followed by ultraviolet renormalisation of the considered correction [162] has recently been completed by the authors of Ref. [48]. An updated SM prediction for the branching ratio is already available and will soon be published [163].

6 First steps to NNLO for $\overline{B} \to X_s\gamma$

Authors: Matthias Steinhauser
Karlsruhe Institute of Technology (KIT)

It was in fall 1998, the start of my second postdoc position at the University of Bern when I got to know Christoph Greub in person. I knew the name but I had never met him before. In Bern, I started to work on B physics and was introduced from Christoph into the process $b \to s\gamma$.

At the end of the 1990s, the most important next-to-leading-order (NLO) contributions to $b \to s\gamma$ were known and Christoph already made several crucial contributions. Among them was the two loop calculation of the matching coefficients C_7 and C_8 [139] and the NLO contribution to the matrix elements for the interference of the four-quark operators $Q_{1,2}$ and the dipole operator Q_7 [33, 135]. Around that time, it was also realised that NLO is not sufficient to match the experimental precision from the theory side and several groups thought about first attempts towards NNLO.

Christoph and I, together with his PhD student, Kai Bieri, started to work on the natural extension of the NLO $Q_{1,2} - Q_7$ interference and considered the corrections induced by closed light fermion loops. After integrating over the momentum running in the fermion loop, one obtains expressions which are close to those obtained at NLO. The difference is a non-integer exponent of the gluon propagator. At this point, one introduces Feynman parameters and one obtains multidimensional integrals which contain denominators which involve both the bottom and charm quark masses, m_b and m_c. Next one introduces a Mellin–Barnes integral which factorises these denominators. Aftwards, all parameter integrals can be solved and one obtains an expansion in m_c/m_b which converges quite fast at the physical point $m_c/m_b \sim 0.3$. The results are published in Ref. [38]. This was the first contribution to the $Q_{1,2} - Q_7$ interference at NNLO, one of the most complex NNLO contributions. Even today, more than 20 years later, the NNLO results are not complete. All fermionic contributions (i.e. also those with a closed charm and bottom quark loop) have been computed in Refs. [164, 165], and for the non-fermionic corrections, the two-particle cut contribution has been published in Refs. [46–48], again with important contributions from Christoph. The three- and four-particle cut contribution are still missing.

The light-fermion contribution from Ref. [38] was an important ingredient to construct an approximation for the NNLO $Q_{1,2} - Q_7$ interference, a central contribution for the first NNLO prediction of the branching ratio for $\bar{B} \to X_s\gamma$ which was published in 2006 [35], my second paper together with Christoph. On the basis of the light-fermion contribution, it was possible to obtain exact results for the so-called BLM contribution [166]. For the remaining part, an extrapolation from large charm quark mass to the physical point was used. Analytic results for $m_c \gg m_b$ were obtained using effective-field theory methods [167].

A few years later, Christoph and I started a second project, together with Tobias Hurth, Volker Pilipp and Christof Schüpbach, about supersymmetric corrections to $b \to s\gamma$ and $b \to sg$. The Wilson coefficients at the high scale for various versions of magnetic and chromomagnetic operators which are induced by a squark–gluino exchange have been computed. The technically most challenging part was the evaluation of the two-loop diagrams since they lead to huge analytic expressions. The results were published in form of a C++ code which can be used as building block for the phenomenological next-to-leading logarithmic analyses of the branching ratio $\bar{B} \to X_s\gamma$ in supersymmetric models beyond minimal flavour violation.

The fourth and to date last common paper with Christoph appeared in 2015 [36]. In this reference, an update of the 2006 analysis was published after several further NNLO ingredients became available. On the one hand, for the $Q_{1,2} - Q_7$ interference, the result for $m_c = 0$ was computed in Ref. [168], and thus, instead of an extrapolation, an interpolation could be used. Furthermore, important contributions have been provided by Christoph and his collaborators, as, e.g. effects of the charm and bottom quark masses in loops on the gluon lines in the $Q_7 - Q_7$ interference [40] and a complete calculation of the $Q_7 - Q_8$ interference [42].

Unfortunately, I could stay in Bern only for 1 year. I have enjoyed my stay a lot. It is extremely pleasant to work with Christoph. I am particularly impressed by his calm and precise way to perform complicated calculations and it happened very rarely (if at all) that he made any mistake.

7 Three-loop $b \rightarrow s\gamma$ calculations

Authors: Julian Eicher and Lukas Born
University of Bern

During 1 year of our master's degree, we supported Christoph's endeavour of calculating the full set of $\mathcal{O}(\alpha_s^2)$ corrections to the decay amplitude $b \rightarrow s\gamma$. This process is of special interest in the search for new physics as it is induced only at one-loop level in the Standard Model (SM), thus providing high sensitivity to heavy new particles running in that loop, such as an additional Higgs doublet. Moreover, the kinematics involved are very simple due to the two-body nature of this process. As a matter of fact, in the approximation where the mass of the strange quark is set to zero, the only remaining kinematic variable is the ratio between the charm and bottom quark mass squared, which we denote as $z = m_c^2/m_b^2$ in the following.

In the SM, the decay $b \rightarrow s\gamma$ involves the full set of particles

$$\{t,\, W,\, b,\, c,\, s,\, u,\, g,\, \gamma\}\,.$$

However, since we are only interested in physics at the bottom quark mass scale m_b, we can integrate out the heavier particles, i.e. the top quark and the W boson. This leads to a new set of particles

$$\{b,\, c,\, s,\, u,\, g,\, \gamma\}\,,$$

which are now the building blocks for an effective theory. UV corrections from the top quark and the W boson are absorbed into Wilson coefficients C_i which multiply the effective operators.

In total, the SM process $b \rightarrow s\gamma$ gives rise to eight effective operators up to dimension six, so the effective Lagrangian can be written as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{QED}} + \frac{4G_F}{\sqrt{2}}\, V_{ts}V_{tb}^* \sum_{i=1}^{8} C_i\, O_i\,, \tag{25}$$

with G_F the Fermi constant and $\mathcal{L}_{\text{QCD,QED}}$ only containing five flavours since the top quark is integrated out. In our master's thesis, we focussed on contributions from the operator

$$O_2 = (\bar{c}_L^\alpha\, \gamma^\mu\, b_L^\alpha)\, (\bar{s}_L^\beta\, \gamma_\mu\, c_L^\beta)\,, \tag{26}$$

since the others are either already calculated or suppressed by a small Wilson coefficient. In order to simplify our calculations, we decompose the total amplitude of the process $b \rightarrow s\gamma$ in terms of form factors F_i,

$$\mathcal{A} \;=\; \bar{u}_s(p_s)\, P_R\, [F_1\, q^\mu + F_2\, p_b^\mu + F_3\, \gamma^\mu]\, u_b(p_b)\, \varepsilon_\mu =: A^\mu\, \varepsilon_\mu\,, \tag{27}$$

with $\bar{u}_s(p_s)$ and $u_b(p_b)$ the spinors of the strange and the bottom quark, respectively. Furthermore, the external photon carries momentum q^μ and is polarised along ε^μ. Using $q^2 = 0$, $q^\mu \varepsilon_\mu = 0$, $q^\mu A_\mu = 0$ and some Dirac algebra, the total amplitude becomes proportional to the second form factor:

$$\mathcal{A} \propto F_2\,. \tag{28}$$

It is therefore sufficient to compute the part of the Feynman diagrams which is proportional to $p_b^\mu \varepsilon_\mu$ in order to obtain the full amplitude.

We extract the contribution to F_2 by contracting the amplitude of any given Feynman diagram with a set of projection operators and then simplifying the Dirac algebra. After this tensor reduction, which was implemented in `REDUCE`, the form factor is given as a linear combination of roughly 1000 scalar three-loop integrals of the type

$$I(\alpha_1, \ldots, \alpha_{12}) = \int \left[\prod_{i=1}^{3} \frac{\mathrm{d}^d k_i}{(2\pi)^d} \right] \frac{1}{P_1^{\alpha_1} \cdots P_{12}^{\alpha_{12}}} \,. \tag{29}$$

However, these scalar integrals are not all independent and they can be reduced to a smaller set of master integrals (MIs) using integration-by-parts relations and Lorentz identities. For this reduction, we use the software `Kira` [169, 170], which implements Laporta's algorithm. The form factor thus becomes a sum of these MIs, which we denote by J_n:

$$F_2 = \sum_n c_n(z, d) J_n \,. \tag{30}$$

We employ the method of differential equations to solve the MIs. By differentiating the MIs with respect to z and then reducing the resulting scalar integrals back to MIs, we obtain a differential equation (DE) of the form

$$\frac{\partial \vec{J}}{\partial z} = A(z, \epsilon)\, \vec{J} \,. \tag{31}$$

Here, $\vec{J} = (J_1, \ldots, J_N)$ is the vector of all MIs, A is an $N \times N$ matrix, and ϵ is related to the spacetime dimension by $d = 4 - 2\epsilon$. Typically such DEs are solved by transforming the system to a *canonical basis*, where the matrix factorises as $A(z, \epsilon) \to \epsilon A(z)$. The solutions are then obtained in a straightforward way in terms of iterated integrals. However, such transformations could not be found for all relevant diagrams and a different method was required to solve the DE. This new method, which is based on obtaining solutions as a series expansion around $z = 0$, became the focal point of our thesis.

We start off by bringing the DE into a more suitable form. Since the matrix A contains poles of the type $1/z^n$ (where $n \in \mathbb{N}$), we perform a first transformation to eliminate all the singularities with $n > 1$. Then, it is clear that the expansion of the matrix starts at $\mathcal{O}(1/z)$:

$$A = \frac{1}{z} A^{(-1)} + A^{(0)} + z A^{(1)} + z^2 A^{(2)} + \ldots \tag{32}$$

We then transform the DE a second time in order to cast the leading-order matrix $A^{(-1)}$ into block-diagonal form. With this setup, it is straightforward to obtain a leading-order solution $\vec{J}_{\mathrm{LO}}$ to the DE by only considering the $\mathcal{O}(1/z)$ part

$$\frac{\partial \vec{J}_{\mathrm{LO}}}{\partial z} = \frac{1}{z} A^{(-1)}\, \vec{J}_{\mathrm{LO}} \,. \tag{33}$$

Based on this leading-order result, we construct higher order solutions by making an power series ansatz in z and $\log(z)$. After substituting it into the DE and comparing coefficients in z and $\log(z)$, we obtain systems of linear equations for the coefficients in the ansatz, which can be readily solved. Lastly, we use `FIESTA5` [171, 172] to numerically evaluate certain parts of the MIs in order to fix the boundary conditions.

Once the MIs are determined, we plug them into the expression for the form factor F_2, from which the full amplitude can be reconstructed. To check the consistency of our approach, we additionally evaluate all MIs numerically at various values of z to obtain the form factor F_2 in an alternative, independent way. The plots in Fig. 15 show the $\mathcal{O}(\epsilon^0)$ parts of F_2 for two different diagrams. The blue lines denote our semi-analytical expansion in z and the black dots mark the numerical checks.

Here, the numbers 26 and 27 refer to the two diagrams shown in Fig. 16, so the gluons only attach to the b-quark leg instead of the s-quark. Within the scope of our thesis, we calculated the diagrams 1, 2, and 26–44. For all these diagrams, we find good agreement between the purely numerical results and our expansion approach. It is important to note that due to the expansion around $z = 0$, our semi-analytical result is of course only valid for small z.

We would like to take this opportunity to thank Christoph once again for his great support, without which our master's thesis would not have been possible. In weekly meetings, he taught us with a lot of patience, enthusiasm and a good dose of humour the technical details of the calculations outlined above. He was there for us when we were desperately trying to make sense of some obscure error messages or when we needed guidance elsewhere.

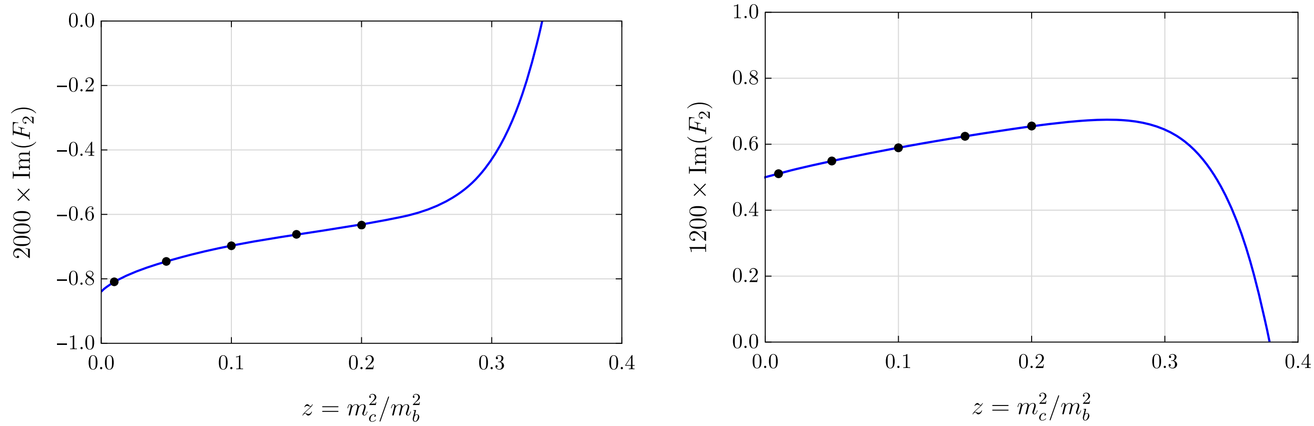
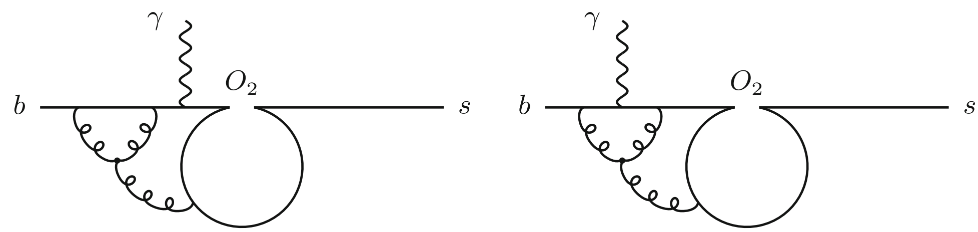


Fig. 15 Finite parts of F_2 for the diagrams 26 (left) and 27 (right) as a function of $z = m_c^2/m_b^2$

Fig. 16 Diagrams 26 (left) and 27 (right) of Ref. [49]



8 NLO-QCD corrections to inclusive non-leptonic decays

Authors: Alexander Lenz

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At the end of 1994, I started working as a student in Andrzej Buras' group at the TU Munich, where I met, among others, Uli Nierste, who was about to complete his PhD, working mainly on NLO-QCD corrections to kaon mixing. Although Uli would leave in 1996 for DESY and then in 1998 for Fermilab, he de-facto became the supervisor of my diploma and my PhD thesis.

The computation of NLO-QCD corrections to different flavour observables was a hot topic at that time, which would soon become the state-of-the-art **and** we also had some *B anomalies* with much better names than R_K, P_5', ... : we had the *missing charm puzzle* [173] and the *baffling semileptonic branching fraction* [174]. These anomalies were related to inclusive decays of B hadrons: i) the average number of charm quarks per inclusive b-quark decay, n_c, ii) the inclusive semileptonic branching fraction, $B_{\rm sl} = \Gamma_{\rm sl}/\Gamma_{\rm tot}$ and in both cases experiment differed from the theory expectation. A few months before my start in Buras' group, Bagan, Ball, Braun and Gosdzinsky had finished the NLO-QCD corrections to the non-leptonic decay $b \to c\bar{u}d$ [175] and again a few months later, they (Volodya left the project and Fiol joined) could also perform the calculation with two massive quarks in the final state [176], i.e. $b \to c\bar{c}s$. With these results at hand, one could calculate n_c

$$n_c = 2 - Br(b \to 1c) - 2Br(b \to 0c)\,, \tag{34}$$

and $B_{\rm sl}$ and these were found to be in disagreement with measurements. Equation (34) emphasises the importance of decays into charm-less final states.

As currently, some people are playing with, e.g. leptoquarks to explain P_5', while others do the hard work to improve the standard model (SM) precision of the hadronic contributions to the $b \to sll$ transitions, also in the 1990s, some people were, e.g. playing with beyond-SM scenarios to increase, e.g. the $b \to sg$ transition, while others were trying to further increase the theoretical precision, by completing the NLO-QCD corrections, beyond the gluon corrections to the tree-level operators $Q_{1,2}$ from the effective Hamiltonian. However, I do not remember if we had a big fitting community in 1995.

Christoph and myself had the first scientific overlap in the determination of important NLO-QCD corrections to $Br(b \to 0c)$: as part of my Diploma thesis Uli, Gaby Ostermaier and myself were determining new NLO-QCD corrections to non-leptonic b-decays steming from insertions of the operators $Q_{1,2}$ into penguin diagrams in the effective theory [177, 178]. We found, e.g. a sizeable modification of the branching ratio for the $b \to c\bar{c}s$, by around 5% of the tree-level contribution. Christoph and his PhD student Patrick Liniger were completing the

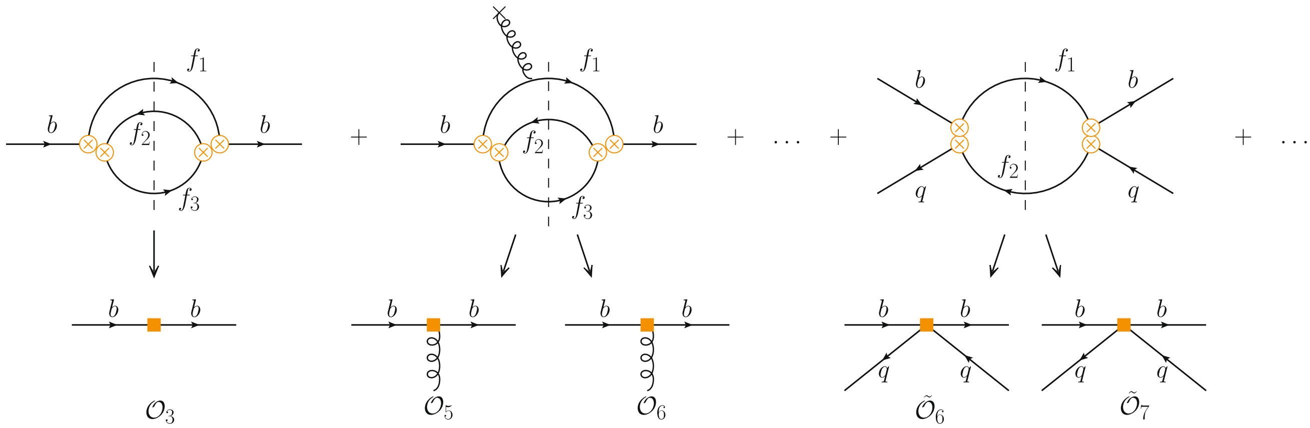

Fig. 17 Schematic representation of the HQE

NLO-QCD corrections to the $b \to sg$ transition [37, 179], where he could use his expertise in the computation of QCD corrections for $b \to s\gamma$. Both calculations softened the missing charm puzzle a little, but it actually never got really resolved, mostly because it was not really clear how to determine the inclusive quantity n_c experimentally. Almost 2 decades later, I was temporarily occupying the chair of Andrzej Buras at TUM for 1 year and I started a project with the Diploma students Fabian Krinner and Thomas Rauh. The scope was to program the NLO-QCD corrections to $b \to c\bar{c}s$; however, we found obvious typos in the equations of the original paper [176] and tried to get in contact with the authors. Unfortunately, they either had left physics, or switched research field so that no documentation of their computation could be found. In the end, we had to recompute these contributions again in 2013 [180]. Another decade later, in summer 2024, Egner, Fael, Schönwald, and Steinhauser managed the tremendous task of calculating the NNLO-QCD corrections to the decays $b \to c\bar{u}d$ and $b \to c\bar{c}s$ [181]; in particular, they determined the α_s^2 corrections to the insertion of the operators $Q_{1,2}$ in tree-level diagrams in the effective theory. NNLO-QCD corrections to the insertion into penguin diagrams and to $b \to sg$ are still missing—maybe a retirement challenge for Christoph?

So far, we have only considered decays of free b-quarks, but actually the b-quark is bound into, e.g. a B_q meson ($q = u$, d, s). The total decay rate, $\Gamma_{tot} = 1/\tau$ of hadrons containing a heavy quark can be described by the heavy quark expansion (HQE)[10] with the free b-quark decay as the leading contribution.

The HQE originates from a double insertion of the effective Hamiltonian, see Fig. 17, and schematically reads

$$\Gamma(B_q) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + ... + 16\pi^2 \left(\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + ... \right), \tag{35}$$

where Γ_i are short-distance functions which can be computed perturbatively in QCD, i.e.

$$\Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_i^{(1)} + \left(\frac{\alpha_s}{4\pi} \right)^2 \Gamma_i^{(2)} + \dots , \tag{36}$$

and $\langle \mathcal{O}_i \rangle \equiv \langle B_q | \mathcal{O}_i | B_q \rangle / (2 m_{B_q})$ denote the matrix elements of the corresponding $\Delta B = 0$ operators $\mathcal{O}_i$ in the effective theory. Γ_3 denotes the free b-quark decay discussed above,[11] and starting from order $1/m_b^3$, which is equivalent to operators with mass dimension six, we can directly involve the spectator quark of the decaying meson, see the right diagram of Fig. 17. This contribution originates from 1-loop diagrams instead of 2-loop diagrams—thus the explicit $16\pi^2$ enhancement in Eq. (35)—and the corresponding operators $\tilde{\mathcal{O}}_6$ consist of two heavy b-quark fields and two light quark fields.

The on-shell contribution to mixing of neutral B_q mesons, i.e. the decay rate difference $\Delta\Gamma_q$, can be described by a similar HQE, containing, however, only the $16\pi^2$ terms in Eq. (35) and instead of $\Delta B = 0$ operators, $\Delta B = 2$ operators. In 1996, Beneke, Buchalla and Dunietz have determined the LO-QCD corrections to $\tilde{\Gamma}_6$ and $\tilde{\Gamma}_7$ for $\Delta\Gamma_s$ [184]. At that point of time, the values of the matrix elements $\langle \tilde{\mathcal{O}}_{6,7} \rangle$ were more or less completely

[10] See, e.g. Ref. [182] for a review of the historical development and Ref. [183] for an overview of the state-of-the-art in lifetimes and mixing.

[11] $\Gamma_5 \langle \mathcal{O}_5 \rangle / m_b^2$ denotes corrections due to the kinetic and chromomagnetic operator and $\Gamma_6 \langle \mathcal{O}_6 \rangle / m_b^3$ corrections due to the Darwin operator. These operators are composed of two b-quark fields and strings of covariant derivatives.

unknown and one mostly had to rely on vacuum insertion approximation. The main part of my PhD thesis was the determination of NLO-QCD corrections to $\tilde{\Gamma}_6$ [185]. Originally Uli planned to do this project with Christoph and me, but since Martin Beneke and Gerhard Buchalla mentioned in their 1996 paper that they also planned to do that calculation, we asked them to join forces. This computation [185] was the first determination of α_s-correction to power suppressed terms in the HQE. We found that $\tilde{\Gamma}_6^{(1)}$ is numerically important, and that $\tilde{\Gamma}_6^{(1)}$ is IR safe, which was of conceptual interest for the consistency of the HQE. For a long time, our calculation was the standard reference for $\Delta\Gamma_s$, the 2024 status for $\Delta\Gamma_s$ is discussed in Ref. [183]. There are now several lattice evaluations of the matrix elements $\langle\tilde{\mathcal{O}}_6\rangle$ [186–188] and also a first study of $\langle\tilde{\mathcal{O}}_7\rangle$ [189]. Together with Thomas Rauh who was from 2016 till 2018 a postdoc in my group in Durham (before moving to Bern) and the PhD students Matthew Kirk and Danny King, we determined the matrix elements $\langle\tilde{\mathcal{O}}_6\rangle$ with 3-loop HQET sum rules [190, 191]. Uli continued working on perturbative calculations, and together with Gerlach, Shtabovenko and Steinhauser, he determined the NNLO-QCD corrections, $\tilde{\Gamma}_6^{(2)}$ [192]. Future improvements can in particular be made by determining $\langle\tilde{\mathcal{O}}_7\rangle$ and $\tilde{\Gamma}_7^{(1)}$—again potential projects for Christoph.

In 1996, the ratios of lifetimes of heavy mesons, like $\tau(B^+)/\tau(B_d)$ were known to LO-QCD, i.e. $\tilde{\Gamma}_6^{(0)}$, e.g. Ref. [193, 194], while there was no first principle calculation of the matrix elements $\langle\tilde{\mathcal{O}}_6\rangle$. In 2001, Martin, Gerhard, Christoph, Uli and myself joined forces again to determine the NLO-QCD corrections $\tilde{\Gamma}_6^{(1)}$ for the $\Delta B = 0$ case [195]. This was particular exciting, since we got to know that our friends in Rome, Franco, Lubicz, Mescia and Tarantino were working on the same calculation. In the end, we submitted on the 12th February 2002, while the Rome group on the 8th March. Therefore, partly we won the race, partly Rome won by adding more topics like the investigation of the ratios $\tau(B_s)/\tau(B_d)$ and $\tau(\Lambda_b)/\tau(B_d)$. The current status for the lifetime ratio $\tau(B^+)/\tau(B_d)$ is also discussed in Ref. [183]: $\tilde{\Gamma}_7^{(0)}$ was calculated in 2003/4 by Gabbiani, Onishchenko and Petrov [196], and the matrix elements $\langle\tilde{\mathcal{O}}_6\rangle$ have been determined with HQET sum rules by Thomas Rauh, Matthew Kirk, Danny King and myself in 2017 and 2021 [190, 197], while first reliable lattice results for these quantities are imminent [198, 199]. Here, the retirement task for Christoph could be the NNLO-QCD corrections $\tilde{\Gamma}_6^{(2)}$.

By now, my work with Christoph has received an appropriate attention in the community. However, in 2004, we did not feel so. In particular, our experimental friends were mostly citing the LO-QCD works of Bigi and Uraltesev for lifetimes and mixing. At that point of time, we got an invitation for a plenary talk at the 3rd edition of *FPCP* in October 2004 in Daegu and Ikaros Bigi (University of Notre Dame, Indiana) was also attending this conference. For whatever reason, my senior colleagues Martin, Gerhard, Christoph and Uli, decided that this might be a good opportunity for the most junior member of the collaboration, to set things clear for the experts in the field: *"Bigi et al. were the pioneers, but our NLO-QCD corrections are the state-of-the-art and should be used and cited"*. I was quite nervous, since I had no idea how Prof. Bigi, probably an American with Greek origins, will react. Therefore, I gave my talk and I noticed an interested and inquisitive member of the audience, who probably was Bigi. I survived and after the talk people were heading to the coffee break and at the exit door Ikaros came next to me and asked in perfect Bavarian dialect: "Kimmsd du aus Obabaian oder Nidabaian?",[12] which was the start of a very nice friendship.

Christoph, I hope you enjoyed this trip of 30 years into the past and you will successfully fulfil all the above-assigned retirement tasks.

9 Contributions of the W-boson propagator to μ and τ leptonic rates

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The collaboration that led to the paper "Contributions of the W-boson propagator to μ and τ leptonic decay rates" [200] started with an email that Christoph sent me in June 2013. Christoph had recalculated the corrections to the muon decay proportional to the ratio m^2/M_W^2 in the Standard Model, where m is the electron mass and M_W the mass of the W boson mediating the tree-level process. These corrections are tiny in comparison to the NLO QED corrections to the decay in the Fermi theory. Nevertheless, these corrections were reported in several places in the literature, in the context of the study of the μ decay or of the closely related τ leptonic decay $\tau^- \to \mu^- + \bar{\nu}_\mu + \nu_\tau$ [201–203]. Christoph found a discrepancy between his calculation and the literature. Following a suggestion by Massimo Passera, Christoph asked me if I had ever evaluated these corrections, since I had been a student of Professor Alberto Sirlin, a leading expert in muon decay.

[12] From my accent in English, he deduced that I originate from Bavaria. It was only not clear to him from which subpart—5 of the 7 subparts he could also exclude.

I had not, but I was in contact with Sirlin and I was visiting him at NYU from time to time, therefore I decided to ask him about this issue. Sirlin and I decided to carry out the calculation that Christoph had done and we confirmed Christoph's result. Independently, also Passera confirmed Christoph's result. In collaboration with Christoph and a graduate student at CUNY, Zhibai Zang, Sirlin and I decided to see if it was possible to extend the calculation to include the full dependence of the tree-level muon decay in the Standard Model (SM) on the masses of the W boson, electron and muon. This turned out to be relatively straightforward. However, surprisingly, the results did not seem to be present in the literature. For this reason, we decided to write a brief paper to illustrate the results [200], which are summarised below.

If one sets the electron mass m to zero, the muon decay width in the SM can be written as

$$\Gamma_{(W)} = \Gamma_0 \left\{ \frac{12}{x^3} \left[1 - \frac{x}{2} - \frac{x^2}{6} + \frac{(1-x)}{x} \ln(1-x) \right] \right\}, \tag{37}$$

where $x = M^2/M_W^2$, M indicates the muon mass and,

$$\Gamma_0 = \frac{G_\mu^2 M^5}{192\pi^3} [1 + \delta_\mu] \,. \tag{38}$$

In addition,

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8M_W^2} (1 + \Delta r) \,, \tag{39}$$

where g is the SU $(2)_L$ gauge coupling constant, Δr the electroweak correction introduced by Sirlin in [204], while δ_μ represents the QED correction to muon decay evaluated in the Fermi V-A theory [205–214]. The result in (37) can be conveniently written as an infinite sum

$$\Gamma_{(W)} = \Gamma_0 \sum_{n=0}^{\infty} \frac{12x^n}{(n+3)(n+4)} = \Gamma_0 \left\{ 1 + \frac{3}{5}x + \frac{2}{5}x^2 + \frac{2}{7}x^3 + \frac{3}{14}x^4 + \frac{x^5}{6} + \cdots \right\}. \tag{40}$$

The second term in the r.h.s. of equation (40) is the well known corrections $3/5M^2/M_W^2$ derived by Lee and Yang in the context of the non-local Fermi Theory [215].

When terms proportional to the electron mass are kept, one finds that decay width can be written as

$$\Gamma_{(W)} = \Gamma_0 \left[F(y) + \frac{3}{5}\frac{M^2}{M_W^2} - \frac{3m^2}{M_W^2} + \mathcal{O}\left(\frac{m^4}{M_W^2 M^2} \right) \right]. \tag{41}$$

where

$$y = \frac{m^2}{M^2} \,, \tag{42}$$

and

$$F(y) = 1 - 8y - 12y^2 \ln y + 8y^3 - y^4 \,, \tag{43}$$

is a well-known tree-level phase-space factor correction to the $\mathcal{O}(x^0)$ decay width. If one decides to factor out the phase-space factor $F(y)$, one finds

$$\Gamma_{(W)} = \Gamma_0 F(y) \left[1 + \frac{3}{5}\frac{M^2}{M_W^2} + \frac{9}{5}\frac{m^2}{M_W^2} + \mathcal{O}\left(\frac{m^4}{M_W^2 M^2} \right) \right]. \tag{44}$$

The term proportional to $9/5m^2/M_W^2$ in (44) is the one where Christoph identified the discrepancy with the literature available at the time, where the factor 9/5 was replaced by a factor -2 [201–203]. However, it must be observed that both for the τ-leptonic decay and for the μ decay terms of order $9/5m^2/M_W^2$ (where m is the mass of the final state lepton, i.e. the electon mass in muon decay and the muon mass in τ decay $\tau^- \to \mu^- + \bar{\nu}_\mu + \nu_\tau$) are smaller than the current experimental errors affecting the lifetime measurements for muon and τ leptons.

It is nevertheless interesting to point out that corrections induced by the tree-level W boson propagator in the SM do play a role in the relation between the quantity G_μ defined in (39) and the traditional definition of the Fermi constant G_F, which is traditionally obtained from the muon lifetime through the relation

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} F(y)[1+\delta_\mu]\,, \tag{45}$$

so that it can be shown that [200]

$$G_F^2 = G_\mu^2\big(1+\delta_{(W)}\big)\,, \tag{46}$$

where $\delta_{(W)}$ indeed indicates the additional correction induced by the W-boson propagator in the SM.

Finally, in the Appendix of [200], it was possible to provide a closed expression for the tree-level muon decay in the SM retaining the full dependence on $M_W > M > m$. It was shown that in order to avoid large cancellations among terms, this result is best expressed as an infinite series:

$$\begin{aligned}\Gamma_{(W)} = \Gamma_0 \Bigg\{ & F(y) + \frac{3}{5}x(1-y)^5 + \frac{x^2}{20}\Big(8 - 27y + 27y^5 \\ & - 8y^6 - 60y^3 \ln y\Big) + 3\sum_{n=3}^{\infty} x^n H_n(y) \Bigg\}\,,\end{aligned} \tag{47}$$

where

$$\begin{aligned} H_n(y) = & \frac{y^3(1-y^{n-2})}{n-2} - \frac{y(1+y^2)(1-y^n)}{n} - \frac{3y(1-y^{n+2})}{n+2} \\ & + \frac{4(1+y)(1-y^{n+3})}{n+3} - \frac{4(1-y^{n+4})}{n+4}\,. \end{aligned} \tag{48}$$

By replacing $(1-y)^5 = 1 - 5y + \cdots$ in the second term in the first line of (47), one recovers the leading terms found in (41).

The impact of the W boson propagator corrections on the differential distributions of muon and τ leptonic decays were studied in Ref. [216]. In that work, a polarised initial-state lepton is considered. In addition, the radiative decay processes, in which one detected photon is emitted in the final state, are also studied.

10 Five-body decay of the muon (with variations)

Authors: Matteo Fael
CERN

The project with Christoph on the five-body decay of the muon [217]

$$\mu^+ \to e^+ e^- e^+ \bar{\nu}_\mu \nu_e, \tag{49}$$

the *rare muon decay* [218] in the PSI dictionary, was initiated by a conversation with Nicklaus Berger at the Lepton Moments conference in 2014 taking place in Centerville, Massachusetts.

At that time, my Ph.D. supervisor Massimo Passera and I were working on the next-to-leading-order (NLO) corrections to the radiative decays of the muon and tau leptons (e.g. $\mu^- \to e^- \gamma \nu_\mu \bar{\nu}_e$). Berger, a collaboration member of the new Mu3e experiment proposed at PSI, Switzerland, pointed out to us that the decay in Eq. (49), closely related to the radiative decay we were working on, is among the main source of background in the search for charged lepton flavour violation (CLFV) in the decay $\mu^+ \to e^+ e^- e^+$. The goal of the Mu3e experiment is to observe the process $\mu^+ \to e^+ e^- e^+$ (if its branching fraction is larger than 10^{-16}) or otherwise to exclude a branching fraction of 10^{-16} at the 90 % confidence level [219].

Berger highlighted that since the SM decay rate of $\mu^+ \to e^+ e^- e^+ \bar{\nu}_\mu \nu_e$ was known only at the leading order (LO) [210, 220–222], it would have been beneficial for Mu3e data analysis to have a prediction at least at NLO accuracy to study the impact of QED radiative corrections in the search for CLFV.

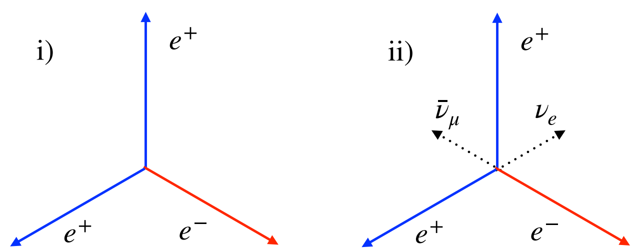
The background in $\mu \to eee$ searches originates from overlays of different processes producing three tracks resembling a $\mu \to eee$ decay (combinatorial background) or from the five-body decay in Eq. (49) (internal conversion background) which is indistinguishable from the signal except for the energy carried out by neutrinos, see Fig. 18. This background can be suppressed only via an excellent momentum resolution and a precise reconstruction of the three-electron total energy, which must be as close as possible to the muon mass. In order to reach a sensitivity of about 2×10^{-15}, the average three-particle mass resolution has to be better than 1.0 MeV.

In October 2014, I started my postdoc with Christoph at the University of Bern and we decided to work on the NLO corrections to $\mu^+ \to e^+e^-e^+\bar{\nu}_\mu\nu_e$. We studied the SM prediction for the differential rate and the branching ratio taking into account the full dependence of the mass ratio $r = m_e/M_\mu$, where M_μ and m_e are the muon and the electron mass, respectively. Virtual and real corrections were computed using the effective four-fermion Fermi Lagrangian plus QED and QCD.

We started by considering the one-loop amplitudes shown in Fig. 19. We decomposed the interference terms between one-loop and tree-level amplitude in terms of Passarino–Veltman tensor integrals and by means of the algebra manipulation program `FORM` [223]. For the numerical evaluation of the tensor-coefficient functions, we employed the very well-tested Fortran library `LoopTools` [224]. For the calculation the contribution of real diagrams, we adopted a phase-space slicing method to regularise the infrared divergence.

One of the main challenges was to find a suitable parametrisation of the phase-space for both $1 \to 5$ (LO kinematics) and $1 \to 6$ (real emission kinematics) to be implemented in the Monte Carlo integration. A generic

Fig. 18 The CLFV decay $\mu^+ \to e^+e^-e^+$: (i) the signal decay, (ii) the SM five-body decay of the muon



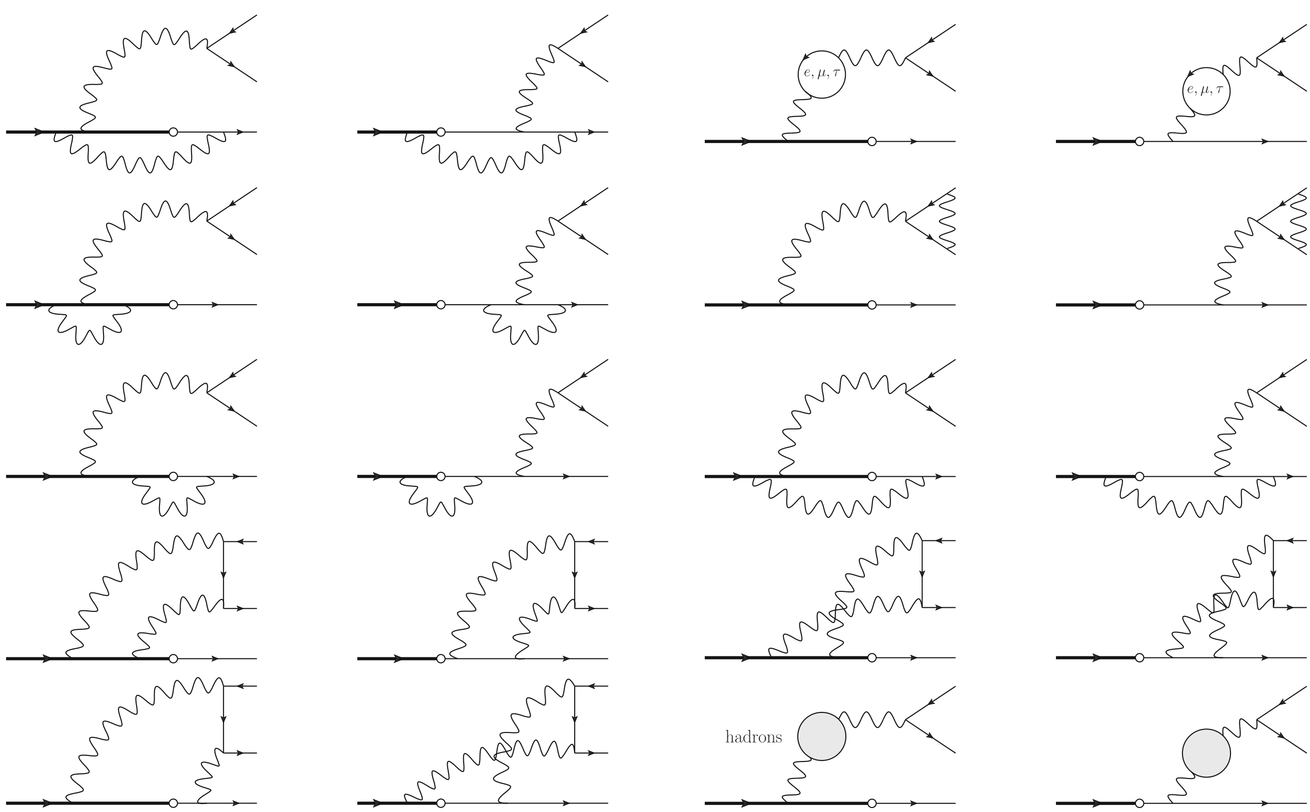


Fig. 19 One-loop diagrams for $\mu \to eee\nu_\mu\bar{\nu}_e$ decay. The muon and the electrons are drawn with bold and thin lines, respectively; the dots represent the Fermi interaction (for simplicity the neutrinos are not drawn). Figure from Ref. [217]

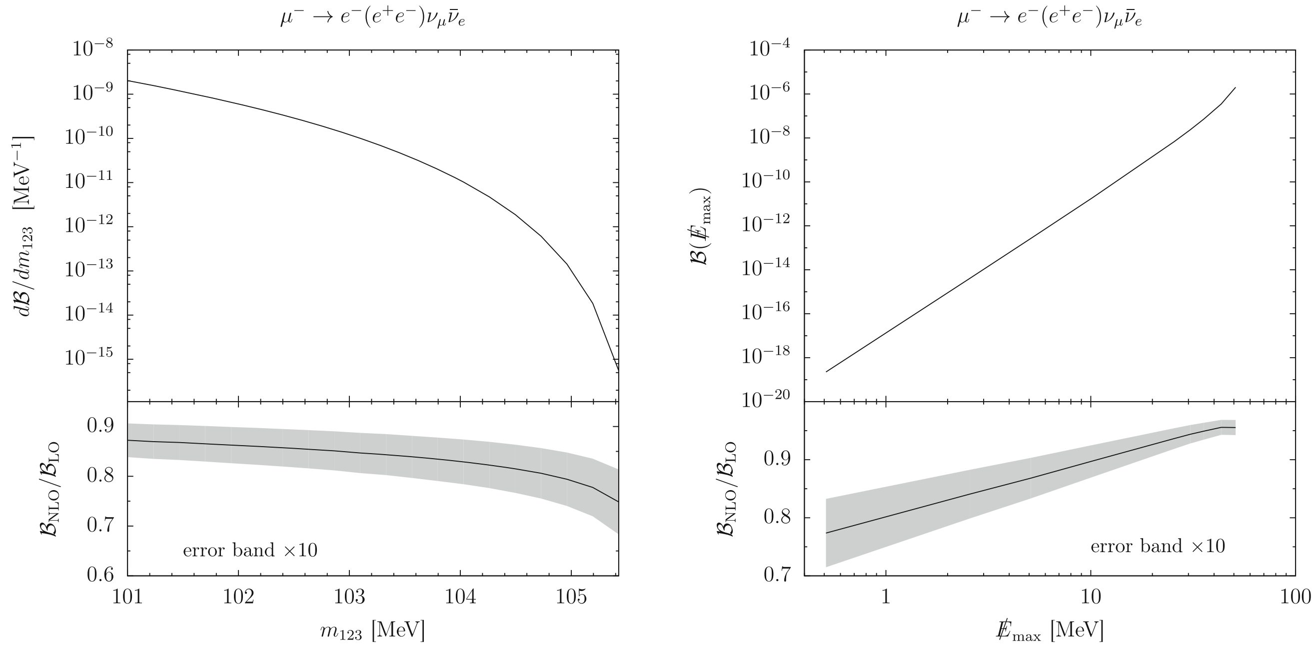


Fig. 20 The $\mu^+ \to e^+e^-e^+\bar{\nu}_\mu\nu_e$ branching ratio at NLO as a function of the three-electron invariant mass m_{123} (left) and the invisible energy cut $\not{E}_{\rm max}$ (right). The ratio between the NLO and LO predictions is depicted in the lower part of each panel. The error band (magnified 10 times) represents the assigned theoretical error due to renormalisation scale variation. Figure from Ref. [217]

parametrisation can be easily constructed via a recursive $1 \to 2$ phase-space splitting; however, our analysis aimed at quantifying the relative magnitude of radiative corrections in the specific final-state configuration of the decay (49) where the neutrino energies are very small and the total energy of the three electrons is close to M_μ.

To this end, we defined the residual branching ratio $\mathcal{B}(\not{E}_{\rm max})$ as the integral of the differential decay rate over the phase-space region satisfying

$$\not{E} = M_\mu - E_{+,1} - E_- - E_{+,2} \leq \not{E}_{\rm cut}\,, \tag{50}$$

where $E_{+,1}$, $E_{+,2}$, E_- are the energies of the two positrons and the electron, and $\not{E}$ is the missing energy in the rest frame of the muon carried away by neutrinos. In [217], we constructed an explicit parametrisation able to sample Monte Carlo events within this region with 100% efficiency, i.e. without applying a veto function.

Our result for the branching ratio of (49) is

$$\mathrm{Br}(\mu^+ \to e^+e^-e^+\bar{\nu}_\mu\nu_e) = 3.6054(1) \times 10^{-5}_{\rm LO} - 6.69(5) \times 10^{-8}_{\rm NLO}. \tag{51}$$

Since in the calculation of the total branching ratio there is no small scale associated, the NLO corrections turn out to be quite small and of order $\alpha/\pi \simeq 0.1\%$ relative to the LO prediction.

The situation changes dramatically once we introduce a constraint on the missing energy. Our main results are presented in Fig. 20. The plot on the left shows the normalised NLO differential rate as a function of the three-electron invariant mass m_{123}, close to the end point region $m_{123} \simeq M_\mu$. The local K-factor is drawn in the lower part. The rate, evaluated at fixed value of m_{123}, is fully inclusive in the bremsstrahlung photon. The plot on the right shows the branching ratio $\mathcal{B}(\not{E}_{\rm max})$ as a function of the cut on the missing energy. In the lower panel, we report the magnitude relative to the LO prediction. The error bands are the assigned theoretical error due to renormalisation scale variation. They are obtained by converting the renormalisation scheme for $\alpha_{\rm em}$ from the on-shell scheme to the $\overline{\rm MS}$ scheme adopted in the NNLO calculation of the muon lifetime. The quoted uncertainty is the difference between the NLO prediction evaluated at the renormalisation scales $\mu = m_e$, corresponding to the on-shell scheme, and $\mu = M_\mu$.

The decay (49) is the most serious background for the $\mu \to eee$ search and its suppression requires three-particle mass resolution better than 1.0 MeV. Our work in Ref. [217] showed that in the configuration where $\not{E}_{\rm max}$ is of the order of 1 MeV, radiative corrections lower the LO prediction by about $10 - 20\%$, thus reducing the background by a sizeable amount. Such a large K-factor enhancement is driven by the smallness of the $\not{E}_{\rm max}$ cut, which forces

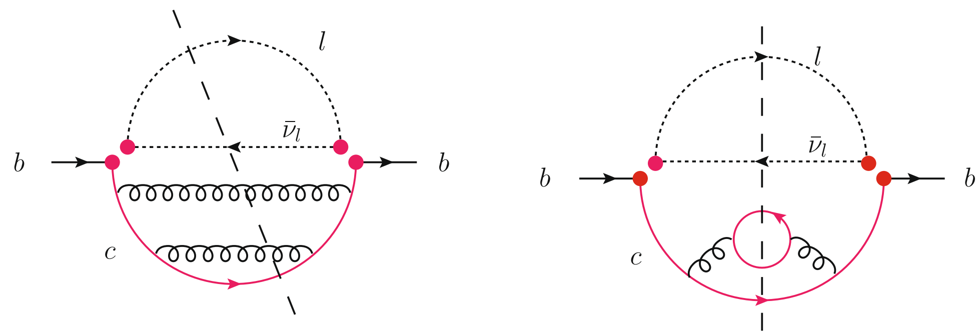


Fig. 21 Sample diagrams whose imaginary part contributes to the semileptonic B meson decays at NNLO. The dashed lines represent the charged lepton neutrino pair. Black solid lines denote bottom quarks, magenta lines denote charm quarks, respectively

the real photon to be soft. While we were close to publication, we were informed by A. Signer that his group was working on the same calculation. After comparing our results prior to publication, we submitted the two paper to the arXiv on the same day [217, 218].

The project on the five-body decay turned out to have significant influence in a series of works published a few years later with M. Egner, K. Schönwald, M. Steinhauser and me focussing on the B-meson lifetime [181, 225]. On the basis of all available correction terms in the heavy quark expansion, one can obtain for the B meson decay rates the following predictions [226]:

$$\Gamma(B^+) = 0.58^{+0.11}_{-0.07}\ \text{ps}^{-1},$$
$$\Gamma(B_d) = 0.63^{+0.11}_{-0.07}\ \text{ps}^{-1}, \tag{52}$$

with an uncertainly of almost 20%. These values are by far dominated by the renormalisation scale dependence of the free-quark decay. The uncertainties arising from CKM elements and quark mass values are significantly smaller. For this reason, it was necessary to determine the NNLO corrections to non-leptonic decays ($b \to cud$ and $b \to ccs$) in the free-quark approximation, including an appropriate choice of the short-distance mass scheme for the heavy quarks.

To calculate the second-order corrections to the width, one can use the optical theorem and consider the imaginary part of four-loop diagrams like those in Fig. 21. This approach requires to tackle the formidable four-loop propagator integrals which depend on two different internal masses, or equivalently one-dimensionless scale: the ratio $\rho = m_c/m_b$, where m_c and m_b are the charm and bottom mass.

In Ref. [227], we developed a method to numerically compute multi-loop integrals depending on one-dimensionless parameter and the dimension d. The method, which we called "Expand and match," is based on differential equations and truncated series expansions around singular and regular points. It provides results well suited for fast numerical evaluation and are sufficiently precise for phenomenological applications to B physics.

Before attacking the complete set of NNLO calculations for $b \to cud$ and $b \to ccs$, whose renormalisation is complicated by γ_5 and the operator mixing in the $\Delta B = \Delta S = 1$ effective Hamiltonian, we turned our attention to a simpler process. We restricted the calculation to the semileptonic B decay in the free-quark approximation,

$$b \to c l \bar{\nu}_l\,, \tag{53}$$

which is the analogous in QCD of the muon decay [225]. To validate our new method for the evaluation of the four-loop propagator integrals, we revisited the calculation of the $O(\alpha_s^2)$ corrections for semileptonic decays. Analytic results were obtained by Czarnecki and Pak [212, 228] via an asymptotic expansion in the limit $\rho \to 0$. We decided in a first step to recalculate the semileptonic width with the "Expand and match" method as a "warm up" and in a second step to extend the calculation to the complete set of diagrams for non-leptonic decays.

To this end after reducing the NNLO amplitude to master integrals, we derived a set of differential equations for the masters contributing to $b \to c l \bar{\nu}_l$ and solved them numerically using [227]. In our application, the master integrals are functions of the ratio $\rho = m_c/m_b$ and the dimensionless scale in the "Expand and match" is played by ρ. As boundary values, we performed an asymptotic expansion in the equal mass limit $m_c \simeq m_b$.

The decay rate of semileptonic decay can be written as

$$\Gamma(b \to X_c l \bar{\nu}_l) = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2\left[X_0(\rho) + \frac{\alpha_s}{\pi}X_1(\rho) + \left(\frac{\alpha_s}{\pi}\right)^2 X_2(\rho) + O(\alpha_s^3)\right]. \tag{54}$$

The function $X_2(\rho)$ is presented in Fig. 22. The red curve shows our first determination for $X_2(\rho)$ as function of $\rho = m_c/m_b$. The result was quite puzzling at the beginning since it manifests a logarithmic singularity close to

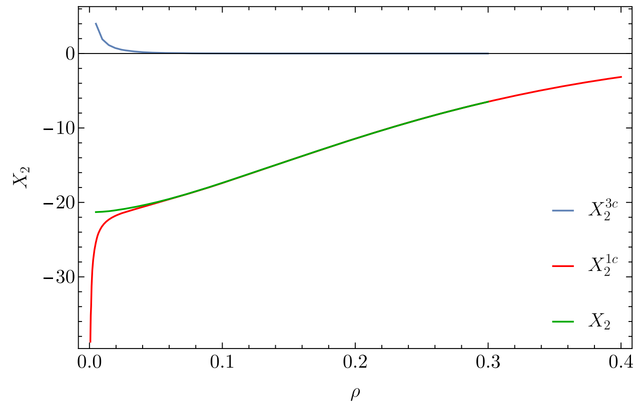


Fig. 22 X_2 is the second-order correction to the width of $b \to c l \bar{\nu}_l$ as a function of the mass ratio $\rho = m_c/m_b$. The red line corresponds to cuts through only one charm quark, the blue line arises from three charm quark lines. The green line is the sum of the two kinds of cuts

$\rho = 0$, in contrast with the cancellation of the mass singularities for inclusive observables. How is this possible? The reason is subtle and related to a decay channel which was missing: the five-body decay of the bottom, $b \to c\bar{c}cl\bar{\nu}_l$, analogous of the rare decay in Eq. (49).

The red line is obtained by considering and solving the system of differential equations only for the imaginary part of the master integrals. Also as boundary conditions, we computed only the imaginary part via the asymptotic expansion around the equal mass limit $m_c \simeq m_b$. However, this limit includes only diagrams with cuts through one (massive) charm quark in the final state but neglects the contributions with cuts through three charm quarks. The latter is accessible in fact only for $m_c < m_b/3$ and thus missing in the equal mass limit $m_c = m_b$. The coefficient X_2 appearing in the decay rate at order α_s^2 can therefore be divided into two parts:

$$X_2 = X_2^{(1c)} + X_2^{(3c)}, \tag{55}$$

which correspond to the contributions with one and three charm quarks in the final state. In Fig. 21, the diagram on the left contains one-charm cut, while the diagram on the right contains a three-charm cut.

The divergent behaviour shown in Fig. 22 by the red curve for $\rho \to 0$ is due to the mass singularity present in $X_2^{(1c)}$ for massless charm quarks which is present since not all possible cuts are included. The blue curve represents the contribution with three charm quarks in the final state (five-body decay). The contribution of this channel could easily be calculated numerically with the code I developed with Christoph in Ref. [217] (up to some trivial reweighting due to QCD colour factors). One can observe that the contribution due to the channel $b \to c c c l \bar{\nu}_l$ contains the same logarithmic behaviour but with an opposite sign such that after adding it to the red curve one obtains the complete NNLO corrections with a smooth limit for $\rho \to 0$ (the green line). Our final prediction agrees with the work by Pak and Czarnecki [212].

We also obtained an analytic expression for the LO decay rate of the five-body muon decay by subtracting the analytic expressions for $X_2^{(1c)}$ in an expansion around $\rho \to 0$ from the complete result for $X_2(\rho)$ computed in Ref. [212]. After specifying the $SU(N)$ colour factors to QED, we obtain for the muon decay [225]

$$\begin{aligned}\Gamma(\mu^+ \to e^+ e^- e^+ \bar{\nu}_\mu \nu_e) &= \frac{M_\mu^5 G_F^2}{192\pi^3}\left(\frac{\alpha_{\rm em}}{\pi}\right)^2 \Bigg\{ -\frac{2l_\rho^3}{9} - \frac{5l_\rho^2}{3} \\ &+ l_\rho\left(-\zeta_3 - \frac{133}{18} + \frac{13\pi^2}{36}\right) - \frac{20953}{2592} - \frac{983\pi^2}{864} - \frac{83\zeta_3}{24} + \frac{19}{6}\pi^2\log(2) - \frac{7\pi^4}{144} \\ &+ \frac{49\pi^2\rho}{24} + \rho^2\Bigg[\frac{2l_\rho^4}{3} + \frac{14l_\rho^3}{3} + \left(\frac{26}{9} - \frac{4\pi^2}{3}\right)l_\rho^2 + \left(-60\zeta_3 + \frac{3001}{54} - \frac{52\pi^2}{9}\right)l_\rho \\ &- \frac{341\zeta_3}{3} - \frac{29\pi^4}{18} - \frac{157\pi^2}{108} + \frac{49303}{648} - \frac{4}{3}\pi^2\log(2)\Bigg] + O(\rho^3)\Bigg\},\end{aligned} \tag{56}$$

where $l_\rho = \log(\rho)$. The formula for the LO width gives perfect agreement with the numerical results from Refs. [217, 218] for the branching ratio in Eq. (51).

After understanding the subtle interplay between the decay channels with one charm and three charms in the final state, I managed together with my collaborators to complete the calculation of the non-leptonic decays at NNLO [181]. Moreover, by observing that the five-body decay of the bottom is numerically small, F. Herren and

Table 1 SMEFT operator and Green's bases for different mass dimensionalities. The operator counting [249] is performed under the assumption of a single fermion generation

Dimension	Number of operators	Basis	Green's basis
5	2	[232]	–
6	84	[233, 234]	[237]
7	30	[238, 239]	[240]
8	993	[241–243]	[244, 245]
9	560	[246, 247]	–
10	15,456	[248]	–
11	11,962	[248]	–
12	261,485	[248]	–

I presented the complete NNLO-QCD corrections to the q^2 spectrum of inclusive semileptonic B decays in an analytic way [229]. In fact, by restricting the calculation to the cuts through only one charm line, we obtained analytic expression for the master integrals in terms of generalised polylogarithms.

In conclusion, the work in collaboration with Christoph allowed me to mature from the scientific point of view and to establish new collaborations and new directions for my research. Even years after I left Bern, I benefited a lot from the project on the five-body decays carried out with Christoph. The experience gained in Bern played a crucial role years later to better handle the computational complexity of semileptonic B decays [225, 229]. Christoph, as you step into retirement, I want to express my deep gratitude for your support and supervision during my years as a postdoc. Your guidance has been invaluable, both professionally and personally. Congratulations for this exciting new journey!

11 Matching and renormalisation in the SMEFT and the WET

Authors: Jason Aebischer
CERN

The Standard Model Effective Field Theory (SMEFT) [230, 231] is one of the most studied Effective Field Theories (EFTs) of our time. It results from promoting the Standard Model (SM) to an EFT, by allowing for higher dimensional operators of arbitrary mass dimension. Its little brother, the Weak Effective Theory (WET), which is obtained when integrating out the W, Z, t and H at the electroweak (EW) scale, is examined just as thoroughly.[13] I encountered these EFTs for the first time during my PhD in Bern in group of Christoph, which I remember as a truly delightful time. Together with Andi Crivellin, Matteo Fael and Javier Virto we had countless discussions on the topic and I learned a lot during that time. In this section we will review the progress made in this field over the past few years, focussing mainly on basis generation, matching as well as renormalisation of these grand theories.

11.1 Operator bases

The story of the SMEFT begins already in the late 1970s [232], when the first non-renormalizable operator of the SMEFT, the so-called Weinberg operator, was first discussed. The next milestone was reached in the 1980s, when the first operator basis for the SMEFT at the dimension-six level was presented in Ref. [233]. This operator basis was then refined by removing all redundant operators, which led to what is now called the Warsaw basis [234]. Since then, a lot of progress has been made in finding higher dimensional SMEFT bases, which are by now known up to mass dimension 12. Besides the regular non-redundant operator bases also Green's bases for the SMEFT have been constructed. They form a complete set of operators up to field redefinitions and therefore include more structures than regular operator bases. Green's bases are particularly useful when performing off-shell calculations. The known operator and Green's bases for the SMEFT, together with the number of independent operators (assuming a single generation), together with the relevant literature are collected in Table 1. Finally, also evanescent bases are known for the SMEFT [235] and WET [236] at the dimension-six level.

[13] The WET is also called Low Energy Effective Theory (LEFT) or Weak Hamiltonian.

11.2 Matching calculation

The matching from the SMEFT onto the WET is known both at the tree- and one-loop level. For the tree-level matching, we mention here in particular the work by Christoph and his collaborators [250], in which the full tree-level matching from the SMEFT onto the WET relevant for B-physics was computed for the first time. In addition, for operators that did not contribute at tree level, also the one-loop matching was presented in [250]. The tree-level matching was then generalised in Ref. [251] to include all possible operators and matching effects. Going beyond the dimension six level, also the (momentum-independent) matching of dim-8 SMEFT operators onto the WET is known since recently [252].

As far as two-loop results are concerned, several partial results are known since a long time: Next-to-leading-order (NLO) matching contributions for $\Delta S = 2$ and $\Delta B = 2$ operators were first presented in Ref. [253] in the context of modified Z-couplings. This one-loop matching was then generalised to arbitrary $\Delta F = 2$ processes in Ref. [254]. Furthermore, NLO matching contributions to dipole operators resulting from the SMEFT were first discussed in Ref. [255]. Finally, the complete matching from four-fermi SMEFT operators invariant under a $U(3)^5$ symmetry was computed in Ref. [256]. Eventually, the complete one-loop matching from the SMEFT onto the WET was worked out in Ref. [257], and since recently even two-loop matching contributions to parts of the operator basis are known [258].

Nowadays, there are also several computational tools available, that allow to perform automated matching computations onto various EFTs. The tree-level matching can for example be performed using `MatchingTools` [259], which is valid for arbitrary theories. At the one-loop level, different Mathematica packages exist, such as `Matchete` [260], `Matchmakereft` [261] and `CoDEx` [262].

11.3 Renormalisation

The renormalisation group equations (RGEs) for the SMEFT at dimension six are fully known at the one-loop level. They were presented in three nominal papers, which include the complete Yukawa [263] gauge [264] and λ-dependence [265] of the RGEs.[14] The complete one-loop RGEs for the Baryon number violating operators was worked out in Ref. [269].

Furthermore, progress has been made in recent years regarding NLO renormalisation: The two-loop QCD anomalous dimension matrix (ADM) for $\Delta F = 2$ transitions can be found in Ref. [270]. Parts of the two-loop RGEs involving semileptonic, four-lepton and Yukawa-type operators mixing into dipole operators were discussed in Ref. [271]. Using on-shell methods, the authors of Ref. [272] have computed various two-loop contributions and in Ref. [273] several zeros in the two-loop SMEFT ADM were pointed out. The scalar sector of the SMEFT has been renormalised recently at the two-loop level [274, 275] and the full bosonic sector in Ref. [276], using functional methods. The fermionic sector is however more challenging due to the shear number of operators, as well as the complications arising in the presence of fermions. In this respect, some progress has been made regarding scheme-independent higher order computations involving four-fermion operators [277]. In this approach, evanescent structures can be omitted altogether, which facilitates the task at hand considerably. Together with the simultaneous scheme- and basis choice at the one-loop level [278–280], this approach is well suited for the complete two-loop renormalisation of the fermion sector in the SMEFT. Finally, two-loop and even three-loop results are known for the gluonic Weinberg operator [281].

Apart from dim-6 contributions, also the renormalisation of the dim-5 Weinberg operator is known [282–284], as well as double insertions of it, mixing into the dimension-six sector [285].

At mass dimension, seven several partial results as well as the complete mixing among the entire operator basis is known [239, 240, 286, 287]. Similarly, at dim-8 level, many results are already known, such as double insertions of dim-6 operators [288, 289], Lepton Number Violating contributions [290], various signs and zeros [291] as well as parts of the bosonic [292–294] and fermionic [295] sector.

Concerning WET running, let us mention another work by Christoph and his collaborators [296]. In this article, the full QCD and QED running of operators relevant for B-physics was presented. Important QED effects were discussed for the first time, which were expanded upon further in Ref. [297]. The complete one-loop running of the WET was then eventually presented in Ref. [298]. Very recently, the two-loop running of the dimension-five sector of the WET was computed in the HV scheme [299]. Finally, the complete two-loop running of the four-fermion sector in the WET was presented in Refs. [300, 301].

Due to the large number of parameters already at the dimension-six level, computing the RG evolution can be quite challenging when performed by hand, although some efforts exist [302]. Luckily, there are several computational tools that allow to perform the RGE running in an automated way, such as the Mathematica package `DsixTools` [303, 304], the Python package `wilson` [305, 306], as well as the C++ library `RGESolver` [307].[15]

[14] Recently, the general one-loop RGEs for a bosonic EFT was derived in Refs [266–268].

[15] For recent reviews on computational tools for the SMEFT and WET, we refer to [308, 309].

I would like to thank Christoph for his support throughout my PhD and also in all the years afterwards. He always had an open door and an open ear and guided me and other members of the group with great patience and thoughtfulness.

12 Analyticity in $b \to s\ell\ell$ at two loops

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In May 2015, when the madness of the *B anomalies* was just taking off, Christoph invited me to give a seminar at the University of Bern. Not about the anomalies, of course: Christoph is always curious, but he has always been more interested in the physics rather than in potentially disruptive social constructions. During my visit I actually gave two talks. The first one was a blackboard (lunch) seminar about $b \to s\gamma$, summarising the current state of the theoretical calculation of the inclusive branching ratio after the papers [36, 168, 310]. The former of these papers is in fact the first paper I have with Christoph (although we did not interact much there...). It was during that blackboard seminar, looking directly at Eiger's north face through the window of the AEC coffee room, that I told myself: "I *need* to come here as a postdoc....".

The second seminar I gave, the next day, was the regular theory seminar. Guess the topic: the B anomalies! I presented the results that would be published later in Ref. [311]. Christoph was always very curious and sceptical, especially about R_K. However, he acknowledged the fact that my primary interest was in the theoretical calculation of the exclusive amplitudes themselves, and thus it was clear that we would understand each other. This chapter will describe our collaboration on this topic, leading to the paper [312].

After my very pleasant visit, Christoph offered me a postdoc at the University of Bern. I wanted to accept immediately, but I had just been awarded a Marie Curie fellowship to go to MIT, and this I could not refuse. Therefore, I asked him if it would be O.K. to come to Bern only for 1 year, which was the maximum I could delay my Marie Curie contract. He very generously said yes. And this would be one of the greatest events in my life.

I was a postdoc with Christoph from June 2016 to June 2017. During that time I became part of Chritoph's *workforce*, with headquaters in an office of the ExWi underground, and composed of Jason Aebischer (then a PhD student), Matteo Fael (then a postdoc), and myself (see Fig. 23). My year in Bern was my most prolific period as a researcher. I will always be thankful to Christoph.

Fig. 23 Christoph's workforce headquaters, June 2017

12.1 Non-local form factors in $b \to s\ell\ell$

When I finally settled in Bern in 2016, the B anomalies were *the thing*. It all began in 2013 with the measurements of the angular distribution in $B \to K^*\mu^+\mu^-$ by the LHCb collaboration [313], and the corresponding deviation of the observable P_5' [314] with respect to the SM prediction [315], which was immediately interpreted as possible New Physics contributing to the low-energy Wilson coefficient C_9, with $C_9^{\rm NP} \sim -1$ [316]. To put this in perspective, the SM value is $C_9^{\rm SM} \sim 4$, so this constituted a 25% effect. (Spoiler alert: this anomaly is still there in 2025 [317]).

Here, we are working with the effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm QED} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \Big[C_1\mathcal{O}_1 + C_2\mathcal{O}_2 + C_7\mathcal{O}_7 + C_9\mathcal{O}_9 + C_{10}\mathcal{O}_{10} \Big] , \tag{57}$$

where

$$\mathcal{O}_1 = (\bar{s}\gamma_\mu P_L T^a c)(\bar{c}\gamma^\mu P_L T^a b) , \quad \mathcal{O}_2 = (\bar{s}\gamma_\mu P_L c)(\bar{c}\gamma^\mu P_L b) ,$$
$$\mathcal{O}_{9(10)} = \frac{\alpha}{4\pi}(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu(\gamma_5)\ell) , \quad \mathcal{O}_7 = \frac{e}{(4\pi)^2} m_b(\bar{s}\sigma_{\mu\nu}P_R b)F^{\mu\nu} .$$

But C_9 is probably the worst place for NP to show up, because certain long-distance QCD effects look very similar. Let me explain this.

The $B \to M\ell^+\ell^-$ amplitude has the form

$$\mathcal{A} = \frac{G_F\,\alpha\,V_{ts}^* V_{tb}}{\sqrt{2}\pi} \bigg[(C_9\,L_V^\mu + C_{10}\,L_A^\mu)\,\mathcal{F}_\mu - \frac{L_V^\mu}{q^2} \Big\{ 2im_b C_7\,\mathcal{F}_\mu^T + \mathcal{H}_\mu \Big\} \bigg] , \tag{58}$$

up to terms of $\mathcal{O}(\alpha^2)$. Here q^2 is the invariant squared mass of the lepton pair and L_i^μ are leptonic currents, $L_{V(A)}^\mu \equiv \bar{u}_\ell(q_1)\gamma^\mu(\gamma_5)v_\ell(q_2)$. We have neglected contributions from other local semileptonic and dipole operators that are not relevant in the SM, as well as higher order QED corrections, but this amplitude is otherwise exact in QCD. All non-perturbative effects are contained in the "local" ($\mathcal{F}_i^{(T)\mu}$) and "non-local" ($\mathcal{H}^\mu$) form factors, with

$$\mathcal{F}_\mu = \langle M(k)|\bar{s}\gamma_\mu P_L\,b|\bar{B}(q+k)\rangle \quad \mathcal{F}_\mu^T = \langle M(k)|\bar{s}\sigma_{\mu\nu}q^\nu P_R\,b|\bar{B}(q+k)\rangle \tag{59}$$

$$\mathcal{H}^\mu(q,\,k) = 16\pi^2\,i\int d^4x\,e^{iq\cdot x}\,\langle M(k)|T\{j_{\rm em}^\mu(x),\,(C_1\mathcal{O}_1 + C_2\mathcal{O}_2)(0)\}|\bar{B}(q+k)\rangle \tag{60}$$

where $j_{\rm em}^\mu = \sum_q Q_q\;\bar{q}\gamma^\mu q$, and $q = \{u,\,d,\,s,\,c,\,b\}$.

The non-local form factors $\mathcal{H}^\mu$ are very difficult to estimate, but there are certain kinematic regions ("OPE" regions) where the integral in Eq. (60) is dominated by the region $x \sim 0$ (or maybe just by $x^2 \sim 0$), and here the bi-local matrix element in the integrand can be expanded for small x (or small x^2), such that [105, 318–320]

$$\mathcal{H}_{\rm OPE}^\mu(q) = \Delta C_9(q^2)(q^\mu q^\nu - q^2 g^{\mu\nu})\mathcal{F}_\nu + 2im_b\,\Delta C_7(q^2)\mathcal{F}^{T\mu} + \cdots , \tag{61}$$

with the ellipsis denoting contributions from subleading terms in the expansion. Then,

$$\mathcal{A} = \frac{G_F\,\alpha\,V_{ts}^* V_{tb}}{\sqrt{2}\pi} \bigg[((C_9 - \Delta C_9)\,L_V^\mu + C_{10}\,L_A^\mu)\,\mathcal{F}_\mu - \frac{L_V^\mu}{q^2} \Big\{ 2im_b(C_7 + \Delta C_7)\,\mathcal{F}_\mu^T \Big\} \bigg] , \tag{62}$$

up to subleading terms. The functions $\Delta C_9(q^2)$ and $\Delta C_7(q^2)$ can be calculated perturbatively in the OPE region (as Christoph has done repetitively to NLO in various ways [43, 312, 321]), but the extrapolation beyond the OPE region introduces long-distance effects that are the subject of heated debate. Since these effects always go together with the short-distance Wilson coefficient C_9, one needs to estimate them properly in order to extract $C_9^{\rm NP}$ from experimental measurements of $B \to K^*\ell^+\ell^-$. All this was of course well known when Ref. [316] was published.

12.2 Analytic structure of the non-local form factors

The strategy, therefore, is to calculate the non-local form factors in the OPE region, and then to extrapolate the result to the phenomenologically interesting region. This can be done by means of a dispersion relation [319], or by means of a z-expansion [322]. The former requires to input information on the spectral density, and the latter

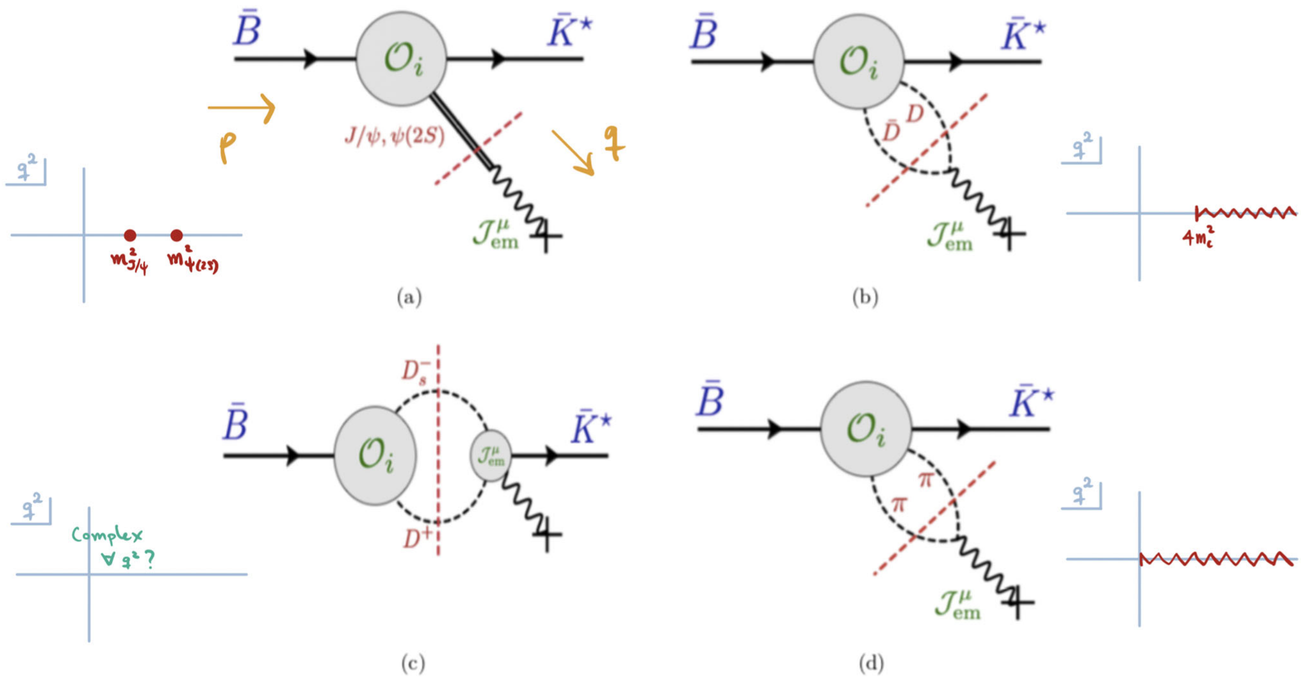

Fig. 24 On-shell cuts contributing to various singularities of the non-local form factors in $B \to K^*\ell^+\ell^-$

makes a few mild assumptions to parametrize the q^2 dependence below the open-charm threshold. But in both cases, one must understand the singularities of the functions $\mathcal{H}(q^2)$ in the complex q^2 plane. These are related by unitarity to on-shell intermediate states, and thus one expects the various singularities depicted in Fig. 24. Particularly, difficult is to understand the effect of the cuts in panel (c).

At one loop, the OPE matching for $\Delta C_{7,9}(q^2)$ is given by a single penguin diagram with a charm loop, resulting in

$$\begin{aligned}\Delta C_7(q^2) =& \mathcal{O}(\alpha_s) \qquad \Delta C_9(q^2) = \frac{2}{3}Q_c(C_F C_1 + C_2) f_9(x,\, y) + \mathcal{O}(\alpha_s)\\ f_9(x,\, y) =& \frac{2}{3} + i\pi + \frac{4m_c^2}{q^2} + \log\frac{4\mu^2}{m_b^2} + \log\frac{x^2}{1-x^2} + \frac{1-3y^2}{2y^3}\log\frac{1+y}{1-y}\end{aligned} \tag{63}$$

with

$$x = \frac{m_b}{\sqrt{m_b^2 - 4m_c^2}}\ , \quad y = \frac{\sqrt{q^2}}{\sqrt{q^2 - 4m_c^2}}\ . \tag{64}$$

This result for $\Delta C_9(q^2)$ contains logarithms that result in singularities of the form of panel (b) in Fig. 24, and will be associated to singularities of the type of panels (a) and (b) in the non-perturbative regime. However, in order to observe the type of singularites in panel (c) one needs to consider the two-loop corrections, calculated in a way such that the analytic continuation to complex q^2 can be studied.

This was the context in which Christoph, Hrachia and I started collaborating on the subject. The idea was to calculate the functions $\Delta C_{7,9}(q^2)$ in the OPE region, fully analytically, at two loops. The objective was twofold:

1. To have the exact expressions, not expanded in m_c or in q^2, as in Refs. [43, 321], and thus to "crack" definitely this problem.
2. To study the analytic structure of the OPE functions in order to gain some insight in the analytic structure of $\mathcal{H}(q^2)$.

As a side result, our calculation would also give the separate contributions to the non-local form factor proportional to different charge factors, which is interesting in order to write separate dispersion relations [323].

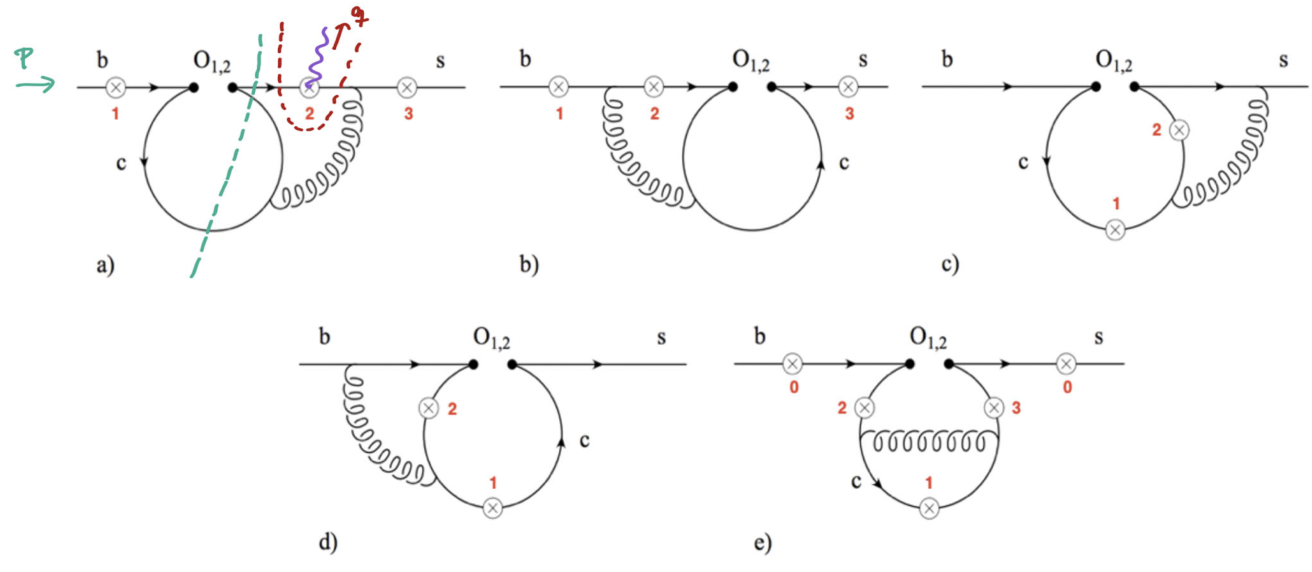

Fig. 25 Two-loop diagrams that contribute to the functions $\Delta C_{7,9}(q^2)$. Crosses denote insertions of the EM current. In diagrams **a** two possible cuts are shown, providing singularities of the type of panels **c** (green cut) and **d** (red cut) in Fig. 24

12.3 OPE matching at two loops

At two loops, the functions $\Delta C_{7,9}(q^2)$ are given by the result of the Feynman diagrams shown in Fig. 25. These diagrams must be calculated fully analytically in the variables q^2 and m_c, (or more specifically, in the variables x, y defined in Eq. (64)) which makes the problem not completely trivial. For this purpose, we followed Ref. [324] and employed the method of differential equations in canonical form [325].

In the calculation of the two-loop diagrams, one ends up reducing all the integrals to a set of two-loop scalar master integrals of the form

$$J_i(\epsilon,\, x,\, y) = (2\pi)^{-2d} \int \frac{(m_b^2)^{N_i-4}(\tilde{\mu}^2)^{2\epsilon}\; d^d\ell\; d^dr}{P_{i_1}^{n_{i_1}} P_{i_2}^{n_{i_2}} P_{i_3}^{n_{i_3}} P_{i_4}^{n_{i_4}} P_{i_5}^{n_{i_5}} P_{i_6}^{n_{i_6}} P_{i_7}^{n_{i_7}}}\ , \tag{65}$$

where P_k are a specific set of propagators raised to some integer (positive or negative) powers n_k, $\tilde{\mu}$ is the $\overline{\text{MS}}$ scale, and N_i is the mass dimension of the integral in $d=4$. A closed set of a minimal number of such master integrals is obtained by the method of IBP reduction using LiteRed [326]. One then takes derivatives of the integrands with respect to the variables x, y, and reapplies the IBP reduction in order to write a set of differential equations. The resulting system of differential equations is in general hard to solve. However, it may be possible [325] to change the basis of master integrals to one defined by

$$M_k(\epsilon,\, x,\, y) = T_{k\ell}(\epsilon,\, x,\, y) J_\ell(\epsilon,\, x,\, y)\ , \tag{66}$$

for some transformation matrix $T_{k\ell}$, such that the differential equations are given by

$$\begin{aligned} \partial_x\, M_k(\epsilon,\, x,\, y) =& \epsilon A_x^{k\ell}(x,\, y)\, M_\ell(\epsilon,\, x,\, y), \\ \partial_y\, M_k(\epsilon,\, x,\, y) =& \epsilon A_y^{k\ell}(x,\, y)\, M_\ell(\epsilon,\, x,\, y). \end{aligned} \tag{67}$$

The basis M_k is called a "canonical basis". This set of differential equations are a solved problem, and can be solved iteratively in terms of generalised polylogarithms (GPLs), in an algorithmic way. The difficult thing is to find the transformation $T_{k\ell}$. This requires sometimes to change the variables x, y to a different set of variables. In our project, diagrams (b) in Fig. 25 seemed uncrackable at this step, until Christoph happily found the appropriate variables:

$$t_b = \frac{-4x_b^2 + 4x_b^2 y_b + 2\sqrt{2}x_b^2(1+y_b)\sqrt{\frac{2x_b^4 - x_b^2 y_b + 2x_b^4 y_b - x_b^6 y_b + x_b^2 y_b^2 + 4x_b^4 y_b^2 + x_b^6 y_b^2}{x_b^4(1+y_b)^2}}}{-1 + 6x_b^2 - x_b^4 + y_b + 2x_b^2 y_b + x_b^4 y_b},$$

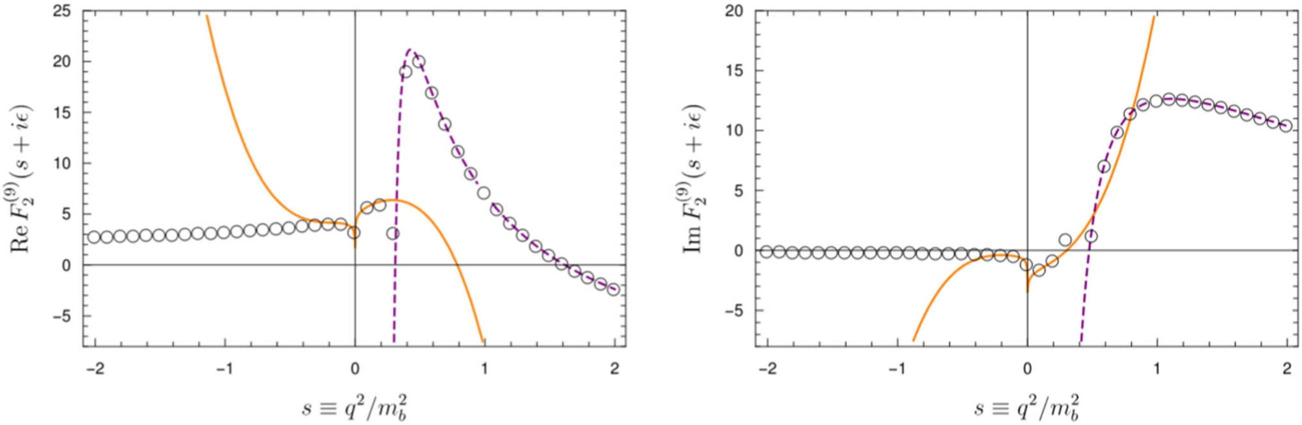

Fig. 26 Comparison of the exact results (black circles), with the expanded results of Ref [43] at low-q^2 (solid orange line) and the ones of Ref. [321] at high-q^2 (dashed purple line). The notation is $\Delta C_{7,9}^{\rm NLO}(s) = -(\alpha_s/4\pi)(C_1 F_1^{(7,9)}(s) + C_2 F_2^{(7,9)}(s))$

$$v_b = \frac{-4x_b^2 - 4x_b^2 y_b + 4\sqrt{2}x_b^2(1-y_b)\sqrt{\frac{2x_b^4 + x_b^2 y_b - 2x_b^4 y_b + x_b^6 y_b + x_b^2 y_b^2 + 4x_b^4 y_b^2 + x_b^6 y_b^2}{x_b^4(1-y_b)^2}}}{1 - 6x_b^2 + x_b^4 + y_b + 2x_b^2 y_b + x_b^4 y_b},$$

with

$$x_b = \frac{2m_c}{m_b} - \frac{\sqrt{4m_c^2 - m_b^2}}{m_b} \ , \quad y_b = \frac{\sqrt{q^2}}{\sqrt{q^2 - 4m_b^2}} \ . \tag{68}$$

These "extremely simple" expressions for t_b and v_b are a perfect example of who Christoph is: a radical physicist. I was the postdoc, I was supposed to figure this out, but Chritoph is Christoph, so he did (I still do not know how).

Our NLO results for $\Delta C_{7,9}(q^2)$ were now able to reproduce perfectly the approximate results in Refs. [43, 321], see Fig. 26.

12.4 Singularities of the two-loop contributions

With our analytic two-loop results at hand, Christoph, Hrachia and I were able to study the analytic structure of the non-local form factors. We found that the NLO functions indeed reproduce the singularities expected from unitarity, as discussed in Sect. 11.2. The examples of diagrams (a) and (d) are shown in Fig. 27.

In order to check the presence of additional singularities, we checked a dispersion relation: for any two points $\{s_0, s_1\}$ in the complex plane, it should hold that

$$F_i^{(j)}(s_1) - F_i^{(j)}(s_0) = \frac{s_1 - s_0}{2\pi i} \int_{s_{th}}^{\infty} dt \ \frac{F_i^{(j)}(t+i0) - F_i^{(j)}(t-i0)}{(t-s_1)(t-s_0)} \ , \tag{69}$$

provided the function $F_i^{(j)}(s)$ only contains a branch cut on the real axis extending from s_{th} to $+\infty$. This turned out to work well in general. For example, in the case of diagrams (b),

$$F_{2,(b)}^{(7)}(-3+i) - F_{2,(b)}^{(7)}(-1-2i) = 0.0894864 - 0.160827\, i \ , \tag{70}$$

$$\frac{-2+3i}{2\pi i} \int_4^{\infty} dt \ \frac{\text{Disc}\, F_{2,(b)}^{(7)}(t)}{(t+3-i)(t+1+2i)} = 0.0894966 - 0.160839\, i \ . \tag{71}$$

We did, however, mention two features that we found noteworthy:

- The discontinuities in diagrams a and c become purely imaginary for $s > 4z$ and $s > 1$, respectively.

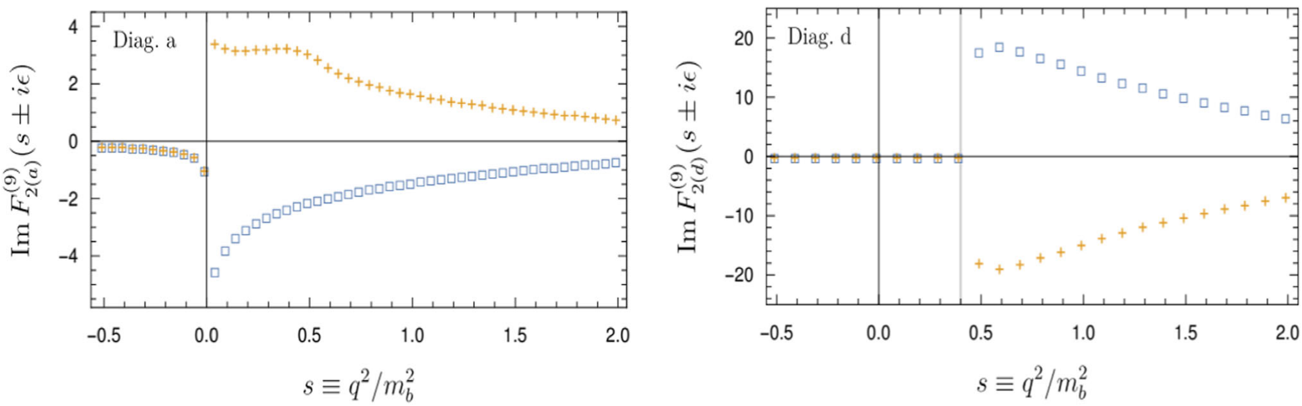

Fig. 27 Imaginary parts in diagrams (**a**) and (**d**), for $q^2 \pm i\epsilon$, showing the expected branch-cut singularites

- The contribution from diagrams c features a pole on the real axis when approaching the point $s = 1$ from the negative imaginary plane.

On the second point, we wrote "This pole is related to an anomalous threshold." However, we did not know very well what was going on.

The issue with the analytic structure of the non-local form factors is an issue under debate, which is crucial in order to understand how to extend the OPE calculations to the physical region. Recently, there has been much discussion on the presence of anomalous cuts (see e.g. [327, 328]), which is an issue that might be related to the findings in our paper with Christoph.

12.5 Outlook

More than a decade after the irruption of the $b \to s\ell\ell$ anomalies in the HEP community, and almost a decade after Christoph invited me for that seminar in Bern, R_K has disappeared and things have cooled down. However, the exclusive $b \to s\ell\ell$ decays are still as interesting as they ever where, and still a golden window to unknown physics. In that respect, Christoph's work on the subject stands high and stands still, because it is robust. Christoph's work is also somehow always coming back, it is always on the frontline. This chapter is an example of that. I want to show my immense gratitude to Christoph for his generosity and his influence, and for inspiring me with his philosophy: the philosophy of a purely radical physicist.

13 Two-Higgs-doublet models and leptoquarks

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As Christoph is very honest and serious, he likes to work on things that are "real". While this is in general difficult to define in theoretical particle physics, for Christoph it means mostly the Standard Model. Therefore, when I started my postdoc with Christoph in the fall of 2010, it was a minor challenge to motivate him to work on the MSSM and 2HDMs. Even though he worked on it before with Francesca, Daniel, Tobias, Youichi, etc., he is not a fan of models with a lot of free parameters and he especially dislikes scatter plots.

Nonetheless, I managed to convince him that the newly appeared $R(D^{(*)})$ anomalies [329] were at least of enough interest to write an article about it [330], even though he prepared me for their early demise. While I was hoping for a confirmation, it is ironic that now, more than 10 years later, the situation is still virtually the same with a significance a bit above the 3σ level [331]. Therefore, the hope is that the situation will be resolved by Christoph's 80th birthday. As Christoph likes to say, hope dies last, but eventually it dies. I still disagree with the latter, but maybe you should ask me again on my 65th birthday.

Anyway, we completed a comprehensive analysis of flavour violation in the 2HDM and a two-loop calculation of effective Higgs vertices in the MSSM [332] before moving to safer grounds, i.e. the study of effective operators in B decays with Jason and Matteo [333].

After I started my Ambizione fellowship at PSI, I was able to persuade Christoph to work on even more exotic new physics: leptoquarks. Combining the forces of my PhD student Dario (who was his master student) and his

PhD student Francesco, we performed an extensive analysis of leptoquarks [334, 335] and with Jason, we calculated the QCD corrections to the matching of leptoquarks for semileptonic B decays [336].

Working with Christoph was always very enjoyable. In particular, his absolute reliability was and is both impressive and reassuring. After his retirement, Christoph promised, to help with the calculation of the QCD corrections to $H \to tt^*$ and $H^\pm \to t^*b$ in 2HDMs, which he considers now "nearly SM".

13.1 Two-Higgs-doublet models

If one takes the MSSM and integrates out all SUSY partners, one arrives at a 2HDM as holomorphicity of the superpotential requires a second Higgs doublet. While at tree level, this is a simple type-II model where one-Higgs-doublet couples to down quarks and leptons and the second Higgs doublet to up-quarks, this Z_2 symmetry is broken at the loop level. Therefore, the decoupling limit of the MSSM is the 2HDM with generic sources of flavour violation, also called the type-III 2HDM.

Since the MSSM was still popular in these days but limits on SUSY masses were already getting higher, it was well motivated to consider this matching in detail. In fact, after my initial 1-loop calculations [337], I was able to convince Christoph to calculate the supersymmetric QCD corrections, i.e. to perform the matching at the 2-loop level. Part of Christoph's hesitation was because he thought this project to be technically too easy, while conceptually challenging. This means the idea of extracting the information on three-point functions from self-energies [338] was a bit suspicious to Christoph, but after detailed discussions he started calling me the "master of self-energies", reflecting his typical humour. Anyway, the 2-loop calculation was very instructive, at least for me, and led to the expected reduction of the scale uncertainty to the percent level [332].

My work trip request to the last "super-B factory" workshop to be held before the project was cancelled was on short notice and despite this, and the luxury holiday setting at the INFN hotel in Elba, was immediately and kindly granted by Christoph. While the conference was much more collider oriented than expected, my overlook of the conference bus schedule led to the nice coincidence that the workshop organizers took me by car from Pisa to Elba and I met Eugenio Paoloni who made me aware of the newly announced $R(D)$ and $R(D^*)$ anomalies [329]. While Cristoph was convinced that the anomalies would not last too long, he agreed to write an article on it after my return, probably also to support his new PhD student Ahmet Kokulu. The result was that the generic 2HDM can explain the excesses if it has sizeable top-charm and tau lepton couplings [330]. In fact, now, more than 12 years later, the situation did not change much. While bounds from the B_c lifetime were pointed out [339], the preferred size of the new physics effect decreased, so that the 2HDM of type-III can still explain the excesses [340]. Therefore, also our comprehensive analysis of flavour violation in the 2HDM [341] has not lost its phenomenological motivation.

13.2 Leptoquarks

Leptoquarks are predicted by Grand Unified Theories and some composite models. However, experimental motivation to study them with TeV scale masses comes from the B anomalies as well as $g-2$ of the muon. Promoted by this, I wanted to explore leptoquarks during my Ambizione fellowship at PSI. For this, I needed a PhD student and asked Christoph if he had any good candidates. He suggested his master student Dario Müller to me who he rated to be very good. Knowing Christoph, it was clear to me that his judgement can be trusted and I offered the position to Dario without hesitation (and never regretted it).

While we first explored LQs without Christoph, he also had a new PhD student, Francesco Saturnino, a friend of Dario. This again offered a nice opportunity to involve both of them into a project and to continue our cherished collaboration. In fact, our work had a great start: we found phenomenologically relevant loop effects in the $U(1)$ leptoquark model [335]. In particular, we pointed out that $B \to K\nu\nu$ and $K \to \pi\nu\nu$ are necessarily affected at the loop level and that $b \to s\ell^+\ell^-$ processes receive a lepton flavour universal contribution via an off-shell photon penguin diagram. In particular, the latter is one of the prime candidates to explain the updated $b \to s\ell^+\ell^-$ data [342] while at the same time accounting for $R(D^{(*)})$. However, we did not stop there and performed a comprehensive analysis of leptoquarks in low-energy precision observables [334, 343].

14 Epilogue

As this tribute comes to a close, we reflect once more on the legacy of Christoph Greub. Christoph's scientific achievements are vast and lasting, from pioneering contributions to the theory of rare B-meson decays and precision QCD corrections, to foundational work in effective-field theories and beyond-the-Standard-Model physics. It is not only his academic achievements that have earned our admiration, but also his integrity, generosity and quiet dedication with which he has conducted his career.

Christoph's work exemplifies the ideals of intellectual rigour, attention to details and a full commitment to advancing our understanding of the Standard Model step by (careful) step. His role in shaping the field of flavour physics is well established. Whether pushing the limits of multi-loop calculations or mentoring new generations of physicists, Christoph has consistently influenced the life of his colleagues and raised the bar for what it means to be a theoretical physicist.

We also celebrate his human side: the friendship, the support he has offered to collaborators across the globe and young postdocs, and the enduring respect he inspires in all who have had the privilege to work with him. Openness, humility and perseverance allowed Christoph to leave a mark not only in research but also inside all of us.

Particle physics has been enriched by his presence and scientific achievements over the last 30 years. We will continue to benefit from the paths he has helped to chart. With deep appreciation and warmest wishes, we thank him.

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