Quantum (hyper)computation through universal quantum gates in water coherent domains

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Abstract. According to quantum electrodynamics (QED), liquid water is composed by two phases, the first one characterized by a coherent state in which all the molecules oscillate in phase, the other by a non-coherent state in which they are all non-correlated. In the coherent state, the phase-correlated oscillations take place within macroscopic spatial regions called "coherent domains" (CD), admitting a spectrum of excited energy levels and generating, at their borders, an evanescent coherent e.m. field. When two or more excited CDs are sufficiently close to each other, the overlapping between their evanescent fields gives rise to a novel type of interaction due to the mutual exchange of virtual photons by quantum tunnel effect. Furthermore, when such water coherent domains are enclosed with waveguides consisting of suitable materials and design, these effects are stabilized and enhanced, allowing for the realization of an extended network of interacting coherent domains. In this paper, we'll discuss how to exploit this dynamics to perform quantum computations by setting-up a set of universal quantum gates and calculate their operational time as a function the main parameters of the proposed physical model. We show this model can represent a basic architecture for a novel kind of quantum hyper-computer, characterized by a very high computational speed and able to overcame some of the main issues currently affecting the quantum computational frameworks so far proposed.

1. Introduction

The fundamental unit of any quantum computer is the logical quantum bit (or simply qubit), namely a mathematical entity, associated to a unit vector state in a two-dimensional complex space. In order to build a quantum computer, it is needed to realize a physical implementation of such logical qubits, called "physical" qubits. A qubit can be built by considering many different two-levels physical systems having certain specific requisites [1] and different proposal in this direction has been advanced so far [2].

Apart its theoretical interest, the importance of realizing quantum computation lies in the common belief it would offer several key advantages with respect its classical counterpart and in the deep impact it would have, for this reason, on a lot of scientific and technological fields such as computer science, cryptography, simulation of very complex systems, artificial intelligence, quantum information, finance and many others.

Nevertheless, despite this enormous potential, many challenges affect its physical implementation so limiting the quantum systems suitable for this scope to, for example, trapped ions, superconductors, quantum dots, molecular spins and optical cavities, each being characterized by specific strengths and weaknesses [3].

A common issue affecting all the above implementations is the decoherence that compromises the stability of the quantum state, namely the effect of environmental noise (due to the interaction of the quantum system with its surrounding) able to destroy the phase information required for quantum computation. On the other hand, the reading of computational results needs for a measurement of the quantum state of the system (namely its interaction with a classical system) thus requiring a delicate and effective compromise between stability and measurability. A failure to achieve such compromise could heavily limit the scalability of any quantum computational system so preventing it from being actually useful.

A key parameter used to evaluate the "merit" of a particular quantum computational scheme is the ratio $\lambda^{-1} = \tau_Q/\tau_{op}$ [4] representing the longest possible quantum calculation, where τ_Q is the "decoherence time" (i.e. the time for which a system remains quantum coherent) and τ_{op} the operation time (namely the time required by the system to perform elementary unitary operations involving at least two qubits). The value of λ^{-1} is larger when the greater is τ_Q and the smaller is τ_{op}. The latter is a measure of the calculation speed, defined as the maximum number of logical operations for unit time achievable by the system. The attainment of very high values of computational speed is then a top priority to achieve for any quantum computational system.

In this paper we discuss a completely novel proposal regarding how to exploit the QED coherent dynamics occurring in liquid water to perform quantum computations.

According to the theory of QED coherence in condensed matter [5], liquid water exhibits a two – phases behavior, due to the presence of a coherent and non-coherent fractions of molecules. The coherent fraction is composed by arrays of so-called "coherent domains" (CDs), namely macroscopic quantum regions in which all the molecules oscillate in phase with each other and with a self-trapped electromagnetic field, while the non-coherent one is made by an ensemble of uncorrelated molecules (vapor phase) filling the interstices between coherent domains. Inside a coherent domain, the electromagnetic field, generated by coherent dynamics, spreads across its boundaries in the form of evanescent field whose tail extends far from it, being therefore able to overlap the analogous evanescent fields produced by the other CDs in the surroundings.

We have yet shown [6-10] this overlapping gives rise to a quantum-type interaction between CDs, we have called "evanescent tunneling coupling" interaction, mediated by the exchange, through tunnel effect, of evanescent (virtual) photons between close CDs, provided the involved CDs have a spectrum of excited states as always occurs in the case of coherent liquid water [9,11].

The resulting transition probability between quantum states of the coupled interacting coherent domains can be described by a "Rabi -like equation" [9]. This suggests the fascinating possibility to use this still unexplored "evanescent tunneling coupling" interaction to achieve quantum computation in liquid water since, as also discussed in this paper, such interaction can be used to realize every type of one- and two-qubit gates and, consequently, every type of quantum computation at all [10].

This possibility is further strengthened by enclosing the water CDs with waveguides made of metamaterials (MTMs) [6,10,12]. This has two important consequences: a) it ensures the stability of coherent fraction of water against the disruptive effect of thermal collisions at room temperature [9] and b) it determines an important amplification of the evanescent e.m. field crossing the boundaries of a given CD in turn allowing the possibility for each of them "to communicate", through the evanescent tunneling-coupling mechanism, with other and farther CDs, so generating an extended network of superfast mutually interacting coherent domains [6-8,10].

On the other hand, the alleged extraordinary computational power of quantum computing with respect its classical counterpart is limited by several factors that meaningfully reduce the achievable calculation speed with respect to the largest theoretically reachable one. According

to some theoretical results [13,14], the "ultimate" limit of computation would be theoretically affected by the energy spread ΔE of the physical system performing computation. Furthermore, as known, quantum computational efficiency is heavily reduced by the loss of coherence of the system due to its interaction with the environment.

In this paper we'll show how the use of water as physical substrate for quantum computation could offer several meaningful advantages with respect the present-day state of the art in quantum computation as, firstly, its stability against the environmental "decoherence" and a very high computational speed and density.

As regards as the calculation speed, that can be define as the maximum number of logical operations the system can perform per unit of time, if we could "indefinitely" increase the value of ΔE we would, in principle, "indefinitely" boost the calculation speed to its theoretical limit.

As we'll prove in this paper, the time required to set-up the elementary quantum gates based on the evanescent tunneling-coupling interaction decreases when the energy spread ΔE increases. A very distinguishing feature of the proposed model is the presence of an energy gap per molecule characterizing water in its coherent state (with respect to the non coherent one). In fact, due to the very high number of molecules contained in a single coherent domain, the energy storable in its excited energy levels could be very high, so that the resulting spectrum would have virtually no upper bound [11]. Such energy gap could be further increased by the occurrence of so-called "supercoherence", namely the onset of a coherence between coherent domains [12,15], that would make available to each quantum logic gate a greater quantity of energy (due to a wider range of possible excited energy levels for each CD).

Supercoherent dynamics could then further boost the computational power of the proposed quantum system based on water CDs [10] by considering, for example, a system composed by an extended network of correlated water CDs, oscillating in phase with each other.

In this kind of system, a high number N of tuned oscillating elementary systems (water CDs) would work in parallel achieving a computational time inversely proportional to N. This reminds, in principle, the concept of quantum "hyper-computing" or "accelerated" quantum computation (namely the execution of a virtually countable infinite number of computational steps within a finite time interval) already investigated in some previous publications [6,7,8,16].

In this paper, we'll particularly focus on a possible scheme to employ coherent dynamics of liquid water to realize a universal set of quantum logic gates, also giving a theoretical estimation of the operational time for each of them. Finally, we'll also discuss some aspects of a possible novel architecture for quantum hypercomputer system that uses liquid water as physical substrate and tunneling interaction between water coherent domains.

2. A brief overview of QED coherence in liquid water

In 1973 mathematical physicists K. Hepp and E. Lieb [17] found that the Dicke Hamiltonian, the first conceptual physical model of the laser, under suitable conditions (almost always verified in the condensed matter) about density and temperature of the system, undergoes a spontaneous phase transition (called "Superradiant Phase Transition" or SPT) to a new state in which a large classic coherent electromagnetic field, oscillating in tune with the atomic transitions between the ground state and a particular excited state, spontaneously emerges and gets trapped inside the atomic or molecular ensemble.

Later, G. Preparata [5], within the framework of Quantum Field Theory (QFT), shown such SPT to be a general phenomenon occurring in a large class of physical systems whose stationary (long-time) dynamics automatically selects the couple of levels, driving the transition itself, characterized by an energy gap given by

$$E = \hbar\omega_0 = \frac{hc}{\lambda} \tag{1}$$

where λ is the wavelength of light, c the speed of light in vacuum and $\hbar = h/2\pi$ is the reduced Planck constant (from this point on, unless differently said, we assume the "natural units" $\hbar = c = 1$). The need to consider the QFT approach to describe such dynamics is rooted in the assumption that the Lagrangian function of a physical system (and then its equations of motion) must be invariant under local transformation of the phase $\phi(\vec{x}, t)$ of the matter field $\psi(\vec{x}, t)$ due to the fluctuations of its components. Such invariance is achieved by introducing a gauge field $A_\mu(\vec{x}, t)$ in the Lagrangian so that the space-time derivative D_μ of the matter field is given by

$$D_\mu \psi = i\partial_\mu \psi - A_\mu \psi \tag{2}$$

so that the Lagrangian is invariant under the gauge transformation

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu X \tag{3}$$

in which $X(\vec{x}, t)$ is an arbitrary function of space-time. When we consider the space-time scales of atoms and molecules, the gauge field $A_\mu(\vec{x}, t)$ is just the usual electromagnetic field so that the description of the system dynamics is just QED.

For a system composed by N elementary units (atoms and /or molecules) there exist a critical value of density $\bar{\rho} = (N/V)_c$, that depends on the "electric polarizability" of the elementary species, and a temperature T_0 such that, when $\rho > \bar{\rho}$ and $T < T_0$) for which all the matter components are phase correlated among them by means of the action of an electromagnetic field oscillating in tune with them too, confined within a defined spatial region, called "Coherence Domain" (CD), associated to the wavelength of the tuning electromagnetic field, given by equation (1). More specifically, we consider a two-levels quantum system, whose matter field is described by the two wavefunctions $\psi_1(\vec{x}, t)$ and $\psi_2(\vec{x}, t)$, respectively associated to the two levels, subjected to the normalization condition

$$|\psi_1|^2 + |\psi_2|^2 = 1 \tag{4}$$

and an electromagnetic filed described by a scalar field $A(\vec{x}, t)$. If now we restrict ourselves to a region whose spatial dimension is of order of λ given by eq. (1) and introduce the adimensional time $\tau = \omega_0 t$, the Eulero – Lagrange equations describing the two - levels matter system interacting with the e.m. field are given by [5]

$$\begin{aligned} i\frac{\partial}{\partial \tau}\psi_1(\tau) &= gA^*(\tau)\psi_2(\tau) \\ i\frac{\partial}{\partial \tau}\psi_2(\tau) &= gA(\tau)\psi_1(\tau) \\ -\frac{1}{2}\ddot{A}(\tau) + i\dot{A}(\tau) - \mu A(\tau) &= g\psi_1^*(\tau)\psi_2(\tau) \end{aligned} \tag{5}$$

with

$$g \equiv 2\pi(\omega_p/\omega_0) f_{01}^{\frac{1}{2}} \tag{6}$$

where ω_p is electron "plasma frequency" (m_e is the electron mass)

$$\omega_p = (e/\sqrt{m_e})(N/V)^{1/2} \tag{7}$$

the term f_{01} is the oscillator strength for the electronic transition from the ground state ψ_1 to the excited state ψ_2 and μ is the photon "mass" term [5]. We search for the solution of the system (5) around $\tau = 0$, where we assume the initial conditions

$$A(0) \sim 0, \ \psi_1(0) \sim 1, \ \psi_2(0) \sim 0 \tag{8}$$

By differentiating the third of Eqs. (5) and substituting it into the second one, we obtain the following equation

$$-\frac{1}{2}\frac{\partial^3}{\partial\tau^3}A + \frac{\partial^2}{\partial\tau^2}A + i\mu\frac{\partial}{\partial\tau}A + gA^2 = 0 \qquad (9)$$

We now look for solutions of Eq. (10) in the form

$$A = \exp\left(ip\tau\right) \qquad (10)$$

that, inserted in Eq. (10), gives

$$\frac{p^3}{2} - p^2 - \mu p + g^2 = 0 \qquad (11)$$

Equation (11) describes two completely different dynamical regimes for our system according as [5] $g > g_{crit}$ or $g < g_{crit}$, where

$$g_{crit} = \left[\frac{8}{27} + \frac{2}{3}\mu + \left(\frac{4}{9} + \frac{2}{3}\mu\right)^{3/2}\right]^{1/2} \qquad (12)$$

In the first case, Eq. (11) admits three real solutions and, by Eq. (10), the system performs zero-point periodic oscillations around a state characterized by the conditions (8). In the second case, Eq. (11) admits two complex conjugate $(p_1, p_2 = p_1^*)$ and one real p_3 roots. This means the field A has a "runaway" solution like

$$A \sim \exp\left[\mathrm{Im}\left(p\right)\tau\right] \qquad (13)$$

where p is the complex solution of Eq. (11) for which $\mathrm{Im}\left(p\right) > 0$. Such a solution is characterized by an exponentially growing amplitude from its "perturbative" point-zero value corresponding to vacuum fluctuations towards a non-vanishing value that forces a similar increase of the matter field component ψ_2. The system then becomes instable and evolves, in a very short time [5], towards a limit cycle defined by $(0 < \gamma < \pi/2)$

$$\begin{aligned}\psi_1\left(\tau\right) &= \cos\gamma\exp\left[i\theta_1\left(\tau\right)\right]\\\psi_2\left(\tau\right) &= \sin\gamma\exp\left[i\theta_2\left(\tau\right)\right]\\A\left(\tau\right) &= A_0\exp\left[i\varphi\left(\tau\right)\right]\end{aligned} \qquad (14)$$

characterized by the **phase-locking** constrain [5]

$$\frac{\partial\varphi}{\partial\tau} = \frac{\partial\theta_1}{\partial\tau} - \frac{\partial\theta_2}{\partial\tau} \qquad (15)$$

Equations (14) and (15) imply the new state of the system is just the coherent state above described, called "coherent ground state" (CGS), characterized by the coherent common oscillation of matter and e.m. fields all assuming non-vanishing amplitudes.

One of the most important consequences of such dynamical evolution is the shift of the frequency of the coherent e.m. field, that becomes renormalized by the coherent interaction to a value $\omega_{coh} < \omega$ [5] given by

$$\omega_{coh} = \omega_0 - \frac{\partial\varphi}{\partial\tau} < 0 \qquad (16)$$

Such frequency shift, not corresponding to a similar change in wavelength λ, ensures the coherent e.m. field to be trapped inside the CD, a process in all similar to that of total reflection of light between two media having different refraction indices.
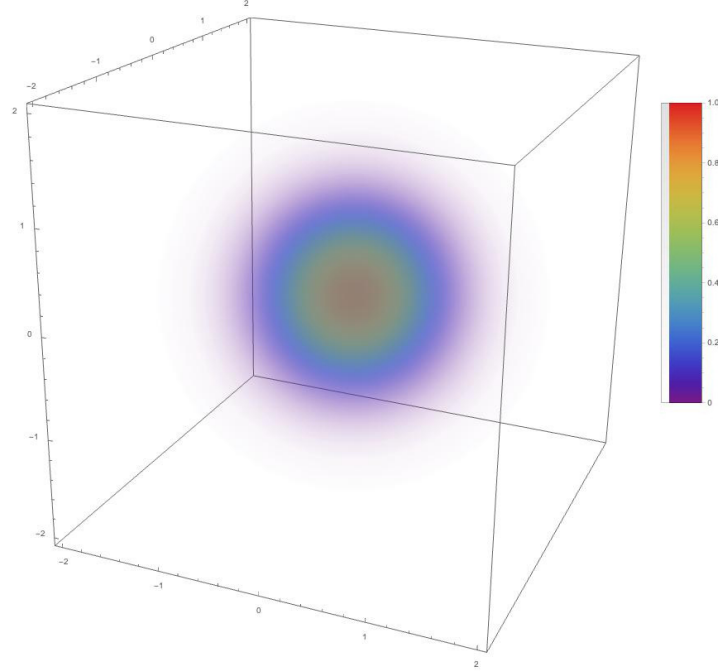
Figure 1. Schematic view of a spherical CD and the coherent evanescent e.m. field escaping from it.

In particular, it can be shown [5] the e.m. field, in the case of a spherical CD, can be described by the following equations

$$A\left(\vec{x}, \tau\right) = A\left(0\right) \frac{\sin\left(\omega_0 \tau\right)}{\omega_0 \tau} \qquad (r < r_0) \tag{17}$$

$$A\left(r\right) \simeq \frac{A\left(0\right)}{\sqrt{2}} \frac{\exp\left[-r\sqrt{\omega_0^2 - \omega_{coh}^2}\left(r - r_0\right)\right]}{\omega_0 r} \qquad (r > r_0) \tag{18}$$

At the interface between CD, whose radius is given in general by [5]

$$R_{CD} \cong \frac{3}{8}\lambda \tag{19}$$

and the surrounding medium, an evanescent e.m. field is then produced (fig. 1).

This evanescent field is characterized by a pulsation ω_{coh} and it crosses the CD's border, falling off at a rate of the order of λ [5] (fig. 2) as given by Eq.(18).

According to the coherent dynamics, the phase agreement between matter and co-resonating e.m. field defines a new macroscopic quantum state (corresponding to a high number N of elementary components) characterized by a well-defined value of phase Φ which defines the rhythm of oscillation of whole the system. From a quantum viewpoint this means the state vector of the system to be an eigenstate of a quantum phase operator [5, 18].

The transition from the non-coherent state to the coherent one can be considered as a "condensation" in which the system releases outwards a given energy, previously "borrowed"
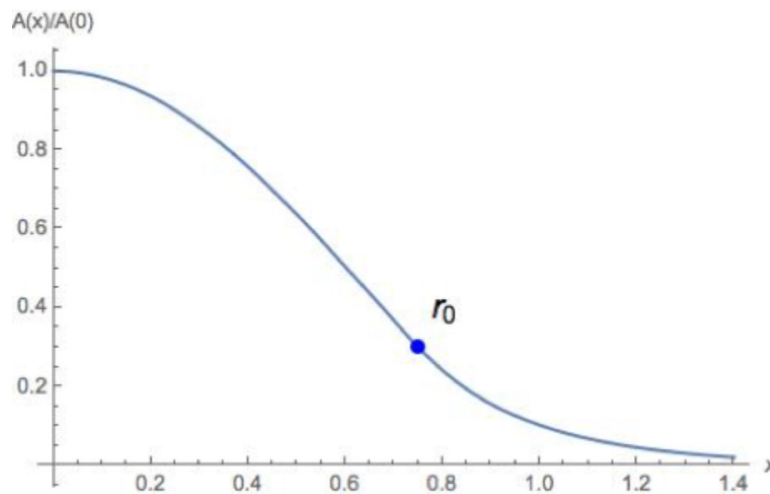
Figure 2. Radial profile of evanescent e.m. field $A(r)$ of a CD, with $r_0 = R_{CD}$. $A(0)$ is the e.m. field intensity at the center of CD.

from quantum vacuum zero-point oscillations. Every coherent domain is then characterized by an energy gap per molecule $\Delta E/N$ that makes it more stable with respect the non-coherent state. The value of this gap depends on the couple of levels involved in the coherent oscillations [5]. For absolute temperatures different than $T = 0$, thermal collisions could transfer to molecules an energy higher than the energy gap so pushing them out of tune. In the general case we can identify a coherent and a non - coherent fraction of the system $F_c(T)$ and $F_{nc}(T)$, respectively indicating the number of molecules belonging, at each instant, to the coherent and non - coherent phases satisfying, at a given temperature, the following constraint:

$$F_c(T) + F_{nc}(T) = 1 \tag{20}$$

For every $T \neq 0$, the system oscillates between the coherent and the non-coherent state, according to whether the coherent electrodynamic attraction is overtaken or not by the thermal collisions transferring to the CD the energy E_{ther}. In the case of liquid water, already studied in several previous works [5,15,19-23], the molecules are characterized by a very rich spectrum of electronic excitations and the coherent oscillations occurs between a ground state and a $5d$ excited level characterized by the transition energy $E_{exc} = 12.06 \, eV$ that lies just below the ionization threshold. The water CD can be then considered as a very huge reservoir of quasi-free electrons (a water CD can include, at the density of liquid water, a number of molecules of the order of $n \simeq 2 \cdot 10^5$, corresponding to a number of quasi-free electrons of about $n_e \simeq 2 \cdot 10^4$) that are very easily excitable in the form of cold electron "vortices", as far as this energy supply is lower than the energy gap per molecule $\Delta E/N \simeq 0.26 \, eV$ associated to the coherent state. These vortices form the metastable coherent excited spectrum of water CD [11] whose energy depends on the quantized value of their angular momentum and on the external magnetic field. A very important feature of such excited states is that, because of the coherence, energy supplied from the outside is absorbed by the CD as a whole and such vortices cannot be excited nor decay thermally since they are coherent and have a quantized magnetic moment that allows them to align to an external static magnetic field. For this reason, they are characterized by a very long lifetime.

Furthermore, being the spectrum $\{E_n\}$ a collective property of the entire CD, it has practically no upper limit since, although every molecule in the coherent state cannot absorb

an energy higher than E_{coh} , all the N resonant molecules can account for a total energy of excitation of the order of NE_{coh} a very high quantity (including frequencies up to visible and HV spectrum) since, at room temperature, $N/V \sim 10^{22}\, cm^{-3}$. Finally, we remark another important feature of coherent water when it is close to a surface. In this case the disruptive effect of thermal collisions is compensated by the attraction of the water molecules to the wall [6,9,24,25]. In this way the coherent fraction of water is stabilized to a value close to the unity so that it can be considered as fully coherent [25] even at room temperature.

3. Tunneling-coupling interaction between coherent domains of water

Our idea to exploit coherent dynamics occurring in liquid water in order to perform quantum computation is based on the chance the water CDs could interact each other [10,11,26]. This is possible if the CDs has a spectrum of excited states and can release outwards the energy stocked in its cold excited vortices. A conceivable chemo-electrodynamical mechanism has been proposed so far [10, 27] to explain the emission of energy from excited CDs.

On the other hand, we have proven [6-9,15,26] the existence of a purely quantum process allowing the water CDs to interact each other through the exchange of virtual photons by quantum tunneling effect.

Such interaction would occur when the evanescent coherent fields produced at the boundaries of two or more CDs (sufficiently close each other) partially or totally overlap, allowing for the exchange of virtual photons among them as schematically shown in fig. 3.

From the standpoint of classical physics, evanescent waves usually originate by the "frustrated total internal reflection" (FTIR) of a light beam at the boundary between two media characterized by different values of refraction index. In this case a "transmitted" wave component is nevertheless present in the second medium, namely just the so-called "evanescent" wave. From the quantum standpoint, the tunneling of evanescent modes of e.m. waves is just interpreted as the tunneling of virtual photons through the corresponding potential barrier. Such tunneling photons are characterized by a negative square mass in the ordinary metric [6,7]. The "quantum-tunneling" interaction above described can be also considered as the result of the process of excitation-deexcitation of the CD energy levels, that determines the tunneling of virtual photons between them (since the CDs are stable in their fundamental state) as the result of the exchange of virtual photons between the quasi-free electrons belonging to the coherent vortices associated to the couple of involved CDs.

From a QFT viewpoint, this process can be described, assuming a not too strong coupling, by the following Hamiltonian (we limit ourselves to the photon dynamics) [10,28]:

$$H = \sum_{\vec{k}} \left[\omega_{\vec{k}} \left(a_{\vec{k},1}^{\dagger} a_{\vec{k},1} + a_{\vec{k},2}^{\dagger} a_{\vec{k},2} \right) + \Gamma \left(a_{\vec{k},1}^{\dagger} a_{\vec{k},1} + a_{\vec{k},2}^{\dagger} a_{\vec{k},2} \right) \right] \tag{21}$$

where $a_{\vec{k},i}$ $(a_{\vec{k},i}^{\dagger})$ is the annihilation (creation) operator for the mode $\vec{k}$ in the i-th CD $(i = 1, 2)$ acting on the vacuum state $|0\rangle$ and Γ is a real parameter quantifying the coupling strength. We then assume:

$$|1\rangle \rightarrow |\psi_1\rangle = a_1^{\dagger} |0\rangle \tag{22}$$

$$|2\rangle \rightarrow |\psi_2\rangle = a_2^{\dagger} |0\rangle \tag{23}$$

and Γ is approximatively independent of the value of $\vec{k}$. From a macroscopic viewpoint, the exchange of a virtual photon can be described as the time evolution of a two – levels system (described by the energy eigenstates $|1\rangle$ and $|2\rangle$ respectively associated to the values E_1 and E_2) whose Hamiltonian has the form:
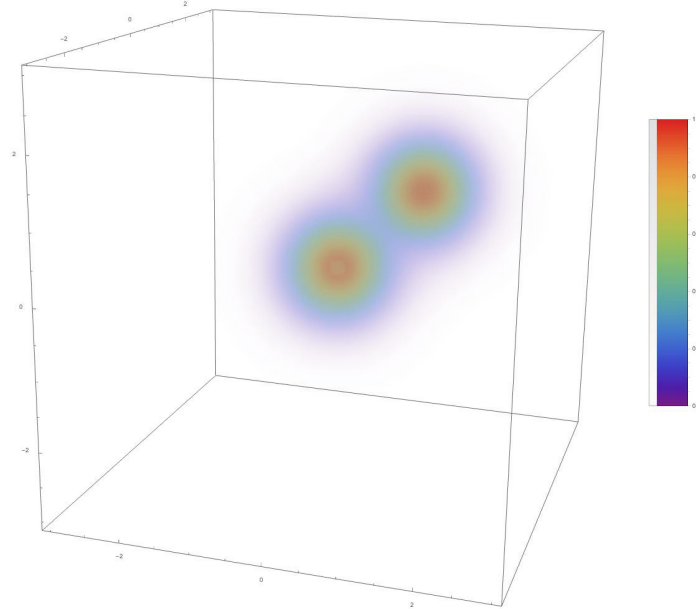
Figure 3. Overlapping between the evanescent fields associated to two nearby CDs.

$$H = E_1 \left|1\right\rangle \left\langle 1\right| + E_2 \left|2\right\rangle \left\langle 2\right| + \Gamma \left(\left|1\right\rangle \left\langle 1\right| + \left|2\right\rangle \left\langle 2\right|\right) \tag{24}$$

where, again, the parameter $\Gamma \in R$ represents the "strength" of the interaction, giving the coupling of the two states, namely the "amplitude" of the tunneling of virtual photons. The transition probability between the two eigenstates $\left|1\right\rangle$ and $\left|2\right\rangle$ is then given by a "Rabi equation" [9,26]:

$$P_{12}\left(t\right) = \frac{\Gamma^2}{\Gamma^2 + \left(E_1 - E_2\right)^2} \sin^2\left(\sqrt{\frac{\left(E_1 - E_2\right)^2 + 4\Gamma^2}{4}} t\right) \tag{25}$$

that describes quantum oscillations in which a photon wave packet of a given wave number is coupled back and forth between two CDs.

In particular, we note the probability of transition between the two states admits the maximum value:

$$P_{\max} = \frac{\Gamma^2}{\Gamma^2 + \left(E_1 - E_2\right)^2} \tag{26}$$

that is equal to 1, namely the excitation is completely transferred from one state to the other (from a CD to the other) when $E_1 = E_2$, whatever the value of $\Gamma \neq 0$. In this case the two CDs are energy degenerate.

The time evolution of such a coupled system, originating from the Hamiltonian (24), is described by the unitary evolution operator [4,9]:

$$U\left(t\right) = \exp\left[-i\frac{\left(E_1 + E_2\right)}{2}t\right]\left[\cos\left(\left|\vec{n}\right|t\right)I - i\sin\left(\left|\vec{n}\right|t\right)\frac{\vec{n}}{\left|\vec{n}\right|}\cdot\vec{\sigma}\right] \tag{27}$$

where $\vec{\sigma} = \left(\sigma_x, \sigma_y, \sigma_z\right)$ are the Pauli matrices, I is the 2×2 identity matrix and the three - dimensional vector $\vec{n}$ has the components:

$$n_1 = \Gamma, \; n_2 = 0, \; n_3 = \frac{1}{2}\left(E_1 - E_2\right) \tag{28}$$

Mathematically, the evolution operator expressed by Eq. (27) is a "rotation" operator of angle θ in the quantum states space:

$$U = \exp\left(i\alpha\right)\exp\left(-i\theta\hat{u}\cdot\frac{\vec{\sigma}}{2}\right) \tag{29}$$

in which, by comparing Eqs (27) and (29):

$$\alpha = -\frac{E_1 + E_2}{2}t; \quad \theta = 2\left|\vec{n}\right|t; \quad \hat{u} = \frac{\vec{n}}{\left|\vec{n}\right|} \tag{30}$$

The time-evolution operator of the system composed by two tunneling - interacting CDs can be then expressed as the product of a rotation of angle θ and an overall phase shift of angle α. As a consequence of the above reasoning, two interacting CDs cannot be further considered as independent entities but as interconnected parts of the same system.

3.1. Tubular coherent domains of water and their interaction
When two CDs are sufficiently close to each other (or when the evanescent field amplitude is sufficiently high even at higher inter-distances), their respective e.m. evanescent fields can overlap. The width of the resulting overlapping zone, through which tunneling of virtual photons takes place, will depend on the distance between the two CDs and on the spreading of the two evanescent fields outside the respective CDs, whose "extension", described by their tales, is of order of $\lambda = 2\pi/\omega_0$ (fig. 3). For a water CD, it has been calculated [6] $d_p \simeq 10^{-7}\,m$, so that, in order to the evanescent fields overlapping to take place, a couple of water CDs should have their mutual intercentrum distance satisfying the condition:

$$d_{CD} \leq 2\left(R_{CD} + d_p\right) \simeq \frac{11}{4}\lambda \cong 137.5\ nm \tag{31}$$

On the other hand, in order to the tunneling coupling interaction to take place effectively, once the coherent state has been achieved, it is needed to ensure: a) the stabilization of the coherent fraction of liquid water against disruptive thermal fluctuations (so that $F_c\left(T\right) \sim 1$) b) a suitable amplification of the evanescent fields generated by each coherent domain. The latter is a necessary condition since, although the condition (31) is satisfied, the evanescent field intensity could be too low for ensure a sufficient coupling between them. On the other hand, such amplification could also allow for the evanescent coupling to spread, for a given field intensity $A\left(0\right)$, even at distances $d >> d_{CD}$.

As already proven [6,9,10,16,24] both the conditions a) and b) can be assured by enclosing water inside the inner volume of (even symmetrical) waveguides whose walls are made up of suitably designed metamaterials (MTMs) (fig. 4), a kind of materials characterized by a purely imaginary value of refraction index. We can also consider the water molecules contained in such waveguide as a whole tubular coherent domain, according to a process we could define as CD "compactification" (whose dynamics and consequences will be studied in detail in a forthcoming paper), at least with respect to the generation of the evanescent field inside the contained CDs [5].
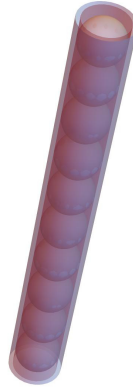
Figure 4. Tubular structure of water CD inside a waveguide made up of metamaterial.
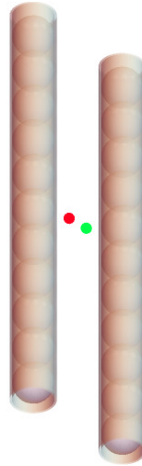

Figure 5. Evanescent coupling by virtual photons (green and red spots) exchange between two waveguides filled with water CDs.

The evanescent field amplification induced by MTMs determines an important increase of the evanescent penetration depth d_p (at least up to four time the original value) allowing a single CD to interact with other ones even if placed far away from it, so forming a spatially extended network of mutually interacting CDs that could be used, as we'll see, to realize superfast parallel quantum computations.

In our theoretical framework the propagation of a photon in a waveguide (namely a tubular water CD) is likened to the presence of a photonic spatial mode, due to tunneling, in one or the other waveguide as sketched in fig. 5.

The previous discussion can be easily generalized to the case of many interacting water CDs, by restricting our analysis to single excited states in which the i-th CD is described by its coherent ground state $|\varphi_{ig}\rangle$ and by an excited state $|\varphi_{ie}\rangle$ so that the overall quantum ground state is given by:

$$|\varphi_0\rangle = \prod_i |\varphi_{ig}\rangle \tag{32}$$

while all possible excited states are described by the state vector:

$$|\varphi_k\rangle = |\varphi_{ke}\rangle \prod_{i \neq k} |\varphi_{ig}\rangle \tag{33}$$

In this picture, the excited state $|\varphi_k\rangle$ is the state in which the k-th tubular CD is in the excited state while all the other are in the coherent ground state. The set $\{|\varphi_k\rangle\}$ is a basis for the space of excited states and satisfies the general orthogonality relation:

$$\langle \phi_{i,k} | \phi_{j,l} \rangle = \delta_{i,j} \delta_{k,l} \tag{34}$$

with $k, l = e, g$ (where "e" stands for "excited" and "g" for "ground"). This basis is often referred to as "site basis" since each state, when expressed in this basis, can be identified by the specific CD where the excitation is localized.

The Hamiltonian of a system composed by N interacting CDs can be then written, in this basis, as:

$$H = \sum_{i=1}^{N} \varepsilon_i |i\rangle \langle i| + \sum_{i \neq j}^{N} \Gamma_{ij} |i\rangle \langle j| \tag{35}$$

where ε_i is the relative excitation energy of the i-th tubular CD, namely $\varepsilon_i = E_i - \omega_{coh}$, and Γ_{ij} is the "strength" of the tunneling interaction between the i-th and j-th CDs. In our system configuration, the values of Γ_{ij} will definitely depend on the mutual distance between tubular units as well as on the structural features (that in turn control the evanescent field amplification) of MTMs tubular waveguides, whose detailed design and features will be studied in a separate work.

4. The mapping of unitary quantum transformations on interacting water CDs and the realization of universal quantum computation

Quantum computation relies on the manipulation of quantum information by exploiting dynamic transformation of quantum states of a physical system.

Each of these transformations is described by unitary linear operator (a matrix when expressed in a suitable basis) defined in the complex vector space associated with the quantum state space. In the framework of classical computation, there exists a small set of gates (like, for example, AND, OR, NOT gates) that can be used to compute an arbitrary classical function and such set is then said to be "universal".

A similar result holds in quantum computation framework where a set of gates can be found to be used to realize universal quantum computation, since it can be shown [4] that any arbitrary unitary transformation can be implemented from a set of primitive transformations, including the two-qubits CNOT (controlled-NOT) gate in addition to three one-qubit gates.

A suitable set of gates that also allows a fault-tolerant construction is [4]:

(1) the Hadamard gate H (to be not confused with the Hamiltonian);

(2) the $\pi/8$ gate T;

(3) the "phase" gate S;

(4) the CNOT gate.

To each of these gates is associated a unitary operator whose matrix representation in the computational basis that is given in table 1. The physical realization of quantum computation algorithms implies the mapping of logical qubits to physical states of the considered quantum system (namely the "physical" qubits), and then the implementation of a suitable set of unitary operators, acting on these states and realizing the universal quantum gates, each corresponding to a unitary time evolution of the system, as described by its Hamiltonian.

Generally, for a given physical quantum system, this mapping is not unique and, also, different mappings could give rise to different values of computational speed and overall fidelity.

As we'll see, the physical implementation of such unitary transformations generally sets some constraints on the parameters of physical system. The possibility to engineering the CDs interaction in order to use them for practical quantum computation relies on the ability to set and control such parameters.

Encoding quantum operations into the dynamics of physical system then implies the ability to state a correspondence between quantum unitary transformations, the system's Hamiltonian and the physical (geometrical and/or structural) properties of the system itself.

For a closed quantum system, the time evolution is driven by a set of unitary transformations that are functions of the system's Hamiltonian H according to:

$$U\left(t\right) = e^{-\frac{i}{\hbar}Ht} \tag{36}$$

We can then "invert" Eq. (36) and search for the Hamiltonian able to originate, after a time interval τ_G, a desired unitary transformation τ_G, namely:

$$H_Q = \frac{i\hbar}{\tau_G}\ln U_Q \tag{37}$$

By using the site basis defined by Eqs (32) and (33), the logical qubit states are mapped to the physical states $|i\rangle$ where i indicates the i-th CD that, in this representation, is directly associated with a state of the qubit register.

4.1. Realization of one-qubit gates

According to the above mapping, an N-dimensional quantum state needs 2^N CDs to be represented by a physical circuit made of water CDs. In our model, a single quantum qubit gate, acting on a two-dimensional quantum space, needs two interacting CDs (labelled by 1 and 2) to be realized.

Then we can write, using the notation of Eqs (32) and (33):

$$\begin{aligned} |0\rangle &\to |\phi_1\rangle = |1_e2_g\rangle \\ |1\rangle &\to |\phi_2\rangle = |1_g2_e\rangle \end{aligned} \tag{38}$$

in which, for further clarity, we have explicitly labeled the single CD (as 1 and 2) and its energy state (ground or excited). For a two-qubits gate, the related quantum space is four dimensional, so we must consider four CDs and adopt the following mapping of the (computational) basis states:

$$\begin{aligned} |00\rangle &\to |\phi_1\rangle = |1_e2_g3_g4_g\rangle \\ |01\rangle &\to |\phi_2\rangle = |1_g2_e3_g4_g\rangle \\ |10\rangle &\to |\phi_3\rangle = |1_g2_g3_e4_g\rangle \\ |11\rangle &\to |\phi_4\rangle = |1_g2_g3_g4_e\rangle \end{aligned} \tag{39}$$

It is worthy to be observed how the adopted mapping can be directly related, within our model, to the underlying system dynamics, since it can be associated to the tunneling exchange of virtual photons between two water-filled waveguides (that is a couple of interacting tubular

CDs of water) by adopting the dual-rail representation for the photon. According to such representation, the two logic states $|01\rangle$ and $|10\rangle$ respectively represent the state in which a photon is absent and present as spatial mode in the i-th waveguide.

We are now in position to study how to realize the elementary quantum transformations needed to perform universal quantum computation.

The simplest gate to be implemented is the T gate whose unitary matrix is given in table 1. The corresponding Hamiltonian is given, using Eq. (38), by the matrix:

$$H_T(\tau) = \frac{\pi\hbar}{4\tau} \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \tag{40}$$

The effect of T gate is to change the relative phase between $|0\rangle$ and $|1\rangle$ by the amount $e^{i\pi/4}$; this transformation then doesn't involve any exchange between the basis states and then no interaction between the corresponding CDs. This is confirmed in the Hamiltonian (40) in which the off-diagonal terms, associated to the terms $\Gamma_{12} = \Gamma_{21} = \Gamma$ are zero. It's easy to obtain the expression of the time τ_T required by the system evolution to realize the T gate. By remembering the difference in the diagonal term of the Hamiltonian corresponds to the difference between the relative energy of excitation of the two states $|0\rangle$ and $|1\rangle$ we have:

$$\frac{\pi\hbar}{4\tau_T} = \Delta\varepsilon_{01} = E_1 - E_2 \qquad \Rightarrow \tau_T = \frac{\pi\hbar}{4(E_1 - E_2)} \tag{41}$$

We would obtain the same result, through a more formal and general approach, by considering the Eqs (27)-(30) and recalling that (see Table 1), in terms of Pauli matrices:

$$U_T = \exp\left(i\frac{\pi}{8}\right)\exp\left(-i\frac{\pi}{8}\sigma_z\right) \tag{42}$$

so, by equating Eqs. (27) and (42) and using Eq. (30) we find [10,26]:

$$u_z = 1; \quad u_x = u_y = 0; \quad \theta = \frac{\pi}{4}; \quad \alpha = \frac{\pi}{8} \tag{43}$$

By inserting these values in the Eq. (30) and using Eq. (28) we find:

$$\frac{\frac{E_1 - E_2}{2}}{\sqrt{\Gamma^2 + \frac{(E_1 - E_2)^2}{4}}} = 1 \tag{44}$$

and

$$t = \frac{\pi\hbar}{8|\vec{n}|} = \frac{\pi\hbar}{8}\frac{1}{\sqrt{\Gamma^2 + \frac{(E_1 - E_2)^2}{4}}} \tag{45}$$

From Eq. (44) we derive $\Gamma = 0$ that, inserted in the Eq. (45) just gives the Eq. (41). The latter indicates the time required to perform T gate is inversely proportional to the difference of the excited energy levels of the two CDs. Equation (41) and the requirement $\Gamma = 0$ show the T gate can be implemented, in our model, by considering a couple of non-interacting ($\Gamma = 0$) tubular CDs, characterized by non-degenerate excited energy states ($E_1 \neq E_2$). The Hamiltonian corresponding to S gate (whose unitary matrix is given in table 1) is given by:

$$H_S(\tau) = \frac{\pi\hbar}{2\tau} \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \tag{46}$$

The effect of S gate is to apply the T gate twice since $S = T^2$. So, like the T gate, we have in this case $\Gamma_{12} = \Gamma_{21} = \Gamma = 0$ and the time required to implement the gate is obtained as above:

$$\tau_S = \frac{\pi\hbar}{2\left(E_1 - E_2\right)} \tag{47}$$

with $E_1 \neq E_2$. The physical implementation of such gate is then realized in the same way as the T gate. The values of the parameters $\hat{u}$, θ and α are found to be [10]:

$$u_z = 1; \quad u_x = u_y = 0; \quad \theta = \frac{\pi}{2}; \quad \alpha = \frac{\pi}{4} \tag{48}$$

As known, the application of the NOT gate to a qubit $|\phi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ interchanges the basis vectors so that we have $U_{NOT}\,|\phi\rangle = \alpha\,|1\rangle + \beta\,|0\rangle$ Its unitary matrix, in computational basis, is given in table 1.

Table 1. Parameters to implement quantum gates and related operational times.

gate	unitary matrix
Hadamard	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
T	$\exp\left(i\pi/8\right)\begin{pmatrix} \exp\left(-i\pi/8\right) & 0 \\ 0 & \exp\left(i\pi/8\right) \end{pmatrix}$
S	$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$
NOT	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
CNOT	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

The corresponding Hamiltonian is:

$$H_{NOT}\left(\tau\right) = \frac{\pi\hbar}{2\tau}\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \tag{49}$$

We note U_{NOT} can be expressed, in terms of Pauli matrices as:

$$U_T = \sigma_x \tag{50}$$

the same analysis of NOT gate performed as above leads us to the constraints [10]:

$$u_x = 1; \quad u_y = u_z = 0; \quad \theta = \pi; \quad \alpha = \frac{\pi}{2} \tag{51}$$

By inserting these values in Eq. (30) and using Eq. (28) we find:

$$\frac{\Gamma}{\sqrt{\Gamma^2 + \frac{(E_1 - E_2)^2}{4}}} = 1 \tag{52}$$

and

$$t = \frac{\pi\hbar}{2\,|\vec{n}|} = \frac{\pi\hbar}{2} \frac{1}{\sqrt{\Gamma^2 + \frac{(E_1-E_2)^2}{4}}} \tag{53}$$

Equation (52) implies $E_1 = E_2$ while, from Eqs. (51) and (53) we obtain:

$$\tau_{NOT} = \frac{\pi\hbar}{2\Gamma} \tag{54}$$

as can be also confirmed by the analysis of the Hamiltonian (49). We note τ_{NOT} is equal to half of the "Rabi time" as calculated through Eq. (26) so that the physical implementation of such gate can be achieved by considering a couple of two interacting tubular CDs ($\Gamma_{12} = \Gamma_{21} = \Gamma \neq 0$) characterized by degenerate excited energy levels ($E_1 = E_2$) as required in order to ensure a fully transfer of excitation between them (corresponding to the transition $|0\rangle \rightleftarrows |1\rangle$). The Hadamard gate, often referred to as "square root of NOT gate", turns $|0\rangle$ into $(|0\rangle + |1\rangle)\big/\sqrt{2}$ and $|1\rangle$ into $(|0\rangle - |1\rangle)\big/\sqrt{2}$ in a sense "halfway" between the states $|0\rangle$ and $|1\rangle$ [4]. Its associated unitary matrix is given in table 1 and the corresponding Hamiltonian is:

$$H_{HAD}(\tau) = \frac{\pi\hbar}{2\sqrt{2}\tau} \begin{pmatrix} 1-\sqrt{2} & 1 \\ 1 & -1-\sqrt{2} \end{pmatrix} \tag{55}$$

The matrix U_{HAD} can be expressed, in terms of Pauli matrices as:

$$U_{HAD} = \frac{\sigma_x + \sigma_z}{\sqrt{2}} \tag{56}$$

this implies [10]

$$u_x = \frac{1}{\sqrt{2}}; \quad u_y = 0; \quad u_z = \frac{1}{\sqrt{2}}; \quad \theta = \pi; \quad \alpha = \frac{\pi}{2} \tag{57}$$

By inserting these values in Eq. (30) and using Eq. (28) we find:

$$\frac{\Gamma}{\sqrt{\Gamma^2 + \frac{(E_1-E_2)^2}{4}}} = \frac{1}{\sqrt{2}}; \qquad \frac{E_1 - E_2}{2\sqrt{\Gamma^2 + \frac{(E_1-E_2)^2}{4}}} = \frac{1}{\sqrt{2}} \tag{58}$$

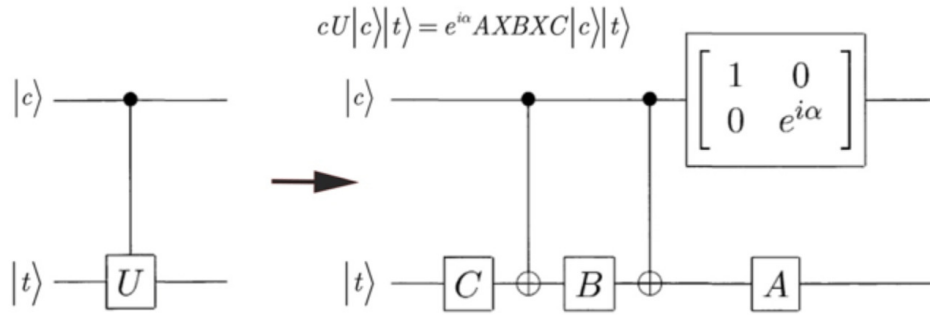leading to the constraint

$$\frac{E_1 - E_2}{2\Gamma} = 1 \tag{59}$$

By using Eq. (59) in Eq. (30) we find:

$$\tau_{HAD} = \frac{\pi\hbar}{2\sqrt{2}\Gamma} = \frac{\pi\hbar}{\sqrt{2}\,(E_1 - E_2)} \tag{60}$$

The Hadamard gate can be physically realized by considering two non-degenerated interacting tubular CDs evolving for a time given by Eq. (60). It's important to note Eq. (59) sets a constraint on the circuital realization of such port in terms of geometrical and/or engineering design of the involved tubular CDs.

Table 2. Parameters to implement one-qubit gates and related operational times.

Gate	α	θ	u_x	u_y	u_z	Δt
Hadamard	$-\pi/2$	π	$1\big/\sqrt{2}$	0	$1\big/\sqrt{2}$	$\pi\hbar\big/\sqrt{2}\Delta E$
T	$-\pi/8$	$\pi/4$	0	0	1	$\pi\hbar/4\Delta E$
S	$-\pi/4$	$\pi/2$	0	0	1	$\pi\hbar/2\Delta E$

Figure 6. Circuit representation of controlled unitary operation on single qubit.

4.2. Controlled operations

The above discussion clearly indicates how, by a suitable selection of the physical parameters E_1, E_2, Γ and by letting the system evolving for an appropriate time Δt, an ensemble of interacting water CDs can realize, in principle, any generic quantum gate. Such dynamics is also able to reproduce any controlled unitary operations cU (such as, for example, CNOT), so allowing the realization of any quantum computing circuits. A generic controlled unitary operation U on a single qubit can be realized by considering the circuit shown in fig. 6, by decomposing U as:

$$U = e^{i\alpha} AXBXC \tag{61}$$

where $X = \hat{\sigma}_X$ and

$$
\begin{aligned}
A &\equiv R_z\left(\beta\right) R_y\left(-\tfrac{\gamma}{2}\right) \\
B &\equiv R_y\left(-\tfrac{\gamma}{2}\right) R_z\left(-\tfrac{\delta+\beta}{2}\right) \\
C &\equiv R_z\left(\tfrac{\delta-\beta}{2}\right)
\end{aligned}
\tag{62}
$$

with the unitary operators A, B, C acting on a single qubit such that $ABC = I$ and for some angles β, δ and γ. To prove how this allows us, within our model of quantum computation, to physically realize controlled operations on qubits, it suffices to note any unitary transformation can be expressed in terms of rotations in the space of quantum states that, in turn, can be simulated through interacting water coherent domains for a suitable choice of the transformation parameters (see table 2). The relations between the parameters in Eq. (62) and the angle θ in table 1 has been calculated in [10]. The previous discussion can be easily generalized to realize multiple - qubits conditioned unitary operations [4,10].

4.3. Realization of two-qubits gates

In order to show how to implement a two-qubits gate, we apply the above framework to the example of the CNOT gate, due to its fundamental importance in quantum computing. The

unitary matrix describing CNOT is given in table 1 and its corresponding Hamiltonian can be written as:

$$H_{CNOT}(\tau) = \frac{\pi\hbar}{2\tau} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \qquad (63)$$

The CNOT gate acts on a two-qubits state $|ct\rangle$, where the first qubit is the "control" and the second one is the "target", in such a way that if $c = 0$ the target qubit is unchanged while, if $c = 1$, it is subject to the NOT operation. According to our model, CNOT gate can be physically constructed by considering four tubular CDs according to the following scheme. We note the upper left quarter of U_{CNOT}, acting on upper part of the two-qubit state, is the identity transformation while the lower right quarter, acting on the lower part of the two-qubit state, is just the NOT gate already considered. So, the CNOT gate can be physically realized by considering a couple of degenerate interacting CDs to perform the NOT gate, and another couple of non-degenerate non-interacting CDs to realize the control. In this case the time required by realize the CNOT operation is simply given by:

$$\tau_{CNOT} = \frac{\pi\hbar}{2\Gamma} \qquad (64)$$

5. Quantum (hyper)computation by means of water coherent domains
Any feasible quantum computing scheme should satisfy some general requirements [4]:

(1) the physical realization of well-defined qubits;
(2) the initialization of the system in a well-defined initial state, such, for example, the qubit $|0\rangle$;
(3) the set-up and "programming" of a set of universal quantum gates;
(4) the efficient storing and retrieving of information;
(5) a sufficiently long decoherence time in order to ensure the quantum computation to finish.

A general treatment of how to address the above requirements within our model is the subject of previous and forthcoming publications [10,26]. Here we'll focus specifically on the point c), limiting ourselves to recall some key previous results related to the other aspects only when needed. According to the quantum coherent dynamics of water, the wavefunction describing a coherent domain, $\Psi = \psi \exp(i\Theta)$, represents an eigenstate $|\Theta\rangle$ of the quantum phase operator (whose eigenvalue is related to the coherent frequency of oscillation including the angular frequency of the excited vortices) [9,10,21,27].

So, any coherent state corresponds to an eigenstate of quantum phase operator and then is always a well-defined quantum state. On the other hand, we have shown the systems composed by tubular CDs can be used as circuits able to hard code the action of specific elementary quantum unitary operations. The actual feasibility of a quantum computer based on the water coherent domains dynamics demands for the "programmability" of such circuits in order to perform sequential and / or parallel operations. This also implies our ability to control and "adjust" the strength of the evanescent photon tunneling interaction between CDs, macroscopically measured by the value of Γ. We can assume such value depends:

(1) on the reciprocal distance between interacting CDs (or, more generally, their geometrical configuration) since, for a given couple of tubular CDs, Γ is inversely proportional to their spatial separation;

(2) for a given spatial separation, on the intensities of the evanescent coherent fields produced by them, in turn related to the specific design of the enclosing shell made up of MTMs.

Many different configurations of two or more tubular CDs could reproduce the same quantum gate, because of the "freedom" in the choice of ΔE and Γ. Each of these configurations is characterized, for a given specific quantum gate they belong to, by a different computational time, depending on the values of ΔE or Γ. The value of quantum phase Θ exactly defines the quantum state $|\Psi\rangle$ of the water tubular CDs (ground or excited) since a change in Θ would imply a change in the value of the coherent oscillation frequency and vice versa. The "identification" of the quantum state $|\Psi\rangle$ with its phase state $|\Theta\rangle$, together with the precise arrangement of the tubular CDs in a given circuit, allows us:

(1) to set the quantum state of each tubular CD, so initializing the quantum qubits register;

(2) to "program" and "control" the quantum gates made by water CDs [10,26];

(3) to achieve a "selective" excitation and a measurement scheme of the phase of such coherent domains capable to address each of them and the qubits they are associated to in order to realize the quantum circuits and operations.

The last point is particularly important in controlled operations in which the setting of the control qubit is crucial. For example, in the case of CNOT gate, the four state $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ have to be selectively addressed, initialized and measured. In the mapping we have defined (as results from the Hamiltonian H_{CNOT}) the two sets $A = \{|00\rangle, |01\rangle\}$ and $B = \{|10\rangle, |11\rangle\}$, corresponding to different values of the control qubit, are characterized by different transition energies $\hbar\omega_A$ and $\hbar\omega_B$ (the set A is implemented by a couple of non-interacting CDs while the set B by interacting ones) so that they can be distinguished and selected by using, for example, a resonating approach [10].

With regard to the performance of a quantum computational system, as we have seen a rough indication is given by the value of $\lambda^{-1} = N_{op}$ that is directly proportional to the decoherence time τ_Q of the system. As already shown [9,24,25], the decoherence time of coherent quantum state of liquid water can be extraordinary long (virtually even "indefinitely" long) provided that we are able to ensure the condition of interfacial water inside tubular CDs. If this condition is satisfied, as we assume for water inside waveguides made by MTMs, then τ_Q could be of many orders of magnitude greater than τ_{op} so overcoming the critical issue related to environmental decoherence that affect all the other quantum computational schemes so far proposed. Equations (41), (47) and (60) also show computing time can be reduced by increasing ΔE (for T, S and Hadamard gates respectively), whereas for the NOT and CNOT gates, according to Eqs. (54) and (64), a reduction of computational time corresponds to increase the interaction strength Γ.

The energy "accumulated" in an excited water CDs, in the form of rotational energy of its cold vortices, can be very high, leading to a value of ΔE up to about $10^5\,eV$ and a corresponding operational time as small as:

$$\tau_G \sim 10^{-39}\,s \tag{65}$$

for T, S and Hadamard gates. In order to estimate the transformation times required to implement the NOT and CNOT gates, we note the value of Γ (which can be increased by a suitable geometrical and structural design of the tubular MTMs shells) is related, for a given coupling configuration of the tubular CDs, to the tunneling time Δt_{tun} of the evanescent photon from one CD to the other [6,8,9,10,16,26], in the following way:

$$\Gamma \propto \frac{1}{\Delta t_{tun}} \tag{66}$$

On the other hand, it is just such tunneling process that allows the excitation to transfer between a couple of interacting CDs so we can assume:

$$\Delta t_{tun} = \frac{\tau_{Rabi}}{2} \tag{67}$$

where τ_{Rabi} is the "Rabi" time, namely the time required by the system of coupled CDs to perform a complete oscillation between its excited states, that is exaclty twice the time to implement the NOT and CNOT gate, leading us to:

$$\tau_{CNOT} = \tau_{NOT} \cong \Delta t_{tun} \tag{68}$$

The tunneling time is related, in turn, to the energy of the tunneling photon [6,10], that is:

$$\Delta t_{tun} \leq \frac{2\pi\hbar}{\omega_{coh}} \tag{69}$$

where ω_{coh} is the frequency of the coherent evanescent field associatd to the water CD. For a single exctited water CDs we can extimate an upper bound for Δt_{tun} as [10]:

$$\Delta t_{tun} \leq \frac{2\pi\hbar}{N_{CD}\Delta E_{mol}} \tag{70}$$

where N_{CD} is the number of water molecules belonging to the the CD and ΔE_{mol} is the energy gap per molecule carachterizing the coherent state. Equation (68) gives for our system [10]:

$$\tau_{CNOT} = \tau_{NOT} \sim 10^{-20} s \tag{71}$$

In general, it can be assumed the overall operational time of a computational system depends on the:

(1) overall gates operational time $\sum\limits_{G} \tau_G$;

(2) time required by a circuit to "communicate" with another τ_C;

(3) decoherence time of the system τ_{dec};

(4) time of writing and retrieval of information τ_{RW};

(5) implementation of parallel and/or serial computational schemes.

For a generic system, several theoretical upper bounds for computational speed (expressed as the number of logical operations performed per unit of time n_{op}) related to energy has been proposed so far, even if the computation if performed reversibly. More specifically, for a quantum system characterized by an energy uncertainty ΔE, the time $T_\perp$ required to evolve to an orthogonal quantum state obeys the bound [13,14]:

$$T_\perp \geq \frac{\pi\hbar}{2\Delta E} \tag{72}$$

that gives the corresponding upper limit for the computational speed n_{op}

$$n_{op} \equiv \frac{1}{T_\perp} \leq \frac{2\Delta E}{\pi\hbar} \tag{73}$$

For a two-levels system, the energy uncertainty of the system can be written as $\Delta E = |E_2 - E_1|$ so that the minimum computational speed is directly proportional to ΔE. The identification of $T_\perp^{-1}$ with a maximum computationl clock speed seems quite reasonable for

some reasons. In quantum circuits each logic gate brings the system to a near-orthogonal state so we can tipically assume $T_\perp$ as the minimum time required, for example, to flip a qubit from $|0\rangle$ to $|1\rangle$ (and vice versa), like in the NOT operation. In our model of computation, the value of $n_{op,G} = 1/\tau_G$ increases, for every quantum gate, with the value of relative excitation energy $\Delta E = E_1 - E_2$, that can also represent the energy uncertainty of the quantum state of the system composed by a couple of interacting CDs. Then we can write, for a given quantum gate:

$$n_{op,G} = \alpha_G \frac{\Delta E}{\pi\hbar} \tag{74}$$

where the coefficient α_G is specific for every quantum gate and can be easily obtained from the values of $\Delta t = 1/n_{op}$ in table 2 and agrees with the general bound (56). More specifically, for the NOT gate, that realizes the flip $|0\rangle \Leftrightarrow |1\rangle$, the computational speed can be extimated by Eq. (70) that, compared with Eq. (73), gives the constraint:

$$\frac{N_{CD}\Delta E}{2\pi\hbar} \leq n_{op} \leq \frac{2\Delta E}{\pi\hbar} \tag{75}$$

One of the most important reason of interest in quantum computing is the common believe it could be intrinsically much more powerful than classical computation. The model of quantum computation based on water coherent domains could be, in principle, even much more powerful of the quantum computation schemes so far proposed.

The discussed model in fact, apart to be able to solve efficiently the critical issue of environmental decoherence, could ensure, if properly implemented, a precise and superfast (theoretically almost instantaneous as already discussed in some previous publications [6,10,26]) set-up and addressing of the quantum states used in computation. A similar mechanism could be also used to perform writing and reading of quantum information as we'll show in a forthcoming publication.

But one of the most important and interesting features of the proposed model of computation concerns its scalability to a large number of quantum circuits composed by many interacting CDs that are able to meaningful increase the computational speed.

According to the commonly accepted picture, the overall speed of a computational system wouldn't depend on the computational architecture, namely if the logical operations are performed in serial or in parallel [10,26].

This can be noticed, for example, by generalizing the Eq. (73) to N logical gates acting in parallel:

$$n_{op}(N) \leq \frac{2}{\pi\hbar} \sum_{l=1}^{N} \Delta E_l(N) \tag{76}$$

if ΔE_{tot} is the total energy available to the system then we can suppose $\Delta E_i = \Delta E_{tot}/N$ and write:

$$n_{op}(N) \leq \frac{2}{\pi\hbar} \sum_{l=1}^{N} \frac{\Delta E_{tot}}{N} = \frac{2N}{\pi\hbar} \frac{\Delta E_{tot}}{N} = \frac{2\Delta E_{tot}}{\pi\hbar} \tag{77}$$

that is independent on the number N of the involved water CDs.

Yet, as we have seen, the onset of the QED coherence is characterized by an energy gap that allows the coherent state to have an energy smaller than the non-coherent one. A further increase of the coherence of the system, as occurring in the so-called "supercoherence" [10,12,26,27], would further increase the stability of the system and the related energy gap. For the proposed water system instead, the use of a greater number of logic gates (associated to an increasing number of water CDs) would be accompanied by a higher energy ΔE available for
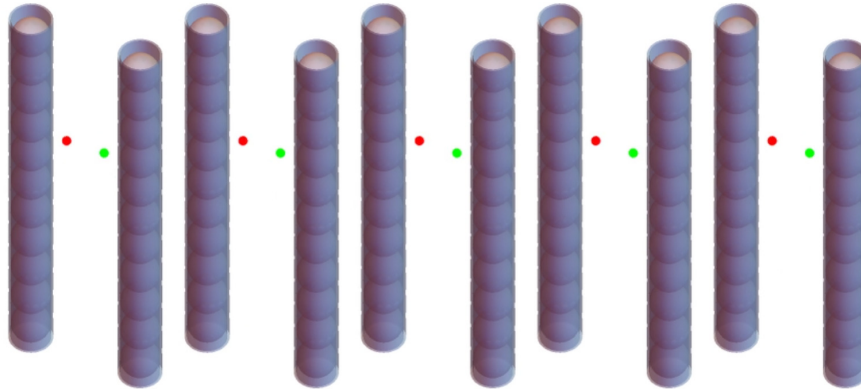
Figure 7. "Synchronized" evanescent coupling and virtual photon exchange between waveguides filled water CDs acting in parallel.

each gate (due to a wider range of possible excited energy levels for each CD associated to the increased number of elementary systems involved in the coherent dynamics) with a consequent reduction of the related computational time for each gate. The underlying dynamics of the proposed computational model then opens a very remarkable perspective to further boost its computational power.

The consideration of an extended network of correlated (through the tunneling and/or phase interaction) water CDs oscillating in phase with each other (fig. 7), is then able to reduce, in principle, the overall parallel computational time, in a way proportional to the number N of the tuned oscillating elementary systems. Such result, that would be impossible to achieve for a closed system, is a natural consequence of the quantum coherent dynamics of liquid water as an open system locally exchanging energy with the fluctuations of quantum vacuum.

So, in the Eq. (77), when N increases, each term of the sum increases as well and the time required to execute each step proportionally reduces, so that, if $N_1 \gg N_2$ we recover, in principle, the concept of "hyper-computing", already analyzed, in a preliminary form, in several previous publications [6-8,9,10,12,16], for which the time required to perform a given number of operations decreases or, at least, remains constant whatever the number of operations.

Finally, in an actual quantum computer architecture, it is very important to consider the issue of memorization and readout of quantum information and the magnitude of the time interval required to perform such operations. This is strictly related to the specific processes used to implement memorization and readout of quantum information that, in our model, are linked to the mutual phase interaction between water CDs and to the interaction between them and their surrounding electromagnetic-thermodynamics environment which they "probe" through e.m. vector potentials. A more detailed study of the role of such interactions in the storage and retrieval of quantum information is not trivial and will discussed in a forthcoming work.

6. Conclusions

Starting from the predictions of QED coherence in water, the realization of quantum circuits, based on the dynamics of energy excitations of coherent water domains, as well as the proposal of a novel architecture of quantum hypercomputing system have been discussed. We have shown such dynamics is able to perform universal quantum gates for one and two-qubits and then, in principle, any type of quantum computation.

The operational speed of the quantum gates so obtained can reach very high values close to the

ultimate limit imposed by the fundamental principles of quantum mechanics. This result can be achieved by suitably "adjusting" the energy levels of the water coherent domains (corresponding to its excited levels), by composing the quantum circuits and controlling the parameters ruling the interaction between them.

Apart from the very high computational speed reachable by each elementary universal gate and the precise control and setting of the quantum states representing the qubits, further very amazing features of the proposed model concern the possibility to overcome the environmental decoherence and to realize "supercoherent" quantum computing. The latter refers to the possibility of implementing an extended network of quantum gates acting in parallel, whose computational speed would be proportional to number of the matter components that join the common coherent oscillation, so achieving a much higher operational speed than of a single quantum gate. "Supercoherence" then opens the door to a sort of "quantum hypercomputing" able to boost the computational speed well beyond the ultimate limit so far assumed in classical and quantum computation.

Despite some features of the proposed model (like, for example, storing and retrieving of the quantum information and the study of other interaction mechanisms apart the tunneling of virtual photons already considered) would obviously need further closer examinations (a lot of which already strongly in progress), we argue it could open very fascinating perspectives towards a new conception of quantum computing using water molecules as physical and computational substrate as well as very intriguing technological applications requiring a very high computational power and stability as, for example, the simulation of very complex quantum systems, environmental and health forecasting, large-scale and strong artificial intelligence, encryption / decryption of data and cybersecurity, big data analysis, financial predictions and many others.

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