Scattering in noncommutative quantum mechanics.

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Abstract. We derive the correction due to noncommutativity of space on Born approximation, then the correction for the case of Yukawa potential is explicitly calculated. The correction depends on the angle of scattering. Using partial wave method it is shown that the conservation of the number of particles in elastic scattering is also valid in noncommutative spaces which means that the unitarity relation is held in noncommutative spaces. We also show that the noncommutativity of space has no effect on the optical theorem.

1. Born approximation in noncommutative spaces.

The matrix elements M_{fi} are defined as follows :

$$M_{fi}^{NC} = \langle \psi_f \mid V \mid \psi_i \rangle_{NC} = \int d^3 r e^{-i\frac{\vec{p}_f.\vec{r}}{\hbar}} \star V(\vec{r}) \star e^{i\frac{\vec{p}_i.\vec{r}}{\hbar}} \tag{1}$$

Using the definition of star product to the first order we have :

$$\langle \psi_f \mid V \mid \psi_i \rangle_{NC} = \langle \psi_f \mid V \mid \psi_i \rangle_C + \tfrac{1}{2\hbar}\theta_{\mu\nu}(p_\mu^i + p_\mu^f)(\nabla V)_\nu e^{i\frac{(\vec{p}_i-\vec{p}_f).\vec{r}}{\hbar}}$$
$$+\frac{i}{2\hbar^2}\theta_{\mu\nu}p_\mu^f p_\nu^i V(\vec{r}) e^{i\frac{(\vec{p}_i-\vec{p}_f).\vec{r}}{\hbar}} \tag{2}$$

If we take the component of $\vec{\theta}$ in the direction perpendicular to the plane of $\vec{p}_f$ and $\vec{p}_i$(z-axis) equal to $\theta_3 = \theta$ and put the rest $\theta-$components to zero(which can be done by a rotation or a redifinition of coordinates), then :

$$\frac{i}{4\hbar^2}(\vec{p}_f \times \vec{p}_i).\vec{\theta} = \frac{i\theta}{4\hbar^2}p^2 \sin\phi. \tag{3}$$

where ϕ is the angle between $\vec{p}_f$ and $\vec{p}_i$. For Yukawa potential $V(r) = Z_1 Z_2 e^2 \frac{e^{-\frac{r}{a}}}{r}$, Eq.(2) leads to:

$$(\frac{d\sigma}{d\Omega})_{NC} = (\frac{Z_1 Z_2 e^2}{4E\sin^2(\frac{\phi}{2}) + \frac{\hbar^2}{2ma^2}})^2[1 + \frac{9\theta^2 p^4}{16\hbar^4}\sin^2\phi]. \tag{4}$$

2. Partial wave method in noncommutative spaces-Partial wave expansion of scattering amplitude.

We consider a sphere of radius $r = R$. Inside the sphere there may be given a potential $V(r)$; outside the sphere the potential may vanish, so :

$$u^{inside}(r, \theta') = \sum_{\ell=0}^{\infty} i^\ell (2\ell+1)[\frac{1}{kr}\chi_\ell(kr)] \star P_\ell(\cos\theta') \tag{5}$$

Outside the sphere $r = R$ we write :

$$u^{outside}(r,\theta') = \sum_{\ell=0}^{\infty} i^{\ell}(2\ell+1)\{\frac{1}{kr}[j_\ell(kr) + \frac{1}{2}\alpha_\ell h_\ell^{(1)}(kr)]\} \star P_\ell(\cos\theta') \tag{6}$$

One can show that the coefficients α_ℓ, satisfy in the following equation :

$$\alpha_\ell + 1 = -\frac{L_\ell e^{\frac{\theta}{2}\frac{\partial}{\partial x}}[\frac{1}{x}h_\ell^{(2)}(x)] - xe^{-\frac{\theta}{2}\frac{\partial}{\partial x}}[\frac{1}{x}h_\ell^{(2)\prime}(x)]}{L_\ell e^{\frac{\theta}{2}\frac{\partial}{\partial x}}[\frac{1}{x}h_\ell^{(1)}(x)] - xe^{-\frac{\theta}{2}\frac{\partial}{\partial x}}[\frac{1}{x}h_\ell^{(1)\prime}(x)]} \tag{7}$$

The numerator of $1+\alpha_\ell$ is complex conjugate of the donominator, so that :

$$\mid 1+\alpha_\ell \mid = 1 \tag{8}$$

This is an important result which shows the conservation of the number of particles in elastic scattering in noncommutative spaces, because the absolute squares of the amplitudes of ingoing and outgoing waves must be equal. This is the unitarity relation for the ℓth partial wave in a noncommutative space.
Finally one can easily show that the noncommutativity of space has no effect on the optical theorem and scattering amplitude of two identical particles,because:

$$(\frac{d\sigma}{d\Omega})_{NC} = \mid f(\phi) \mid^2 = f(\phi) \star f^*(\phi). \tag{9}$$

where :

$$f(\phi) = \frac{1}{k}\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell(k)}\sin\delta_\ell(k)P_\ell(\cos\phi). \tag{10}$$

but ϕ is the angle between the ingoing and outgoing particle momenta. This angle variable has nothing to do with NC vraiables $[\hat{x}_i, \hat{x}_j] = i\theta_{ij}$, so the star product in Eq.(9), reduces to ordinary produt. The same argument is true for the case of two identical particles, we have :

$$(\tfrac{d\sigma}{d\Omega})_{NC} = \mid f(\phi) + f(\pi-\phi) \mid^2 =$$

$$f^\star(\phi)\star f(\phi) + f^\star(\pi-\phi)\star f(\pi-\phi) + f^\star(\phi)\star f(\pi-\phi) + f^\star(\pi-\phi)\star f(\phi) \tag{11}$$

where ϕ is the the angle between the ingoing and outgoing particle momenta(two particles), and again this angle variable has nothing to do with noncommutative variables.

References

[1] Szabo R 2006*Class. Quant. Grav* **23** R199-R242 and 2003 *Phys. Rept.* **378** 207-299.