

Computational mathematics applied to astrophysics: Three cases of study

S Arceo Díaz¹, E E Bricio Barrios¹, K P Schröder², K Zuber^{3,4}, D Jack², and J A Verduzco Ramírez¹

¹ Instituto Tecnológico de Colima, Tecnológico Nacional de México, Colima, México

² División de Ciencias Naturales y Exactas, Universidad de Guanajuato, Guanajuato, México

³ Institut für Kern- und Teilchenphysik, Dresden, Germany

⁴ Institute for Nuclear Research, Debrecen, Hungary

E-mail: santiago.arceo@itcolima.edu.mx

Abstract. The role of computational mathematics on the study of the fundamental properties of neutrinos and axions and their impact on the energy loss within stellar structure is illustrated in this work. The theoretical predictions, done with the aid of a numerical code for simulating stellar evolution, are used in three cases of interest: the number of neutrinos reaching our planet from the closest stellar system, calculated by modelling its two main stars and their corresponding neutrino spectrum; the magnitude of the neutrino magnetic dipole moment and the axion-electron coupling constant is estimated by comparing stellar models to the tip-RGB of fifty globular clusters; and, finally, the survival of the Earth, after the Sun becomes a red giant is tested for an scenario in which axions and neutrinos are being simultaneously produced within the solar core.

1. Introduction

Neutrino astronomy is now well established within particle astrophysics. The detection of solar neutrinos in real-time, in form of direct measurements of the pp [1], pep [2], ⁷Be [3] and ⁸B neutrinos [4], made the solar energy production an experimental topic. Furthermore, the flux of neutrinos detected, coming from the supernova (SN-1987 A) [5] demonstrated the basic picture of core collapse supernovae. However, apart from the Sun or SN-1987 A, there are not more confirmed stellar sources of neutrinos. In this respect, computational astrophysics offers the opportunity of making a preliminary estimation on the magnitude of the neutrino flux coming from the closest stellar system, allowing to infer the probability for a direct detection for the next generation of neutrino telescopes.

Thermonuclear reactions are not the only source of neutrinos within low-mass stars. The so-called "thermal processes" [6], involving the production of neutrino pairs through their weak coupling with electrons are predicted to be active within the stellar core (although their typical energy should be three orders of magnitude lower). The effect of neutrino production is to deplete energy from the stellar core and depends on intrinsic properties of neutrinos. The discovery of neutrino flavour oscillations opened the possibility that neutrinos could have a non-zero magnetic dipole moment and, as it was soon made clear by [7], if neutrinos do possess a non-zero magnetic dipole moment, the amount of energy depleted from the core of red giants could have observable



consequences on their bolometric luminosity (making them a lot brighter than they should be). Even more, axions, particles originally proposed as a way to solve the problem of the breaking of the charge-parity symmetry [8], could also be produced within the core of red giant stars, by the same reactions producing neutrinos, also with observable consequences. Currently, the most restrictive limits on the magnetic dipole moment of neutrinos and the axion-electron coupling constant are set by computational stellar astrophysics [9, 10]. In this work we use the luminosity coming from fifty globular clusters in an attempt to test the validity of these constraints.

The survival of Earth after the red giant phase of the Sun (the so-called doomsday scenario) is a classical topic in stellar astrophysics. Most works relate Earth's fate to the interplay between the expansion rate of the Sun's outer layers and the rate in which mass is expelled from its surface due to the low gravity that it will have as a red giant: If the expansion rate of the solar giant's photosphere is faster than the rate in which mass is lost from it, then Earth inevitably will be engulfed by it. On the contrary, if the mass-loss rate is faster than the expansion rate of the Sun, then the orbit of our planet will grow enough as to escape its engulfment [11]. However, the dissipation of energy from the Sun's core, in the form of neutrinos and axions, has a major role in defining the photospheric radius prior to the helium flash: as long as the critical core temperature is not reached, the solar giant continues to become larger, expanding at a faster rate. The consequences on the simultaneous production of axions and neutrinos with a non-zero magnetic dipole moment, in the context of the future evolution of the solar system are shown below.

2. Methodology

This study used the numerical code created by Eggleton [12] to simulate stellar evolution. The present version is capable of calculating the total number of neutrinos produced by any of the thermonuclear and thermal reactions occurring in the core of low-mass stars [9, 10]. The number of neutrinos reaching our planet from the α -Centauri system was estimated by calculating the neutrino production rate per second of each of the thermonuclear reactions, integrated from the stellar core to its surface. Then, the neutrino flux at Earth was obtained by dividing the total number of neutrinos produced each second by the surface of a sphere whose radius is equal to the distance modulus of the α -Centauri system. Additionally, the neutrino spectrum (i.e. the number of neutrinos produced each second per MeV of energy produced by each reaction) of both stars was obtained. From it, a direct comparison to the standard solar neutrino spectrum was made to analyze what differences can be expected due to the differences in mass and chemical composition.

The effect of the neutrino magnetic dipole moment and axion production was included by following [7] and [13]. Two different scenarios were considered: i) enhanced neutrino emission (due to a non-zero magnetic dipole moment) and ii) normal neutrino emission combined with axion production. The resulting stellar models representing these two anomalous scenarios were compared against the canonical case. As explained by [9] Stellar tracks with $M_i = 0.8 - 1.1 M_\odot$, $Z = 0.0001 - 0.02$ and hydrogen and helium mass fractions according to Pols *et al.* [14] were constructed and the theoretical tip-RGB was defined as the model in which helium luminosity reached $10 L_\odot$ [9].

The doomsday scenario depicted below follows [11], using the formulation of planetary orbits by [15]. The construction of the temporal evolution of planetary orbits was made in two stages: first the Eggleton code was used to evolve a zero age main-sequence (ZAMS) up to the end of the asymptotic giant phase (two sets of models were considered: the canonical scenario and the one in which axion production and enhanced neutrino emission happened simultaneously). Afterwards, the orbital evolution of each planet within the inner solar system was calculated by using a fourth-order Runge-Kutta algorithm.

3. Results

3.1. The neutrino flux coming from the α -Centauri system

α -Centauri is the closest stellar system to Earth. Its two main stars are among the most probable candidates for a future first direct detection of neutrinos produced by main-sequence stars apart from the Sun [16]. The observational calibrations done by [17, 18] were used as references for selecting the best fitting models, and from these models the neutrino luminosity for α -Centauri A and B was calculated (see Table 1). Due to their difference in mass, the neutrino output of each star is different in magnitude: while the smaller star, α -Centauri B, is predicted to have a central temperature and density that should be slightly lower than the estimated temperature and density for the Sun (hence its neutrino luminosity its only around one third of the solar neutrino luminosity), the denser and hotter core of α -Centauri A is predicted to produce almost twice the solar neutrino luminosity. Even more, the models for both stars also show differences in which kind of thermonuclear reactions should be the most prominent source of neutrinos: the pp-chain should be responsible for 96% of the neutrino luminosity of α -Centauri B, while the model for α -Centauri A predicts that already 35% of its neutrino luminosity should be produced by the reactions of the CNO cycle.

Table 1. Stellar parameters and neutrino luminosity for the main stars in the α -Centauri system. Stellar models were calculated by assuming an initial metallicity and hydrogen content of $Z = 0.03$ and $X_H = 0.685$.

Star	α -Centauri A	α -Centauri B
M [M_\odot]	1.10	0.90
T_{eff} [K]	5840	5180
L_{bol} [L_\odot]	1.52	0.46
R_* [R_\odot]	1.23	0.85
T_c [10^6 K]	18.74	13.44
ρ_c [$10^2 \text{g} \cdot \text{cm}^{-3}$]	2.35	1.12
$L_{\nu T}$ [$L_{\nu\odot}$]	1.97	0.33

Figure 1 shows the neutrino spectrum for α -Centauri A and α -Centauri B, as if the stars were placed at 1 astronomical unit (AU) from Earth (this was done to establish a more direct comparison between these two stars and the solar neutrino spectrum [19], represented by the dashed lines on each panel). The hotter core in α -Centauri A should enhance the reactions of the CNO-cycle, making the corresponding neutrino flux an order of magnitude larger than the Sun's, while the flux from the pp-I and p-e-p reactions remains at the same level as in the Sun's core. The total neutrino spectrum for α -Centauri B should be similar to the Sun's. However, there would a difference in the fluxes from the ^8B and ^7Be reactions, as on α -Centauri B they should be several orders of magnitude smaller, due to their high sensitivity on temperature [20].

The integrated neutrino production from both stars amounts to $2.62 \pm 0.28 \times 10^{38} \nu \cdot \text{s}^{-1}$. Taking into account the distance between these stars and the Earth, the flux of neutrinos reaching our planet should be around $1.8 \nu \cdot \text{s}^{-1} \cdot \text{cm}^{-2}$.

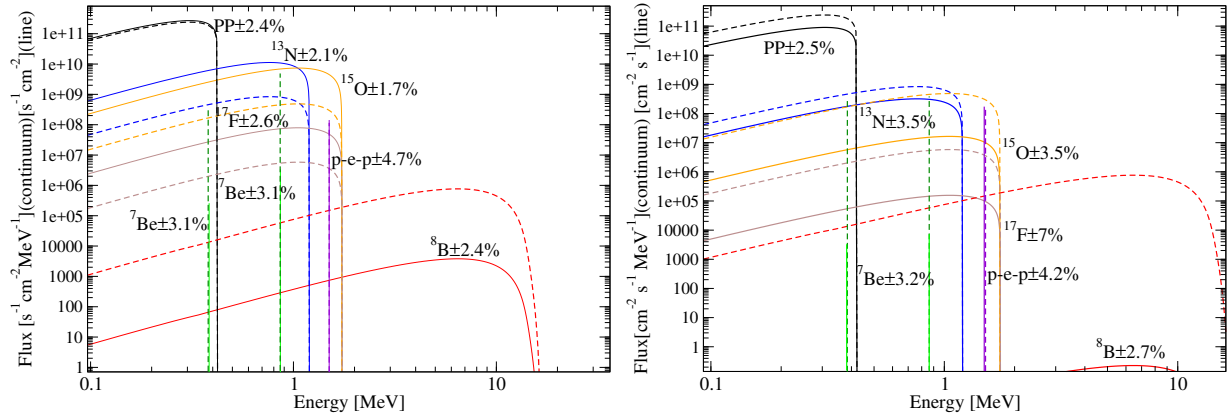


Figure 1. Neutrino spectrum for the two main stars of the α -Centauri system (solid lines), compared to the solar neutrino spectrum (dashed lines) from the standard solar model, as if each star was located at 1 A.U. The numbers represent the variation on the flux related to the choice of initial mass for the stellar model.

3.2. The effect of non-standard neutrino properties and axion production at the end of the red giant branch

The existence of a non-zero magnetic dipole moment for neutrinos has strong consequences on the physical conditions within the core of a star similar to the Sun as soon as it grows old and becomes a red giant. The bolometric luminosity of red giant stars is mostly dominated by the mass of their helium cores: the heavier the core gets, the brighter the star becomes, as the surrounding hydrogen burning shell is forced by hydrostatic equilibrium to produce more energy to compensate for its growing gravitational pull. This leads to a feedback situation in which hydrogen burning produces more helium, making the core to become heavier, and this, in turn, induces a progressive increment on the star's bolometric luminosity. The helium flash, the event that terminates the red giant phase, happens only until the density and temperature of the core reach the critical values necessary for starting helium fusion by the triple-alpha process [21]. The amount of energy lost by the production of weakly interacting particles (like neutrinos or axions) delays the helium flash, implying that the core could have more time to augment its mass and the bolometric luminosity of the star could become even brighter than what is estimated by the canonical stellar evolution theory [7].

A meaningful estimation of the impact of the neutrino magnetic dipole moment and axions on the bolometric luminosity of red giants required to compare to observational evidence. Globular clusters contain the largest populations of red giants in the galaxy, thus providing enough data for a statistically significant constraint on by how much the bolometric luminosity of stellar models could be enhanced, by axions and the magnetic dipole moment of neutrinos, without contradicting the observational evidence. A sample, from the largest homogeneous NIR-database [22–25], was extracted. To select the sample, two restrictions were made: only clusters with at least 30 stars on the two brightest bins below the tip-RGB (the luminosity at which the helium flash is predicted to happen) were chosen, this rendered the statistical uncertainty between this point and the brightest red giant to $\sigma_s \leq 0.16$ magnitude. Also, only clusters with a single RGB were selected (with the exception of ω -Centauri, for which its multiple stellar populations do not affect the tip-RGB [26]). The mean value for the absolute bolometric magnitude of the tip-RGB from the sample is $\langle M_{\text{obs}}^{\text{tip}} \rangle = -3.64$ (with a standard deviation $\sigma = 0.17$).

To compare against observational evidence, stellar tracks, representing stars evolving from the main-sequence to the tip-RGB, were constructed. All the stellar tracks have an initial

mass $M_i = 0.95 M_\odot$ and a metallicity that matched the reported global metallicity $[M/H]$ for each cluster. The tip-RGB models correspond to the particular point in stellar tracks in which $L_{\text{He}} = 10L_\odot$. For each cluster three tracks were considered: the canonical track (without axion productions or neutrinos with a magnetic dipole moment) and two non-standard scenarios: one with $\mu_\nu = 2.2 \times 10^{-12} \mu_B$ and no axions and the other with an axion-electron coupling constant of $\alpha_a = 0.5 \times 10^{-26}$ but no magnetic dipole moment for neutrinos.

Figure 2 compares the absolute bolometric magnitude of the globular clusters in the sample (the observational calibration represented by the black asterisks) with our stellar models (circles symbolize canonical models while triangle and diamonds correspond to models with either $\mu_{12} = 2.2$ or $\alpha_{26} = 0.5$). The canonical models predict bolometric magnitudes closer than 0.05 from the empirical value from observations and, in most cases, these are less than 0.1 magnitude away from the observational calibration (the clusters showing the best agreement are marked by asterisks). Despite their different nature both anomalies raise the tip-RGB by approximately the same amount (triangles almost superposed with diamonds through all the metallicity range). For 38 globular clusters, the absolute magnitude of non-standard tip-RGB surpasses the upper limit of the observational calibration, allowing to support $\mu_\nu \leq 2.2 \times 10^{-12} \mu_B$ and $\alpha_a \leq 0.5 \times 10^{-26}$ as valid constraints.

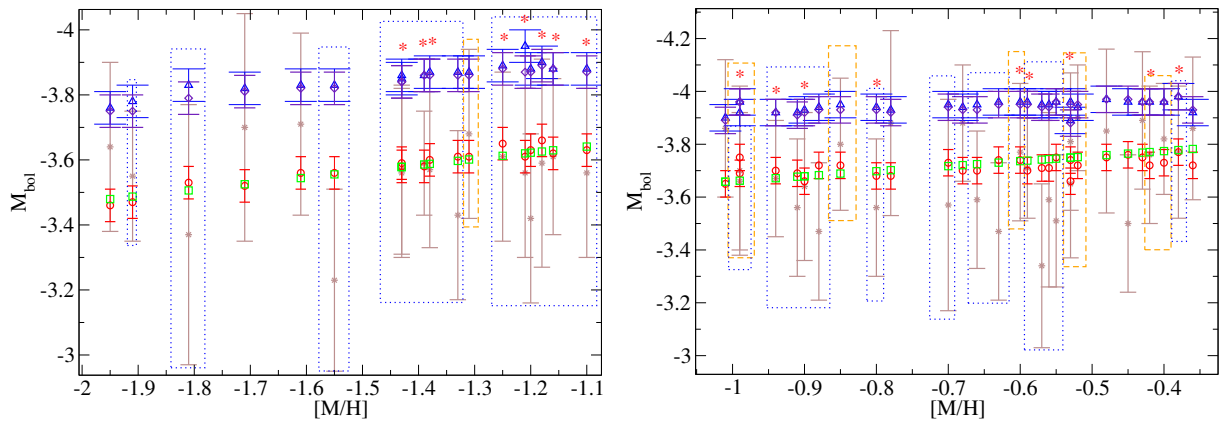


Figure 2. Comparison between the observational tip-RGB (black asterisk), its empirical estimation (green square) and our canonical (red circle) and non-standard models, with $\mu_{12} = 2.2 \times 10^{-12} \mu_B$ (upward-blue triangle) and $\alpha_a = 0.5 \times 10^{-26}$ (purple diamonds).

3.3. The future of the inner solar system

Figure 3 follows the orbital evolution for the inner solar system from the RGB phase to the end of the AGB, under the assumption that neutrinos, with $\mu_\nu = 2.2 \times 10^{-12} \mu_B$, and axions, with $\alpha_a \leq 0.5 \times 10^{-26}$, are simultaneously produced with the stellar core. While in the canonical scenario (studied multiple times by other authors) our planet could survive the first red giant phase of the Sun (see Table 2), on this case, Earth and the innermost planets quickly fall into the Sun during its first time as a red giant: Mercury and Venus spiral inwards 5.06 and 2.69 million years before the end of the red giant phase, while the solar photosphere reaches Earth about one million years after the fall of Venus. The final radius of the solar giant reaches almost to Mars's orbit. However, due to the decreasing gravitational pull caused by the stellar mass-loss, Mars has moved 0.5 AU further away. It remains to be studied if this distance could be considered safe, as the radiation coming from the star should be orders of magnitude larger to what its currently predicted by the canonical stellar evolution theory [27].

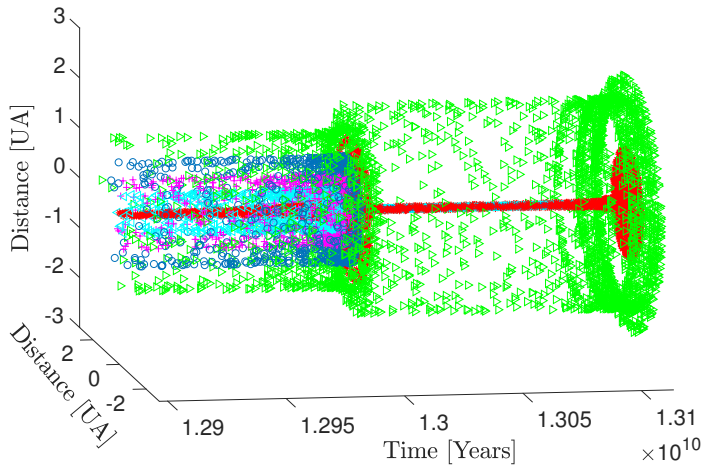


Figure 3. Orbital evolution of the inner solar system from the RGB-phase to the end of the AGB-phase. The solar photosphere is represented by the red marks. The orbits of Mercury (light bluish marks), Venus (magenta marks), Earth (blue marks) and Mars (green marks) correspond to the other symbols.

Table 2. Solar models and orbital distances for the planets within the inner solar system, measured in AU.

Body	$R_{p(\text{Now})}$ [AU]	$R_{p(\text{Tip-RGB})}$ [AU]	$R_{p(\text{Tip-AGB})}$ [AU]	Fall Time
Canonical				
Sun	0.0046	1.0731	0.6458	13.0187
Mercury	0.3886	0.0006	0.0000	13.0129
Venus	0.7219	0.0048	0.0000	13.0167
Earth	1.0000	0.3877	0.0000	13.0187
Mars	1.5241	2.0722	2.6203	-
Non-standard				
Sun	0.0046	1.41711	0.9072	12.9948
Mercury	0.3886	0.00005	0.0000	12.9897
Venus	0.7219	0.00036	0.0000	12.9919
Earth	1.0000	0.00708	0.0000	12.9931
Mars	1.5241	2.05214	2.5114	-

4. Conclusions

In this work three cases related to the use of stellar modelling were shown: the flux of neutrinos reaching Earth from the α -Centauri system was predicted to be less than two neutrinos per square centimeter per second and the neutrino spectrum for the two main stars of the system was obtained and compared to the solar case. The observational calibration for the tip-RGB bolometric luminosity was compared against the predictions of stellar models with and without a neutrino magnetic dipole moment and axion production, yielding $\mu_\nu \leq 2.2 \times 10^{-12} \mu_B$ and $\alpha_a \leq 0.5 \times 10^{-26}$, representing the most restrictive constraints on these physical parameters, obtained from a large population of globular clusters. Finally, using these values for μ_ν and α_a , the doomsday scenario was revisited, showing that the energy loss due to neutrino and axion emission would condemn Earth to falling into the Sun. Even if Mars escapes, its surface could be severely irradiated due to the dramatic increment in the solar bolometric luminosity.

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