Collinear distributions
in double parton scattering

Mikhail G. Ryskin[1], Alexander M. Snigirev[2]*

[1] Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg, 188300, Russia
[2] Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, 119991, Moscow, Russia

In this talk we consider revised formulas which operate with the modified collinear two-parton distributions extracted from deep inelastic scattering to describe the inclusive cross section of a double parton scattering in a hadron collision. The related phenomenological effects are discussed.

1 Introduction and customary formalism

Now it has become clear that multiple parton interactions play an important role in high energy hadronic collisions and are one of the most common, yet poorly understood [1], phenomenon at the LHC. Experimental evidence for double hard scattering has been found in the production of multijets[2, 3, 4] and of single photons associated with three jets [5, 6]. The theoretical investigation of multiple parton interactions has a long history and has experienced a renewed interest in more recent times (see, for instance, [1] and references therein), driven by the need to understand the hadronic activity at the LHC.

Nevertheless, the phenomenology of multiple parton interactions relies on the models which are essentially intuitive and involve substantial simplifying assumptions. Therefore, it is extremely desirable to combine theoretical efforts in order to achieve a better description of multiple interactions, in particular, double scattering, which is very likely to be an important multiple scattering mode at the LHC. In this talk we consider some steps towards this purpose basing on our previous work [7]. The cross section formulas currently used to calculate the double scattering processes are revised basing on the modified collinear two-parton distributions extracted from deep inelastic scattering (DIS).

Using only the assumption of factorization of the two hard parton processes A and B, the inclusive cross section of a double parton scattering process in a hadron collision may be written in the following form

$$\sigma_{(A,B)}^{D} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; Q_1^2, Q_2^2) \times \hat{\sigma}_{ik}^{A}(x_1, x_1^{'}, Q_1^2) \hat{\sigma}_{jl}^{B}(x_2, x_2^{'}, Q_2^2)$$
$$\times \Gamma_{kl}(x_1^{'}, x_2^{'}; \mathbf{b_1} - \mathbf{b}, \mathbf{b_2} - \mathbf{b}; Q_1^2, Q_2^2) dx_1 dx_2 dx_1^{'} dx_2^{'} d^2 b_1 d^2 b_2 d^2 b, \tag{1}$$

*Speaker

where $\mathbf{b}$ is the impact parameter — the distance between centers of colliding hadrons (e.g., the beam and the target) in transverse plane. $\Gamma_{ij}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; Q_1^2, Q_2^2)$ are the double parton distribution functions, which depend on the longitudinal momentum fractions x_1 and x_2, and on the transverse position $\mathbf{b_1}$ and $\mathbf{b_2}$ of the two partons undergoing the hard processes A and B at the scales Q_1 and Q_2. $\hat{\sigma}_{ik}^A$ and $\hat{\sigma}_{jl}^B$ are the parton-level subprocess cross sections. The factor $m/2$ appears due to the symmetry of the expression for interchanging parton species i and j. $m = 1$ if $A = B$, and $m = 2$ otherwise.

It is typically assumed that the double parton distribution functions may be decomposed in terms of longitudinal and transverse components as follows:

$$\Gamma_{ij}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; Q_1^2, Q_2^2) = D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) f(\mathbf{b_1}) f(\mathbf{b_2}), \qquad (2)$$

where $f(\mathbf{b_1})$ is supposed to be a universal function for all kinds of partons with its normalization fixed as

$$\int f(\mathbf{b_1}) f(\mathbf{b_1} - \mathbf{b}) d^2 b_1 d^2 b = \int T(\mathbf{b}) d^2 b = 1, \qquad (3)$$

and $T(\mathbf{b}) = \int f(\mathbf{b_1}) f(\mathbf{b_1} - \mathbf{b}) d^2 b_1$ is the overlap function.

If one also makes the assumption that the longitudinal components $D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2)$ reduce to the product of two independent one parton distributions,

$$D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) = D_h^i(x_1; Q_1^2) D_h^j(x_2; Q_2^2), \qquad (4)$$

the cross section of double parton scattering can be expressed in the simple form

$$\sigma_{(A,B)}^D = \frac{m}{2} \frac{\sigma_{(A)}^S \sigma_{(B)}^S}{\sigma_{\text{eff}}}, \qquad (5)$$

$$\sigma_{\text{eff}} = [\int d^2 b (T(\mathbf{b}))^2]^{-1}. \qquad (6)$$

In this representation and at the factorization of longitudinal and transverse components, the inclusive cross section of single hard scattering is written as

$$
\begin{aligned}
\sigma_{(A)}^S &= \sum_{i,k} \int D_h^i(x_1; Q_1^2) f(\mathbf{b_1}) \hat{\sigma}_{ik}^A(x_1, x_1') D_{h'}^k(x_1'; Q_1^2) f(\mathbf{b_1} - \mathbf{b}) dx_1 dx_1' d^2 b_1 d^2 b \\
&= \sum_{i,k} \int D_h^i(x_1; Q_1^2) \hat{\sigma}_{ik}^A(x_1, x_1') D_{h'}^k(x_1'; Q_1^2) dx_1 dx_1'. \qquad (7)
\end{aligned}
$$

These simplifying assumptions, though rather customary in the literature and quite convenient from a computational point of view, are not sufficiently justified and should be revised [7, 8, 9, 10]. However, the starting cross section formula (1) was found (derived) in many works (see, e.g., Refs. [8, 9, 10]) using the light-cone variables and the same approximations as those applied to the processes with a single hard scattering.

2 Revised formulas in momentum representation

All the previous formulas were written in the mixed (momentum and coordinate) representation. Recall that in general, for the case of the multiple parton interactions, we have to use
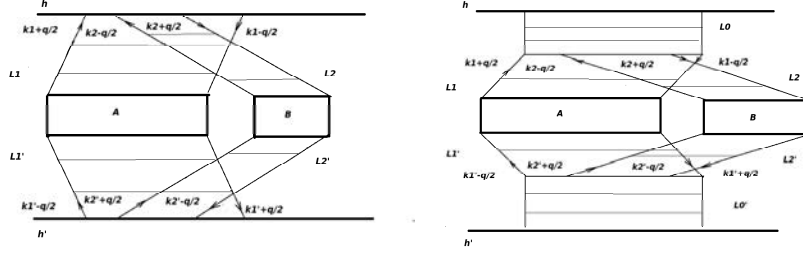
Figure 1: Graphs for double parton scattering according to the first term in Eq. (9) (left) and according to the second evolution term in Eq. (9) (right). A and B are the hard parton subprocesses. q is the momentum transfer through the ladders $L1, L2, L1'$ and $L2'$.

the *Generalized* Parton Distribution Functions (GPDF). In other words, in the Feynman diagram (ladder) which describes the GPDF, the parton momenta k_L (in the left part of diagram corresponding to the amplitude A^*) and k_R (in the right part of the diagram corresponding to amplitude A) may be different. Let us denote $k_L = k + q/2$ and $k_R = k - q/2$, where q is the momentum transfer through the whole ladder. Since the ladders in Figure 1 form a loop we will call q the loop momentum. In the previous formulas, instead of transverse momentum q_t, we used the conjugate coordinate $\mathbf{b}$ [1].

For our further goal the momentum representation is more convenient:

$$\sigma_{(A,B)}^D = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2; \mathbf{q}; Q_1^2, Q_2^2) \hat{\sigma}_{ik}^A(x_1, x_1') \hat{\sigma}_{jl}^B(x_2, x_2')$$

$$\times \Gamma_{kl}(x_1', x_2'; -\mathbf{q}; Q_1^2, Q_2^2) dx_1 dx_2 dx_1' dx_2' \frac{d^2q}{(2\pi)^2}. \tag{8}$$

The hard subprocesses A and B originate from two different branches of the parton cascade. Note that only the sum of the parton momenta (in both branches) is conserved, while in each individual branch there may be some difference, q, between the transverse (parton) momenta in the initial state wave function and the conjugate wave function.

The main problem is to make the correct calculation of $\Gamma_{ij}(x_1, x_2; \mathbf{q}; Q_1^2, Q_2^2)$ without simplifying assumptions (2) and (4). These functions are available in the current literature [11, 12, 13, 14] only for $\mathbf{q} = 0$ in the collinear approximation. In this approximation the two-parton distribution functions, $\Gamma_{ij}(x_1, x_2; \mathbf{q} = 0; Q^2, Q^2) = D_h^{ij}(x_1, x_2; Q^2, Q^2)$ with the two hard scales set equal, satisfy the generalized Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations, derived initially in Refs. [13, 14].

The evolution equation for Γ_{ij} consists of two terms. The first term describes the independent (simultaneous) evolution of two branches of parton cascade: one branch contains the parton x_1, and another branch — the parton x_2. The second term allows for the possibility of splitting of one parton evolution (one branch k) into two different branches, i and j. It contains the usual splitting function $P_{k \to ij}(z)$. The solutions of the generalized DGLAP evolution equations with the given initial conditions at the reference scales μ^2 may be written [7, 15] in

---

[1]In the case of conventional DIS the cross section is given by the integral over $\mathbf{b}$ corresponding to $q_t = 0$.

the form:

$$D_h^{j_1 j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2) = D_{h1}^{j_1 j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2) + D_{h2}^{j_1 j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2) \qquad (9)$$

with

$$D_{h1}^{j_1 j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2)$$

$$= \sum_{j_1' j_2'} \int\limits_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int\limits_{x_2}^{1-z_1} \frac{dz_2}{z_2} \, D_h^{j_1' j_2'}(z_1, z_2; \mu^2) D_{j_1'}^{j_1}(\frac{x_1}{z_1}; \mu^2, Q_1^2) D_{j_2'}^{j_2}(\frac{x_2}{z_2}, \mu^2, Q_2^2) \qquad (10)$$

and

$$D_{h2}^{j_1 j_2}(x_1, x_2; \mu^2, Q_1^2, Q_2^2)$$

$$= \sum_{j' j_1' j_2'} \int\limits_{\mu^2}^{\min(Q_1^2, Q_2^2)} dk^2 \frac{\alpha_s(k^2)}{2\pi k^2} \int\limits_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int\limits_{x_2}^{1-z_1} \frac{dz_2}{z_2} D_h^{j'}(z_1 + z_2; \mu^2, k^2)$$

$$\times \frac{1}{z_1 + z_2} P_{j' \to j_1' j_2'}\left(\frac{z_1}{z_1 + z_2}\right) D_{j_1'}^{j_1}(\frac{x_1}{z_1}; k^2, Q_1^2) D_{j_2'}^{j_2}(\frac{x_2}{z_2}; k^2, Q_2^2), \qquad (11)$$

where $\alpha_s(k^2)$ is the QCD coupling, $D_{j_1'}^{j_1}(z; k^2, Q^2)$ are the known single distribution functions (the Green's functions) at the parton level with the specific δ-like initial conditions at $Q^2 = k^2$. $D_h^{j_1' j_2'}(z_1, z_2, \mu^2)$ is the initial (input) two-parton distribution at the relatively low scale μ. The one parton distribution (before splitting into the two branches at some scale k^2) is given by $D_h^{j'}(z_1 + z_2, \mu^2, k^2)$. Note, that in Eq. (9) we assume that the loop momentum $q < \mu$ is small and due to strong ordering of parton transverse momenta in the collinear DGLAP evolution it may be neglected.

The first term is the solution of homogeneous evolution equation (independent evolution of two branches), where the input two-parton distribution is generally not known at the low scale μ. For this nonperturbative two-parton function at low z_1, z_2 one may assume the factorization $D_h^{j_1' j_2'}(z_1, z_2, \mu^2) \simeq D_h^{j_1'}(z_1, \mu^2) D_h^{j_2'}(z_2, \mu^2)$ neglecting the constraints due to momentum conservation ($z_1 + z_2 < 1$). This leads to

$$D_{h1}^{ij}(x_1, x_2; \mu^2, Q_1^2, Q_2^2) \simeq D_h^i(x_1; \mu^2, Q_1^2) D_h^j(x_2; \mu^2, Q_2^2). \qquad (12)$$

As a rule, the multiple interactions take place at relatively low transverse momenta and low x_1, x_2 where the factorization hypothesis (12) for the first term is a good approximation. In this case, the cross section for double parton scattering can be estimated, using the two-gluon form factor of the nucleon $F_{2g}(q)$ [8, 16] for the dominant gluon-gluon scattering mode (or something similar for other parton scattering modes),

$$\sigma_{(A,B)}^{D,1\times 1} = \frac{m}{2} \sum_{i,j,k,l} \int D_h^i(x_1; \mu^2, Q_1^2) D_h^j(x_2; \mu^2, Q_2^2) \hat{\sigma}_{ik}^A(x_1, x_1') \hat{\sigma}_{jl}^B(x_2, x_2')$$

$$\times D_{h'}^k(x_1'; \mu^2, Q_1^2) D_{h'}^l(x_2'; \mu^2, Q_2^2) dx_1 dx_2 dx_1' dx_2' \int F_{2g}^4(q) \frac{d^2 q}{(2\pi)^2}. \qquad (13)$$

From the dipole fit $F_{2g}(q) = 1/(q^2/m_g^2 + 1)^2$ to the two-gluon form factor follows that the characteristic value of q is of the order of the "effective gluon mass" m_g. Thus the initial conditions for the single distributions can be fixed at some not large reference scale $\mu \sim m_g$, because of the weak logarithmic dependence of these distributions on the scale value. In this approach $\int F_{2g}^4(q)\frac{d^2q}{(2\pi)^2}$ gives the estimation of $[\sigma_{\text{eff}}]^{-1}$.

The second term in Eq. (9) is the solution of complete evolution equation with the evolution originating from one "nonperturbative" parton at the reference scale. Here, the independent evolution of two branches starts at the scale k^2 from a point-like parton j'. In this case the large q_t domain is not suppressed by the form factor $F_{2g}(q)$ and the corresponding contribution to the cross section reads

$$
\begin{aligned}
\sigma_{(A,B)}^{D,2\times2} = \quad & \frac{m}{2}\sum_{i,j,k,l}\int dx_1 dx_2 dx_1' dx_2' \int^{\min(Q_1^2,Q_2^2)} \frac{d^2q}{(2\pi)^2} \\
\times & \sum_{j'j_1'j_2'}\int_{q^2}^{\min(Q_1^2,Q_2^2)} dk^2 \frac{\alpha_s(k^2)}{2\pi k^2}\int_{x_1}^{1-x_2}\frac{dz_1}{z_1}\int_{x_2}^{1-z_1}\frac{dz_2}{z_2}D_h^{j'}(z_1+z_2;\mu^2,k^2) \\
\times & \frac{1}{z_1+z_2}P_{j'\to j_1'j_2'}\left(\frac{z_1}{z_1+z_2}\right)D_{j_1'}^i(\frac{x_1}{z_1};k^2,Q_1^2)D_{j_2'}^j(\frac{x_2}{z_2};k^2,Q_2^2)\hat{\sigma}_{ik}^A(x_1,x_1')\hat{\sigma}_{jl}^B(x_2,x_2') \\
\times & \sum_{j'j_1'j_2'}\int_{q^2}^{\min(Q_1^2,Q_2^2)} dk'^2 \frac{\alpha_s(k'^2)}{2\pi k'^2}\int_{x_1'}^{1-x_2'}\frac{dz_1}{z_1}\int_{x_2'}^{1-z_1}\frac{dz_2}{z_2}D_{h'}^{j'}(z_1+z_2;\mu^2,k'^2) \\
\times & \frac{1}{z_1+z_2}P_{j'\to j_1'j_2'}\left(\frac{z_1}{z_1+z_2}\right)D_{j_1'}^k(\frac{x_1'}{z_1};k'^2,Q_1^2)D_{j_2'}^l(\frac{x_2'}{z_2};k'^2,Q_2^2),
\end{aligned}
\tag{14}
$$

or in substantially shorter yet less transparent form:

$$
\begin{aligned}
\sigma_{(A,B)}^{D,2\times2} = \quad & \frac{m}{2}\sum_{i,j,k,l}\int dx_1 dx_2 dx_1' dx_2' \int^{\min(Q_1^2,Q_2^2)} \frac{d^2q}{(2\pi)^2} \\
\times & D_{h2}^{ij}(x_1,x_2;q^2,Q_1^2,Q_2^2)\hat{\sigma}_{ik}^A(x_1,x_1')\hat{\sigma}_{jl}^B(x_2,x_2')D_{h'2}^{kl}(x_1',x_2';q^2,Q_1^2,Q_2^2).
\end{aligned}
\tag{15}
$$

By analogy, the combined ("interference") contribution may be written as

$$
\begin{aligned}
\sigma_{(A,B)}^{D,1\times2} = \quad & \frac{m}{2}\sum_{i,j,k,l}\int dx_1 dx_2 dx_1' dx_2' \int^{\min(Q_1^2,Q_2^2)} F_{2g}^2(q)\frac{d^2q}{(2\pi)^2} \\
\times & [D_h^i(x_1;\mu^2,Q_1^2)D_h^j(x_2;\mu^2,Q_2^2)\hat{\sigma}_{ik}^A(x_1,x_1')\hat{\sigma}_{jl}^B(x_2,x_2')D_{h'2}^{kl}(x_1',x_2';q^2,Q_1^2,Q_2^2) \\
+ & D_{h2}^{ij}(x_1,x_2;q^2,Q_1^2,Q_2^2)\hat{\sigma}_{ik}^A(x_1,x_1')\hat{\sigma}_{jl}^B(x_2,x_2')D_{h'}^k(x_1';\mu^2,Q_1^2)D_{h'}^l(x_2';\mu^2,Q_2^2)].
\end{aligned}
\tag{16}
$$

The equations (13), (15) and (16) present our solution of the problem — we obtain the estimation of the inclusive cross section for double parton scattering, taking into account the

QCD evolution and based on the well-known collinear distributions extracted from deep inelastic scattering. Similar results were obtained in Ref. [17] with an emphasis on the differential cross sections. However, one should note that the input two-parton distribution $D_h^{j_1', j_2'}(z_1, z_2, \mu^2)$ may be more complicated than that given by factorization ansatz (12). Now, let us discuss in more detail the second term, that is the 2×2 contribution.

3 Discussion and conclusions

The contribution to the cross section from the second term induced by the QCD evolution cannot be reduced to the form (5) with some new constant effective cross section as it was done in earlier estimations [18, 19, 20]. The QCD evolution effects for the cross section are anticipated to be larger than for the two-parton distribution functions. For those such effects were estimated in Refs. [12, 21] on the level of 10% - 30% as compared to the "factorization" components at $x \sim 0.1$ and $Q \sim 100$ GeV. Indeed, in Eq. (14) the integration over q includes no strong suppression factor $F_{2g}(q)$ and the phase space integral may be estimated as

$$\int^{Q^2} dq^2 \int_{q^2}^{Q^2} \frac{dk^2}{k^2} \int_{q^2}^{Q^2} \frac{dk'^2}{k'^2} \simeq 2Q^2, \quad Q^2 \gg \mu^2, \tag{17}$$

where within the leading order (LO) accuracy we take q^2 as the lower limit for k^2 and k'^2 integrations; at $q^2 > k^2$ the loop momentum q_t destroys the logarithmic structure of the integrals for collinear evolution from k^2 to Q^2.

We see that at a large final scale Q^2 the contribution of the second (2×2) component should dominate being proportional to Q^2, while the contributions of the 1×1 or 1×2 components $\sim m_g^2 \sim 1/\sigma_{eff}$ are limited by the nucleon (hadron) form factor F_{2g} [2].

The real gain is, of course, smaller due to the running coupling constant and the fact that at low x distribution functions grow logarithmically on the integration variables. So we have the additional factor in favor of the first factorized term of Eq. (9), which is proportional [22] to the initial gluon and quark multiplicities: the second term evolves from *one* "nonperturbative" parton, while the first term has *two* initial independent "nonperturbative" partons at the reference scale.

As a result, the experimental effective cross section, $\sigma_{\text{eff}}^{\text{exp}}$, which is not measured directly but is extracted by means of the normalization to the product of two single cross sections:

$$\frac{\sigma_{DPS}^{\gamma+3j}}{\sigma^{\gamma j} \sigma^{jj}} = [\sigma_{\text{eff}}^{\text{exp}}]^{-1}, \tag{18}$$

appears to be dependent on the probing hard scale. It should decrease with increasing the resolution scale because all additional contributions to the cross section of double parton scattering are positive and increase. In the above formula, $\sigma^{\gamma j}$ and σ^{jj} are the inclusive $\gamma+$ jet and dijets cross sections, $\sigma_{DPS}^{\gamma+3j}$ is the inclusive cross section of the $\gamma+3$ jets events produced in the double parton process. It is worth noticing that the CDF and D0 Collaborations extract $\sigma_{\text{eff}}^{\text{exp}}$ without any theoretical predictions on the $\gamma+$ jet and dijets cross sections, by comparing the number of observed double parton $\gamma+3$ jets events in one $p\bar{p}$ collision to the number of $\gamma+3$ jets events with hard interactions occurring in the case of two separate $p\bar{p}$ collisions.

---

[2]In terms of impact parameters **b** this means that in the second (2×2) term two pairs of partons are very close to each other; $|\mathbf{b}_1 - \mathbf{b}_2| \sim 1/Q$.

The recent D0 measurements [6] represent this effective cross section, $\sigma_{\mathrm{eff}}^{\mathrm{exp}}$, as a function of the second (ordered in the transverse momentum p_T) jet p_T, p_T^{jet2}, which can serve as a resolution scale. The obtained cross sections reveal a tendency to be dependent on this scale. In Ref. [23] this observation was interpreted as the first indication to the QCD evolution of double parton distributions.

We have to emphasize that the dominant contribution to the phase space integral (17) comes from a large $q^2 \sim Q^2$ and, strictly speaking, the above reasoning makes no allowance for the collinear (DGLAP) evolution of two independent branches of the parton cascade (i.e., in the ladders $L1, L2, L1'$ and $L2'$) in the 2×2 term. Formally, in the framework of collinear approach this contribution should be considered as the result of interaction of *one* pair of partons with the $2 \to 4$ hard subprocess [3]. Recall, however, that when estimating (17) we neglect the anomalous dimension, γ, of the parton distributions $D_j^k(x/z, k^2, Q^2) \propto (Q^2/k^2)^\gamma$. In collinear approach the anomalous dimensions $\gamma \propto \alpha_s << 1$ are assumed to be small. On the other hand, in a low x region the value of anomalous dimension is enhanced by the $\ln(1/x)$ logarithm and may be rather large numerically. So the integral over q^2 is *slowly* convergent and the major contribution to the cross section is expected to come actually from some characteristic intermediate region, $m_g^2 << q^2 << Q_1^2$ ($Q_1 < Q_2$). Thus we do not expect such strong sensitivity to the upper limit of q-integration as in the case of the pure phase space integral (17). Therefore it makes sense to consider the quantitative contribution of the 2×2 term even within the collinear approach as applied to the LHC kinematics, where the large (in comparison with m_g) available values of Q_1 and Q_2 provide wide enough integration region for the characteristic loop momenta q.

Next, in a configuration with two quite different scales (say, $Q_1^2 << Q_2^2$) the upper limit of q^2 integral is given by a smaller scale (at $q > Q_1$ the hard matrix element corresponding to σ^A begins to diminish with q_t). In this case the collinear evolution from the scale $q = Q_1$ up to the scale Q_2 in the ladders (parton branches) $L2$ and $L2'$ seems sufficiently justified .

In summary, we suggest a practical method which makes it possible to estimate the inclusive cross section for double parton scattering, taking into account the QCD evolution and based on the well-known collinear distributions extracted from deep inelastic scattering. We also support the conclusion in Refs. [23, 24] that the experimentally measured effective cross section, $\sigma_{\mathrm{eff}}^{\mathrm{exp}}$, (18) should decrease with increasing the resolution scale Q^2 due to presence of the evolution (correlation) term in the two-parton distributions.

References

[1] P. Bartalini *et al.*, "Multi-Parton interactions at the LHC", (arXiv:1111.0469 [hep-ph]).

[2] **AFS** Collaboration, T. Akesson *et al.*, "Double parton scattering in p p collisions at $\sqrt{S} = 63$ GeV", Z. Phys. C **34** (1987) 163.

---

[3]This is in agreement with the statement [10] that "the structure of right figure should not be included in the leading logarithmic *double* parton scattering cross section".

[3] **UA2** Collaboration, J. Alitti *et al.,* "A Study of multi-jet events at the CERN anti-p p collider and a search for double parton scattering", Phys. Lett. B **268** (1991) 145.

[4] **CDF** Collaboration, F. Abe *et al.,* "Study of four-jet events and evidence for double parton interactions in p p-bar collisions at $\sqrt{s}$ = 1.8 TeV", Phys. Rev. D **47** (1993) 4857.

[5] **CDF** Collaboration F. Abe *et al.,* "Double Parton Scattering in p anti-p Collisions at $\sqrt{s}$ = 1.8 TeV", Phys. Rev. D **56** (1997) 3811.

[6] **D0** Collaboration, V.M. Abazov *et al.,* "Double parton interactions in photon+3 jet events in p $\bar{p}$ collisions $\sqrt{s}$ = 1.96 TeV", Phys. Rev. D **81** (2010) 052012 (arXiv:0912.5104v2 [hep-ex]).

[7] M.G. Ryskin and A.M. Snigirev, "Fresh look at double parton scattering", Phys. Rev. D **83** (2011) 114047 (arXiv:1103.3495v2 [hep-ph]).

[8] B. Blok, Yu. Dokshitzer, L. Frankfurt, and M. Strikman, "The Four jet production at LHC and Tevatron in QCD", Phys. Rev. **D83** (2011) 071501 (arXiv:1009.2714v2 [hep-ph]).

[9] M. Diehl and A. Schafer, "Theoretical considerations on multiparton interactions in QCD", Phys. Lett. **B698** (2011) 389 (arXiv:1102.3081v2 [hep-ph]).

[10] J.R. Gaunt and W.J. Stirling, "Double Parton Scattering Singularity in One-Loop Integrals", JHEP **1106** (2011) 048 (arXiv:1103.1888v2 [hep-ph]).

[11] A.M. Snigirev, ' 'QCD status of factorization ansatz for double parton distributions", Phys. Rev. D **68** (2003) 114012.

[12] V.L. Korotkikh and A.M. Snigirev, "Double parton correlations versus factorized distributions", Phys. Lett. **B 594** (2004) 171 (arXiv:0404155v1 [hep-ph]).

[13] R. Kirschner, "Generalized Lipatov-Altarelli-Parisi equations and jet calculus rules", Phys. Lett. **84B** (1979) 266.

[14] V.P. Shelest, A.M. Snigirev, and G.M. Zinovjev, "Gazing into the multiparton distribution equations in QCD", Phys. Lett. **113B** (1982) 325.

[15] A.M. Snigirev, "Double-Parton Distributions in QCD", Phys. At. Nucl. **74** (2011) 158 [Yad. Fiz. **74** (2011) 158].

[16] L. Frankfurt and M. Strikman, "Two-gluon form factor of the nucleon and J/ψ photoproduction", Phys. Rev. D **66** (2002) 031502.

[17] B. Blok, Yu. Dokshitzer, L. Frankfurt, and M. Strikman, ' 'pQCD physics of multiparton interactions", (arXiv:1106.5533 [hep-ph]).

[18] E. Cattaruzza, A. Del Fabbro, and D. Treleani, "Fractional momentum correlations in multiple production of W bosons and of $b\bar{b}$ pairs in high energy pp collisions", Phys. Rev. D **72** (2005) 034022 (arXiv:hep-ph/0507052v1).

[19] J.R. Gaunt, C.-H. Kom, A. Kulesza, and W.J. Stirling, "Same-sign W pair production as a probe of double parton scattering at the LHC", Eur. Phys. J. C **69** (2010) 53 (arXiv:1003.3953v2 [hep-ph]).

[20] E. Maina, "Multiple Parton Interactions in Z+jets production at the LHC. A comparison of factorized and non–factorized double parton distribution functions", JHEP **1101** (2011) 061 (arXiv:1010.5674v2 [hep-ph]).

[21] J.R. Gaunt and W.J. Stirling, "Double Parton Distributions Incorporating Perturbative QCD Evolution and Momentum and Quark Number Sum Rules", JHEP **1003** (2010) 005 (arXiv:0910.4347v4 [hep-ph]).

[22] A.M. Snigirev, "Asymptotic behavior of double parton distribution functions", Phys. Rev. D **83** (2011) 034028 (arXiv:1010.4874v1 [hep-ph]).

[23] A.M. Snigirev, "Possible indication to the QCD evolution of double parton distributions?", Phys. Rev. D **81** (2010) 065014 (arXiv:1001.0104v3 [hep-ph]).

[24] C. Flensburg, G. Gustafson, L. Lonnblad, and A. Ster, "Correlations in double parton distributions at small x", JHEP **1106** (2011) 066 (arXiv:1103.4320v3 [hep-ph]).