

# Black hole candidates and the Kerr bound

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## Abstract

The specific angular momentum of a Kerr black hole must not be larger than its mass in the geometrical units. The observational confirmation of this bound which we call a Kerr bound directly suggests the existence of a black hole. On the other hand, the violation of this bound may suggest the existence of a superspinning object which might be suggested from a string theory argument. In order to investigate observational testability of this bound by using the X-ray energy spectrum of black hole candidates, we calculate the energy spectra from an optically thick and geometrically thin accretion disk of a superspinning object which is described by a Kerr metric but whose specific angular momentum is larger than its mass, and then compare the spectra of this object with those of a black hole. Based on this calculation, we present that the observational confirmation of the Kerr bound is very hard but the violation of it may be detectable if only the continuum X-ray spectrum of the disk is available.

## 1 Introduction

Black hole candidates will be directly observed by radio interferometers by X-ray satellites and gravitational wave detectors in the near future. Although these candidates are most likely to be black holes, it is not obvious to confirm that. In principle, to prove that black hole candidates are really black holes, we have to show that the observed data cannot be explained by anything else. In this relation, there is an argument that if we can show that a sufficiently large amount of mass, say, above the model-independent maximum mass of neutron stars  $\sim 3M_{\odot}$ , is compactified within the size comparable with its gravitational radius, general relativity assures that it is a black hole. It turns out however that this argument crucially depends on the cosmic censorship hypothesis [1, 2]. The singularity theorems predict the existence of spacetime singularities in generic spacetimes, where the spacetime curvature is singular in typical cases. The cosmic censorship hypothesis is an unproven hypothesis about the properties of spacetimes which are solutions to the Einstein equation with physically reasonable matter fields and with generic initial data. It claims that spacetime singularities formed in physical gravitational collapse are covered by horizons and hidden from any or distant observers. Thus, the cosmic censorship hypothesis ensures the future predictability of the classical theory including general relativity. On the other hand, since quantum gravity should replace classical general relativity in the Planckian regime, singular spacetimes in classical theory might be modified to regular ones in some sense. If this is true, the cosmic censorship hypothesis for the classical theory should be reconsidered in the new context. One of the interesting possibilities is that naked-singular solutions of the classical Einstein equation are physically significant in the low-curvature region but largely modified to be smooth spacetimes around singularities in the classical solutions due to quantum gravity effects. This article is based on Takahashi and Harada [3]. We use the geometrical units, where  $c = G = 1$ .

The Kerr metric is a solution to the vacuum Einstein equation. This solution is stationary, axisymmetric and asymptotically flat. The metric is given by

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right)dt^2 - \frac{4aMr \sin^2 \theta}{\Sigma} dt d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 \\ + \left(r^2 + a^2 + \frac{2Mra^2 \sin^2 \theta}{\Sigma}\right) \sin^2 \theta d\phi^2,$$

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where  $\Sigma = r^2 + a^2 \cos^2 \theta$ ,  $\Delta = r^2 - 2Mr + a^2$ , and  $M$  and  $a$  are the mass and the Kerr parameter, respectively. We put  $a_* \equiv a/M$  as the nondimensional Kerr parameter. The Kerr solution has a curvature singularity at  $r = 0$  and  $\theta = \pi/2$ , where  $\Sigma = 0$  and this is called a ring singularity. For  $|a_*| \leq 1$ , the solution has an event horizon at  $r = r_H$ , where  $r_H$  is a larger root of  $\Delta = 0$ . We can see that the ring singularity is covered by an event horizon. For  $|a_*| > 1$ , there is no event horizon and the ring singularity is therefore not covered by a horizon. A singularity which is not covered by a horizon is called a naked singularity. Therefore, the condition  $|a_*| \leq 1$  must be satisfied for the singularity to be covered and we call this condition the Kerr bound on the Kerr parameter.

Here we should mention an important example where the naked singularity is remedied due to quantum gravity effects in string theory [4]. There is a solution called the BMPV solution of supergravity in (4+1) dimensions. This solution describes a black hole in some region of the parameter space as a sub-extremal case but has a naked singularity in other region as a super-extremal case. This naked singularity is actually excised by a domain wall of strings and D-branes, if stringy effects are taken into account. Although the Kerr solution has not yet been dealt with in the same context, we can assume that the naked singularity in the super-extremal Kerr solution is excised as well due to stringy effects and this regular object is called a superspinar. Figure 1 shows this assumption schematically. In this connection, we should also mention that it is suggested that we might overspin a sub-extremal black hole to a super-extremal object by plunging a test body [5].



Figure 1: The schematic figure of the remedy of a naked singularity due to stringy effects.

## 2 The standard accretion disk in the general Kerr spacetime

We newly invent the consistent accretion disk model for the superspinar. We generalise the general relativistic standard accretion disk model around a black hole [6] to a superspinar. We adopt the following assumptions. The disk is axisymmetric, steady-state, geometrically thin and optically thick. The angular momentum of the fluid is transferred by the viscous torque and the dissipative energy is used to thermalise the disk. The local thermal equilibrium holds in the disk fluid. The inner edge is torque free and given by the innermost stable circular orbit (ISCO). Then, we find the solution of the fluid equation in the background Kerr metric. Given a Kerr spacetime, the accretion flow is fixed by two parameters, the accretion rate  $\dot{M}$  and the radius of the outer edge  $r_{\text{max}}$ . The accretion rate must be sub-Eddington for the consistency of the model. The result is not sensitive to the choice of  $r_{\text{max}}$  if it is chosen to be sufficiently large. Figure 2(a) shows the present accretion disk model schematically.

It is well known that the Kerr black hole has an ISCO. As  $a_*$  is increased from 0 to 1,  $r_{\text{ISCO}}$ , the radius of the ISCO, decreases from  $6M$  to  $M$ , while  $r_H$ , the horizon radius, decreases from  $2M$  to  $M$ . In fact, the Kerr spacetime has an ISCO even for  $a_* > 1$ . As  $a_*$  is increased from 1,  $r_{\text{ISCO}}$  continues to decrease to  $2/3$  at  $a_* = 4\sqrt{2}/(3\sqrt{3}) \simeq 1.09$ , takes a minimum there and turns to increase.  $r_{\text{ISCO}}$  is greater than its Schwarzschild value  $6M$  for  $a_* > 8\sqrt{6}/3 \simeq 6.53$ . This is shown in Fig. 2(b).

The efficiency  $\epsilon$  of emission can be calculated as  $1 - E_{\text{ISCO}}$ , where  $E_{\text{ISCO}}$  is the specific energy of the particle orbiting the ISCO. Figure 3(a) shows the efficiency as a function of the spin parameter  $a_*$ , where only the short-dashed line and the solid line are relevant to the present accretion disk model. The efficiency is  $\sim 5.7\%$  for the Schwarzschild black hole  $a_* = 0$  and increases to  $\sim 42\%$  for  $a_* = 1^-$ . Then, it jumps to the maximum value  $\sim 157.7\%$ . The efficiency is larger than  $100\%$  for  $1 < a_* \lesssim 1.09$ . It then decreases to  $\sim 42\%$  for  $a_* = 5/3 \simeq 1.67$  and continues to decrease as  $a_*$  is increased further. Note that the efficiency becomes  $\sim 5.7\%$  again for  $a_* \simeq 6.53$ . Figure 3(b) shows the temperature profiles of the

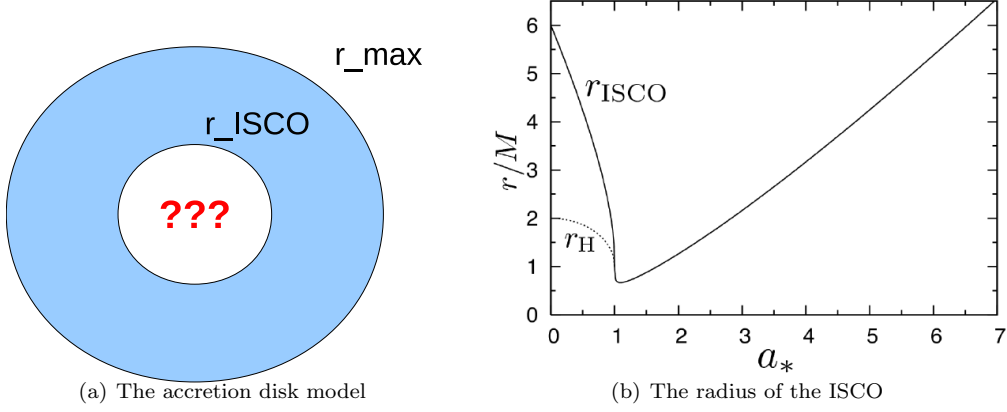


Figure 2: (a) The schematic figure of the accretion disk model in the general Kerr spacetime and (b) the radius of the ISCO as a function of the Kerr parameter [3].

accretion disks. We can see that there appears a peaky spike near the inner edge for  $1 < a_* \lesssim 1.09$  due to the high efficiency of the disk. We should note that for a superspinar with  $1.09 \lesssim a_* \lesssim 6.53$ , there is a black hole counterpart with very similar temperature structure.

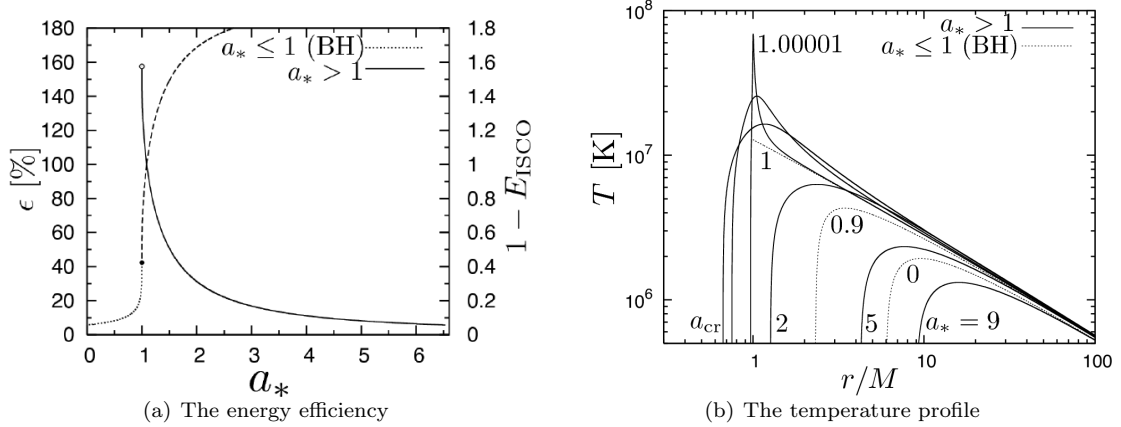


Figure 3: (a) The energy efficiency  $\epsilon = 1 - E_{\text{ISCO}}$  as a function of the Kerr parameter and (b) the temperature profiles of the accretion disks for the different values of the Kerr parameter [3].

### 3 The X-ray spectrum from the accretion disk

In addition to the dynamics and thermodynamics of the accretion disk, we assume that the dissipative energy due to the viscous torque is fully converted to the locally black-body radiation. We solve the general relativistic radiative transfer, including all kinematical general relativistic effects through the spacetime metric and the motion of the fluid. We neglect the absorption, emission and reflection by the surrounding gases for simplicity. We assume there is no emission and no absorption by the superspinar at the centre.

Figure 4(a) shows the predicted energy spectra for the different spin parameter values. For  $1 \lesssim a_* \lesssim 1.67$ , the spectrum extends to higher energy compared to black holes. For  $a_* \gtrsim 6.53$ , the photon energy is lower compared to black holes. However, surprisingly, any black hole has its superspinar counterpart which

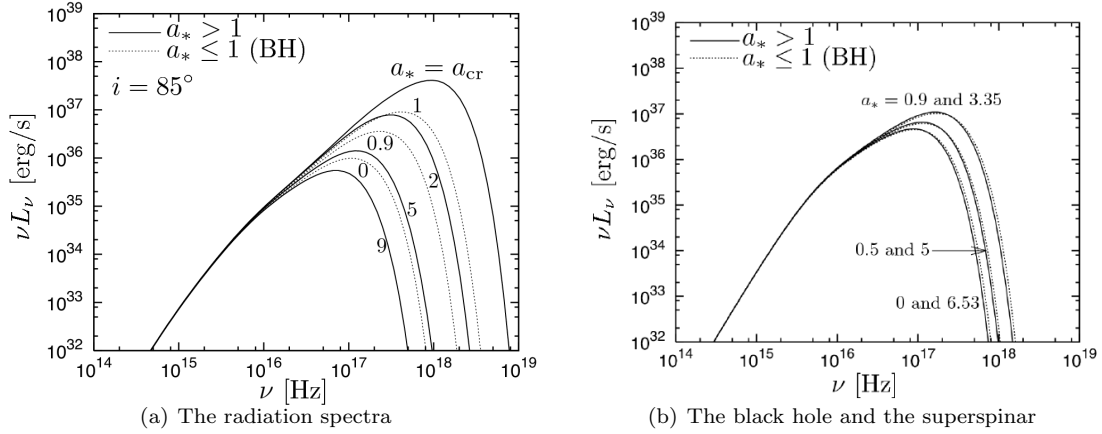


Figure 4: (a) The spectra of the emission from the disk with the different values of the Kerr parameter and (b) the demonstration of the similarity of the spectra between the black hole and the superspinar counterpart [3].

gives a very similar X-ray spectrum. This is demonstrated in Fig. 4(b). In contrast, for a superspinar with  $1 \lesssim a_* \lesssim 1.67$  or  $a_* \gtrsim 6.53$ , there is no black hole counterpart.

## 4 Summary

In the present settings, it is very challenging to distinguish black holes from superspinars only by the X-ray spectrum observation. In contrast, some of the superspinars can be clearly distinguished from black holes. In other words, we can potentially find the violation of the Kerr bound by the X-ray spectrum observation. The present disk and radiation model might be too simple. It is interesting to study the observational testability of the Kerr bound with other accretion models, such as Radiatively Inefficient Accretion Flow (RIAF). See Takahashi and Harada [3] for further details.

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