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Renormalisation of the Higgs sector of the NMSSM. Application to Higgs decays

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Abstract. I will first recollect some memories of my collaboration with Shimizu-sensei, a collaboration that mainly covered the topic of renormalisation and loop corrections. I will then describe the renormalisation of the Higgs sector of the NMSSM, this work could be seen as a natural continuation of the long collaboration I have had with the Minami-Tateya group. This study has been done with Geneviève Bélanger, Guillaume Chalons and Vincent Bizouard. Several on-shell renormalisation schemes as well as a mixed on-shell $\overline{\text{DR}}$ scheme are analysed. Using `SloopS`, a code that can perform automatic computation of one-loop corrections, the one-loop electroweak corrections to the partial widths of the pseudoscalar and scalar Higgses into lighter Higgses and/or neutralino/charginos are computed.

1. Introduction

My first correspondance with Shimizu-sensei was around 1987 when I was graduating from Sussex University, a few months earlier than the time when Norman Dombey and I where preparing the Workshop *Radiative Corrections: Results and Perspectives*[1] in Brighton, UK, in July 1989. This conference, just at the start of LEP1, turned out to be the first in the series that has come to be known as `RADCOR`. The correspondance with Shimizu was about a work on precision calculations, $e^+e^- \to \gamma\gamma$[2]. The first time we met was around 1993 in Annecy, shortly after I joined the CNRS with LAPTh. Our first collaboration was during the year-long *Physics at LEP2* CERN Workshop in 1994-1996 when I was convening the *Standard Model Processes* working group[3]. Shimizu with his Minami-Tateya group had contributed in a major way and showed the prowess of the `GRACE` system. I was not part of the `GRACE` collaboration but its through the CERN Yellow Book (LEP2) that my long discussions with Shimizu-sensei started. In the next year or two that followed, we embarked on a common project, the automation of one-loop electroweak corrections. LEP2 had revealed the importance of four-body final states where many groups competed to provide results for many channels of interest at LEP2. With the concomitant interest in the physics of a high energy e^+e^- linear collider, the project aimed at providing better precision for many processes at the Linear Collider. Many of these processes required the calculation of multi-leg processes. Prime among these processes were Higgs production mechanisms $e^+e^- \to \nu\bar{\nu}H, t\bar{t}H, e^+e^-H, ZHH, \nu\bar{\nu}HH$ and .. 4-fermion process that are needed to go beyond the important resonant contributions from $e^+e^- \to W^+W^-$. Before embarking on this (then) titanic programme, there was no full one-loop calculations to $2 \to 3$ and $2 \to 4$ processes. We wanted to push the theoretical challenge and set these processes as targets. From the end of 1997 the exchanges were intense, as were the visits with which my love for Japan

grew tremendously. The first concrete aspect aimed at finding a powerful internal check of the code. Of course we had to make sure that the renormalisation procedure was implemented correctly by checking for each process that the corrections were ultra-violet finite. But much more powerful was the check on gauge invariance and/or gauge-parameter independence on all physical amplitudes. The problem is that we wanted efficiency, however reverting to the usual ξ 't Hooft-Feynman gauges would have been intractable. The "longitudinal" terms in the vector boson propagators (which are absent in the much used Feynman gauge) not only introduce a high (superficial) degree of divergence in the intermediate steps of the computation but, perhaps worse, generate loop integrals with very high tensorial structures. Reduction of these (very) high rank tensors on the basis of scalar integrals is a tremendous and expensive task. The idea therefore was to have the gauge parameter dependence (in the intermediate stages of the calculations) in each amplitude in a simple polynomial form while keeping the analytic structure of the loop integral evident with only the physical masses showing up in the corresponding diagrams. We used a non-linear gauge fixing[4, 5] with no less than 5 gauge parameters: $\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\kappa}, \tilde{\epsilon}$. Denoting $A, Z, W^{\pm}, H$ for, respectively, the photon, the Z boson, the W and the Higgs boson, this non-linear gauge-fixing writes

$$
\begin{aligned}
\mathcal{L}_{GF} = & -\frac{1}{\xi_W}|(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{\mu+} + \xi_W\frac{g}{2}(v + \tilde{\delta}h + H + i\tilde{\kappa}\chi_3)\chi^+|^2 \\
& -\frac{1}{2\xi_Z}(\partial.Z + \xi_Z\frac{g}{2c_W}(v + \tilde{\epsilon}H)\chi_3)^2 - \frac{1}{2\xi_\gamma}(\partial.A)^2
\end{aligned}
\tag{1}
$$

I retrieved an e-mail from Shimizu-sensei dated 06/10/1997, exactly 19 years ago to the day!, stating that this step of the set-up for the full one-loop computation had been completed and I was invited to KEK so that the implementation of the non-linear gauge be completed in the GRACE system in view of the evaluation of the full one-loop amplitudes in the electroweak theory. This non-linear gauge was implemented in GRACE-Loop together with the full set of counterterms. It took a couple of years for the system to be ready for the first challenging process $e^+e^- \to \nu\bar{\nu}H$. The first preliminary results[6] were shown at RADCOR 2002/LOOP and LEGS 2002 in September by Fujimoto-san in Kloster Banz (Germany). We were all a bit nervous, dreading to be scooped. From Germany Fujimoto was reporting by phone directly to Shimizu who in turn reported to me (from Japan!) by e-mail. In a message I recovered from my e-mail archive, the evening following Fujimoto presentation, Shimizu wrote *".... He (Fujimoto-san) said in last evening session Tarasov has given a talk on nu-nu-Higgs 1-loop calculations but without showing any numbers. ..... It seems that now the world competition is becoming very tough like in 1995 on W-pair for LEP-2. Sayonara,Shimizu Y "*. The mail came with a caring post-scriptum *"Though you will certainly get the true explanation tomorrow, I tried to tell my guess so that you can have a good sleep tonight!*
Our Physics Letters B[7] on $\nu\bar{\nu}H$ appeared soon after. We were not scooped. At that point the collaboration was in full swing, with frequent visits to Japan and for our Minami-Tateya friends to France. The items on our list of priority processes were ticked one after the other[8, 9, 10, 11, 12, 13, 14]. This period corresponded with one of the happiest period in my life, my eldest daughter being born in 2001. The picture of Shimizu-sensei playing with Noor brings some sweet memories to me. At the same time as GRACE-1Loop was developing, Minami-Tateya was also developing the SUSY version and we exchanged also regarding this progress. Later on, it was decided that one should perform a full renormalisation of the MSSM and that the Annecy group should perform independent checks. In Annecy with the help of Andrei Semenov we decided to exploit LANHEP[16] as a model file builder to the FeynArts, FormCalc,LoopTools[17] packages in order to have an independent system which we called SloopS[18, 19, 20, 21]. In retrospect, if I go through my interaction with Shimizu-sensei, the first encounter was on a QED reaction controlled essentially with one parameter, α, then we
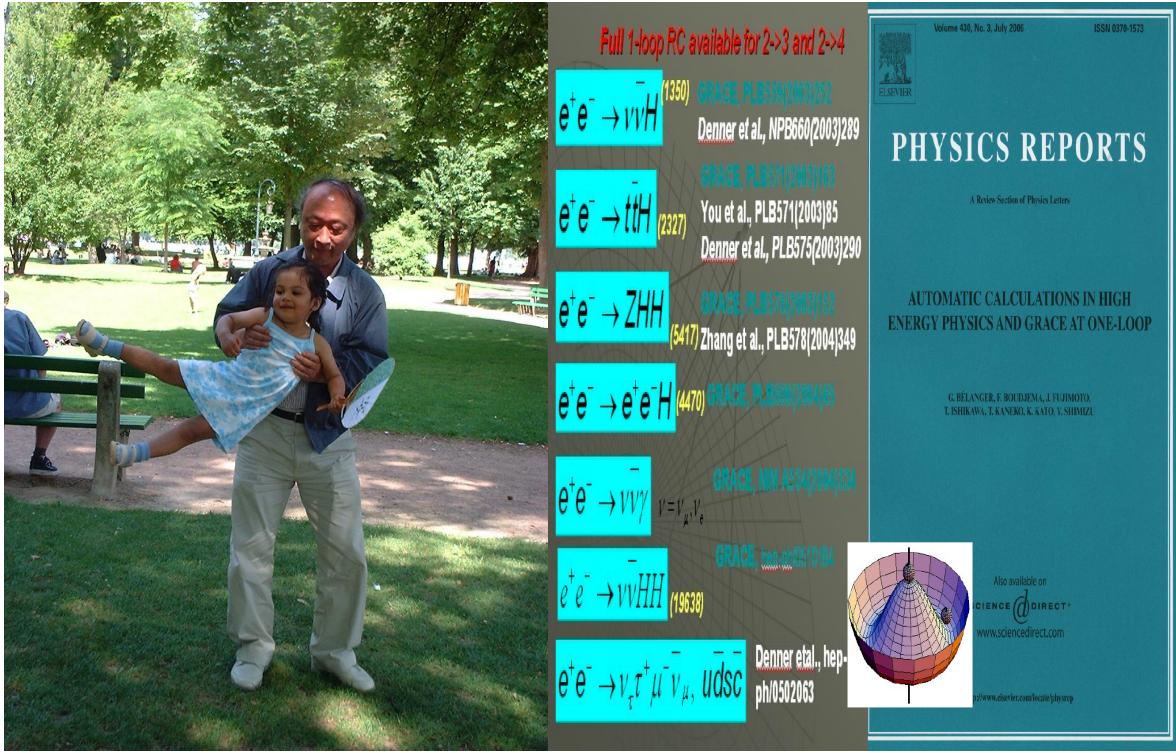
Figure 1. In *full swing*, Shimizu playing with Noor at the time we were writing our Physics Reports[15]

moved to a theory with a few parameters ($SU(2) \times U(1)$), specialising to multi-leg processes and then to a multi-parameter theory like the MSSM. Working with Shimizu-sensei, precision physics was an experience of beauty in different levels of complexity! Being in Japan, I can not help making the connection between our multi-leg calculations in multi-parameter theories with the multiplication of legs and arms within the multitude of statues in Sanjusangendo! Dealing with multi-parameter theories, eventually with multi-arm processes is not the same as what Gudrun referred to (for QCD N(N)LO) processes), mass production of cookies! It's rather recreating the subtlety in the difference contained in the 1000 figures demultiplied through arms (and legs) of the Sanjusangendo hall.

I will now report on one such theory that has even more parameters than the MSSM and the issues one has to face in renormalising this theory. The renormalisation of all the sectors of the NMSSM has been completed in `SloopS`[22, 23] here I will report on the Higgs sector. I will concentrate in particular on the definition/reconstruction of the parameters and the passage from the parameters at the Lagrangian level to the more physical parameters that may be chosen as inputs instead, namely the masses of the particles of the theory. The issue of mixing between the fields is critical, as is the issue of scheme and scale dependence.

2. The NMSSM

The NMSSM[24] provides a natural explanation for the scale of the higgsino parameter, μ by relating it to the vev of a scalar singlet, thus solving the little hierarchy problem of the MSSM. It makes it also much easier to produce a Higgs with a mass of 125GeV as observed at the LHC. For the MSSM such a mass requires large values in the stop sector which in turn creates an unbearable unease with naturalness. The NMSSM contains three Higgs superfields : two

$SU(2)_L$ doublets $\hat{H}_u$ and $\hat{H}_d$, as in the MSSM, and one additional gauge singlet $\hat{S}$

$$\hat{H}_u = \begin{pmatrix} \hat{H}_u^+ \\ \hat{H}_u^0 \end{pmatrix}, \quad \hat{H}_d = \begin{pmatrix} \hat{H}_d^0 \\ \hat{H}_d^- \end{pmatrix}, \quad \hat{S}. \tag{2}$$

In the $\mathbb{Z}_3$ implementation that we will assume, the Higgs superpotential is made up of two operators, each one introduces two dimensionless couplings λ and κ,

$$W_{Higgs} = -\lambda \hat{S} \hat{H}_d \cdot \hat{H}_u + \frac{1}{3}\kappa \hat{S}^3, \tag{3}$$

where $\hat{H}_d \cdot \hat{H}_u = \epsilon_{ab}\hat{H}_d^a \hat{H}_u^b$ and ϵ_{ab} is the two dimensional Levi-Civita symbol with $\epsilon_{12} = 1$. The two parameters λ and κ of the superpotential will, by construction, affect the phenomenology of both the Higgs and chargino/neutralino sector. λ in particular is crucial, not only it is necessary to induce the μ term but it also gives rise to mixing in the neutralino sector as well as in the Higgs sector between the Higgs doublets and the new singlet. For the purpose of parameter-counting and of the renormalisation of the Higgs sector at one-loop there is no need to go over the Yukawa superpotential which can be found in [22]. However, we do need to clearly specify again the soft SUSY breaking Lagrangian, in particular the part relating to the Higgs sector,

$$-\mathcal{L}_{soft,scalar} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2$$
$$+ (\lambda A_\lambda H_u \cdot H_d S + \frac{1}{3}\kappa A_\kappa S^3 + \text{h.c.}) \tag{4}$$

The first two terms in the first line represent the soft mass terms for the Higgs doublets and the third, not present in the MSSM, of the singlet. The second line, not present in the MSSM, represents the NMSSM trilinear Higgs couplings A_κ, A_λ. A_λ affects the mixing between the Higgs doublets and the singlet, beside the mixing introduced by λ. This paramater plays an important role in the phenomenology of the Higgs sector in the NMSSM, note that it gives rise to a Higgs tri-linear coupling $H_u H_d S$. No source of CP violation is assumed.

We are now in a position to write the Higgs potential whose parameters will need to be renormalised. With g, g' being respectively the $SU(2)$ and $U(1)$ gauge couplings and specifying the components of the doublets,

$$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} \quad H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$$

the potential writes

$$V_{Higgs} = |\lambda(H_u^+ H_d^- - H_u^0 H_d^0) + \kappa S^2|^2 + (m_{H_u}^2 + |\lambda S|^2)\left(|H_u^0|^2 + |H_u^+|^2\right)$$
$$+ (m_{H_d}^2 + |\lambda S|^2)\left(|H_d^0|^2 + |H_d^+|^2\right) + \frac{g^2 + g'^2}{8}\left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right)^2$$
$$+ \frac{g^2}{2}|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 + m_S^2|S|^2 + (\lambda A_\lambda(H_u^+ H_d^- - H_u^0 H_d^0)S + \frac{1}{3}\kappa A_\kappa S^3 + \text{h.c.}). \tag{5}$$

The physical scalar fields consist of 3 neutral CP-even Higgs bosons, h_1^0, h_2^0, h_3^0, 2 CP-odd Higgs bosons, A_1^0, A_2^0 and a charged Higgs boson, $H^\pm$. The NMSSM contains of course the SM. In particular the SM gauge parameters g, g' and $v^2 = v_u^2 + v_d^2$ are traded for the input parameters

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e, M_W, M_Z. From the Higgs potential, equation (5), it is clear that the Higgs sector of the NMSSM depends on the parameters:

$$\underbrace{t_\beta, \lambda, \kappa, \mu}_{\text{in } \tilde{\chi} \text{ sector also}}, A_\lambda, A_\kappa, m_{H_d}, m_{H_u}, m_S. \tag{6}$$

The first four are also involved in the characterisation of the neutralino/chargino sector. Alternatively the last three soft Higgs masses can be traded for the tadpoles of the neutral Higgs which need to be constrained to zero to impose that the potential is at its minimum such that the independent parameters can be taken to be

$$\underbrace{t_\beta, \lambda, \kappa(m_\kappa), \mu}_{\text{in } \tilde{\chi} \text{ sector also}}, A_\lambda, A_\kappa, (\mathcal{T}_{H_d}, \mathcal{T}_{H_u}, \mathcal{T}_S). \tag{7}$$

The latter are therefore considered as physical observables. The other six parameters are not unambiguously defined in a simple mapping to an observable. We will have a lot to say about the choice and definitions of the input parameters that will construct the set of these six parameters. This issue is directly related to the renormalisation scheme.

3. Renormalisation

In a nutshell, renormalisation consists in adding (subtracting) the ultra-violet infinite parameters and introducing wave function renormalisation for all fields. Then we need to impose conditions that amount to a definition of the parameters. The procedure consists in the following steps

- From $\mathcal{G}_p = \underbrace{t_\beta, \lambda, \kappa, \mu}_{\text{in } \tilde{\chi} \text{ sector also}}, A_\lambda, A_\kappa, \left(M_{H_u}^2, M_{H_d}^2, M_S^2 \to t_{H_d}, t_{H_u}, t_S \right), M_1, M_2 (\text{and} \underbrace{g, g', v \to e, M_W, M_Z}_{\text{SM}})$
  shift all (independent basic Lagrangian) parameters: $\mathcal{G}_p \to \mathcal{G}_p + \delta \mathcal{G}_p$

- this means that mass mixing will appear: non diagonal transition $A_i^0 Z^0, A_i^0 G^0, h_1 h_2, \ldots$ $(\delta m_{h_i h_j}, \delta m_{H^\pm G^\pm}^2)$ are generated and diagonal masses are shifted $(\delta m_{h_i}^2)$.

- No need to apply shifts to the diagonalising matrices $(S_h, U(\beta), \cdots)$, these are <u>renormalised</u> (no shift), same with gauge-fixing (not physical)

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix}_0 = U(\beta) \begin{pmatrix} h_d^+ \\ h_u^+ \end{pmatrix}_0 \quad \text{implies also} \quad \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = U(\beta) \begin{pmatrix} h_d^+ \\ h_u^+ \end{pmatrix}.$$

- In any case field renormalisation (before or after rotation) still needed

$$\begin{pmatrix} h_1^0 \\ h_2^0 \\ h_3^0 \end{pmatrix}_0 = Z_S \begin{pmatrix} h_1^0 \\ h_2^0 \\ h_3^0 \end{pmatrix}, \begin{pmatrix} A_1^0 \\ A_2^0 \\ G^0 \end{pmatrix}_0 = Z_P \begin{pmatrix} A_1^0 \\ A_2^0 \\ G^0 \end{pmatrix}, \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}_0 = Z_C \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} ((Z_S)_{ij} = 1_{ij} + \delta Z_{h_i h_j}/2)$$

- As an example, the renormalised self-energies of the CP-even Higgs scalars can be cast in the form

$$\begin{cases} \hat{\Sigma}_{h_i^0 h_i^0}(p^2) &= \Sigma_{h_i^0 h_i^0}(p^2) - \delta m_{h_i^0}^2 + (p^2 - m_{h_i^0}^2)\delta Z_{h_i^0} \\ \hat{\Sigma}_{h_i^0 h_j^0}(p^2) &= \Sigma_{h_i^0 h_j^0}(p^2) - \delta m_{h_i^0 h_j^0}^2 + \frac{1}{2}(p^2 - m_{h_i^0}^2)\delta Z_{h_i^0 h_j^0} + \frac{1}{2}(p^2 - m_{h_j^0}^2)\delta Z_{h_j^0 h_i^0} \end{cases} \tag{8}$$

We then require that *i)* Mixing vanishes between physical states when these are on-shell, *essentially* (to solve for δZ_{ij}'s); *ii)* the residue at the pole (mass) of the propagator is 1. The other conditions are set by using/choosing a minimum/sufficient set of physical masses as input

parameters except α_{em} which defined like in QED from the Thomson limit. In this approach only two point-functions are needed. The big question is which minimum set of parameters to choose?

Any choice will require solving a coupled system of counterterms, in other words setting the passage from the underlying Lagrangian parameters to, if possible, physical parameters such as masses. Leaving aside the SM parameter,

$$
\begin{pmatrix} \delta\texttt{input}_1 \\ \cdots \\ \cdots \\ \delta\texttt{input}_8 \end{pmatrix}_{\chi^\pm,\chi_i^0,h_i^0,A_i^0,H^\pm} = \mathcal{P}_{8,\mathrm{param.}} \begin{pmatrix} \delta M_1 \\ \delta M_2 \\ \delta\kappa \\ \delta\mu \\ \delta\lambda \\ \delta t_\beta \\ \delta A_\lambda \\ \delta A_\kappa \end{pmatrix} + \mathcal{R}_{n,\mathrm{residual}}, \tag{9}
$$

$\mathcal{R}_{8,\mathrm{residual}}$ counterterms such as gauge couplings, etc (SM). Inverting the system means that the corresponding determinant$(\to 1/\mathrm{Det}(\mathcal{P}_{m,\mathrm{param.}}))$ better be not too small otherwise the finite part of the counterterms could introduce unnecessarily large radiative *corrections (sic!)*. It is best to break up the system, into smaller subsystem $\mathcal{P}_{n,\mathrm{param.}} = \mathcal{P}_{m,\mathrm{param.}} \oplus \mathcal{P}_{p,\mathrm{param.}} \oplus \cdots$, $m + p + \cdots = n$. At each step one should avoid a choice such that $\mathrm{Det}(\mathcal{P}_{m,\mathrm{param.}}) \to 0$. For example picking up a wino-like neutralino to reconstruct M_1 would be a bad choice if the mixing between the bino and the wino is small. Since t_β is central, the easiest set-up is to define t_β in $\overline{\mathrm{DR}}$ with t_β extracted independently from the Higgs sector (through wave function renormalisation condition) $\delta t_\beta / t_\beta = \left[\frac{1}{2}(\delta Z_{H_u} - \delta Z_{H_d}) \right]_\infty$, ∞ means we only keep the infinite part. In this mixed scheme, μ, M_2 are easily constrained from the charginos masses. Note however that though algebraically this conversion is straighforward, it is not sure that the assignation of μ, M_2 to the correct chargino mass will be correct. Again a knowledge of which is which is essential. One needs more than the masses. We are then left with having to define $M_1, \kappa, (\lambda)$ from neutralino masses again with the caveat that we somehow know the composition. This may not be a big hurdle, since if the masses are to be measured, this means they have been produced. If we know in which proportion this could give a valuable hint on the nature of the particle. $A_\lambda, A_\kappa(\lambda)$ can only be extracted from the Higgs (A, h, H^+). If this were possible with t_β from $\overline{\mathrm{DR}}$ then the break-up corresponds to

$$\mathcal{P}_8 = \mathcal{P}_{1,t_\beta} \oplus \mathcal{P}_{2,\chi_{1,2}^\pm} \oplus \mathcal{P}_{3,\chi^0} \oplus \mathcal{P}_{2,A_1^0,A_2^0} \text{ or } \mathcal{P}_8 = \mathcal{P}_{1,t_\beta} \oplus \mathcal{P}_{2,\chi_{1,2}^\pm} \oplus \mathcal{P}_{2,\chi^0} \oplus \mathcal{P}_{3,H^\pm,A_1^0,A_2^0(h^0)}$$

A complete OS scheme, based on the input provided by 8 masses would look like $\mathcal{P}_{8;\chi_{1,2}^\pm,\chi_{1,2,3}^0,H^\pm,A_1^0,h^0}$. We have used different variants which nonetheless all take the chargino masses as input. $i)\mathrm{OS}_{ijkA_1A_2H^+}$ with the masses of 3 χ^0 preferably $\tilde{b}, \tilde{h}, \tilde{s}$-dominated; $ii)\mathrm{OS}_{ijh_\alpha A_1A_2H^+}$ (only 2 neutralinos) and $iii)\mathrm{OS}_{ih_\alpha h_\beta A_1A_2H^+}$ (only one neutralino,$\tilde{b}$).

In renormalisable theories, most loop calculations involving 1-,2- and 3-point functions give rise to ultraviolet infinities. The latter require a regularisation. In dimensional reduction/regularisation these infinities are contained in the quantity $\widetilde{C}_{UV} = 2/\epsilon - \gamma_E + \ln(4\pi/\bar{\mu}^2) = 2/\epsilon - \gamma_E + \ln(4\pi) + \ln(1/\bar{\mu}^2)$ where the continuation to $n = 4 - \epsilon$ dimension require the introduction of the compensating regularisation scale $\bar{\mu}$. The definition of an underlying parameter p_i at one-loop, say OS scheme, based on a physical observable calls for the corresponding counterterm $\delta p_i / p_i = \beta_{p_i}(C_{UV} + \ln(Q_{p_i}/\bar{\mu}))$ $(\beta_{p_i} = \partial p_i / \partial \ln 1/\bar{\mu})$. Q_{p_i} is a scheme-dependent an effective scale, which depends on the point where the parameter has been

defined and other masses that may appear at one-loop in the definition of the parameter. β_{p_i} is a universal, scheme independent factor. To track down the influence of the scheme on the correction to an observable, it is useful to know the parametric dependence of the observable on the parameter p_i's, $\partial O/\partial p_i = \kappa_{p_i}$. In our so-called $\overline{\text{DR}}$ scheme we will only keep $\beta_{p_i} C_{UV}$, the "finite part" is set to 0. In On-shell (OS) scheme the "finite part" is $\beta_{p_i} \ln(Q_{p_i}/\bar{\mu})$. Then the full one-loop correction to the observable $\delta\mathcal{O}/\mathcal{O} = \overbrace{\Delta(C_{UV} + \ln(Q_\Delta/\bar{\mu}))}^{virtual} + \overbrace{\sum_i \kappa_i \delta p_i/p_i}^{Counterterms}$ leads to

$\delta\mathcal{O}^{\text{OS}}/\mathcal{O} = \sum_i \beta_{p_i}\kappa_{p_i} \ln(Q_{p_i}/Q_\Delta)$ $\delta\mathcal{O}^{\text{DR}}/\mathcal{O} = \ln(\bar{\mu}/Q_\Delta) \sum_i \beta_{p_i}\kappa_{p_i})$. A good choice of $\bar{\mu}$ or Q_{p_i} will minimize the correction. These simple observations show that one can have large corrections when/if $i)\beta_p$ is large, and/or $ii)\kappa_p$ is large and/or $iii)$ when there is a large mismatch/difference between the scales Q_Δ and Q_p.

4. Applications
We have considered two scenarios. We will not, in this short write-up, give the full details of both models by listing all their defining underlying parameters. The salient differences is that point A is MSSM-like with a small $\lambda = 0.1$ while point B has $\lambda = 0.67$. In order for A to reproduce a standard model-like Higgs with mass 125GeV, the SUSY scale set by the masses of the stops as well as, more crucially, A_t need to be large. A large t_β helps also. For example for point A, $A_t/A_\lambda \sim 27$ while for point B $A_t/A_\lambda \sim 2.5$. While a large A_t helps obtain a large correction for the Higgs mass it also provides a very large β function for A_λ. The latter is induced by an enhanced top Yukawa coupling modulated by A_t/A_λ as be seen from the RGE's. With h_t the Yukawa coupling, the dominant contributions to the running of the underlying couplings of the NMSSM can be cast as[24]

$$\frac{1}{h_t^2}\frac{\mathrm{d}h_t^2}{\mathrm{d}\tau} = \frac{1}{A_t}\frac{\mathrm{d}A_t}{\mathrm{d}\tau} = \frac{2}{\lambda^2}\frac{\mathrm{d}\lambda^2}{\mathrm{d}\tau} = \frac{2}{\mu^2}\frac{\mathrm{d}\mu^2}{\mathrm{d}\tau} = 6h_t^2 \quad \text{and} \quad \frac{1}{A_\lambda}\frac{\mathrm{d}A_\lambda}{\mathrm{d}\tau} = 3h_t^2\frac{A_t}{A_\lambda}, \tag{10}$$

with $\tau = \ln\bar{\mu}^2/16\pi^2$. Numerically and with no approximation we find for the beta functions expressed in units of 10^{-3},
Point A: $\beta_\mu = -11.40, \beta_{t_\beta} = 16.9, \beta_\lambda = -11.65, \beta_\kappa = -0.76, \beta_{A_\lambda} = -1097.4$ and
Point B: $\beta_\mu = -14.25, \beta_{t_\beta} = 17.63, \beta_\lambda = -20.45, \beta_\kappa = -18.57, \beta_{A_\lambda} = -122.7$
It is also useful to list the finite parts of the counterterms in some of the schemes for both points A and B.
$\boxed{\text{Point A}}$ The finite parts computed at $\bar{\mu} = \text{Q}_{\text{susy}} = 1117.25\text{GeV}$ give

$$(\delta\mu/\mu, \delta t_\beta/t_\beta, \delta\lambda/\lambda)_{\text{finite}} = \overbrace{(-2.42\%, 0, 62.26\%)}^{t_{134A_1A_2}}; \overbrace{(-1.57\%, -80.69\%, -7.88\%)}^{\text{OS}_{34h_2A_1A_2H^+}} \tag{11}$$

$$(\delta\kappa/\kappa, \delta A_\lambda/A_\lambda, \delta A_\kappa)_{\text{finite}} = \overbrace{(64.01\%, -5.49\%, 0.65)}^{t_{134A_1A_2}}; \overbrace{(-6.01\%, 134\%, 0.66)}^{OS_{34h_2A_1A_2H^+)}}. \tag{12}$$

$\boxed{\text{Point B}}$ Here the finite parts" computed at $\bar{\mu} = \text{Q}_{\text{susy}} = 753.55\text{GeV}$

$$(\delta\mu/\mu, \delta t_\beta/t_\beta, \delta\lambda/\lambda)_{\text{finite}} = \overbrace{(-1.04\%, 0, 3.71\%)}^{t_{123A_1A_2}}; \overbrace{(-1.63\%, 6.49\%, 5.94\%)}^{\text{OS}_{12h_2A_1A_2H^+}} \tag{13}$$

$$(\delta\kappa/\kappa, \delta A_\lambda/A_\lambda, \delta A_\kappa)_{\text{finite}} = \overbrace{(3.25\%, 6.85\%, 10.84\%)}^{t_{123A_1A_2}}; \overbrace{(6.05\%, 3.40\%, 11.54\%)}^{OS_{12h_2A_1A_2H^+}}. \tag{14}$$

All the schemes t where t_β is defined as $\overline{\text{DR}}$ and the other parameters are taken as OS take the chargino masses as inputs. The other parameters are explicit if one note that $1, 2, 3$ stand for the neutralinos ordered by increasing mass. The β functions as well as the finite parts of the counterterms give an insight on the value of the full one-loop corrections and more importantly on the differences between the results of the schemes. The parametric dependence is weighed by perturbing slightly around the values of the model. Fig. 2 shows that for Point A the dependence
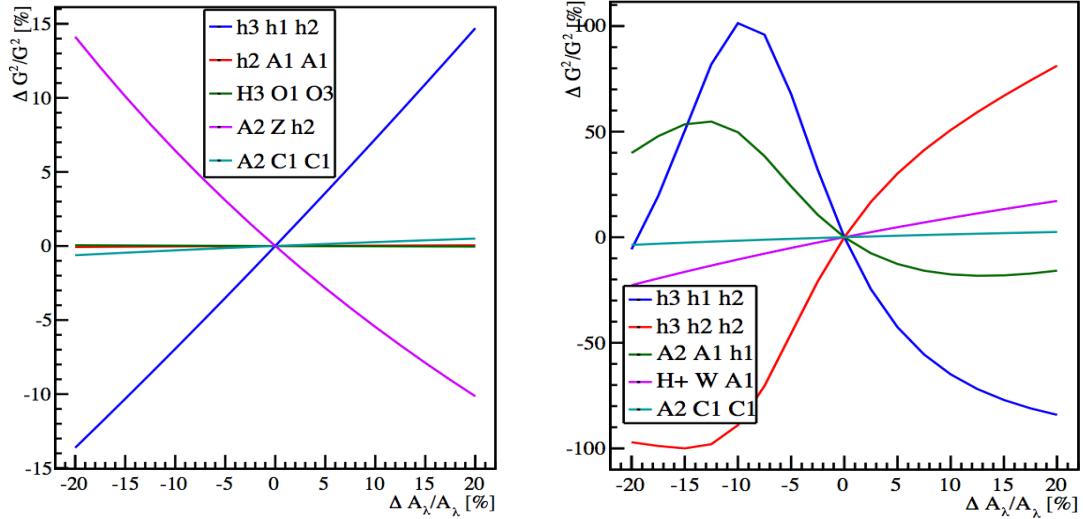

Figure 2. Dependence on A_λ of the square of the couplings, G_{ijk}, shown in the insert of the figures for Point A (left panel) and Point B (right panel). G_{ijk} represents the coupling constant of the interaction between ijk. The variations shown do not take into account the changes in the decay due to phase space that can also change because some of the masses involved in the decay change when the underlying parameters change. h_i stand for the CP-even Higgses. O are for the neutralinos and C the charginos.

one some of the underlying parameters for some of the chosen decays are practically flat or linear in contrast to Point B where the dependence is quite strong. For point B a small change in A_λ can trigger a large difference with a highly non linear variation.

We have computed the full one-loop electroweak corrections to various Higgs decays either to other Higgses or to Higgs gauge bosons or to a pair of neutralinos/charginos. Table 1 is for Point A. To understand the scheme dependence of the majority of the most important decays of the different Higgses, it is important to keep in mind the singlet nature of the states involved. Recall that this scenario is characterised by very small mixing whereby the Higgs states h_2^0, A_1^0 and the neutralino χ_3^0 are essentially singlet states. There are three classes of decays, those among the predominantly singlets ($h_2^0 \to A_1^0 A_1^0$), those not involving any of these states ($A_2^0 \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$, $h_3^0 \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$ and $A_2^0 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$) with characteristics close to the MSSM and with very little dependence on λ. The last class involves both singlet and non singlet states. The latter decays are therefore sensitive to the mixing parameters due to the addition of the singlet in the NMSSM. For most decays this mixing parameter is essentially λ while for $h_3 h_2 h_1$ A_λ is crucial. These observations together with the numerical values of the β functions and the values of the finite terms of the counterterms can explain the differences. The mixed scheme $t_{134A_1A_2}|$ performs quite badly for almost all decays that involve singlet/non singlet states. The latter are very sensitive to λ which is badly reconstructed in this scheme. This also explains the almost equal value of the correction in the many channels, $\sim 120\%$ which is mainly accounted for by

Table 1. Point A. Corrections to the partial decay widths of Higgs bosons in other Higgs bosons and/or neutralinos, charginos and gauge bosons. The column TL represents the tree-level (in units of GeV/100) partial width. The other columns give the percentage relative full one-loop correction, in the mixed scheme where only t_β is taken $\overline{\text{DR}}$ ($t_{134A_1A_2}|$), the full OS scheme and full $\overline{\text{DR}}$. For $t_{134A_1A_2}|$ and $\overline{\text{DR}}|$ the scale $\bar\mu$ is taken at the mass of the decaying Higgs, for $\overline{\text{DR}}\, Q_{\text{SUSY}}$ the scale at the SUSY scale.

Decays	TL $\times 10^{-2}$	$t_{134A_1A_2}|$	OS$_{34h_2A_1A_2H^+}$	$\overline{\text{DR}}|$	$\overline{\text{DR}}\,|Q_{\text{SUSY}}$
$h_2^0 \to A_1^0 A_1^0$	4.79	128%	-12%	0.4%	-0.4%
$h_3^0 \to h_1^0 h_2^0$	2.21	116%	79%	52%	-1.7%
$h_3^0 \to \tilde\chi_1^0 \tilde\chi_3^0$	3.52	122%	-3%	2%	0.3%
$h_3^0 \to \tilde\chi_2^0 \tilde\chi_3^0$	3.38	126%	-35%	3%	1.1%
$h_3^0 \to \tilde\chi_1^+ \tilde\chi_1^-$	4.55	1%	-11%	-9%	-7.4%
$A_2^0 \to Z h_2^0$	1.86	120%	80%	-56%	-14.5%
$A_2^0 \to \tilde\chi_1^0 \tilde\chi_1^0$	3.30	28%	13%	0.3%	-1.6%
$A_2^0 \to \tilde\chi_1^0 \tilde\chi_3^0$	2.44	130%	-31%	8%	6.2%
$A_2^0 \to \tilde\chi_2^0 \tilde\chi_3^0$	3.02	122%	-5%	-0.4%	-1.9%
$A_2^0 \to \tilde\chi_1^+ \tilde\chi_1^-$	5.51	-10%	-1.5%	-6%	-8%
$H^+ \to W^+ h_2^0$	2.01	119%	79%	-56%	-16%
$H^+ \to \tilde\chi_1^+ \tilde\chi_3^0$	6.40	125%	-18%	3%	1.1%

the finite part of $2\delta\lambda$ in this scheme. The decay $h_3^0 \to h_1^0 h_2^0$ (and to some extent $A_2^0 \to Z h_2^0$) stands apart. It is the only channel where there is a large running, as testified by the comparison between the two $\overline{\text{DR}}$ schemes. In fact, this coupling depends on A_λ whose running is quite well approximated by the large value of β_{A_λ}.

For point B, the value of λ is large enough, this is reflected also in the fact the partial widths are for all channels an order of magnitude larger than for point A. A discussion in terms of almost pure or singlet states is not appropriate. For this more genuine NMSSM, the OS scheme performs quite well. It rests that the scheme and scale dependence for $h_3^0 \to h_1^0 h_2^0$ is quite large. This is due not so much to β_{A_λ} bur rather to the strong parametric dependence on A_λ as seen in Fig. 2.

5. Summary and conclusions

Full renormalisation (all sectors) of the NMSSM at one-loop is now completed and implemented in `SloopS`. The set-up allows to choose between different on-shell schemes and also "mixed schemes" with one or several parameters through a $\overline{\text{DR}}$ subtraction. The on-shell schemes are at the moment all based on the usage of physical masses and hence the use of two-point functions in the definition of the parameters. Especially in scenarios where the singlet mixing parameter is small, these schemes are not optimal in extracting this crucial mixing parameter nor in extracting t_β. We must go beyond taking only masses as inputs. When new particles are discovered, not only their masses will be measured but the way they are produced, the strengths of their production and decays offer an important handle that may not need the reconstruction of the whole spectrum. This is technically much more challenging, but it is possible (at least in some manifestations). In the MSSM $A^0 \to \tau\bar\tau$ was shown to be an excellent input for t_β[19].

I would like to end with a personal note. Last time I saw Shimizu-sensei was in February 2015. The last dinner I had with him was in a Fugu restaurant. As always with him, *un*

Table 2. Point B. Corrections to the partial decay widths of Higgs bosons in other Higgs bosons and/or neutralinos, charginos and gauge bosons. TL represents the partial decay widths (in GeV). The percentage relative full one-loop corrections are given as percentage relative corrections in the OS scheme, namely, $\mathrm{OS}_{12h_2A_1A_2H^+}$ as well as a full $\overline{\mathrm{DR}}$ at the SUSY scale and the mass of the decaying Higgs.

	TL	OS	$\overline{\mathrm{DR}}$|	$\overline{\mathrm{DR}}\,Q_{\mathrm{SUSY}}$
$h_3^0 \to \tilde{\chi}_1^0 \tilde{\chi}_2^0$	0.727	14%	5%	3%
$h_3^0 \to A_1^0 Z$	0.613	3%	-3%	-8 %
$h_3^0 \to h_2^0 h_1^0$	0.341	-25%	-106%	-50%
$h_3^0 \to h_2^0 h_2^0$	0.514	6%	13%	-28%
$A_2^0 \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$	1.523	7%	2%	1%
$A_2^0 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$	0.723	32%	2%	2%
$A_2^0 \to Z h_2^0$	0.638	12%	-16%	-9%
$A_2^0 \to A_1^0 h_1^0$	0.415	-0.3%	-32%	-17%
$H^+ \to \tilde{\chi}_1^+ \tilde{\chi}_2^0$	1.056	6%	10%	8%
$H^+ \to W^+ h_2^0$	0.609	11%	-18%	-10%
$H^+ \to W^+ A_1^0$	0.603	2%	-3%	-9%
$H^+ \to \tilde{\chi}_1^+ \tilde{\chi}_1^0$	0.561	21%	9%	9%

fin gourmet, the meal was excellent as were all the subtleties about the dishes. The next morning, in a email, he told me he enjoyed having dinner with me (and with Kato-san and Kuroda-san) but he thought it was the last time we saw each other. I did not believe it, but he was right. Back to France, we exchanged regularly. Most of exchanges were about food and pottery. His celadon gifts to me are priceless. We did discuss physics and of all the exchanges the one which moved me the most was about the visit of Nakawaza-san to the hospital when we told me again about the 2-loop electroweak calculations project. So much about my 2015 trip to Japan.

The last trip of Shimizu to France, Paris and Annecy, was wonderful for my family and myself. I was told by a dear common friend that his dream and one of his wishes had been to take his dearest friends to see Paris. These friends are always welcome to France and I very much hope you will visit. And when you come we will go to `Minami`...in Annecy! Minami is a new, small, genuine Japanese restaurant that opened about 4 or 5 years ago. How wonderful that I can get back to the days of Minami-Tateya by walking 2 minutes from my home and step in *Minami* like Shimizu did in January 2013.

Figure 3. Shimizu-sensei in front of the Japanese restaurant in Annecy, the Minami! January 2013.

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