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On the origin of Poincaré gauge gravity

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ABSTRACT

We argue that the origin of Poincaré gauge gravity (PGG) may be related to spontaneous violation of underlying spacetime symmetries involved and appearance of gauge fields as vector Goldstone bosons. In essence, we start with an arbitrary theory of some vector and fermion fields which possesses only global spacetime symmetries, such as Lorentz and translational invariance, in flat Minkowski space. The two vector field multiplets involved are assumed to belong, respectively, to the adjoint (A_μ^{ij}) and vector (e_μ^i) representations of the starting global Lorentz symmetry. We propose that these prototype vector fields are covariantly constrained, $A_\mu^{ij}A_{ij}^\mu = \pm M_A^2$ and $e_\mu^i e_i^\mu = \pm M_e^2$, that causes a spontaneous violation of the accompanying global symmetries ($M_{A,e}$ are their presumed violation scales). It then follows that the only possible theory compatible with these length-preserving constraints is turned out to be the gauge invariant PGG, while the corresponding massless (pseudo)Goldstone modes are naturally collected in the emergent gauge fields of tetrads and spin-connections. In a minimal theory case being linear in a curvature we unavoidably come to the Einstein–Cartan theory. The extended theories with propagating spin-connection and tetrad modes are also considered and their possible unification with the Standard Model is briefly discussed.

1. Introduction

One can think that Poincaré gauge gravity (PGG) [1,2] (see also [3] and references therein) with the underlying vector fields of tetrads and spin-connections is perhaps the best theory candidate for gravitation to be unified with the other three elementary forces of nature. PGG looks in essence as a gauge field theory in flat Minkowski space which successfully mimics curved space geometry when making the transition to the base world space in terms of general affine connections and metric. Remarkably, there is some clear analogy between a local frame in PGG and a local internal symmetry space in conventional quantum field theories. As a result, the vector fields of the spin-connections gauging the local frame Lorentz group $SO(1,3)_{LF}$ appear in PGG much as photons and gluons appear in the Standard Model. We propose that such an analogy may follow from their common origin related to spontaneous breaking of underlying spacetime symmetries involved (such as relativistic invariance etc.) with all gauge fields appearing as massless Nambu–Goldstone bosons [4]. This rather old idea [5] has gained a further development [6–8] in recent years.

Here we will follow the recently introduced emergence conjecture [9,10] according to which an origin of any gauge symmetry is basically related to some covariant constraint(s) which, for one reason or another, is put on a vector field system possessing only some global internal symmetry. As a matter of fact, the simplest holonomic constraint of this type for vector field (or vector field multiplet) A_μ may be the “length-fixing” condition

$$C(A) = A_\mu A^\mu - n^2 M^2 = 0, \quad n^2 \equiv n_\mu n^\mu = \pm 1 \tag{1}$$

where n_μ is a properly oriented unit Lorentz vector, while M is some high mass scale. We will see that gauge invariance appears unavoidable in the proposed theory, if the equations of motion involved should have enough freedom to allow a constraint like (1) to be fulfilled and preserved over time. Namely, gauge invariance in such theories has to appear in essence as a response of an interacting field system to putting the covariant constraint (1) on its dynamics, provided that we allow parameters in the corresponding Lagrangian density to be adjusted so as to ensure self-consistency without losing too many degrees of freedom. Otherwise, a given field system could get unphysical in a sense that a superfluous reduction in the number of degrees of freedom would make it

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impossible to set the required initial conditions in an appropriate Cauchy problem. Furthermore, in quantum theory, to choose self-consistent equal-time commutation relations would also become impossible [11].

To see how technically a global internal symmetry may be converted into a local one, let us consider the question of consistency of the constraint for vector field (1) with its equations of motion. We propose some arbitrary relativistically invariant Lagrangian $L(A,\psi)$ which only possesses a global Abelian $U(1)$ symmetry, and, apart from the vector field A_μ, contains the charged fermion ψ. In the presence of the constraint (1), it follows that the equations of motion can no longer be independent. This means that there should be some relationship between all the vector and matter field Eulerians (E_A, E_ψ, ...) involved.[1] Such a relationship can quite generally be formulated as a functional – but by locality just a function – of the Eulerians, $F(E_A, E_\psi)$, being put equal to zero at each spacetime point with the configuration space restricted by the constraint $C(A)=0$,

$$F(C=0;E_A,E_\psi)=0 \tag{2}$$

for the one matter fermion case proposed.

Let us consider a "Taylor expansion" of the function F expressed through various combinations of the fields involved, their combinations with the Eulerians, as well as the derivatives acting on them. We basically consider the terms with the lowest mass dimension 4, corresponding to the Lorentz invariant expressions

$$\partial_\mu (E_A)^\mu,\ A_\mu (E_A)^\mu,\ E_\psi \psi,\ \overline{\psi} E_{\overline{\psi}} \tag{3}$$

to eventually have an emergent gauge theory at a renormalizable level. All the other terms in the expansion contain field combinations with higher mass dimensions (presumably related to some Planck mass order scale) and therefore can be neglected.

Now, together with the constraint (1), which has to be preserved under the time development given by the equations of motion,

$$(E_A)^\mu = 0 \tag{4}$$

one has in fact the five equations for the 4-component vector field A^μ. This means that not all of the vector field Eulerian components can be independent. Therefore, there must be a relationship of the form given in the emergence equation (2). When being expressed as a linear combination of the Lorentz invariant terms (3), this equation leads to the identity between the vector and matter field Eulerians of the following type

$$\partial_\mu (E_A)^\mu = itE_\psi \psi - it\overline{\psi} E_{\overline{\psi}} \tag{5}$$

(where t is some constant) which is in fact identically vanished when the equations of motion are satisfied. This identity immediately signals about invariance of the basic Lagrangian $L(A,\psi)$ under vector and fermion field local $U(1)$ transformations whose infinitesimal form is given by

$$\delta A_\mu = \partial_\mu \omega, \quad \delta\psi = it\omega\psi. \tag{6}$$

Conversely, the identity (5) follows from the invariance of the physical Lagrangian $L(A,\psi)$ under the transformations (6). Indeed, both direct and converse assertions are particular cases of Noether's second theorem [12].

So, we have shown how the constraint (1) enforces the choice of the parameters in the starting Lagrangian $L(A,\psi)$, so as to convert its global $U(1)$ charge symmetry into a local one, thus demonstrating an emergence of gauge symmetry (6) that allows the emerged Lagrangian to be completely determined. For a theory with renormalizable couplings, it is in fact the conventional QED Lagrangian supplemented by the constraint (1) imposed on the vector field A_μ. Interestingly, this type of the QED theory with the constrained vector potential was considered by Nambu [13] quite a long ago.

Let us make it clearer what does the constraint (1) mean in the gauge invariant QED framework. This constraint is in fact very similar to the constraint appearing in the nonlinear σ-model for pions [14]. It means, in essence, that the vector field A_μ develops some constant background value, $\langle A_\mu \rangle = n_\mu M$, and the Lorentz symmetry $SO(1,3)$ formally breaks down to $SO(3)$ or $SO(1,2)$ for the time-like ($n^2=1$) or space-like ($n^2=-1$) case, respectively. As a result, the corresponding vector Goldstone mode is produced which may be associated with a photon. Nonetheless, despite an evident similarity with the nonlinear σ-model for pions, which really breaks the corresponding chiral $SU(2)\times SU(2)$ symmetry in hadron physics, the QED theory with the supplementary vector field constraint (1) involved leaves the physical Lorentz invariance intact. Actually, as was shown in the tree [13] and one-loop [15] approximations, there is no physical Lorentz violation in the QED supplemented by the covariant constraint (1). Later this result was also confirmed for many other gauge theories with the supplementary vector field constraints, particularly, in the non-Abelian [16] and supersymmetric theories [10]. So, we conclude with a remark that in contrast to a spontaneous violation of internal symmetries, *a spontaneous Lorentz invariance violation* (SLIV) caused by the length-preserving vector field constraint does not necessarily imply a physical breakdown of Lorentz invariance. Actually, gauge invariance in QED and other gauge theories always leads to a total conversion of SLIV into gauge degrees of freedom of massless vector Goldstone bosons.

In the Section 2 we turn to the construction of an emergent PGG theory. We start with an arbitrary theory of some vector and fermion fields which possesses only global spacetime symmetries, such as Lorentz and translational invariance, in flat Minkowski space M_4. The two vector field multiplets involved are proposed to belong, respectively, to the adjoint (A_μ^{ij}) and vector (e_μ^i) representations of the starting global Lorentz symmetry. We show that if these prototype vector fields are covariantly constrained then the only possible theory compatible with these constraints is turned out to be the standard PGG. In minimal theory case being linear in curvature we unavoidably come to the Einstein–Cartan theory that is thoroughly presented in the Section 3. The extended theories with propagating spin-connection and tetrad modes and their possible unification with the Standard Model is briefly discussed in the final Section 4, where we also conclude.

2. Towards an emergent Poincaré gravity

Conventionally, we have in PGG the world space (WS) symmetry $ISO(1,3)_{WS}$, which includes translations and the orbital part of Lorentz transformations, and a local frame (LF) Lorentz symmetry $SO(1,3)_{LF}$, which only includes the spin part of Lorentz transformations acting on representation indices. Remarkably, this duality is in an automatic accordance with the Einstein equivalence principle which, therefore, need not to be specially postulated in PGG as is in the standard GR. We begin with the entirely global spacetime symmetries, both $ISO(1,3)_{WS}$ and $SO(1,3)_{LF}$, and our starting objects are the two vector field multiplets which are 4-vectors of $ISO(1,3)_{WS}$ and belong, respectively, to the adjoint (A_μ^{ij}) and vector (e_μ^i) representations of the

[1] Hereafter, the notation E_A stands for the vector field Eulerian determined by the corresponding Lagrangian $L(A,\psi)$ $(E_A)^\mu \equiv \partial L/\partial A_\mu - \partial_\nu[\partial L/\partial(\partial_\nu A_\mu)]$. We use similar notations for other field Eulerians as well.

Lorentz group $SO(1,3)_{LF}$ (and an antisymmetry in the Latin indices $(a,b,c,...,i,j,k,...)$ is hereafter imposed). In what follows we will refer to these prototype fields as the spin-connections and tetrads, as they really are turned out once an emergence procedure is applied to them. As a result, the local frame Lorentz symmetry $SO(1,3)_{LF}$ and translation subgroup in $ISO(1,3)_{WS}$ appear gauged, while the orbital Lorentz transformations are actually absorbed by the latter. So, eventually one has the local translations and local $SO(1,3)_{LF}$ transformations being gauged by the emergent tetrad and spin-connection fields, respectively. Again, due to gauge invariance emerged the physical Lorentz (and translation invariance) remains in the final theory.

2.1. Constrained tetrad and spin-connection fields

First of all, as we could learn above from the emergent QED case, the tetrad and spin-connection fields have to be properly constrained to induce an appropriate emergence process. The essential point is, however, that the tetrad field is generically constrained by definition, $e^i_\mu e^\nu_i = \delta^\nu_\mu$. To see clearer what does this constraint mean, let first notice that whereas the spin-connection field A^{ij}_μ has a canonical vector field mass dimension, the tetrad field e^i_μ appears to have zero mass dimension. Treating it as all other boson fields having a canonical dimension of mass we introduce some fundamental mass scale in the definition of tetrad fields e^i_μ (e^μ_i) changing their orthogonality equations to

$$e^i_\mu e^\nu_i = \delta^\nu_\mu M_e^2, \quad e^i_\mu e^\mu_j = \delta^i_j M_e^2, \quad e^i_\mu e^\mu_i = \mathrm{n}^2 M_e^2, \tag{7}$$

where the first two conditions could be considered as those which define the inverse tetrads e^μ_i, whereas the third one is their length-fixing constraint. Here n^2 stands for

$$\mathrm{n}^2 \equiv \delta^\nu_\mu \delta^\mu_\nu = \delta^i_j \delta^j_i = \delta^i_\mu \delta^\mu_i = 4\,. \tag{8}$$

We can readily see that the last constraint in (7) is indeed similar to the constraints we have above for conventional vector fields (1). This constraint actually means that PGG is a spontaneously broken theory that manifests itself at some input mass scale M_e which could be in principle associated with the Plank mass M_P. One can choose this violation in a way that the vacuum of the PGG theory is flat Minkowski space rather than breaks Lorentz invariance.

The similar length-fixing constraint is proposed to be put on the spin-connection fields A^{ij}_μ

$$A^{ij}_\mu A^\mu_{ij} = n^2 M_A^2, \quad n^2 \equiv n^{ij}_\mu n^\mu_{ij} = \pm 1 \tag{9}$$

being analogous to the constraints (1) for ordinary vector fields (here n^{ij}_μ stands now for some properly-oriented 'unit' rectangular matrix). The constraint (9) actually means that we also have a spontaneous Lorentz violation in PGG that appears at some high mass scale M_A which could be in principle close to the Plank mass M_P as well. This will cause, as we confirm later, the generation of Goldstone vector bosons gauging Lorentz symmetry in the local frame, while the physical Lorentz invariance is left intact.

2.2. From global to local symmetries

We start with some prototype theory possessing only global symmetries $ISO(1,3)_{WS}$ and $SO(1,3)_{LF}$ operating in the two flat Minkowski spaces with constant metrics $\eta_{\mu\nu}$ and η_{ij}, respectively. This yet arbitrary theory contains some prototype vector fields having form of spin-connections $A^{ij}_\mu(x)$ and tetrads $e^i_\mu(x)$ and may also contain some matter fields (say, fermions ψ). The theory have in general all possible interactions between all vector and matter fields involved. The corresponding Lagrangian $\mathcal{L}^{tot}$ is supposed to also include the standard Lagrange multiplier terms with the field functions $\lambda_A(x)$ and $\lambda_e(x)$

$$\mathcal{L}^{tot}(e,A,\psi;\lambda_e,\lambda_A) = \mathcal{L}(e,A,\psi) - \frac{\lambda_A}{2}(A^{ij}_\mu A^\mu_{ij} - n^2 M_A^2) - \frac{\lambda_e}{2}(e^i_\mu e^\mu_i - \mathrm{n}^2 M_e^2). \tag{10}$$

The variations under $\lambda_A(x)$ and $\lambda_e(x)$ result, accordingly, in the covariant length-preserving constraints for the spin-connection and tetrad fields

$$\mathcal{C}_A = A^{ij}_\mu A^\mu_{ij} - n^2 M_A^2 = 0, \quad \mathcal{C}_e = e^i_\mu e^\mu_i - \mathrm{n}^2 M_e^2 = 0 \tag{11}$$

in the PGG theory. Therefore, we face the question of consistency of these extra constraint equations with the equations of motion for the vector fields of tetrads e^i_μ and spin-connections A^{ij}_μ

$$(\mathcal{E}^{ij}_A)_\mu = 0, \quad (\mathcal{E}^j_e)_\mu = 0 \quad (i,j = 0,1,2,3;\ \mu = 0,1,2,3). \tag{12}$$

For an arbitrary Lagrangian $\mathcal{L}(e,A,\psi)$, the time development of the fields would not preserve in general the constraints (11). So, the parameters in the Lagrangian $\mathcal{L}$ must be chosen so as to give a relationship between the Eulerians for all the fields involved. The need to preserve the constraints $\mathcal{C}_A = 0$ and $\mathcal{C}_e = 0$ with time implies that the equations of motion for the vector fields of spin-connections A^{ij}_μ and tetrads e^i_μ, respectively, cannot be all independent. As a result, the special emergence equations for spin-connection fields

$$\mathcal{F}^{ij}(\mathcal{C}_A = 0; \mathcal{E}_A, \mathcal{E}_e, \mathcal{E}_\psi, ...) = 0 \quad (i,j = 0,1,2,3) \tag{13}$$

and tetrad fields

$$\mathcal{F}_\mu(\mathcal{C}_e = 0; \mathcal{E}_e, \mathcal{E}_A, \mathcal{E}_\psi, ...) = 0 \quad (\mu = 0,1,2,3), \tag{14}$$

necessarily appear.

Let us consider first the emergence equations (13). Again, when being expressed as a linear combination of the basic mass dimension-4 terms, this equation leads to the identities between all field Eulerians involved

$$\partial^\mu(\mathcal{E}_A)^{ij}_\mu = c^{[ij]}_{[kl][mn]} A^{kl}_\mu (\mathcal{E}_A)^{\mu,mn} + e^{[i}_\mu (\mathcal{E}_e)^{j],\mu} + \mathcal{E}_\psi S^{ij}\psi + \overline{\psi} S^{ij} \mathcal{E}_{\overline{\psi}} \tag{15}$$

which are precisely analogous to those which appear in the emergent Yang–Mills theory [9,10]. An appropriate identification of the Eulerian terms on the right-hand side of the identity (15) with the structure constants $c^{[ij]}_{[kl][mn]}$ and the fermion representation matrices S^{ij} of the Lorentz symmetry group $SO(1,3)_{LF}$ is indeed quite clear. The point is that the right-hand side of this identity must transform in the same way as its left-hand side, which transforms as the adjoint representation of $SO(1,3)_{LF}$. As to their coefficients and other possible terms in the identity (15), there were remained, as usual, only terms which satisfy the Lee bracket operation to close the symmetry algebra once the corresponding field transformations are identified.

As to the basic identities following from the emergence equations for tetrad fields (14), the non-trivial lowest mass dimension terms constructed from the Eulerians for this case will necessarily include the translation operator expression $T_\mu = -\partial_\mu$ for all the fields involved. Consequently they take the following form

$$e^i_\nu\, \partial_\mu (\mathcal{E}_e)^\nu_i + A^{ij}_\nu \partial_\mu (\mathcal{E}_A)^\nu_{ij} + (\partial_\mu \mathcal{E}_\psi)\psi + \overline{\psi}(\partial_\mu \mathcal{E}_{\overline{\psi}}) = 0 \tag{16}$$

which consist of all the terms having mass dimension 5.

Now again, Noether's second theorem [12] can be applied directly to the above identities (15) and (16) in order to derive the gauge invariance of the Lagrangian $\mathcal{L}(e, A, \psi)$ in (10). Indeed, with the constraint (11) implied, this Lagrangian tends to be invariant under local transformations of the spin-connection, tetrad and matter fields of the type

$$\delta A_\mu^{ij} = \varepsilon_k^i A_\mu^{kj} + \varepsilon_k^j A_\mu^{ik} - \partial_\mu \xi^\nu A_\nu^{ij} - \partial_\mu \varepsilon^{ij},$$
$$\delta e_i^\mu = e_i^\nu \partial_\nu \xi^\mu - e_k^\mu \varepsilon_i^k,$$
$$\delta\psi = \frac{1}{2}\varepsilon^{ij}\gamma_{ij}\psi, \quad \gamma_{ij} = [\gamma_i, \gamma_j]/4. \tag{17}$$

Note that the first two terms in δA_μ^{ij} correspond to local Lorentz rotations of the spin-connection fields A_μ^{ij} with parameters $\varepsilon^{ij}(x)$, while the third term is due to the local translations conditioned by the parameters $\xi^\mu(x)$. The last terms in δA_μ^{ij} means that the spin-connection fields A_μ^{ij} gauge just the local Lorentz rotations. The tetrad field in δe_i^μ is Lorentz-rotated (in the local Lorentz frame) and, simultaneously, subject to the coordinate-dependent translations (in the world spacetime). And finally, the transformation of the fermion field ψ in (17) is, as usual, determined by the fermion representation matrices. The local transformations (17) shows that the somewhat arbitrarily introduced prototype vector fields A_μ^{ij} and e_μ^i are really turned out to be the PGG spin-connection and tetrad fields once they satisfy the length-preserving constraints (11). Moreover, the induced gauge symmetry (17) unavoidably leads to the emergent PGG Lagrangian

$$\mathcal{L}^{em}(e, A, \psi; \lambda_e, \lambda_A)$$
$$= \mathcal{L}^{em}(e, A, \psi)_{PGG} - \frac{\lambda_A}{2}(A_\mu^{ij}A_{ij}^\mu - n^2 M_A^2)$$
$$- \frac{\lambda_e}{2}(e_\mu^i e_i^\mu - \mathrm{n}^2 M_e^2) \tag{18}$$

where $\mathcal{L}^{em}(e, A, \psi)_{PGG}$ is solely constructed from the covariant curvature and torsion tensors

$$R_{\mu\nu}^{ij} = \partial_{[\nu}A_{\mu]}^{ij} + \eta_{kl}A_{[\nu}^{ik}A^{lj}{}_{\mu]}, \quad T_{\mu\nu}^{i} = \partial_{[\nu}e_{\mu]}^{i} + \eta_{kl}A_{[\nu}^{ik}e_{\mu]}^{l} \tag{19}$$

and a covariant derivative for the fermion field

$$\bar\psi\gamma^i \overleftrightarrow{D}_\mu\psi = \bar\psi\gamma^i(\partial_\mu\psi) - (\partial_\mu\bar\psi)\gamma^i\psi + \frac{1}{4}A_\mu^{ab}\bar\psi\{\gamma^i, \gamma_{ab}\}\psi. \tag{20}$$

We also included the corresponding Lagrange multiplier terms which, as was mentioned above, do not contribute to the physical field equations of motion. Now, for a theory with the lowest dimension coupling constants, containing at most the quadratic terms in the curvature and torsion one has

$$\mathcal{L}^{em}(e, A, \psi)_{PGG} = \mathcal{L}^{(1)}(e, A, \psi) + \mathcal{L}^{(2)}(e, A, \psi) \tag{21}$$

where the first term correspond to the minimal Einstein–Cartan theory being linear in the curvature

$$\mathcal{L}^{(1)}(e, A, \psi) = \frac{e}{2\kappa}\frac{e_i^\mu e_j^\nu}{M_e^2}R_{\mu\nu}^{ij} + e\frac{e_i{}^\mu}{2M_e}\bar\psi\gamma^i\overleftrightarrow{D}_\mu\psi \tag{22}$$

(where κ stands for the modified Newtonian constant $8\pi G$), while in the second term $\mathcal{L}^{(2)}$ all eight possible quadratic terms [17,18] are generally collected.

2.3. Broken symmetry phase: zero spin-connection modes

We have found above that the presence of the spin-connection and tetrad field constraints (11) in the theory unambiguously convert the global symmetry $ISO(1,3)_{WS} \times SO(1,3)_{LF}$ we started with into the local Poincaré symmetry $T(1,3)_{WS} \times SO(1,3)_{LF}$ that leads to the conventional PGG theory. The point is, however, that these constraints mean at the same time that this global symmetry is spontaneously broken thus inducing the Goldstone spin-connection and tetrad field modes.

To see it in more detail, let us consider first the spin-connection fields. Note above all, whereas the emergent PGG Lagrangian $\mathcal{L}_{PGG}^{em}$ in (18) possesses the local Poincaré symmetry $T(1,3)_{WS} \times SO(1,3)_{LF}$, the accidental global symmetry of the length-fixing spin-connection constraint (9) appears much higher, $ISO(6,18)_{WS}$.[2] This symmetry is indeed spontaneously broken at a scale M_A, $\left\langle A_\mu^{ij}\right\rangle = n_\mu^{ij}M_A$, with the vacuum direction determined now by the matrix n_μ^{ij} (9) which describes simultaneously both of the SLIV cases, time-like or space-like

$$ISO(6,18) \to ISO(5,18), \quad ISO(6,18) \to ISO(6,17) \tag{23}$$

respectively, depending on the sign of $n^2 = \pm 1$. In both cases the matrix n_μ^{ij} has only one non-zero element, subject to the appropriate $ISO(1,3)_{WS}$ and (independently) $SO(1,3)_{LF}$ transformations. They are, specifically, $n_0^{\langle ij\rangle}$ or $n_3^{\langle ij\rangle}$ provided that the VEV is developed along the $\langle ij\rangle$ direction in the local Lorentz frame and along the $\mu = 0$ or $\mu = 3$ direction, respectively, in the world spacetime.

As was argued in the above non-Abelian vector field case, side by side with one true vector Goldstone boson corresponding to spontaneous violation of an actual $ISO(1,3)_{WS} \times SO(1,3)_{LF}$ symmetry of the PGG Lagrangian, the five pseudo-Goldstone vector bosons related to the breakings (23) of the accidental symmetry $ISO(6,18)$ of the constraint (9) per se are also produced.[3] Remarkably, the vector PGBs remain strictly massless being protected by the simultaneously generated Lorentz gauge invariance. Together with the above true vector Goldstone boson, they also come into play thus properly completing the entire adjoint gauge multiplet of spin-connection fields of the local Lorentz symmetry group $SO(1,3)_{LF}$.

Due to the constraint (9), which virtually appears as a single condition put on the spin-connection field multiplet A_μ^{ij}, one can identify the pure Goldstone field modes $\mathcal{A}_\mu^{ij}$ using the parametrization

$$A_\mu^{ij} = \mathcal{A}_\mu^{ij} + n_\mu^{ij}\mathcal{H}, \quad n_\mu^{ij}\mathcal{A}_{ij}^\mu = 0 \quad (\mathcal{A}^2 \equiv \mathcal{A}_\mu^{ij}\mathcal{A}_{ij}^\mu) \tag{24}$$

and an effective "Higgs" mode $\mathcal{H} = \sqrt{M_A^2 - n^2\mathcal{A}^2}$. Note that, apart from the pure vector fields, the general zero modes $\mathcal{A}_\mu^{ij}$ contain the five scalar modes, $\mathcal{A}_0^{ij}$ or $\mathcal{A}_3^{ij}$, for the time-like ($n_\mu^{ij} = n_0^{\langle ij\rangle}g_{\mu 0}\delta^{\langle ij\rangle\langle ij\rangle}$) or space-like ($n_\mu^{ij} = n_3^{\langle ij\rangle}g_{\mu 3}\delta^{\langle ij\rangle\langle ij\rangle}$) SLIV, respectively. They can be eliminated from the theory, if one imposes

[2] This symmetry being treated as the world space symmetry is determined by a proper number of the spacetime directions related to the (local frame) Lorentz group representations of the vector fields involved. In this way, the length-fixing constraint for spin-connection fields (9) possesses the global symmetry $ISO(6,18)_{WS}$, whereas a similar constraint for tetrad fields (7) the lower global symmetry $ISO(4,12)_{WS}$, as is claimed below.

[3] Note that in total there appear the 23 pseudo-Goldstone modes, complying with the number of broken generators of $SO(6,18)$. From these 23 pseudo-Goldstone modes, 18 modes correspond to the six three-component vector states, as will be shown below, while the remaining 5 modes are scalar states which will be excluded from the theory.

appropriate supplementary conditions on the five fields $\mathcal{A}_\mu^{ij}$ which are still free of constraints. Using their overall orthogonality (24) to the physical vacuum direction n_μ^{ij}, one can formulate these supplementary conditions in terms of a general axial gauge for the entire $\mathcal{A}_\mu^{ij}$ multiplet $n^\mu \mathcal{A}_\mu^{ij} = 0$. Here n_μ is the unit world spacetime vector which is oriented so as to be "parallel" to the vacuum unit n_μ^{ij} matrix. This matrix can be taken hereafter in the "two-vector" form $n_\mu^{ij} = n_\mu \epsilon^{ij}$ $(n_\mu n^\mu = 1,\ \epsilon^{ij}\epsilon_{ij} = 1)$ where ϵ^{ij} is the unit Lorentz group tensor belonging to its adjoint representation. As a result, in addition to the "Higgs" mode excluded earlier by the orthogonality condition (24), all the other scalar fields are eliminated. Consequently only the pure vector fields, $\mathcal{A}_{\mu'}^{ij}$ $(\mu' = 1, 2, 3)$ or $\mathcal{A}_{\mu''}^{ij}$ $(\mu'' = 0, 1, 2)$, for the time-like or space-like SLIV respectively, are left in the theory. Clearly, the components $\mathcal{A}_{\mu'}^{(ij)=\langle ij\rangle}$ and $\mathcal{A}_{\mu''}^{(ij)=\langle ij\rangle}$ correspond to the true Goldstone vector boson, for each type of SLIV, respectively, while all the other five ones (with $(ij) \neq \langle ij\rangle$) are vector PGBs. Consequently these six modes altogether represent the fundamental spin-connection field multiplet in the PGG theory in the final symmetry broken phase.

2.4. Broken symmetry phase: zero tetrad modes

Let us now turn to the tetrad fields. Again, as one can readily confirm, the tetrad length-fixing constraint in (7) possesses the high total global symmetry $ISO(4,12)_{WS}$ rather than $ISO(1,3)_{WS} \times SO(1,3)_{LF}$ as other terms in the emergent PGG Lagrangian (18). This symmetry then spontaneously breaks to some its "diagonal" subgroup $ISO(1,3)$ that results in an appearance of the corresponding Goldstone and Higgs modes. Note that this violation precisely looks as the 16-dimensional Poincaré symmetry violation down to the ordinary 4-dimensional one. As it is well known for spontaneously broken spacetime symmetries [19], such a violation can solely lead to the Goldstone modes corresponding to the broken translational generators. There are no additional modes corresponding to the broken Lorentz generators. So, we eventually have only twelve Goldstone modes (according to the number of the broken translation generators) which may be given by the non-diagonal e_μ^i components ($e^0_{1,2,3}$, $e^1_{2,3}$, e^2_3 and their inverse ones), whereas the Higgs mode by some combination of their diagonal ones (e^0_0, e^1_1, e^2_2, e^3_3). Indeed, the above Goldstone modes are in fact pseudo-Goldstone modes since, as was mentioned above, the symmetry of the PGG Lagrangian $\mathcal{L}_{PGG}^{em}$ is much lower than the symmetry of the tetrad field constraint (7).

All that can be readily seen by using the familiar parametrization

$$e_\mu^i = \mathfrak{e}_\mu^i + \mathrm{n}_\mu^i \sqrt{M_e^2 - \mathfrak{e}^2} \quad (\mathfrak{e}^2 \equiv \mathfrak{e}_\mu^i \mathfrak{e}_i^\mu / \mathrm{n}^2) \tag{25}$$

with $\mathfrak{e}_\mu^i$ appearing as the vector Goldstone fields which correspond to the spontaneous violation of the high-dimensional translation invariance. For the unit vacuum direction tensors chosen accordingly as $\mathrm{n}_\mu^i = \delta_\mu^i$ and $\mathrm{n}_i^\mu = \delta_i^\mu$ one therefore has

$$\delta_\mu^i \mathfrak{e}_i^\mu = 0, \quad \delta_i^\mu \mathfrak{e}_\mu^i = 0 \quad (\delta_\mu^i \delta_j^\mu = \delta_j^i,\ \delta_\mu^i \delta_i^\nu = \delta_\mu^\nu,\ \delta_\mu^i \delta_i^\mu = 4). \tag{26}$$

At the same time, the vector Goldstone fields $\mathfrak{e}_\mu^i$ and $\mathfrak{e}_i^\mu$ fields are turn out to be the gauge fields of local translations, as directly follows from the tetrad transformation law in (17). Meanwhile the second (diagonal) term in the parametrization (25) represents the effective Higgs mode, $\mathfrak{h} = \sqrt{M_e^2 - \mathfrak{e}^2}$. Note that with this "mixed" Kronecker symbols δ_μ^i and δ_i^μ one also has some new orthogonality equation

$$e_k^\mu e_i^\nu \delta_\nu^k = e_k^\mu e_i^k = e_k^\mu e_\nu^k \delta_i^\nu = M_e^2 \delta_i^\mu \tag{27}$$

provided that the standard orthogonality conditions (7) work.

For a general metric tensor $g_{\mu\nu}(x)$ which corresponds to the tetrad e_μ^i one consequently has from a conventional metric definition and equations (25)

$$g_{\mu\nu} = \frac{1}{M_e^2}\eta_{ij} e_\mu^i e_\nu^j = \eta_{\mu\nu} + \frac{1}{M_e^2}[\mathfrak{h}(\delta_\mu^i \mathfrak{e}_{i\nu} + \delta_\nu^j \mathfrak{e}_{j\mu}) + \mathfrak{e}_\mu^i \mathfrak{e}_{i\nu} - \eta_{\mu\nu}\mathfrak{e}^2] \tag{28}$$

where $\eta_{\mu\nu}$ stands for a flat metric $\eta_{\mu\nu} = \eta_{ij}\delta_\mu^i \delta_\nu^j$ in the world space and, therefore, the second term in (28) represents a deviation from the flat metric. As is readily seen from (28), the vacuum in the PGG theory is a largely flat Minkowski spacetime that allows to treat gravity as a generically spontaneously broken theory. Though this point was discussed in many different contexts [3], it looks the most transparent just in the emergent PGG framework. Indeed, one can readily see that the above-mentioned deviation from a flat metric is naturally small once the symmetry breaking scale M_e related to the tetrad field e_μ^i is associated with the Planck mass scale M_P. Respectively, an inverse metric tensor $g^{\mu\nu}(x)$ corresponding to the tetrad e_i^μ has a similar form with an extremely small deviation from a flat metric $\eta^{\mu\nu} = \eta^{ij}\delta_i^\mu \delta_j^\nu$ given as in (28) by an appropriate Goldstone tetrad field combinations. Indeed, a conventional relationship between general metrics, $g_{\mu\nu}g^{\nu\rho} = \delta_\mu^\rho$, is automatically satisfied.

3. Emergent Einstein–Cartan theory

We start with the minimal theory part $\mathcal{L}^{(1)}$ (22) in the basic emergent Lagrangian (21). Without kinetic terms, the tetrad and spin-connection Goldstone modes in this minimal Lagrangian are not propagating physical fields, though their variations may lead to some non-trivial constraint equations. We will see below that, varying this Lagrangian under Goldstone tetrad modes $\mathfrak{e}_\mu{}^i$ one comes to the Einstein–Cartan equation, while variation under Goldstone spin-connection modes $\mathcal{A}_\mu^{ij}$ may reveal some spin-spin gravitational interaction trace in this equation.

Let us note first that for a variation of tetrad fields and their determinant we have now taking into account that tetrads are dimensional fields,

$$\delta e_i^\mu = -e_i^\nu e_j^\mu \delta e_\nu^j / M_e^2, \quad \delta e = e e_i^\mu \delta e_\mu^i / M_e^2. \tag{29}$$

Multiplying the both sides of the first equation by δ_μ^i and using the tetrad orthogonality condition (27) in its right side one has

$$\delta(\delta_\mu^i e_i^\mu) = -\delta(\delta_i^\mu e_\mu^i); \quad \delta\mathfrak{h} = 0, \quad \delta e_\mu^i = \delta\mathfrak{e}_\mu^i, \tag{30}$$

where we also used the Goldstone condition (26) for the $\mathfrak{e}_i^\mu$ and $\mathfrak{e}_\mu^i$ modes, respectively. Thus, the effective Higgs field $\mathfrak{h}$ does not vary and a total variation of the starting tetrad fields $e_\mu^i(e_i^\mu)$ amounts to the variation of the pure Goldstone modes $\mathfrak{e}_\mu^i(\mathfrak{e}_i^\mu)$. In terms of these modes the variation equations (29) acquire the simple forms

$$\delta e_i^\mu = \delta\mathfrak{e}_i^\mu = -e_i^\nu e_j^\mu \delta\mathfrak{e}_\nu^j / M_e^2, \quad \delta e = \delta\mathfrak{e} = e e_i^\mu \delta\mathfrak{e}_\mu^i / M_e^2. \tag{31}$$

This in turn means that the variation of the minimal Lagrangian $\mathcal{L}^{(1)}$ (22) under the Goldstone tetrad fields $\mathfrak{e}_\mu{}^i$ will lead to the same equations of motion as the variation under the total tetrad fields $e_\mu{}^i$.

In contrast to tetrads, there is no the similar orthogonality conditions (7), (27) for spin-connection fields A_μ^{ij}. As a result, not only its Goldstone mode $\mathcal{A}_\mu^{ij}$ but also its effective Higgs mode $\mathcal{H}$ in (24) will vary that, therefore, might lead to the corrections of the order $\mathcal{O}(\mathcal{A}^2/M_A^2)$ to the spin-connection constraint equation along the vacuum direction given by the unit tensor n_{ij}^{μ}. However, as we show below, all these corrections are unavoidably canceled in the final Einstein–Cartan equation.

With these preliminary comments, let us now rewrite the minimal PGG theory $\mathcal{L}^{(1)}$ (22) in the symmetry broken phase. Indeed, substituting the spin-connection field parameterization (24), one is led to the Einstein–Cartan theory expressed in terms of the pure emergent modes $\mathcal{A}_{\mu}^{ij}$. At the same time, one can still keep the total tetrad field $e_{\mu}{}^{i}$ in the theory (being properly dimensioned by mass scale M_e) rather than its Goldstone modes $\mathfrak{e}_{\mu}{}^{i}$ since, as was mentioned above, they both lead to the same equations of motion in the minimal theory. However, one should first use the local invariance of the emergent Lagrangian $\mathcal{L}^{\mathfrak{em}}$ (21) to gauge away the apparently large but fictitious Lorentz violating terms (being proportional to the scale M_A) which appear in the symmetry broken phase (24). As one can readily see, they stem from the effective Higgs field $\mathcal{H}$ expansion in (24) when it is applied to some spin-connection field couplings following from the corresponding covariant derivatives in the Lagrangian $\mathcal{L}^{\mathfrak{em}}$. To exclude them we can make some appropriate Lorentz rotations of all the fields involved, namely, spin-connection and tetrad fields and matter fermions

$$\mathcal{A}_{\mu}^{ij} \to \mathcal{A}_{\mu}^{ij} + \varepsilon_{k}^{i}\mathcal{A}_{\mu}^{kj} + \varepsilon_{k}^{j}\mathcal{A}_{\mu}^{ik}, \quad e_{\mu}^{i} \to e_{\mu}^{i} - \varepsilon_{k}^{i}e_{\mu}^{k},$$
$$\psi \to \left(1 + \varepsilon^{ij}\gamma_{ij}/4\right)\psi \tag{32}$$

with a phase $\varepsilon^{ij}(x)$ being linear function in the 4-coordinate, $\varepsilon^{ij} = -(n_{\mu}^{ij}x^{\mu})M_A$. These transformations lead to an exact cancellation of the large constant term in the effective Higgs field $\mathcal{H}$ expansion in (24) so that the transformed Lagrangian appears to contain everywhere just the combination $\mathcal{H} - M_A$ as an effective Higgs field. Thus, the emergent Einstein–Cartan theory following from the minimal Lagrangian (22) in the symmetry broken phase takes the form (we retain the same notations for fields)

$$e^{-1}\mathcal{L}_{EC}^{\mathfrak{em}} = \frac{1}{2\kappa}\frac{e_{i}^{\mu}e_{j}^{\nu}}{M_e^2}\left[\mathcal{R}_{\mu\nu}^{ij} + \overline{\mathcal{R}}_{\mu\nu}^{ij}(\mathcal{H} - M_A)\right] + \frac{1}{2}\delta(n^{\mu}\mathcal{A}_{\mu}^{ij})^2 + \frac{e_i{}^{\mu}}{2M_e}\left\{\bar{\psi}\gamma^{i}(\overleftrightarrow{\mathcal{D}}_{\mu}\psi) + \frac{1}{4}n_{\mu}^{ab}(\mathcal{H} - M_A)\bar{\psi}[\gamma^{i}, \gamma_{ab}]\psi\right\} \tag{33}$$

where $\mathcal{R}_{\mu\nu}^{ij}$ is the stress tensor of emergent spin-connection modes $\mathcal{A}_{\mu}^{ij}$

$$\mathcal{R}_{\mu\nu}^{ij} = \partial_{\nu}\mathcal{A}_{\mu}^{ij} - \partial_{\mu}\mathcal{A}_{\nu}^{ij} + \eta_{kl}(\mathcal{A}_{\nu}^{ik}\mathcal{A}_{\mu}^{lj} - \mathcal{A}_{\mu}^{ik}\mathcal{A}_{\nu}^{lj}) \tag{34}$$

while $\overline{\mathcal{R}}_{\mu\nu}^{ij}$ stands for the new SLIV oriented tensor of the type

$$\overline{\mathcal{R}}_{\mu\nu}^{ij} = n_{\mu}^{ij}\partial_{\nu} - n_{\nu}^{ij}\partial_{\mu} + \eta_{kl}\left[(n_{\nu}^{ik}\mathcal{A}_{\mu}^{lj} + n_{\mu}^{lj}\mathcal{A}_{\nu}^{ik}) - (n_{\mu}^{ik}\mathcal{A}_{\nu}^{lj} + n_{\nu}^{lj}\mathcal{A}_{\mu}^{ik})\right] \tag{35}$$

acting on the effective Higgs field expansion terms in (33). The "standard" Lorentz covariant derivative $\overleftrightarrow{\mathcal{D}}_{\mu}$ for fermion ψ, though written in terms of the emergent $\mathcal{A}$ fields, is defined exactly as in (20). We have also introduced a general axial gauge fixing term for the entire $\mathcal{A}_{\mu}^{ij}$ multiplet to remove all scalar modes from the theory. After variation of the Lagrangian (33) under tetrad field one comes to some extended equation of motion that can be written in the form

$$\mathcal{R}^{\rho\sigma} - g^{\rho\sigma}\mathcal{R}/2 + \kappa\vartheta^{\rho\sigma} = -[\left(\overline{\mathcal{R}}^{\rho\sigma} - g^{\rho\sigma}\overline{\mathcal{R}}/2\right) + \frac{\kappa}{8M_e}(g^{\rho\sigma}e_i{}^{\mu} - g^{\rho\mu}e_i^{\sigma})n_{\mu}^{ab}\bar{\psi}[\gamma^{i}, \gamma_{ab}]\psi](\mathcal{H} - M_{\mathcal{A}}) \tag{36}$$

when going from local to general frame. Here the left side presents the standard Einstein–Cartan equation terms including the energy–momentum tensor

$$\vartheta^{\rho\sigma} = \frac{1}{2M_e}(g^{\rho\sigma}e_i{}^{\mu} - g^{\mu\sigma}e_i^{\rho})\bar{\psi}\gamma^{i}\overleftrightarrow{\mathcal{D}}_{\mu}\psi \tag{37}$$

expressed, however, in terms of the emergent $\mathcal{A}_{\mu}^{ij}$ modes, whereas the right side corresponds to the Lorentz breaking background terms newly appeared. The $\mathcal{R}$ and $\overline{\mathcal{R}}$ tensors are defined as usual

$$(\mathcal{R}, \overline{\mathcal{R}})^{\sigma\rho} = (\mathcal{R}, \overline{\mathcal{R}})_{\mu\nu}^{ij}e_i^{\sigma}e_j^{\nu}g^{\mu\rho}/M_e^2,$$
$$(\mathcal{R}, \overline{\mathcal{R}}) = (\mathcal{R}, \overline{\mathcal{R}})_{\mu\nu}^{ij}e_i^{\mu}e_j^{\nu}/M_e^2. \tag{38}$$

The theory is not yet fully determined until the constraint equations for the spin connection modes $\mathcal{A}_{\mu}^{ij}$ are found by an appropriate variation the Lagrangian (33). They are rather simplified in the limit when the tetrad fields take the constant background value, $e_i^{\mu} = \delta_{\mu}^{i}M_e$ $(e = 1)$. In this approach, which allows to omit all the tetrad derivative terms, and also leaving only terms linear in spin-connection modes $\mathcal{A}_{\mu}^{ij}$ one comes to the "zero-order" constraint equations in the symmetry broken phase

$$\mathcal{A}_{\mu b}^{i}\delta_a^{[\mu}\delta_i^{\rho]} + \mathcal{A}_{\mu a}^{i}\delta_i^{[\mu}\delta_b^{\rho]} = -\frac{\kappa}{4}\left(\delta^{\rho k}\bar{\psi}\{\gamma_k, \gamma_{ab}\}\psi - n^2\frac{\mathcal{A}_{ab}^{\rho}}{2M_A}\delta^{\mu k}n_{\mu}^{ij}\bar{\psi}\{\gamma_k, \gamma_{ij}\}\psi\right), \tag{39}$$

where the second term in the bracket is related to spontaneous Lorentz violation disappearing when its scale M_A goes to infinity. They consequently give the following solution for spin-connection fields expressed in the pure local frame Lorentzian components

$$\mathcal{A}_{abc} = -\frac{\kappa}{4}\bar{\psi}\widehat{\gamma}_{abc}\psi\left(1 + \frac{\kappa}{8M_A}n^2\widehat{n}^{ijk}\bar{\psi}\widehat{\gamma}_{ijk}\psi\right) \tag{40}$$

where the combination of the γ-matrices $\widehat{\gamma}_{ijk}$ and the matrix $\widehat{n}^{ijk}$ are defined according to the following (anti)symmetrization of indices

$$\widehat{\gamma}_{ijk} \equiv \frac{1}{2}\left(\gamma_{i[jk]} - \gamma_{k[ij]} + \gamma_{j[ki]} + \eta_{ik}\eta^{ab}\gamma_{a[bj]} + \eta_{jk}\eta^{ab}\gamma_{b[ia]}\right),$$
$$\gamma_{k[ij]} \equiv \{\gamma_k, \gamma_{ij}\}. \tag{41}$$

Note that the "zero-order" solution (40) holds in fact for the contortion tensor $\mathcal{K}_{jab}$ part in the total spin-connection field $\mathcal{A}_{abc} = \mathcal{A}_{abc}^{0} + \mathcal{K}_{abc}$ since an ordinary part $\mathcal{A}_{abc}^{0}$ vanishes in the absence of the fermion source.

Expanding the effective Higgs field $\mathcal{H}$ in (24) in the Lagrangian (33), one comes to the highly nonlinear theory in terms of the zero spin-connection modes $\mathcal{A}_{\mu}^{ij}$ which contains some properly suppressed Lorentz violating couplings. The point is, however, that all these terms are precisely canceled in the basic equation of motion (36) once the constraint equations (39) are used. Thus, one eventually is led to the standard Einstein–Cartan equation terms given solely by the left side of the equation (36). Indeed, one can readily see how this cancellation works for the largest extra terms in its right side. Putting the "zero-order" solution (40) into the equation of motion (36) taken in the same approximation (the background value for tetrads, no tetrad derivative terms, no terms higher than linear in $\mathcal{A}_\mu^{ij}$) one receives for the right side the vanishing sum of the largest SLIV contributions

$$\pm\frac{e\kappa^2}{128M_A}\widehat{n}^{ijk}\left(\bar{\psi}\widehat{\gamma}_{ijk}\psi\right)\left(\bar{\psi}\widehat{\gamma}_{abc}\psi\right)\left(\bar{\psi}\widehat{\gamma}^{abc}\psi\right) \tag{42}$$

stemming from its first and second terms, respectively. Thereby, one unavoidably comes to the standard Einstein–Cartan equation in (36). Note that these six-fermion contributions are much smaller even compared to the tiny generic four-fermion interaction in the Einstein–Cartan theory itself. Nonetheless, their cancellation is strictly provided by the Poincaré gauge invariance emerged. Applying these arguments order by order in spin-connection modes $\mathcal{A}_\mu^{ij}$ in the equations of motion (36) and constraint equations (39) one may come to the same conclusion in a general case as well.

4. Extended theories: an overlook

After the minimal Einstein–Cartan theory, let us now turn to a general PGG Lagrangian (21) containing in its second part $\mathcal{L}^{(2)}$ all possible quadratic combinations of the Poincaré torsion and curvatures, $T^i_{\mu\nu}$ and $R^{ij}_{\mu\nu}$ (17), respectively. Generally, the quadratic Lagrangians contain ghosts and tachyons but, fortunately, there exist several examples of the unitary PGG theories in the literature [17,18]. They include the models with both torsion and curvature ($\mathcal{R}+\mathcal{R}^2+\mathcal{T}^2$), as well as the models with only torsion-squared ($\mathcal{R}+\mathcal{T}^2$) or curvature-squared ($\mathcal{R}+\mathcal{R}^2$) terms. Substituting the field parametrizations (24) and (25) into them and expanding the square roots there in powers of $\mathcal{A}^2/M_A^2$ and $\mathfrak{e}^2/M_e^2$ one is led, likewise the above minimal model, to a highly nonlinear theory in terms of the propagating tetrad and spin-connection emergent Goldstone modes, $\mathcal{A}_\mu^{ij}$ and $\mathfrak{e}^i_\mu$. Apart from the standard vector field couplings, this theory contains many Lorentz violating couplings stemming from their field strengths $T^i_{\mu\nu}$ and $R^{ij}_{\mu\nu}$ in the symmetry broken phase. However, as can be directly confirmed, all their contributions, similar to conventional gauge theories [13,15,16], are canceled among themselves.

Some of these unitary PGG theories could be also used for an unification with the Standard model. There is some point which may help to choose the right PGG candidate. Actually, one can propose that the spin-connection fields $\mathcal{A}_\mu^{ij}$ could be unified with ordinary SM gauge fields in a framework of some non-compact local symmetry group thus leading to a hyperunification of all gauge forces presented in the local Lorentz frame. As to tetrads $\mathfrak{e}^i_\mu$, however, they transform like as ordinary matter fields being belonged to the fundamental vector multiplet of $SO(1,3)_{LF}$ rather than to its adjoint representation. Note that the ordinary gauge theories do not contain the objects like the tetrads, namely, the fundamental vector field multiplets. In this sense, one may only expect a partial unification of PGG with SM unifying only the spin-connection fields with the SM gauge bosons.

Remarkably, there is such an example of the unitary theory containing only the curvature-squared terms [17] that can be written in our notations as

$$\mathcal{L}^{\mathfrak{em}}(\mathcal{A},\psi)=\mathcal{L}^{\mathfrak{em}}_{EC}-\frac{e}{4\kappa_A}\mathcal{R}_{ijkl}\left(\mathcal{R}^{ijkl}+\mathcal{R}^{klij}-4\mathcal{R}^{ikjl}\right) \tag{43}$$

where the curvature tensors (34) in the second term are properly contracted with tetrads, $\mathcal{R}^{ijkl}=\mathcal{R}^{ij}_{\mu\nu}e^{\mu k}e^{\nu l}$, and (anti)symmetrized. In this theory the tetrads will only give the constraint equations causing some extra terms to the Einstein–Cartan equation (36), whereas the spin-connection fields $\mathcal{A}_\mu^{ij}$ become to propagate like those in the Standard Model. Their entire unification seems to be most obvious inside the pseudo-orthogonal $SO(1,N)$ groups in which a direct embedding of the Lorentz group $SO(1,3)$ can be readily carried out. Requiring then that such unified group has to contain a non-trivial internal symmetry group giving some grand unification theory (GUT) for three other forces, we come to the condition $N-4=4k+2$ ($k=1,2,...$) selecting the $SO(N-4)$ GUTs which have complex representations. So, the minimal possible symmetry group for a hyperunification of all forces appears to be the $SO(1,13)$ symmetry which then spontaneously breaks at some Planck mass order scale into $SO(1,3)\times SO(10)$ so as to naturally lead to PGG, on the one hand, and $SO(10)$ GUT [20] for quarks and leptons, on the other (see also [21] where $SO(1,13)$ was introduced in a somewhat different context). However, apart from the $SO(1,N)$ series, some other hyperunification groups are also possible. Particularly, if one keeps an eye on the $SU(N)$ type GUT, then the hyperunification group could be looked for in the special linear $SL(2N,C)$ groups containing as subgroups the $SL(2,C)$ covering the Lorentz group and some grand unified $SU(N)$ symmetry. Thus, apart from the familiar $SU(5)$ GUT [22], which would stem from the $SL(10,C)$, the higher $SU(N)$ GUTs containing all three quark–lepton families could also emerge from the hyperunified theories.

One might expect that there would be a potential danger for any hyperunified theory due to the Coleman–Mandula "no-go" theorem [23] on the impossibility of combining spacetime and internal symmetries. Nonetheless, regarding to the hyperunified theories we consider here, this "no-go" theorem seems not to be a unavoidable obstacle, as may be seen from the following heuristic arguments. Indeed, the first is that the theorem only works if there is a mass gap in the theory that means difference in energy between the vacuum and the next lowest energy state which is in fact the mass of the lightest particle. However, there is no mass gap in the theory in the hyperunification symmetry limit where all the fields, as gauge bosons so the matter fields, are massless. Apart from an extended gauge invariance in a hyperunified theory, the generic masslessness of gauge fields like as photon, gluons and graviton could also be provided by their Nambu–Goldstone nature that is presumably related to the spontaneous breakdown of global spacetime symmetries [6–8]. The second and rather important point seems to be related to the nature of PGG as a theory where the gauge group involved does not need to be specially linked to the base space manifold. Actually, one may take the space to be either curved [1] or flat [2] being no conditioned by PGG on its own. As usually appears [2], one would have to identify the theory with some space manifold at a later stage. As a result, the local frame Lorentz gauge symmetry rather looks like an internal symmetry in PGG, and as such may then have an unobstructed unification with SM or GUT.

We will return to these interesting issues elsewhere.

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