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Quantum anomaly for spin 3/2 fields from hydrodynamics

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Recently, a new value for the gauge chiral anomaly for Rarita-Schwinger fields has been obtained in the framework of an extended theory that includes interaction with an additional spin 1/2 field. We obtain this anomaly from hydrodynamics of vortical charged fluid by calculating the mean value of the vortical axial current. In the hydrodynamic derivation of the anomaly, in contrast to the quantum-field derivation of the anomaly, the interaction terms with additional spin 1/2 field play central role. However, both results, quantum field and hydrodynamic, are exactly the same.

KEYWORDS: Quantum anomaly, spin 3/2, chiral vortical effect.

1. Introduction

The most common way to describe spin 3/2 fields is based on Rarita-Schwinger (RS) field theory. However, this theory is characterized by a number of problems [1]. In particular, problems arise when interaction with gauge fields is included. At the moment the interaction is switched on, there is a jump in the number of degrees of freedom from 4 to 8 (for the complex RS field describing both polarizations [2]). Furthermore, the Dirac bracket contains a pole in the limit of weak fields – so we cannot construct a perturbation theory.

Both described problems were overcome in the extended Rarita-Schwinger-Adler (RSA) theory [3], in which an interaction with an additional field with spin 1/2 was introduced. The corresponding action is

$$S = \int d^4x \left(-\varepsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_5\gamma_\mu\partial_\nu\psi_\rho + i\bar{\lambda}\gamma^\mu\partial_\mu\lambda - im\bar{\lambda}\gamma^\mu\psi_\mu + im\bar{\psi}_\mu\gamma^\mu\lambda \right), \quad (1)$$

where $-im\bar{\lambda}\gamma^\mu\psi_\mu$ and $im\bar{\psi}_\mu\gamma^\mu\lambda$ are "protomassive" terms describing the interaction of the Rarita-Schwinger field ψ with an additional field λ with spin 1/2 and m is an interaction constant. It is due to the interaction with the additional field that both of the above problems are solved.

The solution of the problem with the pole in the Dirac bracket made it possible to calculate the quantum anomaly in the framework of the perturbation theory using the well-known shift method

$$\langle \partial_\mu \hat{j}^\mu_A \rangle = -\frac{5}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}\,. \quad (2)$$

Thus, for the RSA theory, the anomaly differs by a factor of 5 from the case of spin 1/2. This factor 5 is different from the other estimates.

Recently, in [4], the gravitational chiral anomaly for this theory was also obtained and it was shown that it differs by −19 times from the anomaly for spin 1/2, in contrast to the known factor −21 for the usual RS theory [5].

On the other hand, in [6] the so-called chiral vortical effect (CVE) for spin 3/2 was calculated. We discuss this result as an alternative derivation of the anomaly (2) from a quite different area of physics – hydrodynamics. Below we summarize the key points of this derivation.

2. Anomaly derivation from hydrodynamics

Consider a relativistic quantum fluid. Further, we assume that a vorticity is created in such a fluid, characterized by the vorticity pseudovector $\omega^\mu = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta$, where u_ν is the 4-velocity of the fluid. The properties of such a medium are described by a density operator of the form [7]

$$\hat{\rho} = \frac{1}{Z}\exp\Big\{-\beta_\mu(x)\hat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\hat{J}^{\mu\nu}_x + \zeta\hat{Q}\Big\}, \tag{3}$$

where $\beta_\mu = \frac{u_\mu}{T}$ is the inverse temperature vector, $\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$ is the thermal vorticity tensor, the pseudovector ω^μ is one of the $\varpi_{\mu\nu}$ components, $\zeta = \frac{\mu}{T}$ is the chemical potential divided by the proper temperature, $\hat{P}^\mu$ is the 4-momentum operator, $\hat{Q}$ is the charge operator, and $\hat{J}^{\mu\nu}_x$ are the Lorentz transformation generators shifted by the vector x^μ, which are ultimately expressed in terms of the stress-energy tensor.

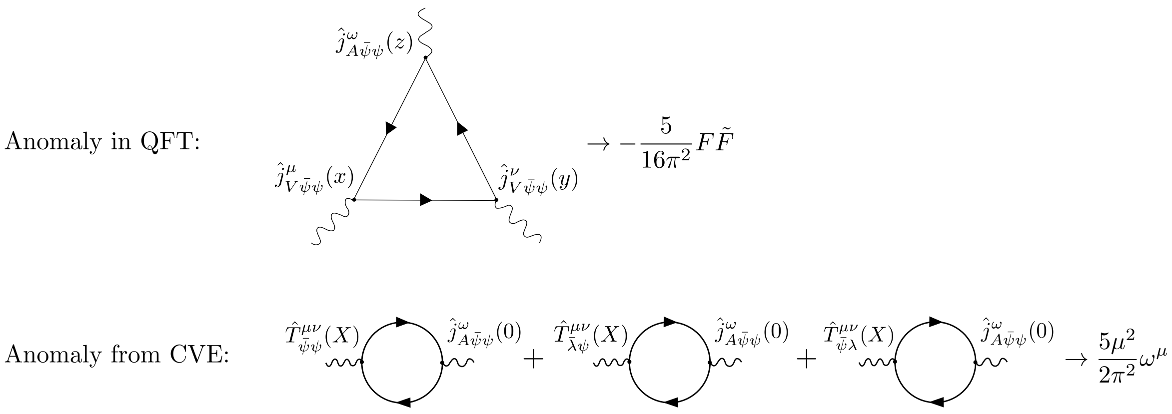

Fig. 1. On the top: derivation of the quantum anomaly with a factor of 5 from quantum field theory. Bottom: derivation of the same anomaly from the consideration of the vortical axial current.

Expanding (3) into a series in $\varpi_{\mu\nu}$ and using finite temperature quantum field theory, we can find the quantum corrections associated with the vorticity ω^μ. In particular, it can be shown that there is a vortical axial current for which, in the first order of perturbation theory, we obtain

$$\langle\hat{j}^\mu_A\rangle^{(1)} = W\,\omega^\mu\,, \quad W = C^{023|1} - C^{013|2}\,, \quad C^{\alpha\beta\gamma|i} = \int_0^{|\beta|}d\tau\int d^3x\, x^i\langle T_\tau\hat{T}^{\alpha\beta}(-i\tau,\mathbf{x})\hat{j}^\gamma_A(0)\rangle_{T,c}\,. \tag{4}$$

where $|\beta| = 1/T$, T_τ corresponds to the imaginary time ordering τ, c in $\langle...\rangle_{T,c}$ selects the connected correlators. This effect is called the Chiral Vortical Effect (CVE) and is now being actively studied at the theoretical and experimental level [8–10].

It turns out that there is a close relationship between vortical current of the type (4) and quantum anomaly (2), which is expressed in the relationship of the factors

$$
\begin{aligned}
CVE: \quad \langle \hat{j}^{\nu}_A \rangle &= (AT^2 + C\mu^2)\omega^{\nu}\,, \\
Anomaly: \langle \partial_\mu \hat{j}^{\mu}_A \rangle &= -\frac{C}{8}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}\,.
\end{aligned} \tag{5}
$$

There are various indications of the validity of the relationship (5). In particular, in [9] it was shown that it is a direct requirement of the second law of thermodynamics and the laws of hydrodynamics, in which we include the effect of the quantum anomaly. A similar conclusion was obtained from the effective field theory in [8] by considering additional vertices in the triangular diagram associated with the effects of the medium.

Eq. (5) can also be looked at from the other side – we can say that by calculating the vortical current, which is given by a two-point correlator with the stress-energy tensor and the axial current operators (4), we find an anomaly. Thus, we can obtain the anomaly by considering a vortical fluid and the corresponding vortical current.

Thus, the problem is to find the correlator in (4). To do this, we first expand the operators $\hat{T}^{\mu\nu}$ and $\hat{j}^{\mu}_A$ depending on the set of the fields

$$
\begin{aligned}
\hat{T}^{\mu\nu} &= \hat{T}^{\mu\nu}_{\bar{\psi}\psi} + \hat{T}^{\mu\nu}_{\bar{\lambda}\lambda} + \hat{T}^{\mu\nu}_{\bar{\psi}\lambda} + \hat{T}^{\mu\nu}_{\bar{\lambda}\psi}\,, \\
\hat{j}^{\mu}_A &= \hat{j}^{\mu}_{A\bar{\psi}\psi} + \hat{j}^{\mu}_{A\bar{\lambda}\lambda}\,.
\end{aligned} \tag{6}
$$

The formulas for the various terms in (6) were given in [6].

Then, we obtain that the mean value of the current is described by the sum of 8 diagrams. 5 of them are equal zero, since the λ field is non-propagating, that is, $\langle \lambda\bar{\lambda}\rangle = 0$ and because we follow the paper [3] and consider the limit $m \to \infty$. As a result, there are only three diagrams shown in the Fig. 1 on the bottom. Direct calculation gives the following expressions for them (indices show the set of fields in the operators)

$$
\begin{aligned}
W_{\bar{\psi}\psi\bar{\psi}\psi} &= -\frac{2}{3\pi^2}\int_0^{\infty} p\,dp - \frac{T^2}{6} - \frac{\mu^2}{2\pi^2}\,, \\
W_{\bar{\psi}\lambda\bar{\psi}\psi} &= -\frac{1}{3\pi^2}\int_0^{\infty} p\,dp + \frac{T^2}{2} + \frac{3\mu^2}{2\pi^2}\,, \\
W_{\bar{\lambda}\psi\bar{\psi}\psi} &= \frac{1}{\pi^2}\int_0^{\infty} p\,dp + \frac{T^2}{2} + \frac{3\mu^2}{2\pi^2}\,.
\end{aligned} \tag{7}
$$

To derive these formulas, it is necessary to use Euclidean propagators, in particular

$$
\langle T_\tau \tilde{\psi}_{a\mu}(X_1)\tilde{\bar{\psi}}_{b\nu}(X_2)\rangle_T = \sum\!\!\!\!\!\!\int_P e^{iP^+_\alpha(X_1-X_2)^\alpha}\frac{i}{2(P^+)^2}\Big(\tilde{\gamma}_\nu \not{P}^+\tilde{\gamma}_\mu + 2\Big[\frac{1}{m^2} - \frac{2}{(P^+)^2}\Big]P^+_\mu P^+_\nu \not{P}^+\Big)_{ab}\,, \tag{8}
$$

where μ, ν are Lorentzian indices and a, b are bispinor indices, and the following notations, convenient in the Euclidean case, were used

$$
t = -i\tau\,, \quad \gamma_\mu = i^{\delta_{0\mu}-1}\tilde{\gamma}_\mu\,, \quad \psi_\mu = i^{\delta_{0\mu}}\tilde{\psi}_\mu\,, \quad X_\mu = (\tau, -\mathbf{x})\,, \quad \sum\!\!\!\!\!\!\int_P = \frac{1}{|\beta|}\sum_{n=-\infty}^{\infty}\int\frac{d^3p}{(2\pi)^3}\,, \tag{9}
$$

$$
P^\pm_\mu = (p^\pm_n, -\mathbf{p})\,, \quad p^\pm_n = \pi(2n+1)/|\beta| \pm i\mu \quad (n = 0, \pm 1, \pm 2, \cdots)\,, \quad \not{P} = P_\mu\tilde{\gamma}_\mu\,, \quad (P^+)^2 = P^+_\mu P^+_\mu\,.
$$

This propagator contains the poles $\frac{1}{(P^+)^2}$ and $\frac{1}{(P^+)^4}$. The appearance of the fourth-order poles distinguishes the case of field with spin 3/2 from the simpler case of Dirac field. Summation over the Matsubara frequencies in the terms with $\frac{1}{(P^+)^2}$ is described, e.g., in [7], while the summation in the terms with $\frac{1}{(P^+)^4}$ needs a more complex formula [11]

$$\frac{1}{|\beta|}\sum_{\omega_n=\pi(2n+1)/|\beta|}\frac{f(\omega_n\pm i\mu)e^{i(\omega_n\pm i\mu)\tau}}{[(\omega_n\pm i\mu)^2+E^2]} = \sum_{s=\pm1}e^{\tau sE}\Bigg(\frac{(1-\tau sE)f(-isE)+iEf'(-isE)}{4E^3}[\theta(-s\tau) -n_F(E\pm s\mu)]+\frac{f(-isE)}{4E^2}n'_F(E\pm s\mu)\Bigg),$$

where $f(x)$ is some analytic function, $\theta(x)$ is the Heaviside function, and $n_F(E)$ is the Fermi distribution.

In (7), each term contains ultraviolet divergency of the form $\int p\,dp$. However, summing all the terms, we see that the sum of these terms is finite. Thus, due to the terms of interaction with the λ field, it is possible to cancel the ultraviolet divergences. As a result, we have the finite expression for CVE in the RSA theory

$$\langle \hat{j}^{\nu}_A\rangle = \left(\frac{5T^2}{6}+\frac{5\mu^2}{2\pi^2}\right)\omega^{\nu}\,. \tag{10}$$

The coefficient in the current in front of the term μ^2 turned out to be exactly equal to the factor 5 in the anomaly (2). Thus, using (5) we can say that we have obtained the anomaly (2) from the hydrodynamics of a vortical fluid. We also note the role of the interaction terms – they give a contribution to the vortical conductivity equal to 6, which, together with the contribution of the Rarita-Schwinger field equal to -1, lead to the factor 5, expected from the anomaly. This once again emphasizes the key role of the interaction terms.

Summarizing, we can say that there are at least two different ways to find chiral quantum anomaly: the first one is based on the standard quantum field theory and calculation of a 3-point diagram with two vector and one axial current $\langle VVA\rangle_{T=\mu=0}$, as shown in the Fig. 1 on the top. Another method is based on considering a vortical medium with finite T and μ and calculation of the vortical current given by the correlator $\langle T^{\mu\nu}A\rangle_{T,\mu=const}$. In the first case, diagrams with the additional field λ turn out to be insignificant, while in the second case they play a key role. However, the results are the same as a surprise.

3. Conclusion

The phenomenon at the intersection of different areas – quantum field theory and hydrodynamics – is considered. It turns out that for a vortical fluid, the transport coefficients are to be directly related to the most fundamental quantum phenomenon – quantum anomalies.

We directly tested the relationship between the gauge quantum anomaly and the vortical axial current at a finite chemical potential for the case of the extended theory of spin 3/2 field interacting with spin 1/2, and derived the anomalous factor 5 from the hydrodynamics.

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