Pion–Nucleon Charge-Exchange Scattering and High-Energy Dynamics

II. Polarization Effects

As described in an earlier note[1] quoted hereafter as I, the differential cross section for $\pi^- p \to \pi^0 n$ scattering at fixed momentum transfer $\sqrt{-t}$ is well described for large laboratory momentum p_{lab} by a power law

$$d\sigma/dt = A(t)p_{\text{lab}}^{2\alpha(t)-2} \tag{1}$$

and this p_{lab} dependence is very nicely explained by the Regge model in terms of exchange of the ρ-meson Regge trajectory $\alpha(t)$ between pion and nucleon. This model gives to the spin-non-flip and spin-flip amplitudes f and g the high p_{lab} dependence

$$f(t) = f_0(t)z^{\alpha(t)}, \qquad g(t) = g_0(t)z^{\alpha(t)}, \tag{2}$$

where z is related to p_{lab} and t by

$$z = 2M\sqrt{m^2 + p_{\text{lab}}^2} + \tfrac{1}{2}t \tag{3}$$

and reduces to $2Mp_{\text{lab}}$ for large p_{lab}. M and m are the proton and pion masses, respectively; in terms of the Mandelstam variables z is $\frac{1}{2}(s - u)$.

It now happens that for the particular reaction $\pi^- p \to \pi^0 n$, general theorems on analyticity of the scattering amplitudes allow to predict the phases of f and g from their asymptotic p_{lab} dependence.[2] The reason is that, if a dynamical system E is exchanged between pion and nucleon to produce the reaction $\pi^- p \to \pi^0 n$, it must be of odd spin [see Fig. 2 and Eq. (3) in I; in more technical terms one says that the process has negative signature]. The predicted phases are [remember that $\alpha(t)$ is real]

$$f_0 = \pm \, |f_0| \, \exp\!\left[-i\frac{\pi}{2}\alpha(t) \right], \qquad g_0 = \pm \, |g_0| \, \exp\!\left[-i\frac{\pi}{2}\alpha(t) \right]. \tag{4}$$

Since they are the same for f and g, the resulting polarization parameter P,‡

$$P = \frac{2 \operatorname{Im}(f^*g)}{|f|^2 + |g|^2},\tag{5}$$

vanishes. This clear-cut prediction is contradicted by the measurements of a Orsay–Pisa–Saclay collaboration at CERN,[3] which found the following average polarizations:

$p_{\text{lab}} = 5.9 \ \text{GeV}/c, \quad \langle P \rangle = 16.4 \pm 3.1\%$ over $0.045 < -t < 0.255 \ (\text{GeV}/c)^2$,

$p_{\text{lab}} = 11.2 \ \text{GeV}/c, \quad \langle P \rangle = 12.3 \pm 2.1\%$ over $0.015 < -t < 0.345 \ (\text{GeV}/c)^2$.

Although small compared to the maximum of 100%, these polarizations must be regarded as sizable, because they are of the same order as the values of P found in other reactions like $\pi^\pm p$ elastic scattering where theory predicts nonvanishing P.

The most natural conclusion of the P measurement is that the form (2) predicted by the Regge model for the amplitudes is too simple. One needs a more complicated p_{lab} dependence. The various possibilities can be illustrated by considering two simple cases, corresponding to a further power or a logarithmic dependence in p_{lab}. In the first one, one adds to (2) small correction terms

$$f = f_0 z^{\alpha(t)} + \bar{f}, \qquad g = g_0 z^{\alpha(t)} + \bar{g},\tag{6}$$

$$\bar{f} = \bar{f}_0 z^{\bar{\alpha}(t)}, \qquad \bar{g} = \bar{g}_0 z^{\bar{\alpha}(t)},\tag{7}$$

with $\bar{f}_0$, $\bar{g}_0$ functions of t. From general analyticity properties, the phases of f_0, g_0 are again given by (4), whereas those of $\bar{f}_0$, $\bar{g}_0$ are given by similar expressions [$\bar{\alpha}(t)$ is assumed real][4]

$$\bar{f}_0 = \pm |\bar{f}_0| \exp\left[-i\frac{\pi}{2}\bar{\alpha}(t)\right], \qquad \bar{g}_0 = \pm |\bar{g}_0| \exp\left[-i\frac{\pi}{2}\bar{\alpha}(t)\right].$$

While the terms $\bar{f}$, $\bar{g}$ in (6) will affect only slightly the differential cross section given by

$$d\sigma/dt \propto |f|^2 + |g|^2,$$

‡ P is the average polarization of the recoil neutron in the direction $\mathbf{n}$ normal to the scattering plane, when the target proton is unpolarized. The actual experiment (Ref. 3) determined P by scattering pions on protons polarized in the direction $\mathbf{n}$ and by measuring the left-right asymmetry of the scattered particles.

they determine completely the polarization,

$$\{|f|^2 + |g|^2\}P$$

$$= \mathrm{Im}(f^*g) = 4z^{\alpha(t)+\bar{\alpha}(t)}\{|f_0\bar{g}_0| - |\bar{f}_0 g_0|\}\sin\left\{\frac{\pi}{2}[\alpha(t) - \bar{\alpha}(t)]\right\}.$$

Clearly, if the polarization is to be sizable for small $\bar{f}/f$ and $\bar{g}/g$, the difference $\alpha(t) - \bar{\alpha}(t)$ cannot be too small compared to 1. If $\bar{\alpha}(t) < \alpha(t)$, this means that P decreases with p_{lab} like a power $p_{\mathrm{lab}}^{\bar{\alpha}(t)-\bar{\alpha}(t)}$. If on the contrary $\bar{\alpha}(t) > \alpha(t)$, the correction terms $\bar{f}$, $\bar{g}$ will eventually become the dominant contributions in f and g at very large p_{lab}, so that $\bar{\alpha}(t)$ rather than $\alpha(t)$ will then become the leading exponent when $d\sigma/dt$ is given the form (1). This would mean that the ρ trajectory either would be $\bar{\alpha}(t)$ and would then not have the simple linear shape described in I, or would be $\alpha(t)$ but would no longer dominate the behavior of $\pi^- p \to \pi^0 n$ scattering as the energy increases beyond the range measured.

Our second example of modified p_{lab} dependence introduces logarithmic factors in Eq.(2),

$$f = f_0 z^{\alpha(t)}\left[\ln\frac{z}{z_0} - i\frac{\pi}{2}\right]^{\beta}, \qquad g = g_0 z^{\alpha(t)}\left[\ln\frac{z}{z_0'} - i\frac{\pi}{2}\right]^{\beta'}, \qquad (8)$$

with z_0, z_0', β, β' real functions of t. These expressions are so chosen that f_0 and g_0 have again the phases of Eq. (4) as a consequence of analyticity.[5] The resulting polarization is

$$\{|f|^2 + |g|^2\}P = 2\left[\zeta^2 + \frac{\pi^2}{4}\right]^{\beta/2}\left[\zeta'^2 + \frac{\pi^2}{4}\right]^{\beta'/2}\sin(\beta\phi - \beta'\phi'),$$

$$\zeta = \ln\frac{z}{z_0}, \qquad \zeta' = \ln\frac{z}{z_0'}, \qquad \phi = \arg\left(\zeta - i\frac{\pi}{2}\right), \qquad \phi' = \arg\left(\zeta' - i\frac{\pi}{2}\right).$$

It can have sizable values, varies logarithmically with p_{lab} and affects the p_{lab} dependence (1) of $d\sigma/dt$ through a logarithmic factor.

Many other modifications of the simple Regge behavior (2) are of course possible in order to produce nonvanishing polarization P. The above examples are nevertheless sufficient to illustrate that measurement of the p_{lab} dependence of P is of great importance. The Orsay–Pisa–Saclay experiment gives[3]

$$\frac{\langle P\rangle_{p_{\mathrm{lab}}=11.2\ \mathrm{GeV}/c}}{\langle P\rangle_{p_{\mathrm{lab}}=\ 5.9\ \mathrm{GeV}/c}} = 0.8 \pm 0.2 \qquad \text{for} \qquad 0.045 < -t < 0.255\ (\mathrm{GeV}/c)^2.$$

The uncertainty is still too large to draw definite conclusions on the rapidity of this variation.

The theoretical interpretation of the nonvanishing polarization is a completely open problem. The simple Regge model does not predict the effect, and more complicated versions of the model allow for further power terms like in Eqs. (6) and (7), preferably with $\bar{\alpha}(t) < \alpha(t)$ (further Regge trajectory), or for logarithmic factors of type (8) (Regge cut instead of Regge pole), or for combinations of them. In addition, at least at energies of order $p_{lab} \sim 6$ GeV/c, the pion–nucleon resonances might still give a sufficient contribution to f and g in order to affect appreciably the value of P.

Many possibilities of this type have been worked out in the recent literature, most of which have no difficulty in fitting the data. Rather than reviewing them, we would like to point out that this lack of unambiguous prediction is characteristic of the Regge model as soon as one goes beyond its simplest qualitative features. It occurs to various degrees in many other processes, including elastic scattering and proton–neutron charge-exchange scattering. Also many two-body collisions having one or two resonances in the final state, as well as photo-production reactions fall in this category, especially when pion exchange is allowed. For all such cases, it becomes increasingly clear that, in order to lift the ambiguities of the Regge model, more precise dynamical mechanisms will have to be invented and tested against experiment, and it is hoped that one will find again particular reactions allowing to test individual features of such mechanisms, just as $\pi^- p \to \pi^0 n$ has provided a direct verification for the usefulness of the Regge-trajectory concept in describing the energy dependence of $d\sigma/dt$, as recalled in our previous communication.[1]

The desirability of more definite dynamical models for high-energy collisions is also indicated by recent theoretical work applying the Regge model to two-body processes with particles of unequal masses and arbitrary spins. To keep the scattering amplitudes free of unwanted singularities without making them too special to fit the data, it appears that certain groups of Regge trajectories must exist with singularities in the contributions of individual trajectories and constraints between the trajectories in the same group, leading to a rather general but non-singular total amplitude.[6] The constraints are imposed by Lorentz invariance and analyticity alone, so that they are quite weak. As a result, the number of free parameters in the Regge-model treatment of general reactions grows so large that the model becomes more a frame-work than a predictive tool, calling again for additional dynamical assumptions.

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